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Two-Stage Supply-Chain Optimization Considering Consumer Low-Carbon Awareness under Cap-and-Trade Regulation

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Abstract: Cap-and-trade regulation is an effective mechanism to control carbon emissions. The optimization problem for a two-stage supply chain consisting of a manufacturer and a retailer under cap-and-trade regulation was investigated in this paper. Consumers' low-carbon awareness level was considered in the decision models. Optimal decision policies, corresponding emissions, and profits were calculated for decentralized and centralized decision-making modes. Under a decentralized mode, the two-stage supply-chain optimization problem was formulated as a Stackelberg game model, where the manufacturer and retailer were the leader and follower, respectively. The manufacturer decides the emission-reduction levels per product unit and the retailer decides the retail price per unit product. The optimal decisions are derived using the reverse-solution method. By contrast, the two-stage supply-chain optimization problem under a decentralized mode was formulated as a single-level optimization model. The nonlinear model is handled by KKT optimality conditions. The influence of the regulation parameters (caps and carbon prices) and consumers' low-carbon awareness on the optimal decision policies, the corresponding emissions, and profits is discussed in detail. A comparison between the two modes implies that the decentralized mode is dominated by the centralized mode in terms of profit and emissions. In order to provoke the decision makers under decentralized modes to make the decisions under the decentralized mode, a profit-sharing contract was designed. This study shows that higher consumer low-carbon awareness and carbon prices can improve the manufacturer-decision flexibility when there exists a profit-sharing contract. Finally, numerical experiments confirmed the analytical results.

Keywords: two-stage supply-chain optimization; cap-and-trade regulation; low-carbon awareness; Stackelberg game

1. Introduction

Increasing carbon emissions (emissions from greenhouse gases) have become a global issue due to their serious consequences. According to the report of Intergovernmental Panel on Climate Change (IPCC), industrial carbon dioxide emissions in 2050 must be 75% to 90% lower than those in 2010 to achieve the goal of controlling the rise of temperature within 1.5 °C. To achieve the 1.5 °C temperature control target, global climate action urgently needs to be accelerated. Faced with this grim situation, governments have to formulate various regulations to control emissions. Among these regulations, the cap-and-trade mechanism driven by market power is regarded as the most widely used policy tool [1–3]. The European Union Emissions Trading System (EU ETS) was launched in 2005 and it is the first international system for trading greenhouse-gas-emission allowances. Moreover, China established a national carbon-trading market for power-generation companies at the end of 2017.

The cap-and-trade mechanism, which was put forward by the Kyoto Protocol [4], integrates regulatory and market power. This mechanism makes full use of market power to price carbon-emission permits so that superemission firms pay a price, while less-emission firms are profitable, and then guides low-carbon technological progress and investment in low-carbon projects, ultimately achieving low-carbon economic transformation.

As important sources of carbon emissions, firms must act in response to government emission regulations; otherwise, they are severely punished. Firms can achieve emission reduction in two aspects, technology upgrades and operation optimization. On the aspect of technology, firms can use more energy-efficient equipment and facilities, cleaner energy, and more environmentally friendly raw materials. Firms can also dispose of carbon emissions by postprocessing, such as Carbon Capture and Sequestration (CSS) technologies [5]. By March 2012, the Global CCS Institute had identified 75 large-scale integrated projects globally (http://www.ccsassociation.org/why-ccs/industry-experience/). All these emission-reduction technologies need extra investment. Apart from the aspect of technology, it is possible for firms to reduce emissions through individually optimizing operations [6]. For example, a logistics firm can change its carbon emissions by adjusting transport routes, the location of distribution centers, and delivering frequency. It should be noted that the effect of self-interested emission-reduction action of a firm is limited since a firm is part of a supply chain. The actions of members in a supply chain, individually reducing emissions without coordination with other members, are unlikely to achieve the target of minimizing the emissions of the entire supply chain. Taking a cold chain as an example, a retailer requires rapid delivering to address the growing expectations of consumers. In order to satisfy retailer requirements, a supplier has to build more cold-storage facilities and more vehicles with refrigeration systems, leading to more carbon emissions. Therefore, coordination among members in a supply chain should not be ignored. Within a supply chain, the self-interested behavior of decision makers can be bridged to some extent by contracts [7].

Faced with climate change caused by increasing carbon emissions, not only governments and firms but also ordinary consumers are changing their perceptions. Their environmental awareness is growing and they are increasingly inclined toward environmentally friendly products, which means that they are willing to pay more for these products in contrast with ordinary products. Toyota reported that its hybrid cars contribute to carbon dioxide reduction and are priced more than 1.5 times the price of their gasoline-powered counterparts [8]. Consumers are willing to pay higher prices for environmentally friendly products because they believe that their actions can benefit them, e.g., by promoting a healthy environment. In this study, environmental awareness is expressed as low-carbon awareness. To obtain premiums from consumer low-carbon awareness, carbon-labeling projects have been or are being developed in many countries. For example, Carbon Trust, a UK company, launched the first batch of carbon-labeling projects in 2007, including potato chips and shampoos. China has also formulated the General Rules for the Evaluation of Carbon Labels for Electrical and Electronic Products in China. Liu et al. [9] used literature review to discuss the evolution of the carbon-labeling concept, and different measurement methodologies and standards for carbon labels.

From this background, the decision makers in supply chains must take emission regulations and consumers' low-carbon awareness into account. This study establishes two-stage supply-chain optimization models involving cap-and-trade regulation and consumers' low-carbon awareness. Specifically, the following issues are investigated: (1) What are the optimal decisions for decision maker(s) under different modes? (2) How are optimal decisions, profits, and emissions influenced by regulation parameters and low-carbon awareness level? (3) How are decisions transferred from decentralized to centralized mode?

In this study, consumers' demand was set as an endogenous variable that depends on emission levels and consumers' low-carbon awareness. Optimal decisions were derived from mathematical models under different modes. On this basis, this study analyzes the influence of regulation parameters and low-carbon awareness level on profits and emissions. In addition, a comparison of the two decision-making modes illustrates the effect of coordination in low-carbon supply-chain operations, which may provide guidelines to firms and governments.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature on supply-chain operations under cap-and-trade regulations and low-carbon (environmental) awareness. Section 3 proposes necessary assumptions and notations in preparation for mathematical-model formulation. Mathematical models and analytical results are presented in Section 4. Section 5 presents a series of numerical experiments to confirm the analytical results. Conclusions and future research are provided in Section 6.

2. Literature Review

This study is mainly related to two literature categories, supply-chain operations under cap-and-trade regulations and low-carbon (environmental) awareness. Researchers investigated the influence of cap-and-trade regulations on supply-chain operations. Adam [10] identified five key implications for transportation planners of extending cap and trade for greenhouse-gas emissions to the transportation sector. Several papers focused on inventory management under cap-and-trade regulations. Hua et al. [11] developed the EOQ model under cap-and-trade regulation, derived optimal order quantity, and examined the influence of regulation on order decisions, carbon emissions, and total cost. Chen et al. [12] compared EOQ models under different regulations, including strict carbon caps, carbon tax, cap and offset, and cap and price (trade). Tao and Xu [13] investigated the influence of regulation policies and consumers' low-carbon awareness on optimal order size, emission levels, and total costs. Apart from EOQ models with a single decision variable, Toptal et al. [14] introduced investment for emission reduction as another decision variable. Production-optimization problems under cap-and-trade regulations are another research hotpot. Du et al. [15] investigated the influence of the carbon footprint and low-carbon preference on the production decision of emission-dependent firms under cap-and-trade regulations. Zhang and Xu [16] developed a profit-maximization model for the multi-item production-planning problem with a carbon cap-and-trade mechanism. Du et al. [17] dealt with the manufacturer's multiproduct joint pricing and production problem with a low-carbon premium under cap-and-trade regulation. Some researchers also studied the influence of cap-and-trade regulations on supply chains with multiple decision makers. Du et al. [18] used a game-theoretical analytical model to characterize the behavior and decision-making of each member in an emission-dependent supply chain under cap-and-trade regulations. Xu et al. [19] focused on the production and pricing problems in a make-to-order (MTO) supply chain containing an upstream manufacturer who produces two products based on MTO production and a downstream retailer. Xu [20] studied decision and coordination in a dual-channel supply chain arising out of low-carbon preference and channel substitution under the cap-and-trade regulation.

Under carbon-emission regulation, customers' low-carbon awareness (more generally, environmental awareness) has significant influence on supply-chain operations. The influence is from changes in consumer purchasing behavior. Some papers confirmed that consumers are willing to pay higher prices for environmentally friendly products [21–24]. To meet consumers' willingness, the Ministry of Environmental Protection of China has organized and formulated the development plan of Environmental Certification Center Carries out Low Carbon Product Certification (http://www.mee.gov.cn/). Conrad [25] used a spatial duopoly model to determine how environmental concerns affect prices, product characteristics, and market shares of competing firms. Ji et al. [26] developed a detailed model for emission-reduction behaviors of chain members in retail- and dual-channel cases, which incorporates both cap-and-trade regulations and consumers' low-carbon preference. Taking into account consumers' low-carbon preferences and stochastic market demand, Wang et al. [27] derived a revenue model of retailer and manufacturer in decentralized and centralized supply chains when the supply chain reduces emissions or is not under stochastic market demand.

The majority of the literature assumed that product demand and price are usually exogenous parameters. When consumers' low-carbon awareness is introduced in models, demand and price are no longer exogenous. In addition, the stream of these studies focused on a single-level decision-making structure. This study is devoted to integrating consumers' low-carbon awareness and cap-and-trade regulations into two-stage supply-chain optimization models under different decision-making modes.

3. Related Notations and Assumptions

This study focuses on a simple two-stage supply chain with single-item product. This supply chain comprises three members, the manufacturer, the retailer, and the customer. The manufacturer and retailer are decision makers regulated by a cap-and-trade mechanism. The manufacturer generates carbon emissions during the production process, while the retailer's carbon emissions originate from logistics. In the cap-and-trade mechanism, the government (the regulator) sets emission caps to firms. If actual emissions exceed the caps, firms need to buy quotas in the carbon-emission trading market to shield themselves from penalties. On the contrary, they sell surplus quotas and profit if their actual emissions are less than the caps. In this context, consumers' low-carbon awareness can influence demand. Figure 1 shows the concept model of the problem investigated in this study.

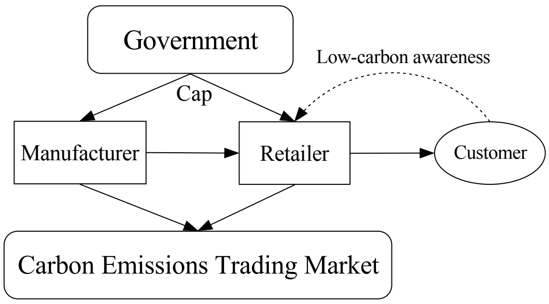


Figure 1. Concept model.

In order to formulate the mathematical models, some key assumptions are presented as follows.

Assumption 1. *Retailer orders from manufacturer according to demand. No consideration is given to inventory.*

Assumption 2. *Potential maximum market demand is fixed.*

Assumption 3. *Both production emissions and logistics emissions linearly decrease in quantity [6].*

Assumption 4. *Only manufacturer has the ability and opportunity to reduce emissions.*

Assumption 5. *Lower production emissions can increase demand.*

Moreover, notations that are used in the models are summarized in Table 1.

Table 1. Notations.

Parameters	
a	Potential maximum market demand;
C_M	annual carbon-emission cap for manufacturer;
C_R	annual carbon-emission cap for retailer;
p_e	unit carbon-emission price;
e_M	initial per unit-product emission by manufacturer;
e_R	initial per unit-product emission by retailer;
p_M	price per unit product paid by retailer to manufacturer according to contract;
h	consumers' low-carbon awareness level (LCAL); and
k	cost coefficient for emission reduction.
Decision variable	
L	Reduction level of emissions per unit product ($0 \leq L \leq 1$) and
p_R	retail price per unit product

4. Analytical Models

According to the relationship between manufacturer and retailer, two decision-making modes are considered: decentralized and centralized mode. Additionally, the profit-sharing contract is also investigated in this section to fuse these two modes.

4.1. Decentralized Decision-Making Mode

In this subsection, the decentralized decision-making mode is investigated. Under this mode, retailer and manufacturer are independent decision makers. The decision-making process follows the Stackelberg game rule [28]. In this study, the manufacturer is considered as the leader, while the retailer is the follower. In fact, retailer-leading supply chains are widespread, e.g., the Apple Inc.-based supply chain. The retailer determines their selling price to optimize their profit, while the manufacturer supplies them with the product at agreed price p_M. The manufacturer determines carbon-emission reduction level L, which can influence demand since the customer is low-carbon-sensitive. The demand for the product at price p_R and emission-reduction level L for the customer with LCAL h is

$$a - p_R + hL. \tag{1}$$

Unit cost paid on emission reduction with L is kL^2 [21]. Here, the quadratic means that, as L grows, the cost increases faster. In other words, investment in emission reduction has a declining marginal effect. Hence, the optimization problem confronted by the manufacturer is formulated as:

$$\max_{L\in[0,1]} \pi_M = (p_M - c_M)(a - p_R + hL) + (C_M - e_M(a - p_R + hL)(1-L))p_e - k(a - p_R + hL)L^2, \tag{2}$$

where p_R is optimal solution for the following model:

$$\max_{p_R} \pi_R = (a - p_R + hL)(p_R - p_M) + (C_R - e_R(a - p_R + hL))p_e \tag{3}$$

The second items in Equations (2) and (3) are profits from carbon trading by the manufacturer and retailer, respectively. If $C_M - e_M(a - p_R + hL)(1-L)(C_R - e_R(a - p_R + hL)) > 0$, the manufacturer (retailer) can profit from selling surplus carbon quotas. On the contrary, the manufacturer (retailer) has to purchase carbon quotas from the trading market to make up for a quota gap when $C_M - e_M(a - p_R + hL)(1-L)(C_R - e_R(a - p_R + hL)) \le 0$.

The following proposition proposes the optimal decisions under the decentralized decision-making mode.

Proposition 1. *Under the decentralized decision-making mode, let*

$$\Delta = (k\,(p_M - a + e_R p_e) + e_M\,h\,p_e)^2 - 3hk\,(p_e e_M\,(h + p_M - a + e_R p_e) + h\,(c_M - p_M))\,,$$

$$L_1 = \frac{kp_M - ak + he_M p_e - ke_R p_e \sqrt{\Delta}}{3hk},$$

$$L_2 = \frac{kp_M - ak + he_M p_e + ke_R p_e \sqrt{\Delta}}{3hk}.$$

If $\Delta > 0$, the optimal L, denoted by L_D^*, is shown in Table 2:

Table 2. Values of L_D^* when $\Delta > 0$.

	$\boldsymbol{L_2 - L_1 \geq 1}$
$1 \leq L_1$	$L_D^* = 0$
$0 \leq L_1 < 1$	$L_D^* = 0$, if $\pi_M(L = 0) > \pi_M(L = 1)$
	$L_D^* = 1$, otherwise
$L_1 < 0 < 1 \leq L_2$	$L_C^* = 1$
$0 < L_2 < 1$	$L_D^* = L_2$
$L_2 \leq 0$	$L_D^* = 0$
	$\boldsymbol{L_2 - L_1 < 1}$
$1 \leq L_1$	$L_D^* = 0$
$0 \leq L_1 < 1 < L_2$	$L_D^* = 0$, if $\pi_M(L = 0) > \pi_M(L = 1)$
	$L_D^* = 1$, otherwise
$0 < L_2 \leq 1$	$L_D^* = L_2$

and optimal p_R, denoted by $p_{R,D}^*$, is

$$p_{R,D}^* = \frac{1}{2}(a + p_M + hL_D^* + p_e e_R). \tag{4}$$

Otherwise, i.e., $\Delta \leq 0$, the optimal solution is

$$(L_D^*, p_{R,D}^*) = \left(0, \frac{1}{2}(a + p_M + p_e e_R)\right). \tag{5}$$

Proof. The reverse-solution method for the Stackelberg game was used to solve the problem. For a fixed L determined by the manufacturer, let the first-order derivative of π_R with respect to p_R be zero, i.e.,

$$a + p_M + hL + p_e e_R - 2p_R = 0, \tag{6}$$

which leads to

$$p_R = \frac{1}{2}(a + p_M + hL + p_e e_R).$$

Plugging $\frac{1}{2}(a + p_M + hL + p_e e_R)$ in the place of p_R in Equation (2) and the first-order derivative of π_M in L was calculated as

$$\frac{\partial \pi_M}{\partial L} = -\frac{3hk}{2}L^2 + \left(k\left(p_M - a + e_R p_e\right) + e_M h p_e\right)L - \frac{1}{2}\left(p_e e_M\left(h + p_M - a + e_R p_e\right) + h\left(c_M - p_M\right)\right),$$

which is a quadratic function of L with discriminant Δ.

If $\Delta > 0$, the two real roots of $\frac{\partial \pi_M}{\partial L} = 0$ are denoted as L_1 and L_2, respectively. There are two cases based on a comparison of intervals $[L_1, L_2]$ and $[0, 1]$.

Case 1. $L_2 - L_1 \geq 1$. In this case, there are five subcases according to the comparison between $[L_1, L_2]$ and $[0, 1]$.

Case 1.1. $1 \leq L_1$. $\frac{\partial \pi_M}{\partial L} \leq 0$ for any $L \in [0, 1]$, which indicates that $L_D^* = 0$.

Case 1.2. $0 \leq L_1 < 1$. In order to determine the optimal L, it is necessary to compare $\pi_M(L = 0)$ with $\pi_M(L = 1)$. If $\pi_M(L = 0)$ is greater than $\pi_M(L = 1)$, π_M takes the maximum at $L = 0$; otherwise, π_M takes the maximum at $L = 1$.

Case 1.3. $L_1 < 0 < 1 \leq L_2$. π_M increases in L in interval $[L_1, L_2]$; hence, π_M takes the maximum at $L = 1$.

Case 1.4. $0 < L_2 < 1$. π_M increases in $[0, L_2]$ and decreases in $[L_2, 1]$, so π_M takes the maximum at $L = L_2$.

Case 1.5. $L_2 \leq 0$. π_M decreases in $[0, 1]$, so π_M takes the maximum at $L = 0$.

Case 2. $L_2 - L_1 < 1$. In this case, there are three subcases according to the comparison between $[L_1, L_2]$ and $[0, 1]$.

Case 2.1. $1 \leq L_1$. Similar to *Case 2.1*, π_M takes the maximum at $L = 0$.

Case 2.2. $0 \leq L_1 < 1 < L_2$. If $\pi_M(L = 0)$ is greater than $\pi_M(L = 1)$, π_M maximizes at $L = 0$; otherwise, π_M maximizes at $L = 1$.

Case 2.3. $L_1 < 0 \leq L_2 \leq 1$. π_M increases in $[0, L_2]$ and decreases in $[L_2, 1]$, so π_M takes the maximum at $L = L_2$.

If $\Delta \leq 0$, $\frac{\partial \pi_M}{\partial L} \leq 0$ for any $L \in [0, 1]$, which implies that π_M decreases in L. Thus, π_M takes the maximum at $L = 0$.

Plugging L_D^* in the expression of p_R, $p_{R,D}^*$ is obtained. Thus, this proof is completed. □

Corollary 1. *The manufacturer and retailer's optimal operation policies (L_D^* and $p_{R,D}^*$) are independent of allocated caps C_M and C_R.*

The corollary is trivial from the expression of L_D^* and $p_{R,D}^*$ in Proposition 1. Corollary 1 indicates that the allocated caps do not directly influence the optimal decisions. L_D^* and $p_{R,D}^*$ are influenced by carbon price p_e, which is stated in Corollary 2.

Corollary 2. *Optimal reduction level L_D^* and retail price $p_{R,D}^*$ increase in p_e if*

$$p_e \geq \frac{2ae_R k^2 + 3e_M h^2 k - 2e_R k^2 p_M + e_M hkp_M - ae_M hk}{2{e_M}^2 h^2 - 2e_M e_R hk + 2{e_R}^2 k^2}. \tag{7}$$

Proof. When $L_D^* = 0$ or 1, L_D^* is independent of p_e. Next, consider the case that $L_D^* = L_2$. The first-order derivative of Δ with respect to p_e is $\left(2{e_M}^2 h^2 - 2e_M e_R hk + 2{e_R}^2 k^2\right) p_e + \left(2e_R k^2 p_M - 3e_M h^2 k - 2ae_R k^2 - e_M hkp_M + ae_M hk\right)$. When p_e is not less than $\frac{2ae_R k^2 + 3e_M h^2 k - 2e_R k^2 p_M + e_M hkp_M - ae_M hk}{2{e_M}^2 h^2 - 2e_M e_R hk + 2{e_R}^2 k^2}$, $\frac{\partial \Delta}{\partial p_e} \geq 0$. Combining the expression of L_2, L_2 increases in p_e. The monotonicity of $p_{R,D}^*$ in p_e is immediately derived from the expression of $p_{R,D}^*$. This completes the proof. □

Corollary 2 provides a sufficient condition under which the L_D^* and $p_{R,D}^*$ increase in p_e. Corollary 2 also implies that higher carbon prices provoke investment in emission-reduction technologies and higher retail prices, which coincides with real practices. This may be explained by the fact that higher carbon prices make emission reductions profitable. At the same time, in order to transfer higher carbon prices to consumers, retailers also set higher retail prices. Although caps do not directly influence optimal decisions of the manufacturer and the retailer, they work by influencing prices. In fact, if generous caps are allocated to firms, decreased carbon prices are inevitable.

Corollary 3. *Optimal reduction level L_D^* and retail price $p_{R,D}^*$ increase in h if h satisfies*

$$(p_M - a + e_R p_e)(2ak - 2kp_M + e_M hp_e - 2e_R kp_e) \geq 0. \tag{8}$$

Proof. When $L_D^* = 0$ or 1, L_D^* is independent of h. Since the first-order derivative of L_2 in h, rewrite L_2 as

$$L_2 = \frac{p_M - a}{3h} + \frac{e_M p_e}{3k} + \frac{e_R p_e}{3}\sqrt{\frac{\Delta}{h^2}},$$

where the first item $\frac{p_M - a}{3h}$ increases in h from $a > p_M$, the second item is constant. The first-order derivative of $\frac{\Delta}{h^2}$ in h is

$$\frac{\partial \frac{\Delta}{h^2}}{\partial h} = \frac{k\,(p_M - a + e_R p_e)\,(2ak - 2kp_M + e_M hp_e - 2e_R kp_e)}{h^3}.$$

Inequality $(p_M - a + e_R p_e)(2ak - 2kp_M + e_M h p_e - 2e_R k p_e) \geq 0$ leads to $\frac{\partial \frac{\Delta}{h^2}}{\partial h} \geq 0$ and then $\frac{\partial L_2}{\partial h} \geq 0$. In addition,

$$\frac{\partial p_{R,D}(L_2)}{\partial h} = \frac{1}{2}\left(h\frac{\partial L_2}{\partial h} + L_2\right).$$

This completes the proof. □

The values of π_M with different L_D^* are

$$\pi_M(L=0) = \frac{1}{2}(p_M - c_M)(a - p_M - p_e e_R) + \left(C_M - \frac{1}{2}e_m(a - p_M - p_e e_R)\right)p_e, \tag{9}$$

$$\pi_M(L=1) = \frac{1}{2}(p_M - c_M)(a + h - p_M - p_e e_R) + C_M p_e - \frac{1}{2}k(a + h - p_M - p_e e_R), \tag{10}$$

and

$$\begin{aligned}\pi_M(L=L_2) = &\tfrac{1}{2}(p_M - c_M)(a + hL - p_M - p_e e_R) + \left(C_M - \tfrac{1}{2}e_M(a + hL_2 - p_M - p_e e_R)(1 - L_2)\right)p_e \\ &- \tfrac{1}{2}k(a + hL_2 - p_M - p_e e_R)L_2^2.\end{aligned} \tag{11}$$

The changes of π_M with parameters C_M, p_e and h are stated by Proposition 2 as follows.

Proposition 2. *(1) $\pi_M(L = L_D^*)$ linearly increases in C_M; (2) If the manufacturer has no chance to reduce emissions, their profit $\pi_M(L=0)$ increases in p_e when*

$$p_e \geq \frac{e_M(a - p_M) + e_R(p_M - c_M) - 2C_M}{2e_M e_R}.$$

Meanwhile, $\pi_M(L=0)$ is independent of h. (3) If the manufacturer eliminates all emissions, their profit $\pi_M(L=1)$ increases in p_e when $C_M + \frac{1}{2}e_R(k + c_M - p_M) \geq 0$; otherwise, $\pi_M(L=1)$ decreases in p_e. Meanwhile, $\pi_M(L=1)$ increases in h if $p_M - c_M - k \geq 0$. (4) $\pi_M(L=L_2)$ increases in p_e when

$$-\frac{e_M e_R {p_e}^2}{2}\frac{\partial L_2}{\partial p_e} + \left(\frac{(1-L)e_R}{2} - \frac{e_M e_R\,(L_2 - 1)}{2} + (L_2 p_e k + \frac{(L_2-1)h}{2} + e_M(a - p_M + L_2 h))\frac{\partial L_2}{\partial p_e}\right)p_e \geq 0.$$

Meanwhile, $\pi_M(L=L_2)$ increases in h when

$$\begin{aligned}&\tfrac{\partial \pi_M(L=L_2)}{\partial h}(p_e\,(e_M\,(a - p_M + L_2 h - e_R p_e) + e_M h(L_2 - 1)) - h(c_M - p_M) - 2Lk(a - p_M + L_2 h - e_R p_e) \\ &- L_2^2 hk) \geq 0.\end{aligned}$$

Proof. (1) is derived from the expressions of $\pi_M(L = L_D^*)$; (2), (3), and (4) are derived from solving the inequalities $\frac{\partial \pi_M(L=0)}{\partial p_e} \geq 0, \frac{\partial \pi_M(L=0)}{\partial h} \geq 0$, $\frac{\partial \pi_M(L=1)}{\partial p_e} \geq 0, \frac{\partial \pi_M(L=1)}{\partial h} \geq 0$, $\frac{\partial \pi_M(L=L_2)}{\partial p_e} \geq 0$ and $\frac{\partial \pi_M(L=L_2)}{\partial h} \geq 0$, respectively. □

Similarly, the profits of the retailer with different L_C^* are calculated as

$$\pi_R(L=0) = \frac{1}{4}{e_R}^2 p_e^2 + \left(C_R - \frac{e_R}{2}\,(a - p_M)\right)p_e + \frac{1}{4}(a - p_M)^2,$$

$$\pi_R(L=1) = \frac{1}{4}{e_R}^2 p_e^2 + \left(C_R - \frac{e_R}{4}\,(a - p_M + h)\right)p_e + \frac{1}{4}(a - p_M + h)^2,$$

and

$$\pi_R(L=L_2) = \frac{1}{4}{e_R}^2 p_e^2 + \left(C_R - \frac{e_R}{2}\,(a - p_M + L_2 h)\right)p_e + \frac{1}{4}(a - p_M + L_2 h)^2.$$

Proposition 3 states the changes of π_R with parameters C_M, p_e, and h.

Proposition 3. *(1) $\pi_R(L_D^*)$ increases in C_R. (2) $\pi_R(L=0)$ increases in p_e if $p_e \geq \frac{e_R(a-p_M)-2C_R}{e_R^2}$. Meanwhile, $\pi_R(L=0)$ is independent of h. (3) $\pi_R(L=1)$ increases in p_e if $p_e \geq \frac{e_R(a-p_M+h)-2C_R}{e_R^2}$. Meanwhile, $\pi_R(L=1)$ increases in h if $h \geq 2(p_M+e_R p_e-a)$. (4) $\pi_R(L=L_2)$ increases in p_e if $p_e\,(2C_R-e_R(a-p_M-e_R+L_2h))+h(a-p_M-e_R+L_2h)\frac{\partial L_2}{\partial p_e}>0$. Meanwhile, $\pi_R(L=L_2)$ increases in h if $\left(\frac{h\,(a-p_M+L\,h-e_R\,p_e)}{2}\right)\frac{\partial L_2}{\partial h}>0$*

Proof. The proof is the same as Proposition 2. □

As emissions are considered, emission amounts from manufacturer and retailer are

$$E_M=\frac{1}{2}e_m(a+hL-p_M-p_e e_R)(1-L), \tag{12}$$

and

$$E_R=\frac{1}{2}e_R(a+hL-p_M-p_e e_R). \tag{13}$$

It is noted that if the manufacturer does not reduce emissions (i.e., $L=0$) and has the same emission intensity as the retailer (i.e., $e_M=e_R$), then emission amounts the manufacturer and retailer are the same. The expressions of E_M and E_R imply that the emission amounts with L_D^* are independent of caps C_M and C_R. However, C_M and C_R can influence E_M and E_R if C_M and C_R are functions of p_e.

When $L_D^*=0$, $E_M(L=0)=\frac{1}{2}e_M(a-p_M-p_e e_R)$ and $E_R(L=0)=\frac{1}{2}e_R(a-p_M-p_e e_R)$. Both of $E_M(L=0)$ and $E_R(L=0)$ increase in p_e and independent of h. This is likely because an increasing p_e inhibits emissions. If the manufacturer eliminates all emissions, i.e., $L_D^*=1$, emissions from the manufacturer are zero. However, emission amount from the retailer $E_R(L=1)=\frac{1}{2}e_R(a+h-p_M-p_e e_R)$ is not zero. $E_R(L=1)$ decreases in p_e but increases in h, which is because increasing carbon prices restrain demand, but increasing LCAL stimulates demand.

The influences of p_e and h on $E_M(L=L_2)$ and $E_R(L=L_2)$ are stated by Proposition 4.

Proposition 4. *(1) $E_M(L=L_2)$ decreases in p_e if*

$$(h+2hL_2-a+p_M+p_e e_R)\frac{\partial L_2}{\partial p_e}-e_R(1-L_2)\leq 0.$$

Meanwhile, $E_M(L=L_2)$ increases in h if

$$(h+2hL_2-a+p_M+p_e e_R)\frac{\partial L_2}{\partial p_e}+L_2(1-L_2)\geq 0.$$

(2) $E_R(L=L_2)$ decreases in p_e if

$$\frac{\partial L_2}{\partial p_e}\leq\frac{h}{e_R}.$$

Meanwhile, $E_R(L=L_2)$ increases in h if

$$\frac{\partial L_2}{\partial h}\geq-\frac{h}{e_R}.$$

Proof. This proof is derived from solving inequalities $\frac{\partial E_M(L=L_2)}{\partial p_e}\leq 0$, $\frac{\partial E_M(L=L_2)}{\partial h}\geq 0$, $\frac{\partial E_R(L=L_2)}{\partial p_e}\leq 0$ and $\frac{\partial E_R(L=L_2)}{\partial h}\geq 0$, respectively. □

4.2. Centralized Decision-Making

This subsection focuses on the centralized decision-making mode. When the manufacturer and retailer belong to the same company, they adopt a centralized decision-making mode. Under this mode, the manufacturer and retailer merge into a unified decision maker to optimize the two-stage supply chain. In large-scale companies, this decision-making mode is very common. Under this mode, the allocated emission cap is $C_M + C_R$, and the objective function denoted by π is the sum of π_M and π_R proposed in the decentralized decision-making mode, i.e.,

$$\begin{aligned} \pi = \; & (p_M - c_M)(a - p_R + hL) + (C_M - e_m(a - p_R + hL)(1-L))p_e - k(a - p_R + hL)L^2 \\ & +(a - p_R + hL)(p_R - p_M) + (C_R - (a - p_R + hL)e_R)p_e \end{aligned} \tag{14}$$

Since $L \in [0,1]$, the optimization model for the decision-making problem under the decentralized mode can be formulated by

$$\begin{cases} \min \pi = (p_M - c_M)(a - p_R + hL) + (C_M - e_m(a - p_R + hL)(1-L))p_e - k(a - p_R + hL)L^2 \\ +(a - p_R + hL)(p_R - p_M) + (C_R - (a - p_R + hL)e_R)p_e \\ s.t. \begin{cases} 0 \le L \le 1 \\ p_R > 0 \end{cases} \end{cases} \tag{15}$$

Optimal solutions for Model (15) are proposed with the following:

Proposition 5. *If* $a > \max\{k + c_M + e_R p_e - h, c_M + (e_M + e_R)p_e, c_M + k + e_R p_e + \frac{2he_M p_e}{k} - 5h, 3k + c_M - 3h - 2e_M p_e + e_R p_e + \frac{(h+e_M p_e)^2}{2k}\}$*, the unique solution for Problem (15), denoted by* $(L_C^*, p_{R,C}^*)$*, is*

$$\begin{cases} L_C^* = 1 \\ p_{R,C}^* = \frac{1}{2}(a + c_M + h + e_R p_e + k) \end{cases} \tag{16}$$

when $\frac{h+e_M p_e}{2k} > 1$*; or*

$$\begin{cases} L_C^* = \frac{h+e_M p_e}{2k} \\ p_{R,C}^* = \frac{-e_M{}^2 p_e{}^2 + 2e_M h p_e + 4ke_M p_e + 3h^2 + 4e_R k p_e + 4ak + 4c_M k}{8k} \end{cases} \tag{17}$$

when $\frac{h+e_M p_e}{2k} \le 1$.

Proof. The proof of this proposition is derived from KKT optimality conditions [29]. Then, optimal solution $(L_C^*, p_{R,C}^*)$ for Problem (15) satisfies KKT optimality conditions, listed as follows:

$$\begin{cases} \frac{\partial \pi}{\partial L}|_{(L_C^*, p_{R,C}^*)} + \lambda_1 - \lambda_2 = 0 \\ \frac{\partial \pi}{\partial p_R}|_{(L_C^*, p_{R,C}^*)} + \lambda_3 = 0 \\ \lambda_1 L_C^* = 0 \\ \lambda_2 (1 - L_C^*) = 0 \\ \lambda_3 p_{R,C}^* = 0 \\ \lambda_1, \lambda_2, \lambda_3 \ge 0 \end{cases} \tag{18}$$

where

$$\begin{aligned} \frac{\partial \pi}{\partial L} = & (-3hk)L^2 + (2e_M h p_e - 2k(a - p_R))L - (h(c_M - p_M) + h(p_M - p_R) + p_e(e_M h - e_M(a - p_R)) \\ & + e_R h p_e), \end{aligned} \tag{19}$$

$$\frac{\partial \pi}{\partial p_R} = a + c_M - 2p_R + Lh + e_R p_e + L^2 k - e_M p_e (L-1), \tag{20}$$

and $\lambda_1, \lambda_2, \lambda_3$ are Lagrange multipliers.

Multipliers $\lambda_1, \lambda_2, \lambda_3$ are equal to zero or positive. There are eight possible cases in theory. However, cases with $\lambda_3 > 0$ violate feasibility since $p_{R,C}^*$ is strictly greater than zero. Moreover, either

λ_1 and λ_2 must be zero; otherwise, $\lambda_1 L_C^* = 0$ contradicts with $\lambda_2(1 - L_C^*) = 0$. The remaining three cases may lead to feasible solutions.

Case 1. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0$.

It follows from $\lambda_2 > 0$ that $L_C^* = 1$. The two equations in (18) can be written as

$$\begin{cases} \left(2e_M h p_e - 2k\left(a - p_{R,C}^*\right)\right) - \left(h\left(c_M - p_M\right) + h\left(p_M - p_{R,C}^*\right) + p_e\left(e_M h - e_M\left(a - p_{R,C}^*\right)\right) + e_R h p_e\right) \\ -3hk - \lambda_2 = 0 \\ a + c_M - 2p_{R,C}^* + h + e_R p_e + k = 0 \end{cases} \tag{21}$$

Solving for $p_{R,C}^*$ and λ_2 in (21) results in

$$\begin{cases} p_{R,C}^* = \frac{1}{2}(a + c_M + h + e_R p_e + k) \\ \lambda_2 = \frac{1}{2}(h - 2k + e_M p_e)(a + h - c_M - k - e_R p_e) \end{cases} \tag{22}$$

Since $a > k + c_M + e_R p_e - h$, $\lambda_2 > 0$ induces $\frac{h + e_M p_e}{2k} > 0$.

Case 2. $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 = 0$.

It follows from $\lambda_1 > 0$ that $L_C^* = 0$. The former two equations in (18) can be written as

$$\begin{cases} -\left(h\left(c_M - p_M\right) + h\left(p_M - p_{R,C}^*\right) + p_e\left(e_M h - e_M\left(a - p_{R,C}^*\right)\right) + e_R h p_e\right) + \lambda_1 = 0 \\ a + c_M - 2p_{R,C}^* + e_R p_e + e_M p_e = 0 \end{cases} \tag{23}$$

Solving for $p_{R,C}^*$ and λ_1 in Equation (23) leads to

$$\begin{cases} p_{R,C}^* = \frac{1}{2}(a + c_M + (e_M + e_R)p_e) \\ \lambda_1 = \frac{1}{2}(h + e_M p_e)(c_M - a + e_M p_e + e_R p_e) \end{cases} \tag{24}$$

Assumption $a > c_M + (e_M + e_R)p_e$ leads to $\lambda_1 < 0$, a contradiction.

Case 3. $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0$.

In this case, the former two equations in (18) can be written as

$$\begin{cases} (-3hk)\, L_C^{*2} + \left(2e_M h p_e - 2k\left(a - p_{R,C}^*\right)\right) L_C^* \\ -\left(h\left(c_M - p_M\right) + h\left(p_M - p_{R,C}^*\right) + p_e\left(e_M h - e_M\left(a - p_{R,C}^*\right)\right) + e_R h p_e\right) = 0 \\ a + c_M - 2p_{R,C}^* + L_C^* h + e_R p_e + L_C^{*2} k - e_M p_e\left(L_C^* - 1\right) = 0 \end{cases} \tag{25}$$

The second equation in (25) results in

$$p_{R,C}^* = \frac{1}{2}(a + c_M + L_C^* h + e_R p_e + L_C^{*2} k - e_M p_e\,(L_C^* - 1)). \tag{26}$$

Substituting $p_{R,C}^*$ by (26), the first equation in (25) can be written as

$$-\frac{(h - 2L_C^* k + e_M p_e)\,(kL_C^{*2} + (-h - e_M p_e)L_C^* - a + c_M + e_M p_e + e_R p_e)}{2} = 0. \tag{27}$$

It follows from $a > k + c_M + e_R p_e - h$ and $L_C^* \le 1$ that $kL_C^{*2} + (-h - e_M p_e)L_C^* - a + c_M + e_M p_e + e_R p_e$ is strictly greater than zero. Thus, the solution of L_C^* in Equation (27) is

$$L_C^* = \frac{h + e_M p_e}{2k} \tag{28}$$

It should be noted that Equation (28) makes practical sense only when $\frac{h+e_M p_e}{2k} \leq 1$. Substituting L_C^* by Equation (28) in Equation (26) leads to

$$p_{R,C}^* = \frac{-e_M{}^2 p_e{}^2 + 2e_M h p_e + 4ke_M p_e + 3h^2 + 4e_R k p_e + 4ak + 4c_M k}{8k} \tag{29}$$

The second-order derivatives of π are calculated as

$$\frac{\partial^2 \pi}{\partial L^2} = 2\, e_M h p_e - 4Lhk - 2k\,(a - p_R + Lh)\,, \tag{30}$$

$$\frac{\partial^2 \pi}{\partial L \partial p_R} = \frac{\partial^2 \pi}{\partial p_R \partial L} = h + 2Lk - e_M p_e, \tag{31}$$

and

$$\frac{\partial^2 \pi}{\partial p_R^2} = -2. \tag{32}$$

The assumption about a ensures that the following inequalities hold:

$$\begin{cases} \frac{\partial^2 \pi}{\partial L^2} < 0 \\ |H| = \begin{vmatrix} \frac{\partial^2 \pi}{\partial L^2} & \frac{\partial^2 \pi}{\partial L \partial p_R} \\ \frac{\partial^2 \pi}{\partial p_R \partial L} & \frac{\partial^2 \pi}{\partial p_R^2} \end{vmatrix} > 0 \end{cases} \tag{33}$$

where $|H|$ is the determinant of Hessian matrix H with respect to L and p_R. Group of inequalities (33) indicates that π is a strictly concave function, which verifies uniqueness. This completes the proof. □

Proposition 5 states that there exist a unique reduction level and retail price to maximize the supply chain's total profit when the potential maximum market demand a is large enough. Because most firms subject to emission regulation are large-scale and emission-intensive, the assumptions about a in Proposition 5 are reasonable. Proposition 5 also states that optimal reduction level L_C^* increases in carbon-emission price p_e when p_e is at a low level ($p_e < \frac{2k-h}{e_M}$). When p_e is greater than $\frac{2k-h}{e_M}$, the manufacturer eliminates all emissions, i.e., $L_C^* = 1$. That means increasing carbon-emission price is conducive to provoke decision makers to reduce emissions, which coincides with Du et al. [17]. When the price reaches a certain threshold, the incentive effect no longer increases. Here, the threshold is named as *critical price* and denoted by $\tilde{p}_e$. Obviously, the critical price increases in k and decreases in h and e_M.

As reduction level and retail price are L_C^* and $p_{R,C}^*$ respectively, the corresponding profit and emission are

$$\pi_C^* = \begin{cases} \frac{A_1 p_e{}^4 + A_2 p_e{}^3 + A_3 p_e{}^2 + A_4 p_e + A_5}{64k^2}, \; if \; \frac{h+e_M p_e}{2k} \leq 1 \\ \frac{1}{4}(a + h - c_M - k)^2 - (e_R p_e)^2, \; otherwise \end{cases} \tag{34}$$

and

$$E_C^* = \begin{cases} C_M + C_R + \frac{1}{16k^2}(e_M(e_M p_e + h)^2 - 4k((e_M + e_R)p_e - a + c_M)(h - 2k + e_M p_e) - 2ke_R), \\ if \; \frac{h+e_M p_e}{2k} \leq 1 \\ \frac{1}{2}(a + h - c_M - e_R p_e - k), \; otherwise. \end{cases} \tag{35}$$

where $A_1 = e_M{}^4, A_2 = 4e_M{}^2\,(e_M h - 2e_M k - 2e_R k)\,, A_3 = 16k^2(e_M + e_R)^2 + 2e_M(3e_M h^2 + 4ae_M k - 4c_M e_M k - 8e_M hk - 8e_R hk), A_4 = 4(16k^2(C_M + C_R) + 8k^2((c_M - a)(e_M + e_R)) + e_M h^3 - e_M h^2 k - e_R h^2 k + 2ae_M hk - 2c_M e_M hk), A_5 = 8(k(a - c_M)(2k(a - c_M) + h^2)) + h^4$.

The following proposition states the influences of regulation parameters and LCAL on the maximal profit.

Proposition 6. *(1) π_C^* increases in total emission cap $C_M + C_R$; (2) When p_e is greater than critical price $\tilde{p}_e$, π_C^* decreases in p_e; otherwise, π_C^* increases in p_e if $B_1 p_e{}^3 + B_2 p_e{}^2 + B_3 p_e + B_4 \geq 0$, where $B_1 = e_M{}^4, B_2 =$*

$3e_M{}^2(e_M h - 2(e_M + e_R)k), B_3 = 3e_M{}^2h^2 + 8k^2(e_M + e_R)^2 + 4e_M{}^2k(a - c_M) - 8e_M hk(e_M + e_R), B_4 = 16k^2(C_M + C_R) + 8k^2(c_M - a)(e_M + e_R) + h^2(e_M h - 2k(e_M + e_R)) + 4e_M h(a - c_M)$. *(3) When market demand a satisfies the assumption in Proposition 5, π_C^* increases in h.*

Proof. (1) The conclusion is easily derived from the expression of π_C^*. (2) When $p_e \le \tilde{p}_e$, $\pi_C^* = \frac{1}{4}(a + h - c_M - k)^2 - (e_R p_e)^2$. Obviously, it decreases in p_e. When $p_e > \tilde{p}_e$, notice that the first-order derivative of π_C^* in p_e is

$$\frac{\partial \pi_c^*}{\partial p_e} = \frac{B_1 p_e{}^3 + B_2 p_e{}^2 + B_3 p_e + B_4}{16k^2},$$

so π_c^* increases in p_e if $B_1 p_e{}^3 + B_2 p_e{}^2 + B_3 p_e + B_4$. (3) This first-order derivative of π_C^* in h listed as follows:

$$\frac{\partial \pi_C^*}{\partial h} = \begin{cases} \frac{(h+e_M p_e)\left((h+e_M p_e)^2 - 4k((e_M+e_R)p_e - (a-c_M))\right)}{16k^2}, \; if \; \frac{h+e_M p_e}{2k} \le 1 \\ \frac{1}{2}(a + h - c_M - k), \; otherwise. \end{cases}$$

It follows from assumption about the market demand a that $\frac{\partial \pi_C^*}{\partial h} \ge 0$. This completes the proof. □

From Proposition 6, some interesting observations are obtained. When p_e is at a relative low level ($\le \tilde{p}_e$), profit with respect to p_e varies indefinitely. As p_e is greater than $\tilde{p}_e$, profit decreases in p_e. Sufficiently high carbon prices prompt the manufacturer to cut all their emissions, i.e., $L_C^* = 0$. However, the retailer's emissions still exist and the retailer pays more with an increasing p_e. Proposition 6(3) implies that the supply chain benefits from a higher LCAL as long as market demand a is big enough.

In order to investigate emission changes with respect to regulation parameters and LCAL, the following is proposed:

Proposition 7. *(1) E_C^* increases in total emission cap C_M and C_R; (2) When $p_e \le \tilde{p}_e$, E_C^* decreases in p_e if $3e_M{}^4 p_e{}^2 + 6e_M^2(e_M h - 2e_M k - 2e_R k)p_e + 8k^2(e_M + e_R)^2 + e_M^2(3h^2 + 4k(a - c_M)) - 8e_M hk(e_M + e_R) \le 0$; otherwise, E_C^* decreases in p_e. (3) When $a \ge \frac{-3h^2}{4k} + (-\frac{3p_e e_M}{2k} + 1 + \frac{e_R}{e_M})h + c_M - \frac{3e_M^2 p_e^2}{4k} + 2(e_M + e_R)p_e$, E_C^* increases in h.*

Proof. (1) The conclusion is trivial from the expression of E_C^*. (2) When $p_e \le \tilde{p}_e$, the first-order derivative of E_C^* with respect to p_e is

$$\frac{\partial E_C^*}{\partial p_e} = \frac{3e_M{}^4 p_e{}^2 + 6e_M^2(e_M h - 2e_M k - 2e_R k)p_e + 8k^2(e_M + e_R)^2 + e_M^2(3h^2 + 4k(a - c_M)) - 8e_M hk(e_M + e_R)}{16k^2}.$$

$\frac{\partial E_C^*}{\partial p_e} \le 0$ leads to $3e_M{}^4 p_e{}^2 + 6e_M^2(e_M h - 2e_M k - 2e_R k)p_e + 8k^2(e_M + e_R)^2 + e_M^2(3h^2 + 4k(a - c_M)) - 8e_M hk(e_M + e_R) \le 0$; When $p_e > \tilde{p}_e$, $E_C^* = \frac{1}{2}(a + h - c_M - e_R p_e - k)$, which decreases in p_e clearly. (3) When $p_e > \tilde{p}_e$, the conclusion is trivial. When $p_e \le \tilde{p}_e$, the conclusion is derived from the first-order derivative of E_C^* with respect to h:

$$\frac{\partial E_C^*}{\partial h} = \frac{3e_M h^2 - \left(-6p_e e_M{}^2 + 4ke_M + 4e_R k\right)h - \left(4c_M e_M k - 4ae_M k - 3e_M{}^3 p_e{}^2 + 8e_M{}^2 kp_e + 8e_M e_R kp_e\right)}{16k^2}$$

$\frac{\partial E_C^*}{\partial h} \ge 0$ is equivalent to $a \ge \frac{-3h^2}{4k} + (-\frac{3p_e e_M}{2k} + 1 + \frac{e_R}{e_M})h + c_M - \frac{3e_M^2 p_e^2}{4k} + 2(e_M + e_R)p_e$. This completes the proof. □

Proposition 6(1) and Proposition 7(1) imply that both total profit and emission increase in total cap $C_M + C_R$. The conclusions are easy to understand because the increase of $C_M + C_R$ means more relaxed regulations, and more arbitrage and emission opportunities for supply-chain decision makers. It also reminds regulators (governments) that too-loose quotas are not conducive to achieving emission-reduction targets. When $p_e \le \tilde{p}_e$, the denominator of $\frac{\partial E_c^*}{\partial p_e}$ is a quadratic function of p_e in Proposition 7(2). If there exist two roots for the quadratic function, E_C^* only decreases in p_e when

p_e lies between the two roots. This means that, if p_e is too low or too high, the increasing p_e results in increasing emissions. When $p_e > \tilde{p}_e$, excessive carbon prices inhibit production and then profit. This suggests that the regulator, who wants to reduce carbon emissions, should control the price within a certain range by setting a reasonable cap. Proposition 7(3) indicates that increasing h does not necessarily result in emission reduction. One possible explanation is that a higher low-carbon premium motivates more production.

4.3. Profit-Sharing Contract

Generally, the decentralized decision-making mode leads to less profit than the centralized decision-making mode due to the effect of double marginalization [30]. To motivate decision makers under the decentralized decision-making mode to make decisions like those in the centralized decision-making mode, one way is to make a profit-sharing contract. As decision makers accept the contract, their decisions change from decentralized to centralized mode. Whether a contract can be executed depends on the proportion of profit being transferred. The following proposition states the range of proportion.

Proposition 8. *Let θ be the proportion of profit shared by the manufacturer. If θ satisfies*

$$\frac{\pi_M^*}{\pi_C^*} \le \theta \le 1 - \frac{\pi_C^*}{\pi_R^*}, \tag{36}$$

both the manufacturer and retailer are willing to accept the profit-sharing contract, where $\pi_M^* = \pi_M\left(L_D^*\right), \pi_R^* = \pi_R\left(L_D^*\right)$.

Proof. In order to promote acceptance of the contract by both parties, the profits they obtained from the contract should not be less than the profits they obtained under the decentralized mode, i.e.,

$$\begin{cases} \theta\pi_C^* \ge \pi_M^*, \\ (1-\theta)\pi_C^* \ge \pi_R^*. \end{cases}$$

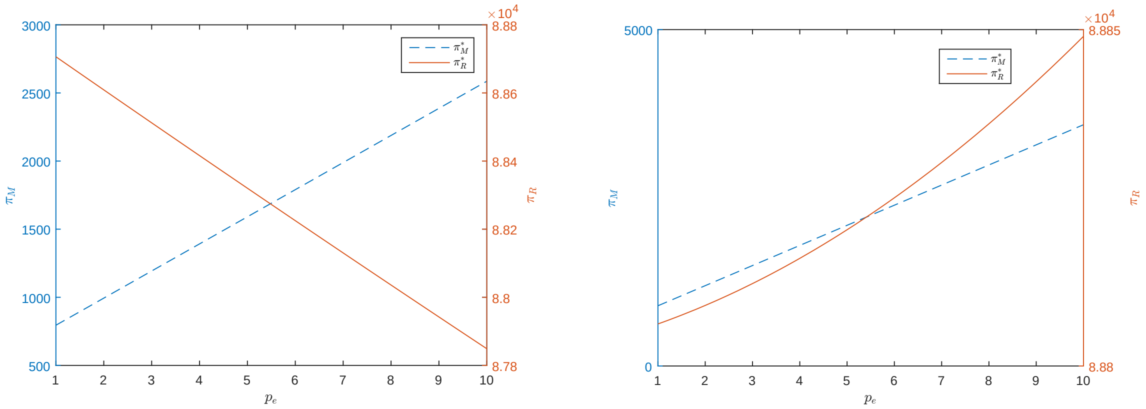
Hence, Equation (36) is easily derived. □

Under the profit-sharing contract, retailer and manufacturer can simultaneously improve their profit levels. The value of θ represent the retailer/manufacturer's bargaining power. The smaller θ means more bargaining power than the manufacturer has. As shown in Equation (36), $\frac{\pi_M^*}{\pi_C^*}$ and $1 - \frac{\pi_R^*}{\pi_C^*}$ are the minimum and maximum that θ can take. In what follows, they are denoted by $\underline{\theta}$ and $\bar{\theta}$, respectively.

5. Numerical Experiment

In this section, a series of numerical examples are presented to illustrate the theoretical results proposed in this study. As shown in Proposition 1, there are several L_D^* values under different scenarios. Only some specific scenarios are illustrated in this section, while others can be similarly implemented.

Let $k = 1, a = 600, p_M = 5, c_M = 2, e_M = e_R = 1, h = 1$. Under this setting, $\Delta > 0$ for any positive p_e, where Δ is defined in Proposition 1. In addition, assume that p_e lies in the [1, 10] interval. Thus L_1 and L_2, defined in Proposition 1, satisfy $L_2 > 1 > 0 > L_1$. Based on the parameter setting, the influences of p_e on π_M and π_R are shown in Figure 2.



(**a**) $C_M = C_R = 200$ (**b**) $C_M = C_R = 300$

Figure 2. Influence of p_e on profits under the decentralized decision-making mode.

Figure 2 shows some interesting phenomena. As shown in Figure 2a, both π_M^* and π_R^* increase in p_e when $C_M = C_R = 300$. However, π_R^* increases in p_e when $C_R = C_M = 200$. This is because condition $p_e \geq \frac{e_R(a-p_M+h)-2C_R}{e_R^2}$ in Proposition 3(3) is not satisfied for any $p_e \in [1, 10]$ when $C_R = 200$. Figure 2 indicates that a higher p_e does not necessarily generate more profit with relatively low emission caps. That is likely because relatively low emission caps restrain the space of arbitrage through the carbon-trading market. It should be noted that π_M^* also decreases in p_e when C_M is less than a threshold.

Let $C_M = C_R = 300, p_e = 1$, and $h \in [0, 10]$. The influence of h on profits is shown in Figure 3. Figure 3 shows that π_M^* and π_R^* increase in h. The product obtains more of a low-carbon premium with an increasing h, which means that the consumer is willing to pay more for low-carbon products.

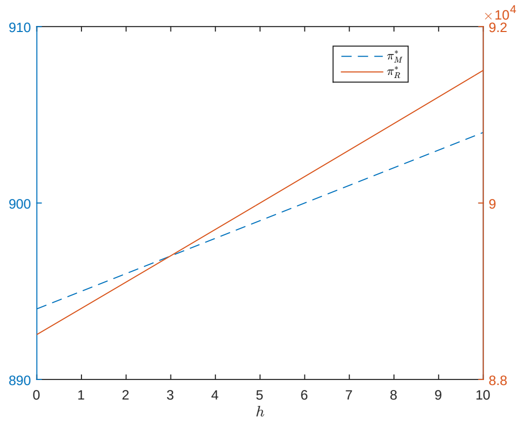


Figure 3. Influence of h on profits under decentralized decision-making mode.

As stated in Proposition 1, there are several situations for L_D^*. Let $p_e \in [0.01, 0.99]$. Under this setting, $\Delta = 0, L_2 - L_1 \geq 1, L_2 = 0$, implies that $L_D^* = 0$. In addition, $\frac{h+e_M p_e}{2k} \leq 1$ for any $p_e \in [0.01, 0.99]$ leads to $L_C^* = \frac{h+e_M p_e}{2k}$. Profits under the two modes with respect to p_e are illustrated in Figure 4. As shown in Figure 4, the changing trends of π_D^* and π_C^* with respect to p_e are different. The curve of π_D^* is always below curve π_C^*. This difference exists due to the effect of double marginalization. In order to investigate the influence of h on profits, let $p_e = 1$ and $h \in [1, 5]$. In this situation, $L_D^* = L_2$ and $L_C^* = 1$. The curves of π_D^* and π_C^* with respect to h are plotted in Figure 5. As shown in Figure 5, both π_D^* and π_C^* increase in h, and always $\pi_D^* < \pi_C^*$.

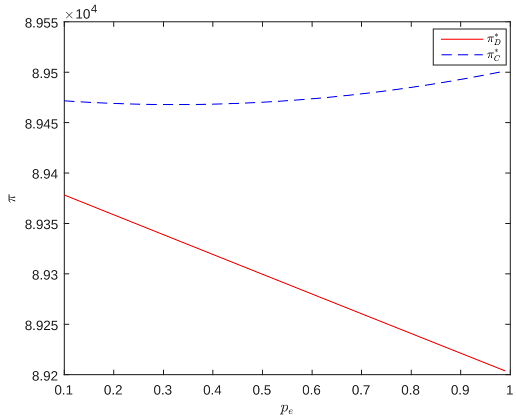


Figure 4. Influence of p_e on total profits under different modes ($\pi_D^* = \pi_M(L_D^*) + \pi_R(L_D^*)$).

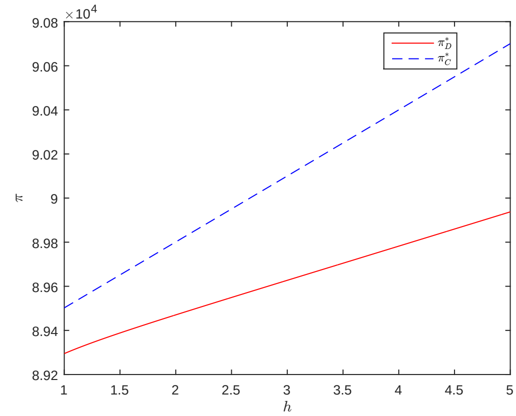


Figure 5. Influence of h on total profits under different modes.

Let $p_e \in [0.01, 0.99]$ and $h = 1$. It follows from the statement above, $L_D^* = 0$ and $L_C^* = \frac{h+e_M p_e}{2k}$. The curves of E_D^* and E_C^* with respect to p_e are plotted in Figure 6. As shown in Figure 6, emissions under both modes decrease in p_e. Emissions under a decentralized mode are always less than emissions under a centralized mode. When $p_e = 1$ and $h \in [1, 5]$, $L_D^* = L_2$ and $L_C^* = 1$. The curves of emissions with respect to h under the two modes are plotted in Figure 7. As shown in Figure 7, both of E_C^* and E_D^* decrease in h. Moreover, E_D^* is greater than E_C^* for any h. Figures 4–7 indicate that the centralized mode dominates the decentralized mode, with higher profits and fewer emissions.

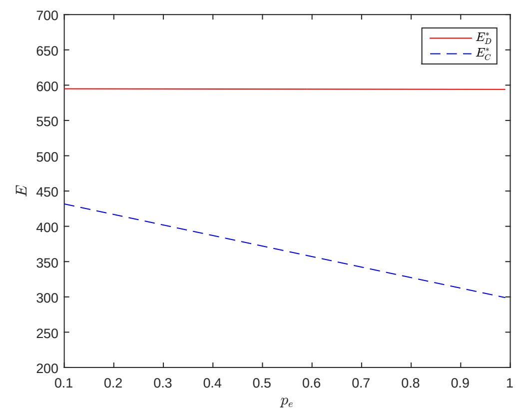


Figure 6. Influence of p_e on total emissions under different modes ($E_D^* = E_M(L_D^*) + E_R(L_D^*)$).

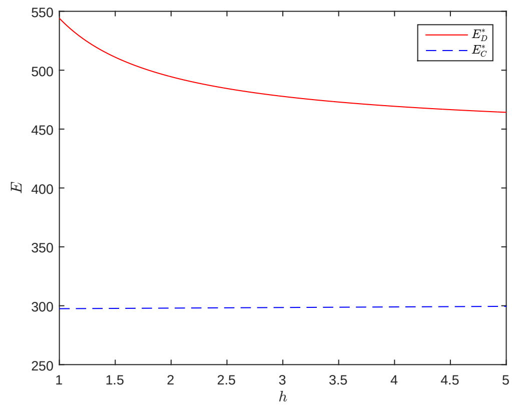


Figure 7. Influence of h on total emissions under different modes.

At the end of this section, the influence of p_e and h on $\underline{\theta}$ and $\bar{\theta}$ is investigated. Since $[\underline{\theta}, \bar{\theta}]$ is the feasible interval for θ, the length of $[\underline{\theta}, \bar{\theta}]$ represents the manufacturer's decision flexibility in profit-sharing mode. Let $p_e \in [0.01, 0.99]$ and $h = 1$. The influence of p_e on $\underline{\theta}$ and $\bar{\theta}$ is plotted by Figure 8. Figure 8 shows the distance of $\underline{\theta}$, and $\bar{\theta}$ increases in p_e, which the manufacturer has more decision flexibility with higher p_e under contract mode. Let $p_e = 1$ and $h \in [1, 5]$, The influence of h on $\underline{\theta}$ and $\bar{\theta}$ is plotted by Figure 9. In Figure 9, the distance of $\underline{\theta}$ and $\bar{\theta}$ expands with an increasing h. In other words, the manufacturer has more decision flexibility with a higher h.

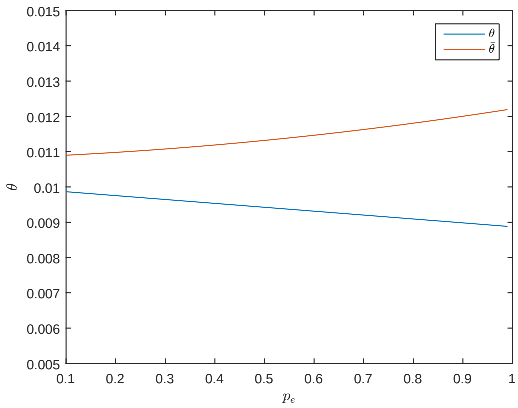


Figure 8. Influence of p_e on $\underline{\theta}$ and $\bar{\theta}$.

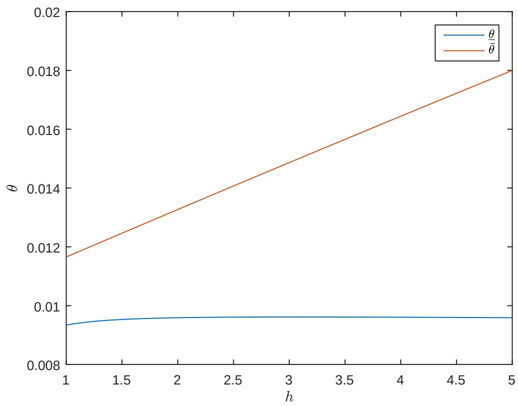


Figure 9. Influence of h on $\underline{\theta}$ and $\bar{\theta}$.

6. Concluding Remarks and Future Research

This study focused on two-stage supply-chain optimization with carbon-emission consideration. The cap-and-trade regulations and customers' LCAL were integrated into this problem. The manufacturer decides the emission-reduction level, and the retailer decides the retail price. Considering the different possible relationships between manufacturer and retailer, this study developed two mathematical models for the optimization problem under different decision-making modes (decentralized and centralized modes). The model under the decentralized mode was formulated as a Stackelberg game model and solved by the reverse-solution method. KKT optimality conditions were adopted to deal with the model under the centralized mode. The optimal decisions, corresponding profits, and emissions were derived by solving the models. Several propositions (corollaries) and numerical examples showed the influence of regulation parameters on supply-chain operations. In addition, the profit-sharing contract mode built a bridge between decentralized and centralized mode.

The analytic results and numerical experiments provided some meaningful insights that have realistic significance for regulators and the decision makers in supply chains. First, the optimal solution for decision makers is dependent on parameter settings regardless of decentralized and centralized mode (Propositions 1 and 5). Under a centralized mode, p_e plays an important role in deciding the optimal emission-reduction level when potential demand a is great enough. The cases of an optimal solution under decentralized mode are more complicated. Second, optimal solutions are not influenced by caps allocated by the regulator. However, generous caps lead to more emissions. This reminds the regulator that they should allocate a relatively low cap level. Third, the supply chain can benefit from increasing LCAL from customers. Finally, both higher carbon price p_e and LCAL h can improve manufacturer decision flexibility when there exists a profit-sharing contract.

Compared with existing studies, the contributions of this study are mainly reflected in the following areas: (1) Cap-and-trade regulation and LCAL were integrated in the same supply-chain operation-optimization models; (2) different decision-making modes were introduced in the modelling; (3) the extent to which a manufacturer is willing to transfer profit was investigated when a profit-sharing contract was available. Theoretical results provided meaningful managerial insights for regulators who could control profits and emissions by adjusting regulation parameter settings. For example, if $\frac{\partial L_2}{\partial h} \geq -\frac{h}{e_R}$ is satisfied, emissions are decreased in p_e (Proposition 4). A regulator can then raise prices by reducing the caps, and the amount of emission reduction can be calculated by theoretical results. In addition, the theoretical results in this study imply that a higher LCAL brings more profits under both modes. Thus, it is beneficial for decision makers of firms if customers' LCAL is promoted through education and publicity.

Future research can extend this study in the following four aspects. First, there can be more members in the supply chain, rather than only the two in this study. When there are more than one manufacturer or retailer, models are more complex (the Stackelberg–Nash model), and the solution method must be accordingly updated. Second, the study focused on a single-item product-supply chain. How heterogeneous products influence supply-chain operations can be considered in the future. Third, more a precise demand function should be formulated. The demand function affected by LCAL was simplified. The real demand function of LCAL should be derived through statistical techniques, e.g., regression. Fourth, future work should consider the influence of other regulations, such as the carbon tax. Future research expanding from these aspects can help researchers develop more reality-approaching models and provide more managerial insights for regulators and firms.

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