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A Dynamic Decision Making Method Based on GM(1,1) Model with Pythagorean Fuzzy Numbers for Selecting Waste Disposal Enterprises

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Abstract: With the rapid development of society and the economy, most cities have to face a serious problem of "Garbage Siege". The garbage classification is imperative because the traditional disposal method for household solid waste is not suitable for this situation. The Chinese government proposed a public private partnership (PPP) style to increase the efficiency of garbage disposal in 2013. An effective method to evaluate the waste disposal enterprises is essential to choose suitable ones. A reasonable evaluation method should consider enterprises' performance not only now but also in the future. This paper aims to propose a dynamic decision making method to evaluate the enterprises' performance based on a GM(1,1) model and regret theory with Pythagorean fuzzy numbers (PFNs). First, we proposed a GM(1,1) model for predicting score function of PFNs. Then, we put forward a method to obtain the prediction of grey degree using OWA operator. Based on the prediction of score function and grey degree, we established a novel GM(1,1) model of PFNs. Furthermore, we utilized the grey incidence method to obtain the criteria weights with Pythagorean fuzzy information. We used the regret theory to aggregate information and rank the alternatives. Finally, we applied our proposed method to solve the selecting waste disposal enterprises problem in Shanghai. By the case study we can obtain that our method is effective to solve this problem.

Keywords: decision making; dynamic; GM(1,1) model; Pythagorean fuzzy number; garbage classification

1. Introduction

With the fast urbanization of China, more and more cities are faced with a serious social problem: "Garbage Siege", which means cities are surrounded by a large amount of garbage [1]. Owing to incorrect disposal, the garbage has severely polluted the soil and underground water [2]. Most cities are devoured with wastes largely because of household solid waste (HSW) and classification of HSW is urgent to reduce the pressure on the environment [3]. Some cities including Shanghai, Beijing and Guangzhou have established some policies about garbage classification [4]. Recycling and reusing garbage is an effective way to solve the garbage problem. However, the utility rate of garbage is not high at present. The public private partnership (PPP) style can improve the efficiency of disposing HSW [2]. Because the evaluation of enterprises involves many aspects, including economic and societal, the selection of excellent enterprises can be described a multi-criteria decision-making problem (MCDM).

In the decision-making process, decision makers (DMs) may feel it is difficult to evaluate the enterprises with crisp numbers owing to the complexity of decision-making problems and DMs' fuzzy thinking. To address this problem, Zadeh [5] put forward the definition of a fuzzy set (FS) utilizing a variable named membership to capture the fuzziness. However, there are some limitations in solving real problems when using FS, because it cannot express the hesitance of DMs. Atanassov [6] proposed the definition of intuitionistic fuzzy set (IFS) to describe the hesitance of DMs using membership and non-membership. In IFS, the sum of membership and non-membership cannot be larger than one. Yager [7] put forward the definition of Pythagorean fuzzy set (PFS), in which the sum of membership and non-membership can be larger than one but the quadratic sum of them not larger than one. PFS is more flexible than IFS and attracts many researchers. The studies of aggregating operators mainly include mean aggregation operators [8], aggregation operators considering division and subtraction [9], Choquet integral aggregation operators [10] and symmetric aggregation operators [11]. The ranking methods mainly include TOPSIS [12], VIKOR [13] and TODIM [14].

In fact, in the process of selecting excellent enterprises of disposing HSW, DMs should think of not only the past data but also the possible future data [15]. In other words, the process should be a dynamic decision making one. An important dynamic MCDM method is dynamic weighted averaging operator [16]. However, this method does not consider the future data. The GM(1,1) model can use the past data to predict the future data [17]. Small sample and poor information are the advantages of the GM(1,1) model. Because the MCDM problems usually include a small number of criteria and alternatives, the GM(1,1) model can effectively be applied to solve the MCDM problems. Traditional GM(1,1) model can effectively solve the prediction problems with crisp numbers [18–21], interval-valued numbers [22] and intuitionistic fuzzy numbers [23]. However, there are few studies for the GM(1,1) model with PFNs.

Meng et al. [3] analyzed the key factors influencing residents' HSW disposal behavior based on a structural equation model, and pointed out that government should strengthen construction of recycling facilities. Fan et al. [24] made a comparative analysis between China and Singapore for the HSW sorting problem, and indicated that waste management system was very important for influencing behavior of HSW sorting. Chauhan and Singh [25] proposed a hybrid MCDM method combining AHP and TOPSIS to solve the choosing location of healthcare waste disposal facility. Qazi et al. [26] proposed an AHP method to analyze the management problem of municipal solid waste in Oman. Beskese et al. [27] proposed a method for selecting a landfill site using AHP and TOPSIS. Liu et al. [28] indicated that the PPP model was effective and had been promoted in the garbage disposal industry. In the process of PPP, it is vitally important to select excellent enterprises in disposing HSW. In the process of evaluating enterprises, not only the past data but also the future data of the enterprises' performance should be considered. To some extent, the future data may be more important than that of the past. The GM(1,1) model is an effective method to predict the future data. Therefore, in this paper, we will establish a novel GM(1,1) model with PFNs. In consideration of DMs' bounded rationality, we propose a dynamic decision making method using the GM(1,1) model and regret theory (RT) to solve the problem of selecting excellent enterprises in disposing HSW.

The contributions of our paper are as follows:

(1) Establish a novel GM(1,1) model for PFNs using score function and grey degree of PFNs, which can realize the prediction of PFNs while avoiding the fact that using membership, non-membership and indeterminacy degrees directly to predict can lead to an unreasonable result.

(2) Propose a new dynamic MCDM method for PFNs based on GM(1,1) model and RT, which can make the most of past and future decision data by GM(1,1) model, and bounded rationality of human beings by RT.

(3) Put forward a new decision-making method considering the future data for choosing excellent enterprises in disposing HSW, which can effectively solve the serious problem: "Garbage Siege".

The paper is organized as follows. Section 2 reviews some definitions and basic rules about PFS and RT. Section 3 proposes a novel GM(1,1) model for PFNs using score function and grey degree of PFNs. Section 4 puts forward a dynamic decision making method using RT and GM(1,1) for PFNs. Section 5 studies a real case about selecting excellent enterprises in disposing HSW in Shanghai using our proposed method. Section 6 makes a summary for our paper.

2. Preliminaries

In this section, some basic definitions and operational laws relative to PFS and RT are reviewed.

2.1. Pythagorean Fuzzy Sets

Yager and Abbasov [8] proposed the definition of PFS, which is very flexible in coping with uncertain information [29].

Definition 1 [29]**.** *Let X be a universe of discourse, x be the element of X. Then, a Pythagorean fuzzy set P in X can be defined as*

$$P = \{x, \mu_P(x), \upsilon_P(x) \big| x \in X\} \tag{1}$$

where $\mu_P(x)$ *and* $\upsilon_P(x)$ *are called the membership degree and non-membership degree of x to P respectively, and hold that* $0 \le \mu_P(x) \le 1$, $0 \le \upsilon_P(x) \le 1$, *and* $(\mu_P(x))^2 + (\upsilon_P(x))^2 \le 1$. *Furthermore,* $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 - (\upsilon_P(x))^2}$ *is called the indeterminacy degree of x to P.*

For convenience, we call $\beta = P(\mu_\beta, \upsilon_\beta)$ a Pythagorean fuzzy number (PFN).

Definition 2 [29]**.** *Let* $\beta_1 = P\left(\mu_{\beta_1}, \upsilon_{\beta_1}\right)$, $\beta_2 = P\left(\mu_{\beta_2}, \upsilon_{\beta_2}\right)$ *and* $\beta = P\left(\mu_\beta, \upsilon_\beta\right)$ *be three PFNs, then the following operational rules hold:*

$$\beta_1 \oplus \beta_2 = P\left(\sqrt{u_{\beta_1}^2 + u_{\beta_2}^2 - u_{\beta_1}^2 u_{\beta_2}^2}, \upsilon_{\beta_1}\upsilon_{\beta_2}\right).$$
$$\beta_1 \otimes \beta_2 = P\left(\mu_{\beta_1}\mu_{\beta_2}, \sqrt{\upsilon_{\beta_1}{}^2 + \upsilon_{\beta_2}{}^2 - \upsilon_{\beta_1}{}^2\upsilon_{\beta_2}{}^2}\right).$$
$$\lambda\beta = P\left(\sqrt{1 - \left(1 - \left(\mu_\beta\right)^2\right)^\lambda}, \left(\upsilon_\beta\right)^\lambda\right), \lambda > 0.$$
$$\beta^\lambda = P\left(\left(\mu_\beta\right)^\lambda, \sqrt{1 - \left(1 - \left(\upsilon_\beta\right)^2\right)^\lambda}\right), \lambda > 0.$$

Definition 3 [30]**.** *Let* $\beta_1 = P\left(\mu_{\beta_1}, \upsilon_{\beta_1}\right)$ *and* $\beta_2 = P\left(\mu_{\beta_2}, \upsilon_{\beta_2}\right)$ *be two PFNs, then the distance between* β_1 *and* β_2 *can be defined as*

$$d(\beta_1, \beta_2) = \frac{1}{2}\left(\left|\left(\mu_{\beta_1}\right)^2 - \left(\mu_{\beta_2}\right)^2\right| + \left|\left(\upsilon_{\beta_1}\right)^2 - \left(\upsilon_{\beta_2}\right)^2\right| + \left|\left(\pi_{\beta_1}\right)^2 - \left(\pi_{\beta_2}\right)^2\right|\right) \tag{2}$$

Definition 4 [29]**.** *Let* $\beta = P\left(\mu_\beta, \upsilon_\beta\right)$ *be a PFN, then the score function of* β *is defined as*

$$S(\beta) = \left(u_\beta\right)^2 - \left(\upsilon_\beta\right)^2. \tag{3}$$

The score function can be used to compare two PFNs [29]. The comparison rules can be concluded as follows:

Let $\beta_1 = P\left(\mu_{\beta_1}, \upsilon_{\beta_1}\right)$ and $\beta_2 = P\left(\mu_{\beta_2}, \upsilon_{\beta_2}\right)$ be two PFNs, then

(1) If $S(\beta_1) > S(\beta_2)$, then $\beta_1 > \beta_2$;
(2) If $S(\beta_1) < S(\beta_2)$, then $\beta_1 < \beta_2$;
(3) If $S(\beta_1) = S(\beta_2)$, then $\beta_1 \sim \beta_2$.

2.2. Regret Theory

In real decision-making process, DMs are usually faced with some situations, such as information insufficient, time pressure and limited knowledge of DMs. In this case, DMs may be boundedly rational.

Regret theory (RT), as a behavioral theory proposed by Loomes and Sugden [31], can effectively deal with uncertain information.

Definition 5 [32]**.** *Let x be a variable of benefit attribute, then the utility function $v(x)$ is defined as*

$$v(x) = \frac{1-e^{-\alpha x}}{\alpha}, 0 < \alpha < 1, \quad (4)$$

where α is a risk aversion coefficient, and the greater the α, the greater the degree of risk aversion.

Definition 6 [32]**.** *Let x be a variable of cost attribute, then the utility function $v(x)$ is defined as*

$$v(x) = \frac{1-e^{\beta x}}{\beta}, 0 < \beta < 1 \quad (5)$$

where β is a risk aversion coefficient, and the greater the β, the greater the degree of risk aversion.

Definition 7 [33]**.** *Let $X = [X^-, X^+]$ be an interval-valued variable, then the utility value of X is defined as*

$$V = \int_{X^-}^{X^+} v(x) f(x) dx \quad (6)$$

where $f(x)$ is probability density function, $v(x)$ is the utility function.

Definition 8 [33]**.** *The regret–rejoice function $R(\Delta V)$ can be defined as*

$$R(\Delta V) = 1 - e^{-r\Delta V} \quad (7)$$

where r is the risk aversion coefficient of DM.

3. A Novel GM(1,1) Model Based on Score Function and Grey Degree for PFNs

In this section, we will introduce the traditional GM(1,1) model and then propose a novel model for PFNs using score function and grey degree.

3.1. Traditional GM(1,1) Model

GM(1,1) model is a very important part of grey system theory, and can effectively solve some prediction problems with a limited number of samples or poor information [17]. It has been applied to many fields, such as electronic technique [18], energy [19,20] and emergency management [21].

Definition 9 [34]**.** *Let $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))$ be a positive original sequence. Then $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n))$ is called the first-order accumulating generation (1-AGO) sequence, where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$; and $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n))$ is called the mean generated sequence of consecutive neighbors of $X^{(1)}$, where $z^{(1)}(k) = 0.5x^{(1)}(k-1) + 0.5x^{(1)}(k)$.*

Definition 10 [34]**.** *Let $Z^{(1)}$, $X^{(1)}$ and $X^{(0)}$ be three sequences showed as Definition 9. Then the following equation is called the fundamental form of the GM(1,1) model.*

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (8)$$

The values of a and b in Equation (8) can be calculated by the equation $[a,b]^T = [B^TB]^{-1}B^TY$, where $B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$, $Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$.

Traditional GM(1,1) models are usually used to solve prediction problems in which the information is expressed by crisp numbers. Zeng [22] proposed a GM(1,1) model for interval-valued information. Li et al. [23] proposed a GM(1,1) model for intuitionistic fuzzy numbers. However, there are few GM(1,1) models for PFNs. In the next section, we will propose a novel GM(1,1) model for PFNs using score function and grey degree of PFN.

3.2. A Novel GM(1,1) Model for PFNs

Given a PFN sequence $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$, we want to predict the next PFN β_{n+1} according to the GM(1,1) model. Every PFN $\beta_i (i = 1, 2, \cdots, n)$ includes three key factors: membership degree μ_{β_i}, non-membership degree v_{β_i} and indeterminacy degree π_{β_i}. The membership degree sequence, non-membership degree sequence and indeterminacy degree sequence are $\mu = (\mu_{\beta_1}, \mu_{\beta_2}, \cdots, \mu_{\beta_n})$, $v = (v_{\beta_1}, v_{\beta_2}, \cdots, v_{\beta_n})$ and $\pi = (\pi_{\beta_1}, \pi_{\beta_2}, \cdots, \pi_{\beta_n})$ respectively.

A straightforward idea is to predict the membership degree $\mu_{\beta_{i+1}}$, $v_{\beta_{i+1}}$ and $\pi_{\beta_{i+1}}$ based on $\mu = (\mu_{\beta_1}, \mu_{\beta_2}, \cdots, \mu_{\beta_n})$, $v = (v_{\beta_1}, v_{\beta_2}, \cdots, v_{\beta_n})$ and $\pi = (\pi_{\beta_1}, \pi_{\beta_2}, \cdots, \pi_{\beta_n})$ respectively. However, in this way, the prediction value $\beta_{n+1} = P(\mu_{\beta_{i+1}}, v_{\beta_{i+1}})$ may result in a problem that $\mu^2_{\beta_{i+1}} + v^2_{\beta_{i+1}} + \pi^2_{\beta_{i+1}} > 1$. In other words, $\beta_{n+1} = P(\mu_{\beta_{i+1}}, v_{\beta_{i+1}})$ may not be a PFN. To overcome this drawback, we use the score function and grey degree to predict the value $\beta_{n+1} = P(\mu_{\beta_{i+1}}, v_{\beta_{i+1}})$.

Grey degree represents the uncertain measure of a number. Therefore, the indeterminacy degree of PFN can be defined by its grey degree.

Definition 11. *Let $\beta = P\left(\mu_\beta, v_\beta\right)$ be a PFN, then the grey degree of β can be defined as*

$$g(\beta) = \sqrt{1 - \mu^2_\beta - v^2_\beta} \tag{9}$$

Theorem 1. *Given a PFN $\beta = P\left(\mu_\beta, v_\beta\right)$, the information of the score function $S(\beta)$ and the grey degree $g(\beta)$ equal to that of PFN β.*

Proof. For a PFN $\beta = P\left(\mu_\beta, v_\beta\right)$, we can obtain the unique score function $S(\beta)$ by Equation (3) and grey degree $g(\beta)$ by Equation (9). Conversely, if we have a score function $S(\beta)$ and a grey degree $g(\beta)$, we obtain $\mu_\beta = \sqrt{\frac{S(\beta) - g(\beta)^2 + 1}{2}}$ and $v_\beta = \sqrt{\frac{1 - S(\beta) - g(\beta)^2}{2}}$. Therefore, the information of score function $S(\beta)$ and grey degree $g(\beta)$ equal to that of PFN β. □

According to the above theorem, if we can obtain the score function and grey degree of PFN β, we can easily obtain the PFN β. This theorem can guarantee that we can use the score function and grey degree to predict the PFN.

Example 1. *Given a PFN $\beta = (0.9, 0.2)$, we can obtain $S(\beta) = 0.9^2 - 0.2^2 = 0.77$ and $g(\beta) = \sqrt{1 - 0.9^2 - 0.2^2} = 0.3873$. Conversely, if $S(\beta) = 0.77$ and $g(\beta) = 0.3873$, we can get $\beta = (0.9, 0.2)$.*

3.2.1. The Prediction of Score Function

For a PFN sequence $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$, the score function sequence $S(\beta) = (S(\beta_1), S(\beta_2), \cdots, S(\beta_n))$ can be obtained according to Equation (3).

Let $S^{(1)}(\beta) = (S^{(1)}(\beta_1), S^{(1)}(\beta_2), \cdots, S^{(1)}(\beta_n))$ be the 1-AGO sequence of $S(\beta)$ and $z^{(1)}(k) = (S^{(1)}(\beta k) + S^{(1)}(\beta k-1))/2$ $(k = 2, 3, \cdots, n)$ be the elements of the mean generated sequence of consecutive neighbors of $S(\beta)$. Then the time response function can be obtained as

$$S(\beta_{k+1}) = (1-e^a)(S^{(1)}(\beta_1) - \frac{b}{a})e^{-ak} \quad (10)$$

Here a and b can be obtained by the equation $[a, b]^T = (B^T B)^{-1} B^T Y$, where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} S(\beta_2) \\ S(\beta_3) \\ \vdots \\ S(\beta_n) \end{bmatrix}.$$

Especially, when $k = n$, we can predict the next number $S(\beta_{n+1})$ by Equation (10).

3.2.2. Obtaining the Grey Degree of Prediction Value

According to the PFN sequence $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$, the grey degree sequence can be calculated as $g(\beta) = (g(\beta_1), g(\beta_2), \cdots, g(\beta_n))$ by Equation (8). The grey degree of prediction PFN β_{n+1} can be written as $g(\beta_{n+1})$. According to [34], the value of $g(\beta_{n+1})$ is related to $g(\beta_1), g(\beta_2), \cdots, g(\beta_n)$. We use an OWA operator to aggregate the values of $g(\beta_1), g(\beta_2), \cdots, g(\beta_n)$ to obtain $g(\beta_{n+1})$.

The OWA operator was proposed by Yager [35] and has been applied to decision making problems [36,37]. An OWA operator can aggregate information according the risk preference of DMs. The basic definition is shown as follows.

Definition 12 [35]**.** *An OWA operator of dimension n is a mapping OWA:* $R^n \rightarrow R$ *that has an associated weighting vector W of dimension n with* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0, 1]$, *such that*

$$OWA(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \omega_j b_j, \quad (11)$$

where b_j *is the j th largest of* a_i.

To obtain the weighting vector W, orness measure representing the optimistic coefficient of DMs was proposed by [35] as

$$orness(w) = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j \quad (12)$$

We can use the following programming to compute the weights

$$\begin{aligned} & Maximize - \sum_{j=1}^{n} w_j ln w_j \\ & s.t \ \ orness(w) = \tfrac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j = \alpha \\ & \sum_{j=1}^{n} w_j = 1 \\ & 0 \le \alpha \le 1; w_j \in [0, 1], j = (1, 2, ..., n) \end{aligned} \quad (P1)$$

Based on Equaion (11), we can obtain the grey degree of β_{n+1} as

$$g(\beta_{n+1}) = \sum_{j=1}^{n} w_j g(\beta_i). \quad (13)$$

3.2.3. Predicting the PFNs Based on Score Function and Grey Degree

For the PFN sequence $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$, based on Equations (10) and (13) respectively, we can compute the score function $S(\beta_{n+1})$ and the grey degree $g(\beta_{n+1})$ of β_{n+1}. According to Theorem 1, we can obtain the predicting PFN as $\beta_{n+1} = P(\mu_{\beta_{n+1}}, v_{\beta_{n+1}})$,

$$\text{where } \mu_{\beta_{n+1}} = \sqrt{\frac{S(\beta_{n+1}) - g(\beta_{n+1})^2 - 1}{2}}, \tag{14}$$

$$v_{\beta_{n+1}} = \sqrt{\frac{1 - S(\beta_{n+1}) - g(\beta_{n+1})^2}{2}} \tag{15}$$

4. A Dynamic Decision Making Method using RT and GM(1,1) for PFNs

In this section, we will proposed a dynamic decision-making method based on RT and GM(1,1) for PFNs.

4.1. Decision Making Problem

For a decision making problem, let $C = (C_1, C_2, \cdots, C_n)$ be the criteria set, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ be its weighting vector satisfying $\sum_{j=1}^{n} \omega_j = 1$, $\omega_j \in [0,1]$, and $A = (A_1, A_2, \cdots, A_m)$ be the alternative set. DM gives s decision making matrices $D^k = [d_{ij}^k]_{m \times n}$ $(k = 1, 2, \cdots, s, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n)$ at s time nodes, where $d_{ij}^k = P(\mu_{d_{ij}^k}, v_{d_{ij}^k})$ is a PFN and indicates the evaluation of alternative A_i under the criterion C_j at the time node k.

4.2. Predicting the Decision-Making Matrix $D^{s+1} = [d_{ij}^{s+1}]_{m \times n}$ at the Time Node $s+1$

In some cases, when DMs make a decision, they consider not only the existing evaluation but also that of future. We use the GM(1,1) model to predict the decision matrix $D^{s+1} = [d_{ij}^{s+1}]_{m \times n}$ at the next time node $s+1$.

For PFN sequence $d_{ij} = (d_{ij}^1, d_{ij}^2, \cdots d_{ij}^s)$, we predict the next PFN d_{ij}^{s+1} as follows.

(1) Compute the score function of $d_{ij}^1, d_{ij}^2, \cdots, d_{ij}^s$ as $S(d_{ij}^1), S(d_{ij}^2), \cdots, S(d_{ij}^s)$ using Equation (3). Based on Equation (10), we can obtain the prediction of score function $S(d_{ij}^{s+1})$.

(2) Based on Equation (9), calculate the grey degree of $d_{ij}^1, d_{ij}^2, \cdots, d_{ij}^s$ as $g(d_{ij}^1), g(d_{ij}^2), \cdots, g(d_{ij}^s)$. According the decision making environment, choose a suitable α and calculate the weights of OWA $w_1, w_2, \cdots, w_s$ based on P1. Based on Equation (11), we can obtain the grey degree of d_{ij}^{s+1} as

$$g(d_{ij}^{s+1}) = \sum_{k=1}^{s} w_k g(d_{ij}^k) \tag{16}$$

Based on Theorem 1, we can get the prediction PFN $d_{ij}^{s+1} = P(\mu_{d_{ij}^{s+1}}, v_{d_{ij}^{s+1}})$ as follows

$$\mu_{d_{ij}^{s+1}} = \sqrt{\frac{S(d_{ij}^{s+1}) - g(d_{ij}^{s+1})^2 + 1}{2}}, \tag{17}$$

$$v_{d_{ij}^{s+1}} = \sqrt{\frac{1 - S(d_{ij}^{s+1}) - g(d_{ij}^{s+1})^2}{2}}. \tag{18}$$

4.3. Aggregating Decision Information Based on RT

Based on Section 4.2, we can obtain the prediction matrix $D^{s+1} = [d_{ij}^{s+1}]_{m\times n}$.

(1) Determining the criteria weights

We here use grey incidence method to compute the criteria weights $\omega_1, \omega_2, \cdots, \omega_n$.

Given the decision matrix $D^{s+1} = [d_{ij}^{s+1}]_{m\times n}$, based on Definition 2, we can compute $r_i = \sum_{j=1}^{n} \frac{1}{n} d_{ij}^{s+1}$ $(i = 1, 2, \cdots, m)$ and obtain a reference column vector $R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix}$. We rewrite the matrix as $D^{s+1} = [D_1, D_2, \cdots, D_n]$, where $D_j = \begin{bmatrix} d_{1j}^{s+1} \\ d_{2j}^{s+1} \\ \vdots \\ d_{mj}^{s+1} \end{bmatrix}$. Seeing $D_1, D_2, \cdots, D_n$ as comparison vectors, according to [34], we compute the grey incidence between D_j $(j = 1, 2, \cdots, n)$ and R as

$r(D_j, R) = \sum_{i=1}^{m} \zeta_{ij}$, where $\zeta_{ij} = \frac{\min_i \min_j \left\{ d(d_{ij}^{s+1}, r_i) \right\} + 0.5 \max_i \max_j \left\{ d(d_{ij}^{s+1}, r_i) \right\}}{d(d_{ij}^{s+1}, r_i) + 0.5 \max_i \max_j \left\{ d(d_{ij}^{s+1}, r_i) \right\}}$.

Then the criteria weights can be obtained as

$$\omega_j = \frac{r(D_j, R)}{\sum_{j=1}^{n} r(D_j, R)} \tag{19}$$

(2) Aggregating information based on RT

Firstly, compute the utility value decision matrix $V = [V_{ij}]_{m\times n}$ based on matrix $D^{s+1} = [d_{ij}^{s+1}]_{m\times n}$. According to [33], for interval-valued variable $X = [X^-, X^+]$, its utility value can be obtained as $V = \int_{X^-}^{X^+} v(x) f(x) dx$ based on Equation (6). For PFN $d_{ij}^{s+1} = P(\mu_{d_{ij}^{s+1}}, v_{d_{ij}^{s+1}})$, its membership degree can be seen as an interval-valued number $[\mu_{d_{ij}^{s+1}}, \sqrt{1 - (v_{d_{ij}^{s+1}})^2}]$. Therefore, for PFN $d_{ij}^{s+1} = P(\mu_{d_{ij}^{s+1}}, v_{d_{ij}^{s+1}})$, according to Equations (4) and (5), its utility value can be written as

$$V_{ij} = \begin{cases} \int_{\mu_{d_{ij}^{s+1}}}^{\sqrt{1-(v_{d_{ij}^{s+1}})^2}} \frac{1-e^{-\alpha x}}{\alpha} f_{ij}(x) dx, \text{ if } C_j \text{ is a benifit criterion} \\ \int_{\mu_{d_{ij}^{s+1}}}^{\sqrt{1-(v_{d_{ij}^{s+1}})^2}} \frac{1-e^{\beta x}}{\beta} f_{ij}(x) dx, \text{ if } C_j \text{ is a cost criterion} \end{cases} \tag{20}$$

In this paper, we assume probability density function $f_{ij}(x)$ as a normal distribution $N(\theta_{ij}, \sigma_{ij}{}^2)$. According 3σ principle [33], we can obtain $\theta_{ij} = \frac{1}{2}(\mu_{d_{ij}^{s+1}} + \sqrt{1 - (v_{d_{ij}^{s+1}})^2})$, $\sigma_{ij} = \frac{1}{6}(\sqrt{1 - (v_{d_{ij}^{s+1}})^2} - \mu_{d_{ij}^{s+1}})$.

Then we can obtain $f_{ij}(x) = \frac{1}{\sqrt{2\pi} \frac{1}{6}(\sqrt{1-(v_{d_{ij}^{s+1}})^2} - \mu_{d_{ij}^{s+1}})} e^{-\frac{\left(x - \frac{1}{2}(\mu_{d_{ij}^{s+1}} + \sqrt{1-(v_{d_{ij}^{s+1}})^2})\right)^2}{2\left(\frac{1}{6}(\sqrt{1-(v_{d_{ij}^{s+1}})^2} - \mu_{d_{ij}^{s+1}})\right)^2}}$.

Furthermore, based on Equation (6), for criterion C_j, the regret–rejoice value alternative A_i related to A_k can be calculated as

$$R_{ikj} = 1 - e^{-r(V_{ij}-V_{kj})} \tag{21}$$

According to the criteria weights computed by Equation (19), the regret–rejoice value of alternative A_i related to A_k can be obtained as

$$R_{ik} = \sum_{j=1}^{n} \omega_j R_{ikj} \tag{22}$$

The overall regret–rejoice value of alternative A_i related to other alternatives can be obtained as

$$R_i = \sum_{k=1}^{m} R_{ik} \tag{23}$$

Then rank the alternatives $A_1, A_2, \cdots, A_m$ according to the values R_i $(i = 1, 2, \cdots, m)$.

Based on the above analysis, we conclude the main decision steps as follows.

Step 1. Based on decision matrices $D^k = [d_{ij}^k]_{m\times n}$ $(k = 1, 2, \cdots, s,\ i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n)$, compute the score function $S(d_{ij}^1), S(d_{ij}^2), \cdots, S(d_{ij}^s)$ using Equation (3) and compute the prediction of score function $S(d_{ij}^{s+1})$ at the next time node $s+1$ according to Equation (10).

Step 2. Choose a suitable α to compute the OWA weights $w_1, w_2, \cdots, w_s$ based on P1, and obtain the grey degree of d_{ij}^{s+1} as $g(d_{ij}^{s+1})$ based on Equation (16).

Step 3. Based on Equations (17) and (18), get the prediction PFN $d_{ij}^{s+1} = P(\mu_{d_{ij}^{s+1}}, v_{d_{ij}^{s+1}})$.

Step 4. Based on prediction matrix D^{s+1} and Equation (19), compute the criteria weights $\omega_1, \omega_2, \cdots, \omega_n$.

Step 5. Based on Equation (20), compute the utility value decision matrix V for D^{s+1}.

Step 6. Based on Equation (23), compute the overall regret–rejoice value R_i $(i = 1, 2, \cdots, m)$.

Step 7. Rank the alternatives A_i $(i = 1, 2, \cdots, m)$ based on R_i.

5. Case Study

In this section, we will use our proposed method to resolve a real case in Shanghai and make comparisons with the traditional TOPSIS method and dynamic decision method based on prospect theory. Furthermore, we will study a case of selecting location for disposing household solid waste after determining the best enterprise, and make a comparison with the TODIM method.

5.1. Description of Case Background

Shanghai, as an economic center of China, has a population more than 24 million and has been surrounded with a great deal of garbage for many years. Mountains of garbage have serious impacts on the physical and mental health of local residents. Recently, the Shanghai government put forward a new policy for mandatory garbage sorting from July 1, 2019. According to the new policy, the household garbage can be sorted into four categories: recyclable wastes, hazardous wastes, wet wastes and dry wastes. The process of disposing recyclable wastes can be decomposed into some PPP projects. There is a PPP project needing some enterprise bids. Based on the process of preliminary screening, there are eight enterprises selected for the list: $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$. Four criteria are considered:

C_1 (Technical level): the ability of the enterprise to effectively dispose the household garbage;

C_2 (Revenue situation): the revenue situation of the enterprise's normal development;

C_3 (Social responsibility): the enterprise's responsibility for protecting environment and contribution to society;

C_4 (Development potential): the ability of the enterprise for rapid development and creating value.

Based on data in the past four years (2015–2018), an evaluation expert team makes an evaluation for the eight enterprises under the four criteria and gives four decision matrices according to their experiences as shown in Tables 1–4. In detail, for example, the element $d_{12}^{1} = P(0.90, 0.20)$ means that the expert team thinks that the score of alternative A_1 under criterion C_2 for "good" is 0.90 and for "bad" 0.20 in 2015. It is worth note that the sum of score for "good" and "bad" may be larger than one because of experts' uncertain judgements.

Table 1. Decision matrix D^1 based on the data in 2015.

	C_1	C_2	C_3	C_4
A_1	P(0.90,0.30)	P(0.90,0.20)	P(0.70,0.50)	P(0.90,0.30)
A_2	P(0.90,0.30)	P(0.90,0.20)	P(0.70,0.50)	P(0.90,0.30)
A_3	P(0.90,0.20)	P(0.80,0.30)	P(0.80,0.50)	P(0.90,0.20)
A_4	P(0.90,0.20)	P(0.80,0.30)	P(0.80,0.50)	P(0.90,0.20)
A_5	P(0.90,0.20)	P(0.90,0.20)	P(0.60,0.50)	P(0.90,0.30)
A_6	P(0.90,0.30)	P(0.90,0.20)	P(0.70,0.50)	P(0.90,0.30)
A_7	P(0.85,0.20)	P(0.75,0.30)	P(0.80,0.45)	P(0.90,0.15)
A_8	P(0.65,0.60)	P(0.70,0.50)	P(0.60,0.60)	P(0.90,0.30)

Table 2. Decision matrix D^2 based on the data in 2016.

	C_1	C_2	C_3	C_4
A_1	P(0.80,0.30)	P(0.80,0.30)	P(0.60,0.60)	P(0.70,0.30)
A_2	P(0.70,0.30)	P(0.60,0.50)	P(0.90,0.20)	P(0.90,0.10)
A_3	P(0.80,0.50)	P(0.90,0.20)	P(0.60,0.50)	P(0.80,0.30)
A_4	P(0.80,0.20)	P(0.60,0.60)	P(0.80,0.20)	P(0.70,0.30)
A_5	P(0.80,0.30)	P(0.80,0.30)	P(0.60,0.50)	P(0.70,0.30)
A_6	P(0.70,0.30)	P(0.80,0.30)	P(0.60,0.60)	P(0.70,0.35)
A_7	P(0.80,0.50)	P(0.85,0.20)	P(0.60,0.50)	P(0.85,0.30)
A_8	P(0.80,0.20)	P(0.65,0.55)	P(0.80,0.20)	P(0.70,0.30)

Table 3. Decision matrix D^3 based on the data in 2017.

	C_1	C_2	C_3	C_4
A_1	P(0.80,0.50)	P(0.60,0.70)	P(0.80,0.40)	P(0.80,0.30)
A_2	P(0.80,0.30)	P(0.90,0.30)	P(0.70,0.60)	P(0.70,0.50)
A_3	P(0.70,0.70)	P(0.80,0.40)	P(0.90,0.30)	P(0.60,0.70)
A_4	P(0.70,0.40)	P(0.80,0.40)	P(0.70,0.40)	P(0.80,0.30)
A_5	P(0.80,0.40)	P(0.70,0.70)	P(0.80,0.40)	P(0.80,0.20)
A_6	P(0.80,0.20)	P(0.60,0.65)	P(0.80,0.40)	P(0.80,0.30)
A_7	P(0.70,0.70)	P(0.80,0.40)	P(0.90,0.25)	P(0.60,0.70)
A_8	P(0.75,0.35)	P(0.80,0.40)	P(0.75,0.40)	P(0.85,0.25)

Table 4. Decision matrix D^4 based on the data in 2018.

	C_1	C_2	C_3	C_4
A_1	P(0.90,0.30)	P(0.70,0.40)	P(0.50,0.60)	P(0.70,0.40)
A_2	P(0.70,0.40)	P(0.70,0.40)	P(0.60,0.50)	P(0.80,0.40)
A_3	P(0.80,0.50)	P(0.80,0.30)	P(0.70,0.40)	P(0.70,0.40)
A_4	P(0.80,0.50)	P(0.50,0.60)	P(0.80,0.50)	P(0.70,0.40)
A_5	P(0.70,0.30)	P(0.70,0.40)	P(0.50,0.60)	P(0.70,0.40)
A_6	P(0.70,0.40)	P(0.70,0.40)	P(0.65,0.60)	P(0.75,0.40)
A_7	P(0.80,0.50)	P(0.75,0.30)	P(0.70,0.35)	P(0.70,0.40)
A_8	P(0.80,0.50)	P(0.55,0.60)	P(0.80,0.50)	P(0.70,0.40)

5.2. Decision Procedure

Step 1. Based on decision matrices D^1, D^2, D^3 and D^4, prediction of score function $S(d_{ij}^5)$ can be obtained as Table 5 according to Equations (3) and (10).

Table 5. Prediction of score function at time node 5.

	C_1	C_2	C_3	C_4
A_1	0.777	0.017	0.093	0.369
A_2	0.369	0.567	0.018	0.205
A_3	0.297	0.393	0.567	0.017
A_4	0.248	0.093	0.248	0.369
A_5	0.345	0.065	0.090	0.388
A_6	0.388	0.039	0.216	0.472
A_7	0.260	0.356	0.628	0.011
A_8	0.296	0.112	0.280	0.409

Step 2. Choose $\alpha = 0.5$, based on Equation (16), obtain the grey degree $g(d_{ij}^5)$ as the following Table 6.

Table 6. Grey degree of prediction values at time node 5.

	C_1	C_2	C_3	C_4
A_1	0.371	0.471	0.528	0.519
A_2	0.519	0.466	0.433	0.501
A_3	0.298	0.468	0.466	0.471
A_4	0.505	0.528	0.505	0.519
A_5	0.501	0.410	0.580	0.530
A_6	0.530	0.491	0.488	0.496
A_7	0.323	0.528	0.500	0.455
A_8	0.481	0.516	0.488	0.505

Step 3. Based on Equations (17) and (18), get the prediction Pythagorean fuzzy number (PFN) matrix as the following Table 7.

Table 7. Prediction PFN matrix D^5.

	C_1	C_2	C_3	C_4
A_1	P(0.905,0.207)	P(0.630,0.617)	P(0.638,0.561)	P(0.742,0.425)
A_2	P(0.742,0.425)	P(0.822,0.328)	P(0.644,0.630)	P(0.690,0.522)
A_3	P(0.777,0.554)	P(0.766,0.440)	P(0.822,0.328)	P(0.630,0.617)
A_4	P(0.705,0.499)	P(0.638,0.561)	P(0.705,0.499)	P(0.742,0.425)
A_5	P(0.740,0.450)	P(0.670,0.619)	P(0.614,0.536)	P(0.744,0.407)
A_6	P(0.744,0.407)	P(0.632,0.600)	P(0.699,0.522)	P(0.783,0.375)
A_7	P(0.760,0.564)	P(0.734,0.427)	P(0.830,0.246)	P(0.634,0.625)
A_8	P(0.729,0.486)	P(0.650,0.558)	P(0.722,0.491)	P(0.760,0.410)

Step 4. Based on prediction matrix D^5 and Equation (19), compute the criteria weights as $(\omega_1, \omega_2, \omega_3, \omega_4) = (0.273, 0.234, 0.251, 0.242)$.

Step 5. Based on Equation (20), choose $\alpha = \beta = 0.02$ [33] and compute the utility value decision matrix V as the following Table 8.

Table 8. Utility matrix V.

	C_1	C_2	C_3	C_4
A_1	0.931	0.702	0.726	0.814
A_2	0.814	0.873	0.703	0.764
A_3	0.797	0.823	0.873	0.702
A_4	0.778	0.726	0.778	0.814
A_5	0.808	0.720	0.722	0.820
A_6	0.820	0.709	0.768	0.845
A_7	0.785	0.810	0.890	0.701
A_8	0.794	0.733	0.788	0.827

Step 6. Based on Equation (23) and the criteria weights, compute the overall regret–rejoice value R_i ($r = 0.5$) as

$$R_1 = 0.033, R_2 = -0.004, R_3 = 0.040, R_4 = -0.056, R_5 = -0.079, R_6 = -0.005,$$
$$R_7 = 0.029, R_8 = -0.008.$$

Step 7. The ranking result is $A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$.

5.3. Further Discussions

In the above decision procedure, risk aversion coefficient r and orness value α play an important role. We next discuss the two parameters.

(1) Decision making result analysis based on variation of parameter r

To make the analysis easy, we assume the orness value $\alpha = 0.5$. The overall regret–rejoice value of A_i ($i = 1, 2, \cdots, 8$) under different r can be seen as Table 9 and Figure 1.

Table 9. Overall regret–rejoice value of A_i under different r.

	r = 0.1	r = 0.2	r = 0.3	r = 0.4	r = 0.5	r = 0.6	r = 0.7	r = 0.8	r = 0.9
A_1	0.0080	0.0153	0.0220	0.0279	0.0332	0.0378	0.0417	0.0450	0.0475
A_2	0.0003	0.0001	−0.0008	−0.0023	−0.0044	−0.0071	−0.0104	−0.0144	−0.0189
A_3	0.0093	0.0179	0.0259	0.0332	0.0400	0.0461	0.0516	0.0564	0.0606
A_4	−0.0105	−0.0212	−0.0324	−0.0438	−0.0556	−0.0677	−0.0801	−0.0929	−0.1060
A_5	−0.0150	−0.0305	−0.0463	−0.0625	−0.0790	−0.0960	−0.1134	−0.1312	−0.1494
A_6	−0.0003	−0.0010	−0.0021	−0.0035	−0.0054	−0.0076	−0.0102	−0.0131	−0.0165
A_7	0.0088	0.0149	0.0195	0.0250	0.0291	0.0324	0.0359	0.0383	0.0400
A_8	−0.0010	−0.0023	−0.0040	−0.0059	−0.0082	−0.0107	−0.0136	−0.0169	−0.0204

The ranking result can be seen in Table 10.

Table 10. Ranking result with different r.

r	Ranking Result
0.1	$A_3 \succ A_7 \succ A_1 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.2	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.3	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.4	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.5	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.6	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
0.7	$A_3 \succ A_1 \succ A_7 \succ A_6 \succ A_2 \succ A_8 \succ A_4 \succ A_5$
0.8	$A_3 \succ A_1 \succ A_7 \succ A_6 \succ A_2 \succ A_8 \succ A_4 \succ A_5$
0.9	$A_3 \succ A_1 \succ A_7 \succ A_6 \succ A_2 \succ A_8 \succ A_4 \succ A_5$

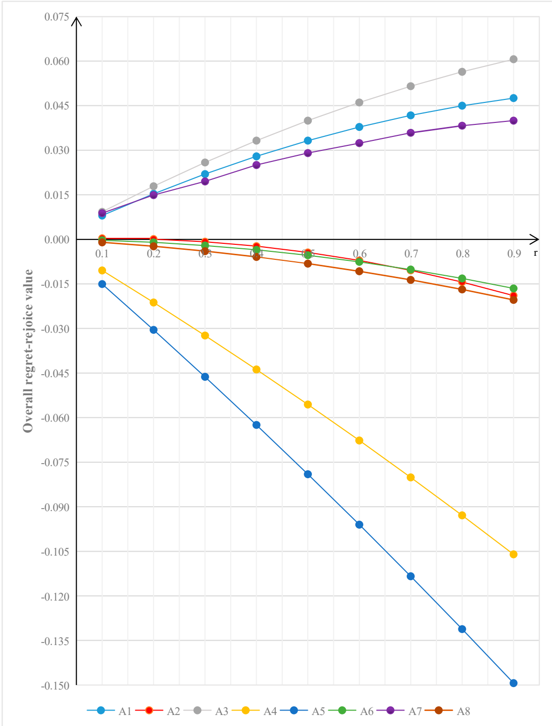

Figure 1. Overall regret–rejoice value of A_i under different r.

From Table 10, we can see that the ranking results are somewhat different with different r, and the alternative A_3 is the best enterprise to invest.

(2) Decision making result analysis based on variation of parameter α

We here assume $r = 0.5$. The overall regret–rejoice value of A_i $(i = 1, 2, \cdots, 8)$ under $\alpha = 0.3$, $\alpha = 0.5$ and $\alpha = 0.8$ can be seen as the following Figure 2.

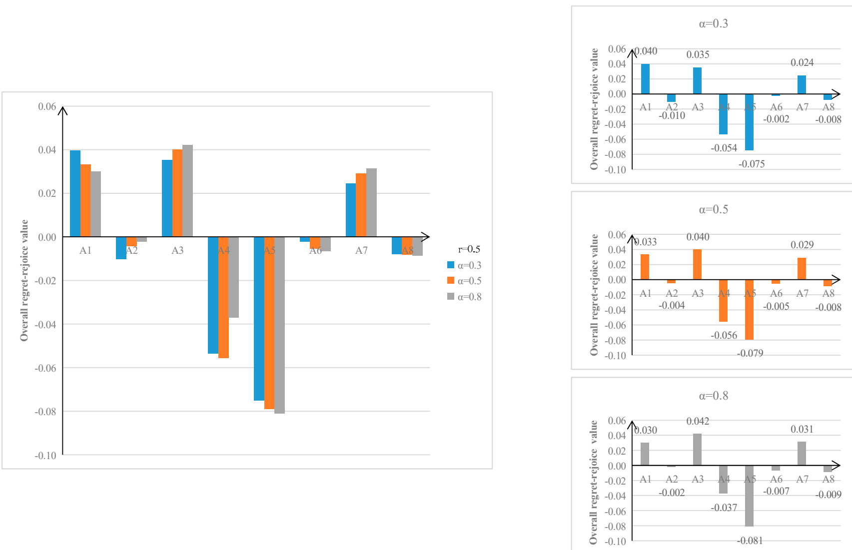

Figure 2. Overall regret–rejoice value of A_i under $\alpha = 0.3$, $\alpha = 0.5$ and $\alpha = 0.8$.

The ranking result under $\alpha = 0.3$, $\alpha = 0.5$ and $\alpha = 0.8$ can be seen as Table 11.

Table 11. Ranking result under $\alpha = 0.3$, $\alpha = 0.5$ and $\alpha = 0.8$.

	Ranking Result
$\alpha = 0.3$	$A_1 \succ A_3 \succ A_7 \succ A_6 \succ A_8 \succ A_2 \succ A_4 \succ A_5$
$\alpha = 0.5$	$A_3 \succ A_1 \succ A_7 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$
$\alpha = 0.8$	$A_3 \succ A_7 \succ A_1 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_5$

From Table 11, we can see that the ranking results are different with different α. The alternative A_3 is the best enterprise to invest when $\alpha = 0.5$ and $\alpha = 0.8$, and alternative A_1 is the best enterprise to invest when $\alpha = 0.3$.

5.4. Comparison Analysis

(1) Comparison with the TOPSIS method proposed in [29]

To embody the effectiveness of our method, we make a comparison analysis with the TOPSIS method proposed by Zhang and Xu [29]. The TOPSIS method can be described as the following steps.

(1) Determine the PIS A^+ and NIS A^- based on decision matrix D^5;
(2) Compute the distance between A_i and A^+ as $D(A_i, A^+)$;
(3) Compute the distance between A_i and A^- as $D(A_i, A^-)$;
(4) Calculate the revised closeness $\xi(A_i)$ for alternative A_i;
(5) Rank the alternatives according to the value $\xi(A_i)$.

We use the TOPSIS method to solve our decision problem and obtain the decision results as follows.

The ranking result is $A_1 \succ A_6 \succ A_3 \succ A_7 \succ A_8 \succ A_4 \succ A_2 \succ A_5$.

From Tables 11 and 12, we can see that alternative A_1 is the best enterprise both in TOPSIS method and our proposed method when $r = 0.5$ and $\alpha = 0.3$, and in most cases the ranking results are different between the two methods. TOPSIS method mainly focuses on the distances between alternatives and an ideal solution, and usually does not consider the DMs' preference. In real decision-making problems, DMs' preference is very important and influences the decision-making results. Our proposed method uses the RT considering the DMs' preference, which is close to actual decision-making processes. Our proposed method may be flexible by setting different parameters r according to the DMs' preference in practical decision-making problems. Furthermore, the TOPSIS method can hardly solve the dynamic decision making problems while our proposed can solve them by a GM(1,1) model. Therefore, our proposed method is more generally applicable than the TOPSIS method.

Table 12. Decision results using TOPSIS method in [29].

	$D(A_i,A^+)$	$D(A_i,A^-)$	$\xi(A_i)$	Ranking
A_1	0.151	0.251	0	1
A_2	0.191	0.131	−0.746	7
A_3	0.163	0.131	−0.559	3
A_4	0.214	0.170	−0.742	6
A_5	0.232	0.184	−0.803	8
A_6	0.191	0.200	−0.464	2
A_7	0.168	0.137	−0.566	4
A_8	0.191	0.251	−0.583	5

(2) Comparison with the dynamic decision method based on prospect theory proposed in [38]

Ding et al. [38] proposed a dynamic method based on prospect theory. The main decision steps are concluded as follows.

(1) Compute the criteria weights;
(2) Compute the prospect values P_i^k for A_i $(i = 1, 2, \cdots, m)$ at different time node k $(k = 1, 2, \cdots, s)$;

(3) Rank the alternatives according to P_i;
(4) Let $k = k + 1$. If $k < s$, then go to Step 1; otherwise, stop.

We use the dynamic decision method to solve our decision problem. We use the same parameters in prospect theory as Ding et al. [38].

(1) To make the comparison effective, we use the criteria weights $(\omega_1, \omega_2, \omega_3, \omega_4) = (0.273, 0.234, 0.251, 0.242)$.

(2)–(3) We assume the probabilities of these states is equal. We obtain

(1) At the time node 1

$P_1^1 = -0.143$, $P_2^1 = -0.143$, $P_3^1 = -0.048$,$P_4^1 = -0.048$, $P_5^1 = -0.069$, $P_6^1 = -0.059$, $P_7^1 = -0.066$, $P_8^1 = -0.203$.

The ranking result is $A_3 \sim A_4 > A_6 > A_7 > A_5 > A_1 \sim A_2 > A_8$.

(2) At time node 2

$$P_1^2 = -0.185,\ P_2^2 = -0.11,\ P_3^2 = -0.161,\ P_4^2 = -0.169,\ P_5^2 = -0.185,\ P_6^2 = -0.206$$
$$P_7^2 = -0.164,\ P_8^2 = -0.16$$

The ranking result is $A_2 > A_8 > A_3 > A_7 > A_4 > A_1 \sim A_5 > A_6$

(3) At time node 3

$$P_1^3 = -0.174,\ P_2^3 = -0.117,\ P_3^3 = -0.206,\ P_4^3 = -0.144,\ P_5^3 = -0.153,\ P_6^3 = -0.129$$
$$P_7^3 = -0.196, P_8^3 = -0.101$$

The ranking result is $A_8 > A_2 > A_6 > A_4 > A_5 > A_1 \sim A_7 > A_3$

(4) At time node 4

$$P_1^4 = -0.131,\ P_2^4 = -0.149,\ P_3^4 = -0.166,\ P_4^4 = -0.135,\ P_5^4 = -0.197,\ P_6^4 = -0.155$$
$$P_7^4 = -0.137,\ P_8^4 = -0.127$$

The ranking result is $A_8 > A_1 > A_4 > A_7 > A_2 > A_6 \sim A_3 > A_5$.

We can see that the ranking results are different between our proposed method and in [38]. According to the method proposed in [38], alternative A_3 is the best enterprise only at time node 1. In our proposed method, alternative A_3 is the best enterprise in most cases. Both the two methods consider the DM's preference and are in a dynamic decision-making style. However, the method proposed by Ding et al. [38] ranks the alternatives in different time nodes and cannot predict the decision-making data in future. As we know, an excellent enterprise should have a good development prospect. In other words, the decision data in future are essential to predict. Our proposed method can use the GM(1,1) model to predict the decision matrix at the next time node 5. DMs can obtain the possible information in the future using our method. Furthermore, the method proposed by Ding et al. [38] gives different ranking results at different time nodes, which will make DMs confused to rank alternatives. Our proposed method can obtain certain ranking result according to the DM's preference and the decision-making data in future.

5.5. Selecting Location for Disposing Household Solid Waste

Decision Making Problem Description

From the above analysis, we can obtain that the alternative A_3 is the best enterprise to dispose HSW. Another problem is how to select the most suitable location for disposing HSW, which can be seen as an MCDM problem. Through the investigation and research, three locations are considered: A_1', A_2' and A_3'. According to Beskese et al. [27], there are four criteria: C_1' (Available land), C_2' (Soil conditions and topography), C_3' (Climatologic and hydrologic conditions) and C_4' (Economic considerations). Experts make an evaluation for the three alternatives under the four criteria according to the data in the past four years (2015–2018), and give four decision making matrices D_1', D_2', D_3' and D_4' as the following Tables 13–16.

Table 13. Decision matrix D'_1 based on the data in 2015.

	C'_1	C'_2	C'_3	C'_4
A'_1	P(0.65,0.20)	P(0.80,0.40)	P(0.60,0.20)	P(0.75,0.30)
A'_2	P(0.75,0.40)	P(0.50,0.20)	P(0.90,0.35)	P(0.60,0.40)
A'_3	P(0.80,0.30)	P(0.50,0.45)	P(0.70,0.35)	P(0.55,0.35)

Table 14. Decision matrix D'_2 based on the data in 2016.

	C'_1	C'_2	C'_3	C'_4
A'_1	P(0.50,0.10)	P(0.75,0.30)	P(0.60,0.15)	P(0.70,0.25)
A'_2	P(0.80,0.30)	P(0.65,0.20)	P(0.60,0.30)	P(0.50,0.10)
A'_3	P(0.55,0.10)	P(0.85,0.30)	P(0.60,0.25)	P(0.60,0.20)

Table 15. Decision matrix D'_3 based on the data in 2017.

	C'_1	C'_2	C'_3	C'_4
A'_1	P(0.85,0.35)	P(0.70,0.30)	P(0.75,0.30)	P(0.65,0.35)
A'_2	P(0.70,0.40)	P(0.60,0.10)	P(0.70,0.40)	P(0.80,0.50)
A'_3	P(0.70,0.40)	P(0.70,0.25)	P(0.85,0.40)	P(0.60,0.25)

Table 16. Decision matrix D'_4 based on the data in 2018.

	C'_1	C'_2	C'_3	C'_4
A'_1	P(0.70,0.30)	P(0.80,0.50)	P(0.50,0.50)	P(0.80,0.30)
A'_2	P(0.70,0.25)	P(0.70,0.40)	P(0.80,0.60)	P(0.60,0.15)
A'_3	P(0.50,0.25)	P(0.40,0.10)	P(0.85,0.35)	P(0.70,0.40)

We solve this decision-making problem using our proposed method as follows.

Based on Equations (17) and (18), we get the prediction PFN matrix as showed in Table 17 ($\alpha = 0.5$).

Table 17. Prediction PFN matrix D'_5.

	C'_1	C'_2	C'_3	C'_4
A'_1	P(0.65,0.20)	P(0.80,0.40)	P(0.60,0.20)	P(0.75,0.30)
A'_2	P(0.75,0.40)	P(0.50,0.20)	P(0.90,0.35)	P(0.60,0.40)
A'_3	P(0.80,0.30)	P(0.50,0.45)	P(0.70,0.35)	P(0.55,0.35)

Based on prediction matrix D'_5 and Equation (19), the criteria weights can be obtained as $\left(\omega'_1, \omega'_2, \omega'_3, \omega'_4\right) = (0.301, 0.242, 0.152, 0.305)$.

Based on Equation (20), choose $\alpha = \beta = 0.02$ and compute the utility value decision matrix V' as the following Table 18.

Table 18. Utility matrix V'.

	C_1	C_2	C_3	C_4
A_1	0.865	0.807	0.717	0.874
A_2	0.796	0.791	0.798	0.823
A_3	0.746	0.723	0.897	0.781

Compute the overall regret–rejoice value ($r = 0.5$) as $R'_1 = 0.041,\ R'_2 = 0,\ R'_3 = -0.045$.

The ranking result is $A'_1 \succ A'_2 \succ A'_3$. The alternative A'_1 is the most suitable location.

(2) Comparison with Pythagorean fuzzy TODIM method proposed by Ren et al. [14]

Pythagorean fuzzy TODIM method proposed by Ren et al. [14] can be concluded as follows:

Step 1. Compute the relative weight for C_j $(j = 1, 2, \cdots, n)$ as $\omega_{jr} = \frac{\omega_j}{\omega_r}$, where $\omega_r = \max\{\omega_1, \omega_2, \cdots, \omega_n\}$.

Step 2. Calculate dominance degree A_i $(i = 1, 2, \cdots, m)$ over A_k under criterion C_j as $\phi_j(A_i, A_k) = \begin{cases} \sqrt{\omega_{jr} d(d_{ij}, d_{kj}) / \sum_{j=1}^{n} \omega_{jr}}, if d_{ij} > d_{kj} \\ -\frac{1}{\theta} \sqrt{(\sum_{j=1}^{n} \omega_{jr}) d(d_{ij}, d_{kj}) / \omega_{jr}}, if d_{ij} < d_{kj} \end{cases}$

Step 3. Compute the overall dominance degree of the alternative A_i over A_k as $\Phi(A_i, A_k) = \sum_{j=1}^{n} \phi_j(A_i, A_k)$

Step 4. Compute the overall value of alternative A_i over other alternatives as $\varsigma_i = \sum_{k=1}^{n} \Phi(A_i, A_k)$.

Step 5. Rank the alternatives according to ξ_i.

We apply this method to solve our decision-making problem.

Step 1. The relative weights can be obtained as $\omega_{1r} = 0.99$, $\omega_{2r} = 0.79$, $\omega_{3r} = 0.5$, $\omega_{4r} = 1$.

Step 2–Step 3. According to prediction PFN matrix, the overall dominance degree can be computed as

$\Phi(A'_1, A'_2) = -2$, $\Phi(A'_1, A'_3) = -1.34$, $\Phi(A'_2, A'_1) = -1.76$, $\Phi(A'_2, A'_3) = 0.09$, $\Phi(A'_3, A'_1) = -1.79$, $\Phi(A'_3, A'_2) = -2.68$.

Step 4. The overall values of alternatives can be computed as $\varsigma_1 = -3.35$, $\varsigma_2 = -1.67$, $\varsigma_3 = -4.47$.

Step 5. The ranking result is $A'_2 > A'_1 > A'_3$.

If we use a traditional linear weighting method, we can obtain $A'_1 > A'_2 > A'_3$, which is the same as our proposed method.

The ranking results are different between our proposed method and the TODIM method proposed by Ren et al. [14]. The main reason lies in the fact that our proposed method uses the RT while the TODIM method proposed by Ren et al. [14] uses the TODIM method. Both the two methods are behavior decision-making methods. However, the former method can solve the dynamic decision-making problems while the latter one cannot. Our proposed method is more generally applicable than the TODIM method proposed by Ren et al. [14]. Furthermore, the ranking result using traditional linear weighting method is the same as our proposed method, which illustrates the effectiveness of our method.

6. Conclusions

With the fast development of China, living standard of people have increased at an express speed. However, meanwhile, HSW has become a serious social problem. Most cities are surrounded by a good deal of household solid waste, which affects the people's health. Garbage classification and recycling can be an effective way to cope with this issue. Selecting suitable enterprises to dispose the HSW is very important to solve the garbage problem. When we evaluate the enterprises, we should consider not only the present performance but also that of the future. Therefore, we proposed a dynamic decision making method based on regret theory and the GM(1,1) model with Pythagorean fuzzy numbers. A novel GM(1,1) was proposed to predict the performance in future of the enterprises. Regret theory was used to embody the bounded rationality of decision makers. By the case study, we can see that the ranking results in our proposed are different to the TOPSIS method proposed by Zhang and Xu [29] and the dynamic method proposed by Ding et al. [38]. In our method, the best alternative is A_3 in most cases. In the method proposed by Zhang and Xu [29], alternative A_1 is the best enterprise. The TOPSIS method does not consider the DM's preference, which is vitally important to influence the ranking results. Additionally, in the method proposed by Ding et al. [38] the ranking results are different at different time nodes. The dynamic method proposed by Ding et al. [38] can hardly predict the decision data in future. It can be seen that our proposed method can solve the dynamic decision making problems considering both DM's preference and the decision data in the future based on the GM(1,1) model. Furthermore, a case of selecting location for disposing HSW after determining the best enterprise was used to illustrate the effectiveness of our proposed method. Therefore, we can draw the conclusion that our proposed method is feasible and effective to solve the problem of choosing suitable enterprises disposing household solid waste.

Future research will consider the inconsistency of decision makers and make a conflict analysis in the choosing between alternative enterprises in household solid waste field.

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