Optimization of the Product–Service System Configuration Based on a Multilayer Network

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Abstract: Product–service systems (PSS) accelerate the transition of value creation patterns for manufacturing industries, from product design and production to the delivery of overall solution integrating products and services. Existing PSS configuration solutions provide customers with preferable product modules and service modules characterized by the module granularity. Every service module is essentially a whole service flow. However, the performance of the PSS configuration solution is greatly influenced by service details. In summary, this paper studied the configuration optimization of product-oriented PSS using a fine-grained perspective. A multilayer network composed of (i) a product layer, (ii) a service layer, and (iii) a resource layer was constructed to represent the elements (product parts, service activities, resources) and relationships in PSS. Service activities selection and resource allocation were considered jointly to construct the mathematical model of PSS configuration optimization, thus enabling the calculation of optimizing objectives (time, cost, and reliability) under constraints closer to the actual implementation. The importance degree of service activity was considered to improve the performance of service activities with higher importance. Corresponding algorithms were improved and applied for obtaining the optimal solutions. The case study in the automotive industry shows the various advantages of the proposed method.

Keywords: product service system; configuration optimization; multilayer network; service activities selection; resource allocation

1. Introduction

Manufacturing industries are experiencing a transition of value creation patterns from the design and production of products to the design and the delivery of services based on products, to accommodate increasingly global competition and customer-centered business settings [1]. As an overall solution that effectively integrates products and services [2], product–service systems (PSS) [3–5] play a vital role in accelerating this transition for manufacturers. By prolonging the service life of products and commoditizing the related services, PSS can change the revenue models and pricing capabilities for manufacturers [6,7]. This exemplifies a new source of competitive advantage and differentiation. Furthermore, compared to traditional business models, PSS has the potential to reduce environmental impact, thus receiving much attention as one strategic alternative for the sustainable development of firms [8–10].

PSS can be divided into three types: (a) product-oriented PSS, (b) use-oriented PSS, and (c) result-oriented PSS [5]. In product-oriented PSS, customers buy the product as usual; a service is also sold to customers additionally, which then will be operated on products or provided to customers [11].
This means that the product is the principal purpose, so the service is rendered to help customers achieve a better user experience. This paper focuses extensively on product-oriented PSS.

Services help to attain higher margins, but performing the transition or maintaining the success of service strategy is still a challenge for companies [12]. With the increase in service offers, higher costs or failures in achieving expected returns sometimes occur [13,14]. To overcome this glitch in the profit generation and commercial success, companies should target the role of PSS design [15,16]. Many studies have focused on PSS design to provide support, such as design frames [17–19], design methods [20], modularization approaches [21,22], configuration and evaluation methods [23–25].

Product–service integration solutions provided to customers include product modules and service modules [26]. There are usually many possible PSS solutions. A PSS configuration is the construction of technical systems by selecting and assembling preferable modules from a predefined product and service library according to customers’ needs under certain constraints [27]. Thus, PSS configuration is an imperative part of PSS design. Studies on PSS configuration have mainly focused on the PSS configuration framework [24,28], customer perception in PSS configuration [29], the extraction of PSS configuration rules [30–33], and PSS configuration optimization methods [23,34]. Most studies on PSS configuration, especially research on PSS configuration optimization, commonly deal with some predefined level of module granularity. Overall, the product module granularity is introduced to achieve flexible manufacturing systems for product functions realization. With regard to service module granularity, every service module is essentially a whole service flow, depending on the specific module’s granularity level. However, it presently lacks the PSS configuration on a finer granularity level, which can inspire the final configuration’s optimization.

Considering the balance between differentiated customization and mass production, inappropriate product module granularity will increase managerial difficulty and manufacturing cost [35]. Services are generally under-designed in comparison to physical products [36]. Also, service modules are not restricted to physical artefacts. Thus, service modules should be designed considering a finer granularity level, so they can integrate tangible and non-tangible artefacts. Recent studies on service modules in PSS have mainly focused on the identification and evaluation of customer requirements [37,38], modularization and related quantification [39,40], and the evaluation of internal and external performance (such as cost, customer satisfaction, service cycle time, availability, and reliability) [11,41]. Modularization can normalize the PSS configuration and improve its suitability. However, a service module in PSS is essentially a composition of service activities. It is coarse-grained to use a whole service flow as a service module unit in PSS configuration. Therefore, configuring a PSS in finer granularity by using service activity as the minimum configuration unit is proposed and developed in this paper.

There are various methods to optimize the composition of service activities, which can be embodied in the following three approaches [42]:

1. Service composition solver based on artificial intelligence planning, which can automatically generate service activities composition according to customer demands;
2. The method based on semantics, studying the service matching and composability;
3. The service workflow-based semiautomatic service composition method, which is the most widely used one.

In this context, Zhu et al. [43] proposed an ontology-based service process model to support decision-making for maintenance, repair, and overhaul services in the aerospace industry. Cao et al. [44] transformed the service process model into a process structure tree to consider service process structuration and service activities selection simultaneously. Wang et al. [45] advocated a model of service activities composition based on stochastic Petri net. Hu et al. [46] modeled the service process by logical Petri net to study the structural transformation of the service process.

The above-mentioned studies only focused on service activities selection. The impact of resource allocation on service activity has not been considered during the service activities selection. Service activities selection and resource allocation were consequently divided into two independent processes.
In a practical situation, the performance of service activity operation will be altered based on different resource allocation instances. It is necessary to combine service activities selection and resource allocation into a whole process during the PSS configuration optimization. Furthermore, most of the previous studies highlighted multi-objective optimization of the overall solution, but high-importance service activities usually exert a greater impact on the solution, compared to low-importance services. Therefore, the importance degree of service activity is also taken into consideration in this paper, to improve the performance of service activities with higher importance.

Based on the above analysis, it was found that: (a) PSS configuration is an imperative part of PSS design; (b) existing PSS configuration solutions are obtained in the module granularity; and (c) the significant impact of service details on the performance of a PSS configuration solution has not been given sufficient attention in recent research studies. PSS configuration optimization is, therefore, studied from a fine-grained perspective in this paper. The optimization is performed by combining service activities selection and resource allocation. As a result, the PSS configuration solutions provided to the customer will be more refined and precise. In particular, the optimizing objectives, which present the performance of the solutions, will be calculated closer to the actual implementation.

The complex network is chosen as the framework for the PSS modeling because its structural properties enable the exploration of the interactions between products, resources, and service activities. The mathematical characteristics of complex networks are very useful for analyzing the characteristics of complex systems. However, it is difficult for a general complex network to effectively reveal the multilayered or multidimensional relationships in a complex system when the scale of the system increases substantially. Many approaches have been proposed to enrich the representation of networks. Hypernetworks [47–49] are developed based on the definition of the hypergraphs to generalize the multidimensional relationships, in which multi-elements are connected rather than only two elements. Multilayer networks [50] are applied to describe the complex structures of systems, which include different types of relationships between their components. Originally, multilayer networks were proposed to express different types of relationships between one single type of elements in different layers [51,52]. More recently, they have been used to model other systems, including those with different types of elements. Omodei et al. [53] proposed a method based on the multilayer networks of citations and disciplines to assess the interdisciplinary importance of scholars, institutions, and countries. Additionally, Pasqual and Weck [54] introduced a multilayer network model integrating three coupled layers (product layer, change layer, and social layer) to analyze and manage engineering change propagation. Leng and Jiang [55] also offered a multilayer network model comprising processes, machines, and work in processes to assess dynamic scheduling in a radio frequency identification-driven discrete manufacturing system.

Based upon the comparison of the general complex network, hypernetwork, and multilayer network, it is obvious that a multilayer network can perfectly accommodate multiple heterogeneous elements and provide a better description of relationships between heterogeneous elements than a general complex network or hypernetwork. Therefore, a multilayer network, as a special type of complex network, is applied for the modeling of PSS in this paper.

The organization of this paper is as follows. In Section 2, this paper’s research problem is described and the framework is then introduced. The PSS multilayer network model is constructed in Section 3. Section 4 establishes the mathematical model of PSS configuration. This section also delineates the improved optimization algorithms and their applications for obtaining the optimal solutions. Moreover, a case is studied in Section 5 to discuss the feasibility and effectiveness of this method. Section 6 highlights the benefits and the contributions to the sustainability of the proposed approach. Finally, conclusions are stated in Section 7.

2. Problem Description and Framework

In product-oriented PSS, products are delivered to customers as tangible objects in a definite time, just after the PSS configuration solution is accepted. However, that is not the case with services.
Services are provided to customers only when the customer requests or the service premises are satisfied and operation of services is affected by the real-time conditions of PSS. Service details, which include service activities selection and resource allocation, are consequently not strictly determined when the PSS configuration solution is accepted. Contracts to implement corresponding service functions are provided to the customer. But service details definitely impact the performance of the PSS configuration solution. Service time depends on service activities selection, cost and reliability are accurately influenced by resource allocation. Thus, the PSS configuration should be optimized by combining service activities selection and resource allocation. In this way, the PSS solutions obtained after optimization will be more precise, and the results based on the optimizing objectives (executing time, executing cost, and executing reliability) will be closer to the actual implementation.

The framework of this fine-grained configuration optimization of the product-oriented PSS is shown in Figure 1. A PSS model is constructed based on a three-layer network, which contains product and services selected by the customer, and the resources in the system. A node in every layer (product layer, service layer, resource layer) of the multilayer network respectively represents an element (product, service, resource). The relationships between homogeneous elements in a layer and the relationships between heterogeneous elements in different layers are represented as edges. With appropriate decomposition, PSS can be clearly described by the proposed multilayer network. Elements and relationships in PSS can be expressed by connection matrixes, which make the description of the mathematical model and realization of optimization more convenient. By using this PSS multilayer network model, the importance degrees of service activities are analyzed based on the attributes of the multilayer network. A mathematical model based on the connection matrixes is constructed to express the objectives and constraints of PSS configuration optimization, and the corresponding solution algorithm is proposed to solve the mathematical model of optimization.

Figure 1. Framework of the proposed method.
3. PSS Multilayer Network Model

The PSS multilayer network (NET_{PSS}) is composed of the product layer (Net_P), service layer (Net_S), resource layer (Net_R), and set of inter-layer edges (E_{ij}) which connect layers to each other. Therefore, there are three types of nodes and five kinds of edges considered in NET_{PSS}. Specifically, nodes in Net_P, nodes in Net_S, and nodes in Net_R are denoted as sets P, S, and R, respectively; edges in Net_P, edges in Net_S, edges in Net_R, inter-layer edges between Net_S and Net_P, and inter-layer edges between Net_S and Net_R are denoted as sets E_P, E_S, E_R, E_{P,S}, and E_{R,S}, respectively. The inter-layer edges between Net_R and Net_P are not considered in this model.

Then, NET_{PSS} is described by Formula (1):

\[
\]  

A product can be decomposed into parts according to required granularity. Net_P is formed by part nodes (p) connecting to each other by interrelationships between parts. Net_S consists of service flows, each of which is described by a service trigger node (ss), numbers of service activity nodes (sa), and a service end node (se) logically associated with each other. Net_R contains resources required by service activities, including personnel resources (pr) and equipment resources (er), as resource types are denoted as kr. E_l contains E_{P,S}, E_{R,S}, and the weights on inter-layer edges. Detailed descriptions of Net_P, Net_S, Net_R, and E_l are given in the following subsections.

3.1. Product Layer

A node p_i in Net_P is described by Formula (2):

\[
p_i = \langle p_{_BAS}, p_{_REC}, p_{_ST}\rangle,
\]  

where p_{_BAS} is the set of basic information, including information such as name, quality, and material. p_{_REC} is the set of accepted service records. p_{_ST} is the set of working statuses, including various data of physical status, material status, and environmental status. All statuses vary with time t. For any p_i whose status data cannot be monitored, p_{_ST} = \emptyset.

There are four types of interactions among parts [56]: spatial-type interaction, energy-type interaction, information-type interaction, and material-type interaction. These interactions could be extracted from the product design manually. If p_i provides spatial reference (adjacency or orientation) to p_j, then se_{ij} = 1. If p_i provides or transmits energy to p_j, then ee_{ij} = 1. If p_i provides or transmits information to p_j, then ie_{ij} = 1. If p_i determines or impacts the material selection of p_j, then me_{ij} = 1. If there are one or more interactions from p_i to p_j, then e_{ij} = 1, denotes the directed edge from p_i to p_j; otherwise, e_{ij} = 0. Obviously, e_{ij} \neq e_{ji}.

In order to describe the overall strength of the relationship between p_i and p_j, the weight on e_{ij}^P is represented by the value w_{ij}^P following Formula (3):

\[
w_{ij}^P = w_s \cdot se_{ij} + w_e \cdot ee_{ij} + w_i \cdot ie_{ij} + w_m \cdot me_{ij},
\]  

where w_s, w_e, w_i, and w_m, respectively, represent the proportion of se_{ij}, ee_{ij}, ie_{ij}, and me_{ij} in the edge e_{ij}^P. w_{ij}^P is defined in [0,1]. If the four types of interactions are equal in importance, then w_s = w_e = w_i = w_m = 0.25.
Net_P is consequently represented by Formula (4):
\[
Net_P = (P, E_P);
\]
\[
P = \{p_1, p_2, \ldots, p_n\};
\]
\[
E_P = \{e^p_{ij}\};
\]
\[
W_P = \{w^p_{ij}\}.
\] (4)

3.2. Service Layer

Services in a product lifecycle can be classified into two types: production services and product services. All services in Net_S are product services.

3.2.1. Service Trigger Node

A node ss_i in Net_S is as described by Formula (5):
\[
\begin{align*}
ss_i &= \{ssP_i, Tri_i, s_st_i\}; \\
Tri_i &= \{tri(p_i)|p_i \in ssP_i\}; \\
s_st_i &= \{on(start_{t_i}, ddl_{t_i}) \lor off\};
\end{align*}
\] (5)

where ssP_i is the set of parts whose working statuses are monitored by ss_i. Tri_i is the set of triggering conditions of ss_i, tri(p_i) is the triggering condition for working statuses p_ST_i of p_i to trigger ss_i. s_st_i denotes the status of ss_i: if triggering conditions are satisfied, the status will be on(start_{t_i}, ddl_{t_i}), where start_{t_i} and ddl_{t_i} are, respectively, the start time and the deadline of the entire service set according to customer requirements; otherwise, the status will be off.

3.2.2. Service Activity Node

The function of service activity is to change the status of the service activity receiver [57]. Service activity is operated by the specified resources and executed on the specified component. By decomposing the service flow at specified granularity, the corresponding set of service activities could be obtained. A service activity node sa_i in Net_S is as described by Formula (6):
\[
\begin{align*}
sa_i &= \{a_{REC_i}, T_i, C_i, RE_i, O_i, a_{st_i}\}; \\
a_{st_i} &= \{off \lor on(bt_{t_i}, f_{t_i})\};
\end{align*}
\] (6)

where a_{REC_i} is the set of service execution records of sa_i. T_i and C_i are the theoretical executing time and fixed cost for operating sa_i, which are extracted from historical data. RE_i denotes the reliability of that sa_i, which can be completed in time T_i. It is represented by the ratio between the number of times that sa_i has been completed in time T_i and the total number of times that sa_i has been executed. RE_i \in (0, 1). O_i is set of service objects of sa_i. a_{st_i} denotes the status of sa_i: off is idle status, on(bt_{t_i}, f_{t_i}) denotes working status. bt_{t_i} is the beginning time of sa_i, ending time is defined as f_{t_i} = bt_{t_i} + T_i.

3.2.3. Service End Node

A service end node se_0 is put in Net_S to distinguish whether the service flow is finished at the end of sa_i. An edge between sa_i and se_0 means that sa_i is the last service activity in the service flow.

3.2.4. Edges in Service Layer

Regarding ss_j and se_0 as the activities of “trigger service” and “end service”, respectively, then ss_j, se_0, and sa_i are homogeneous. Edges between sa_i and sa_j, edges between sa_i and ss_j, and edges between sa_i and se_0 are all denoted by \(e^S\); \(e^S_{ij} \neq e^S_{ji}\). \(e^S_{ij} = 0\). \(w^S_{ij}\) defined in the interval \([0,1]\) denotes weight on \(e^S_{ij}\). \(w^S_{ij} = 0\) when \(e^S_{ij} = 0\). The description of an edge \(e^S\) between two nodes considers the following cases:
1. Between \( ss_j \) and \( sa_i \): if \( sa_i \) is the first one executed in the service flow triggered by \( ss_j \), then \( e_{ij}^S = 1 \), \( bt_l = start_{ss_j} \); \( sa_i \) can participate in different service flows triggered by \( ss_j \), denoted as \( e_{ij}^S = 1 \) with \( w_{ij} \to 0 \); \( sa_i \) will not output any weight to \( ss_j \); \( e_{ij}^S = 0 \) between \( ss_j \) and \( ss_j' \);

2. Between \( sa_i \) and \( sa_j \): if \( sa_i \) is the last one in the service flow being executed, then \( e_{ij}^S = 1 \); \( e_{ij}^S = 0 \) for any \( sa_i \) and \( e_{ij}^S = e_{ij}^S = 0 \) between \( sa_0 \) and \( ss_j' \);

3. Between \( sa_i \) and \( sa_j \): if \( sa_i \) and \( sa_j \) are in the same service flow and are executed in time sequence, then \( e_{ij}^S = 1 \). Otherwise, \( e_{ij}^S = 0 \).

There are three types of correlations among service activities [58]: correlation of function similarity, correlation of function complementary, and no correlation. If two service activities have a correlation of function similarity, their function might be identical, inclusive, or partially similar. If two service activities have a correlation of function complementary, the composable situation might be accurate composition, inclusive composition, blocked composition, or crossed composition.

According to the above correlations, logical relationships among service activity nodes are divided into serial relationship, parallel relationship, and selective relationship. Two activities in a serial relationship are sequentially executed in chronological order. Two activities in a parallel relationship are executed at the same time, and only when they are fully completed will the execution of subsequent activities begin. In a selective relationship, there are multiple activities for selection: if only one of them is executed, then the execution of subsequent activities will begin.

The in-degree and out-degree of \( sa_i \) in NetS are denoted as \( inw_i \) and \( outw_i \), which are described by Formulas (7) and (8):

\[
\text{inw}_i = \sum_j e_{ij}^S \cdot w_{ij}^S \quad (7)
\]

\[
\text{outw}_i = \sum_j e_{ij}^S \cdot w_{ij}^S \quad (8)
\]

Then, the description of the three types of logical relationships is expanded as follows:

- If \( sa_h \) and \( sa_i \) are in a serial relationship, and \( inw_h = x \), then \( e_{hi}^S = 1 \), \( w_{hi}^S = x \), \( bt_l = f_{t_h} \);
- If \( n \) nodes \( (sa_1, sa_2, \ldots, sa_n) \) are in a selective relationship, and \( inw_h = x \), then \( e_{hi}^S = e_{hj}^S = e_{il}^S = e_{jl}^S = 1 \), \( w_{hi}^S = w_{hj}^S = w_{il}^S = w_{jl}^S = x \), \( outw_i = x \). In this case, \( outw_h > inw_h \) and \( outw_i < inw_i \). If \( sa_i \) is selected, then \( bt_l = f_{t_h}, bt_l = f_{t_i} \);
- If \( n \) nodes \( (sa_1, sa_2, \ldots, sa_n) \) are in a parallel relationship, and \( inw_h = x \), then \( e_{hi}^S = e_{hj}^S = e_{il}^S = e_{jl}^S = 1 \), \( w_{hi}^S = w_{hj}^S = w_{il}^S = w_{jl}^S = x/n \), \( outw_i = x \), and \( bt_l = f_{t_h}, bt_l = f_{t_i} \). In this case, \( outw_h = inw_h \) and \( outw_i = inw_i \).

Connections of these logical relationships are shown in Figure 2. If \( sa_h \) is replaced by a service trigger node, then \( x = 1 \).

![Figure 2. Logical relationships among service activity nodes. (a) Serial relationship; (b) selective relationship; (c) parallel relationship.](image-url)
Then, the logical relationship between nodes in Net$_S$ can be distinguished by values of $inv$ and $outr$. However, if $sa_k$ has multi inputs and multi outputs, as depicted in Figure 3a, the logical relationship between $sa_j$ and other nodes will be uncertain. The virtual service activity node $sa_m$ is applied to prevent that. Then, $sa_k$ will become a node with a single input or single output, as shown in Figure 3b,c. There is not any essential service content in a virtual service activity node, so $T_m = 0$, $C_m = 0$, $RE_m = 1$, $a_{\text{REC}_m} = \emptyset$, $O_m = \emptyset$, and $sa_m$ does not require any resources.

![Figure 3](image)

**Figure 3.** Different connection situations of $sa_k$. (a) Multiple inputs and multiple outputs; (b) Multiple inputs and single output; (c) Single input and multiple outputs.

If $sa_k$ connects the other nodes, which are in selective relationships and parallel relationships, intricately, it will be impractical to distinguish different relationships by values of $inv$ and $outr$. As shown in Figure 4a, when $w_{hm}^S = w_{hi}^S = w_{bk}^S = w_{bk}^S = x/2$, relationships among $sa_i$, $sa_j$, $sa_k$, and $sa_g$ cannot be distinguished directly. In order to solve this situation, there will be two situations to integrate in the virtual service activity nodes $sa_m$ and $sa_n$, as follows:

1. $w_{hm}^S = w_{hi}^S = w_{bk}^S = w_{bk}^S = x/2$ means that $sa_i$ and $sa_j$ are in a selective relationship; $sa_k$ and $sa_g$ are in a selective relationship. Meanwhile, $sa_i$ (or $sa_j$) and $sa_k$ (or $sa_g$) are in a parallel relationship, as expressed in Figure 4b.
2. $w_{hm}^S = w_{hi}^S = x$, $w_{bk}^S = w_{bk}^S = w_{bk}^S = w_{bk}^S = x/2$ means that $sa_i$ and $sa_j$ are in a parallel relationship; $sa_k$ and $sa_g$ are in a parallel relationship. Meanwhile, two parallel clusters are in a selective relationship, as indicated in Figure 4c.

![Figure 4](image)

**Figure 4.** The logical relationship between $sa_i$ and $sa_j$. (a) It cannot be distinguished; (b) It is selective relationship; (c) It is parallel relationship.

Assuming that the number of $sa$ and $ss$ in Net$_S$ are indicated, respectively, by $\delta$ and $\gamma$, and there is only one $sa$, then the description of Net$_S$ is consequently shown by Formula (9):

$$
S = \{s_0, s_1, s_2, \ldots, s_{\delta+\gamma}\} = \{s_0, sa_1, sa_2, \ldots, sa_\delta, ss_{\delta+1}, ss_{\delta+2}, \ldots, ss_{\delta+\gamma}\};
$$

$$
E_S = \{e_{ij}^S\};
$$

$$
W_S = \{w_{ij}^S\}.
$$

(9)
3.3. Resource Layer

The node \( pr_i \) or \( er_i \) in \( Net_R \) is described by Formula (10):

\[
\begin{align*}
\dot{r}_i &= pr_i \land er_i = \langle r_{REC_i}, c_i, r_{st_i} \rangle; \\
r_{st_i} &= \text{idle} \lor \text{busy};
\end{align*}
\]

where \( r_{REC_i} \) is the set of records of services operated by \( r_i \), \( c_i \) is the cost for \( r_i \) working in one unit of time, \( r_{st_i} \) denotes the status of \( r_i \). If \( r_{st_i} = \text{idle} \), then \( r_i \) could accept a new assignment of the service activity; if \( r_{st_i} = \text{busy} \), it means \( r_i \) is in working status and will not accept any new assignments until the service activity \( r_i \) is working on is finished. If there are not any resource contents in \( kr_j \), then \( r_{REC_j} = \emptyset \), \( c_j = 0 \), \( r_{st_j} = \emptyset \).

If there is a subordinate relationship between \( pr_i \lor er_i \) and \( kr_j \), which means that the resource type of \( pr_i \lor er_i \) is \( kr_j \), then \( e_{ji}^R = 1 \). This denotes that there is a directed edge from the father node \( kr_j \) to the child node \( pr_i \lor er_i \). Otherwise, \( e_{ji}^R = 0 \). For any \( pr_i \lor er_i \), \( \sum_{j} e_{ji}^R = 1 \).

Assuming that quantities of \( pr_i \), \( er_i \) and \( kr_j \) in \( Net_R \) are indicated by \( \alpha \), \( \beta \), and \( b \), respectively, then the description of \( Net_R \) is shown by Formula (11):

\[
\begin{align*}
Net_R &= \langle R, E_R \rangle; \\
R &= \{ pr_{1_i}, \cdots, pr_{n_i}, er_{a+1_i}, \cdots, er_{a+b_i}, kr_{a+\beta+1_i}, \cdots, kr_{a+b+1_i} \} = \{ r_1, r_2, \cdots, r_{a+\beta+b} \} \\
E_R &= \{ e_{ji}^R \}, (i = 1, 2, \cdots, a + \beta; j = a + \beta + 1, \cdots, a + \beta + b). 
\end{align*}
\]

3.4. Inter-Layer Edges

The service content is determined by the working status of products, while service activities are executed on products by required resources. In the PSS multilayer network, some inter-layer edges exist between \( Net_S \) and \( Net_P \), the rest exist between \( Net_S \) and \( Net_R \).

3.4.1. Inter-Layer Edges Between \( Net_S \) and \( Net_P \)

For \( p_i \) and \( sa_j \), if \( p_i \in O_j \), then \( e_{ji}^{SP} = 1 \), which denotes the directed inter-layer edge from \( sa_j \) to \( p_i \). This means that \( p_i \) is one of the service objects of \( sa_j \). Otherwise, \( e_{ji}^{SP} = 0 \). Meanwhile, \( e_{ji}^{SP} = 1 \) only makes sense between \( p \) and \( sa \); for \( SSR \) or \( se \), \( e_{ji}^{SP} = 0 \).

Therefore, the set of inter-layer edges between \( Net_S \) and \( Net_P \) is as described by Formula (12):

\[
E_{PS} = \{ e_{ji}^{SP} \}. 
\]

3.4.2. Inter-Layer Edges Between \( Net_S \) and \( Net_R \)

Inter-layer edges between \( Net_S \) and \( Net_R \) include directed inter-layer edges from service nodes to resource nodes and directed inter-layer edges from resource nodes to service nodes. Their descriptions are depicted below:

- The execution of service activities demands the resources be in corresponding specific kinds and quantities. If \( sa_j \) demands the resources whose type is \( kr_j \), then \( e_{ji}^{SR} = 1 \), which denotes a directed inter-layer edge from \( sa_j \) to \( kr_j \) and the quantity demand is represented by integer weight \( w_{ji}^{SR} \). Otherwise, \( e_{ji}^{SR} = 0 \), \( w_{ji}^{SR} = 0 \). Meanwhile, \( e_{ji}^{SR} = 1 \) only makes sense between \( kr \) and \( sa \); for other nodes, \( e_{ji}^{SR} = 0 \);

- If \( e_{ji}^{SR} = 1 \), \( e_{ji}^R = 1 \), and \( r_{st_i} = \text{idle} \), it means that \( pr_i \lor er_i \) is assignable to \( sa_j \), then \( e_{ij}^{RS} = 1 \), which denotes a directed inter-layer edge from \( pr_i \lor er_i \) to \( sa_j \). Otherwise, \( e_{ij}^{RS} = 0 \). Meanwhile, \( e_{ij}^{RS} = 1 \) only makes sense between \( pr \lor er \) and \( sa \); for other nodes, \( e_{ij}^{RS} = 0 \). The reliability of \( sa_j \) being completed in time \( T_j \) by \( pr_i \lor er_i \) is denoted by weight \( w_{ij}^{RS} \) in \([0,1]\).
Every pr$_i$ has a different knowledge level and proficiency level about different sa. Assuming $PA_i$ denotes the number of times that $sa_j$ has been completed in the time $T_j$ by pr$_i$, $PB_i$ denotes the total number of times that pr$_i$ has participated in the execution of $sa_j$, then $w^{RS}_{ij}$ between pr$_i$ and $sa_j$ is represented by $PA_i/PB_i$. Every er$_i$ has a constant function in a different sa. Assuming $EA_i$ denotes the number of times that any $sa$ has been completed in time by er$_i$, $EB_i$ denotes the total number of times that er$_i$ has participated in the execution of any $sa$, then $w^{RS}_{ij}$ between er$_i$ and $sa_j$ is represented by $EA_i/EB_i$.

The set of inter-layer edges between $Net_S$ and $Net_R$, and the set of weights on these inter-layer edges are described by Formula (13):

$$E_{RS}, S = \\left\{ e^{RS}_{ij}, e^{SR}_{ji} \right\};$$
$$W_{RS} = \left\{ w^{RS}_{ij}, w^{SR}_{ji} \right\}.$$  

Therefore, $E_i$ is as given by Formula (14):

$$E_i = \left\{ E_{P,S}, E_{R,S} \right\}.\quad (14)$$

### 4. PSS Configuration Optimization

In most practical cases, many services are usually triggered at the same time. Consequently, the configuration optimization should be designed by considering multi-services. After any ss is triggered, service activities selection and resource allocation should be accomplished under given constraints to obtain the PSS configuration under constraint conditions.

Let us assume that the number of triggered ss is $\sigma$ up to the point of time $t$. If ss$\phi$ is triggered and is represented by $u_\phi$ following Formula (15), then the situation of all ss is represented by trigger matrix $[u_{\delta+1}, u_{\delta+2}, \ldots, u_{\delta+\gamma}]$ with $\sigma = \sum_{\phi=\delta+1}^{\delta+\gamma} u_\phi$:

$$u_\phi = \begin{cases} 
1, & \text{ss}_\phi \text{ is triggered} \\
0, & \text{otherwise} 
\end{cases}.$$  \quad (15)

As a result, service activities selection is regarded as a selection of the paths with multi ss as start points in $Net_S$. It is necessary to consider the executing time, cost, and reliability of $sa$ in the selective relationship. The allocation of resources is viewed as resource allocation for multitasks (service activities), in which the cost and reliability of different $r_i$ are considered. Therefore, the services solution is optimized with the goal of executing time minimization, cost minimization, and reliability maximization.

#### 4.1. Importance Degree of Service Activity

When a certain type of resource is required by several service activities simultaneously, then those resources with higher reliability should be allocated to more important service activities. Product parts are the objects of service activities; hence, the importance degree of product parts exerts a decisive influence on the importance degrees of service activities. Consequently, the importance degree of product parts is taken as the base of calculation for the importance degree of service activities, which provides a theoretical reference for the resource allocation.

Today, network-based importance measurement is thoroughly developed and widely applied. Various measure methods can be summarized as:

1. Importance calculation methods based on neighbor relationships between nodes, such as degree centrality or semi-local centrality [59];
2. Path-based importance calculation methods, such as closeness centrality and betweenness centrality;
(3) Importance calculation methods based on eigenvectors, such as search algorithm LeaderRank [60] and SRank [61] based on algorithm PageRank [62];

(4) Importance calculation methods based on node removal and contraction, in which the importance degree of nodes is measured by the variation in network attributes after node removal or contraction [63].

4.1.1. Importance Degree of Product Parts

Importance calculation methods based on eigenvector are applied in this paper. In the concept of PageRank and LeaderRank, the importance of a page in a network is determined by the number and the quality of other pages pointing to it. However, the importance of $p_i$ in $Net_P$ is ascertained by the number and quality of other nodes it points to. This makes PageRank and LeaderRank unsuitable for calculating the importance degree of product parts. Referring to the weighted LeaderRank [64], PartRank is proposed to calculate the importance degree of product parts. Details are illustrated as follows.

A ground node $p_g$, which connects with every part node by a bidirectional edge, is introduced. Then $e_{P_{gi}} = 1$ and $e_{P_{ig}} = 1$, $\forall i \neq g$. Based on the concept of weighted LeaderRank, nodes with higher input intensity embody a higher probability of being visited by $p_g$. Inversely, $p_i$ with a higher output intensity is more likely to visit $p_g$ in $Net_P$. $w_{P_{gi}}$ is consequently described by formula (16), where $\theta$ is a free parameter, $n$ is total number of nodes in $Net_P$ (excluding $p_g$). $w_{P_{gi}}$ is described as Formula (17):

$$w_{P_{gi}} = \left( \sum_{h=1}^{n+1} e_{P_{hi}} \cdot w_{P_{hi}} \right)^{\theta};$$  \hspace{1cm} (16)

$$w_{P_{gi}} = \frac{\sum_{h=1}^{n} e_{P_{hi}} \cdot w_{P_{hi}}}{\sum_{h=1}^{n} e_{P_{hi}}}. \hspace{1cm} (17)$$

The importance degrees are calculated iteratively until they reach a steady state. Assuming that calculation reaches a steady state at step $k$, then the importance degree of $p_g$ will be allotted to every $p_i$. The importance degree of $p_i$ is denoted as $PR_i$, calculated as Formula (18),

$$\begin{cases}
PR_i(0) = 0 \\
PR_i(0) = 1, \forall i \neq g \\
PR_i(t) = \sum_{j=1}^{n+1} e_{SP_{ji}} \cdot w_{SP_{ji}} \cdot PR_j(t-1), t = 1, 2, \cdots , k \\
PR_i = PR_i(k) + \frac{w_{P_{gi}}}{k_g} \cdot PR_g(k)
\end{cases} \hspace{1cm} (18)$$

where $k_{ji}^{SP} = \sum_{h=1}^{n+1} \left( e_{SP_{hi}} \cdot w_{SP_{hi}} \right)$ is the input intensity of $p_i$ in $Net_P$.

4.1.2. Importance Degree of Service Activities

Knowing that the service object of $sa$ is $p$ and if $sa$ is operated on $p$ with a high importance degree, a designer should pay more attention to that $sa$. CNI$_j$ denotes the cross layer importance degree of $sa_j$, which is related to the product parts on which $sa_j$ operates. The value of CNI$_j$ is determined by $p_i$ with the highest $PR_i$ in the set of service objects of $sa_j$. CNI$_j$ is consequently calculated by Formula (19):

$$CNI_j = \max_{1 \leq i \leq n} \left( e_{SP_{ji}} \cdot PR_i \right). \hspace{1cm} (19)$$

After being normalized, the importance degree of $sa_j$ is denoted as $D_j = CNI_j / \max$, where max is the maximum of all $CNI_j$ in $Net_S$. Therein, the virtual service activity node does not have any service object, so the importance degree of every virtual service activity node is consequently 0.
4.2. Mathematical Model

Define the binary variable $y_j$ to indicate whether $sa_j$ is selected, and define the binary variable $ax_{ji}$ to indicate whether to allocate $r_i$ to $sa_j$, by the Formulas (20) and (21), respectively:

\[
y_j = \begin{cases} 1, & sa_j \text{ is selected} \\ 0, & \text{otherwise} \end{cases}, \quad (20)
\]

\[
ax_{ji} = \begin{cases} 1, & \text{allocate } r_i \text{ to } sa_j \\ 0, & \text{otherwise} \end{cases}. \quad (21)
\]

The mathematical model of PSS configuration optimization is constructed by Formulas (A1)–(A15) in Appendix A. The objective functions are given by Formulas (A1)–(A3) and constraints are given by the Formulas (A4)–(A15).

The objective function of executing time minimization is described by Formula (A1). The executing times of service activities are assumed to be fixed parameters, and they are not impacted by resource allocation. $T_s$ denotes the sum of executing times for all selected $sa$.

The objective function of cost minimization is described by Formula (A2). The executing cost of $sa_j$ includes the fixed cost $C_j$ and the work costs of all resources allocated to $sa_j$, which are related to the executing times of $sa_j$. $C_s$ denotes the sum of executing costs of all selected $sa$.

The objective function of reliability maximization is described by Formula (A3). The executing reliability of $sa_j$ is determined by $w^{RS}_{ji}$ of all $r_i$ that are allocated to $sa_j$. Therefore, a logic function $z_{ji}$, which depends on $x_{ji}$ as the formula (A12) is defined. Then, $exre_{ji}$, denoting the executing reliability of $sa_j$, is described as $exre_{ji} = \prod_{i=1}^{\delta} z_{ji}$. The target of reliability is always to maximize the executing reliability of the whole solution, which is related to the $exre_{ji}$ of all selected $sa_j$. However, if a certain type of resource is required by several service activities with overlapping service times, the allocation result of that type tends to make a $sa_j$ with a higher importance degree for reaching a higher $exre_{ji}$. Therefore, $AVRE_S$ is defined as the average executing reliability of all selected $sa_j$, which is shown by Formula (22):

\[
AVRE_S = \frac{\sum_{j=1}^{\delta}(y_{ji} \cdot D_j \cdot exre_{ji})}{\sum_{j=1}^{\delta} y_{ji} \cdot D_j}. \quad (22)
\]

The $exre_{ji}$ of a $sa_j$ with higher $D_j$ has a greater impact on the value of $AVRE_S$.

When $y_{ji} = 1$ and $r_i$ is assigned to $sa_j$, allocating $r_i$ to $sa_j$ might be feasible. Therefore, $x_{ji}$ is defined by the formula (A13). Here, $x_{ji} = 1$ means $r_i$ is allocated to $sa_j$ successfully, $x_{ji} = 0$ means $r_i$ is not allocated to $sa_j$. Constraints for $y_{ji}$ and $ax_{ji}$ are shown by Formulas (A14) and (A15). Constraints for the three objective functions are developed as indicated below:

1. All selected service activities must be part of the construction for the triggered service flows. They are represented by Formula (A4);
2. Every service flow starts with the triggered $ss_p$, and the out-degree of $ss_p$ is 1. This is expressed by Formula (A5);
3. The number of triggered $ss$ is $\sigma$. All triggered service flows end at $s_0$. Consequently, the in-degree of $s_0$ is $\sigma$. This is shown by Formula (A6);
4. If all selected service activities constitute complete service flows, there will not be any selective relationship between selected $sa_i$ and $sa_j$. As a result, the sum of weights on edges from triggered $ss_p$ and selected $sa_j$ to $sa_i$ will equal to the sum of weights on edges from $sa_i$ to $s_0$ and selected $sa_j$. This is shown by Formula (A7);
5. For every triggered $ss_p$, $start_{ss_p} = t$. Every triggered flow must be completed within the service time range $[start_{ss_p}, ddl_{ss_p}]$ requested by a customer, as illustrated in Formula (A8);
6. The resources allocated to the selected \( sa_j \) must satisfy the constraints as the resource type and the quantity demands of \( sa_j \). This constraint is denoted by Formula (A9);

7. Every \( r_i \) can only participate in a maximum of one service activity at the same time. For all service activities that \( r_i \) is allocated to, their service time intervals cannot overlap. Defining \( ol_{rh} \) indicates whether there is any overlap between the service time intervals of \( sa_j \) and \( sa_{lh} \), as shown in Formula (A11). That constraint is described by Formula (A10).

### 4.3. Obtaining the Optimal Solutions

To obtain the optimal solutions by the trigger matrix \([u_{b+1}, u_{b+2}, \cdots, u_{b+\gamma}]\) and the proposed mathematical model, the solving procedures are as follows.

**Step 1:** Obtaining SAS\(_i\) as the service activities solution set for \( ss_{ij} \), the number of solutions is denoted as \( n_i \). If \( u_i = 1 \), obtain SAS\(_i\) = \( \{SA_{1i}, \cdots, SA_{xi}, \cdots, SA_{ui}\} \) by Depth-First Traversal (DFT), wherein \( SA_{xi} \) is the set of all service activities selected in the \( x_i \) time traversal; if \( u_i = 0 \), \( ss_{ij} \) is not activated, then \( n_i = 1 \), SAS\(_i\) = \( \{SA_{xi}\} \), wherein \( SA_{xi} = \varnothing \). Net\(_{s}\) is different from ordinary complex networks, and as a consequence, the traditional DFT is unsuitable for Net\(_{s}\). Depth-First Traversal based on service layer (SL-DFT) is designed and detailed in Section 4.3.1.

**Step 2:** Solutions of service activities for each triggered \( ss \) are arranged by permutation and combination. Obtaining a solutions set of service activities as SAS\(_{all}\) = \( \{SA_{1i}, \cdots, SA_{xi}, \cdots, SA_{ui}\} \) for system, \( n_{all} = \prod n_i \), SAS\(_{all}\) = \( \{SA_{S_{i+1}}, \cdots, SA_{S_{i+2}}, \cdots, SA_{S_{i+\gamma}}\} \).

**Step 3:** Using the Pareto-improved multiple-objective discrete cuckoo search algorithm (Pareto-IMODCS) [23] to obtain the set of optimal solutions of resource allocation for each \( SA_{all} \) as RAS\(_i\) = \{RA\(_{1i}\), \cdots, RA\(_{mi}\)\}, the number of solutions is denoted as \( m_i \). Pareto optimality and Pareto-IMODCS are detailed in Sections 4.3.2 and 4.3.3, respectively.

**Step 4:** Merging all RAS\(_i\) into one as RAS\(_{all}\) = \{RA\(_{1i}^{i+1}\), \cdots, RA\(_{mi}^{i+1}\), RA\(_{1i}^{i+\gamma}\), \cdots, RA\(_{mi}^{i+\gamma}\)\}, filter RAS\(_{all}\) to obtain the set of non-dominated solutions as RAS\(_{pareto}\) based on Pareto optimality. RAS\(_{pareto}\) is the final set of optimal solutions.

### 4.3.1. Obtaining Solutions of Service Activities by SL-DFT

Obtaining SAS\(_i\) by SL-DFT involves searching all paths from \( ss_{i0} \) to \( ss_{ij0} \) in Net\(_{s}\). Assuming the current visited node is \( s_j \):

- If \( \sum^i_{j=0} e_{yi}^j = 1 \) and \( e_{xy}^j = 1 \), then add node \( s_y \) into the path and continue searching from \( s_y \);
- If \( \sum^i_{j=0} e_{xy}^j > 1 \) and \( outw_x > invw_x \), nodes satisfying \( e_{xi}^j = 1 \) are in a selective relationship, then select one of these to visit. Assuming that the selected node is \( s_y \), continue to search from \( s_y \) until all paths from \( s_y \) to \( ss_{i0} \) are obtained, then backtrack to \( s_x \), selecting an unvisited node from nodes satisfying \( e_{xi}^j = 1 \) to visit. This new search forms a new path, so nodes visited in the search from \( s_y \) can still be visited. Repeat this step until all nodes satisfying \( e_{xi}^j = 1 \) are visited;
- If \( \sum^i_{j=0} e_{xi}^j > 1 \) and \( outw_x = invw_x \), it means nodes satisfying \( e_{xi}^j = 1 \) are in a parallel relationship. Then, select one to visit and continue searching until the visited node \( s_z \) satisfies \( \sum^i_{j=0} e_{xi}^j > 1 \) and \( invw_z = outw_z \), which means \( s_z \) is the confluence node of these parallel paths. Then, backtracking to \( s_y \), select an unvisited node from nodes satisfying \( e_{xi}^j = 1 \) to visit. Repeat this step until nodes satisfying \( e_{xi}^j = 1 \) are all visited. Add all nodes visited in this step into the path and continue to search from \( s_z \).

After all paths from \( s_x \) to \( ss_{i0} \) are obtained, if \( s_y \) is a service trigger node, then end the search. Otherwise, backtrack to the node before \( s_x \). Based on the above search, service activity compositions obtained by SL-DFT satisfy the constraints as given by Formulas (A4)–(A7).
4.3.2. Pareto Optimality

In the optimizing process, the solution with smaller executing time, smaller executing cost, and greater executing reliability is considered better. However, objectives in one solution might conflict with each other in reality. Traditional linear weighting methods are unsuitable for solving such multi-objective optimization, which could be solved based on Pareto optimality [65].

If \( x_i \) and \( x_j \) satisfy that \( T_S(x_i) \leq T_S(x_j), C_S(x_i) \leq C_S(x_j) \) and \( AVRE_S(x_i) \geq AVRE_S(x_j) \), and at least one of these inequalities is a strict inequality, then \( x_i \) dominates \( x_j \). If \( x_m \) is not dominated by any \( x \), then \( x_m \) is a non-dominated solution, which is the Pareto optimal solution.

4.3.3. Obtaining Optimal Solutions of Resource Allocation by Pareto-IMODCS

Let us assume that the number of \( sa \) in \( SA^{all} \) is \( v \) regardless of \( ss, se \), and virtual service activity nodes. The types and quantities of resources that every \( sa \) demands can be determined based on the multilayer network model. Then, each possible resource allocation solution is represented by a cuckoo nest: the dimension \( L(x_{K_i} N_{x_{K_i}}) \) of nest represents the \( N_{x_{K_i}} \)th resource instance for the \( K_i \)th resource type demand of the \( x \)th \( sa \) in set \( SA^{all} \); various instances in each dimension are all represented by decimal integers. The subscript number of the resource type node corresponding to the \( x \)th \( sa \) is denoted as \( nsa(x_{K_i}) \). The subscript number of the resource type node corresponding to the \( K_i \)th resource type is denoted as \( nr(x_{K_i}) \). \( nsa(x_{K_i}) \) and \( nr(x_{K_i}) \) facilitate in decoding the nest into the corresponding resource allocation solutions, which satisfy the constraint defined by Formula (A9).

The original Pareto-IMODCS algorithm is detailed in reference [23]. If multiple \( sa \) in \( SA^{all} \) demand same type of resources in the same time interval, the solution obtained by the original Pareto-IMODCS algorithm might allocate the same resource instance to multiple \( sa \) simultaneously. This does not comply with the constraint described by Formula (A10). To avoid this, a “nest pre-processing” step is added to Pareto-IMODCS. The specific procedures of the improved Pareto-IMODCS algorithm to obtain Pareto optimal solutions for each \( SA_i^{all} \) are shown in Figure 5.

![Figure 5](image-url)

**Figure 5.** Procedures of the improved Pareto-improved multiple-objective discrete cuckoo search algorithm (Pareto-IMODCS) algorithm.
The procedure “nest pre-processing” is specified as: for any two dimensions as $L(hij)$ and $L(lmn)$, when $nr(hi) = nr(lm)$ and $L(hij) = L(lmn) = x$, it means that the same resource instance is allocated to multiple dimensions. If $nsa(hi) = nsa(lm)$ or $ol_{nsa(hi)} nsa(lm) = 1$, it denotes that service activities related to $L(hij)$ and $L(lmn)$ are the same one or their working time intervals overlap, then $L(hij) = x$ and $L(lmn) = x + 1$.

5. Case Study

In this section, a case study for the PSS of an automobile enterprise is provided to validate the proposed multilayer network-based PSS model and the fine-grained PSS configuration optimization method. A product was delivered to a customer with a contract to implement eight selected services (running system maintenance, steering system maintenance, braking system repair, etc.). These services could be decomposed as service flows consisting of service activities. For example, running system maintenance was decomposed as the maintenance of axles, maintenance of suspensions, maintenance of shock absorbers, maintenance of wheels, and maintenance of frames. As indicated, the PSS of the automobile enterprise was modeled as a multilayer network, which included 193 product nodes, 89 service nodes (1 se, 80 ss, and 8 ss), and 140 resource nodes (70 pr, 50 er, and 20 kr).

When services are triggered, a set of PSS configuration solutions should be provided to the customer for making a final decision. The customer’s desired performances for the final solution were the minimal executing time and cost and maximal executing reliability. The designer had a PSS configuration optimization task at time $t$, which contained the configuration of services as running system maintenance ($ss_g1$) and steering system maintenance ($ss_g2$). The network status of the case at time $t$ is modeled in Figure 6, in which the sparse matrixes of edges denoting corresponding relationships among nodes in multilayer network are detailed in Figure 7.
Figure 7. The sparse matrixes of edges at time $t$. (a) Sparse matrixes of edges in $Net_P$; (b) sparse matrixes of edges in $Net_S$; (c) sparse matrixes of edges in $Net_R$; (d) sparse matrixes of inter-layer edges from $Net_S$ to $Net_P$; (e) sparse matrixes of inter-layer edges from $Net_S$ to $Net_R$; (f) sparse matrixes of inter-layer edges from $Net_R$ to $Net_S$.

According to the PSS configuration optimization task, the trigger matrix at time $t$ was $[1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$. Structures of triggered service flows in service layer are detailed in Figure 8. The executing time, cost, and reliability of related $sa_j$ are detailed in Table 1. It is evident that $sa_1$ and $sa_5$ were virtual service activity nodes. The cost for all $r_i$ working in one unit of time is shown in Figure 9.

Figure 8. Triggered service flows in the service layer at time $t$. 
Table 1. The executing time, cost, and reliability of related service activities.

<table>
<thead>
<tr>
<th>sa_i</th>
<th>T_i (h)</th>
<th>C_i (10^3 yuan)</th>
<th>RE_i</th>
<th>sa_i</th>
<th>T_i (h)</th>
<th>C_i (10^3 yuan)</th>
<th>RE_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.000</td>
<td>13</td>
<td>2.6</td>
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<td>0.9721</td>
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</tr>
<tr>
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<td>1.9</td>
<td>4.24</td>
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</tr>
<tr>
<td>4</td>
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<td>16</td>
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<td>2.31</td>
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<td>0</td>
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<td>0.9996</td>
</tr>
<tr>
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<td>3.73</td>
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</tr>
<tr>
<td>7</td>
<td>3.5</td>
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<td>4.19</td>
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</tr>
<tr>
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<td>0.98</td>
<td>0.9840</td>
<td>20</td>
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<td>1.91</td>
<td>0.9904</td>
</tr>
<tr>
<td>9</td>
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<td>0.9949</td>
<td>21</td>
<td>1.3</td>
<td>3.56</td>
<td>0.9880</td>
</tr>
<tr>
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<td>0.99</td>
<td>0.9974</td>
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<td>2.3</td>
<td>2.87</td>
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<tr>
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<td>...</td>
</tr>
<tr>
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<td>1.81</td>
<td>0.9881</td>
<td></td>
<td></td>
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<td>...</td>
</tr>
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</table>

Figure 9. Cost for all resources.

In the above-mentioned network status and solution constraints, all paths from triggered ss_{S1} to sa_0 and all paths from triggered ss_{S2} to sa_0 were searched by SL-DFT. Then, one from both sets of paths was selected to form service activity compositions. Pareto optimal solutions of resource allocation for every service activity composition were obtained by using the mentioned improved Pareto-IMODCS algorithm. Relevant parameters of the algorithm were as follows: the number of initial nests was 200, the probability of cuckoo eggs being found was 0.25, the maximum number of iterations for a nest was 3000, and the number of times of an iteration run was 10. Then, solutions of all compositions were filtered according to Pareto optimality. As a result, 14 non-dominated solutions were obtained, which were recorded as set A (Table 2).

Table 2. Solutions in set A.

<table>
<thead>
<tr>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2(6,10,52,73,102)</td>
<td>3(2,10,51,75,105)</td>
<td>3(2,9,52,75,105)</td>
<td>3(5,8,51,72,102)</td>
<td>3(1,10,51,73,102)</td>
</tr>
<tr>
<td>5</td>
<td>6(4,15,35,72,93)</td>
<td>6(9,15,39,73,94)</td>
<td>6(4,16,39,73,95)</td>
<td>6(10,16,35,74,95)</td>
<td>6(2,14,33,75,94)</td>
</tr>
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<td>22(29,44,47,100)</td>
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</table>

\[ T_5 \]

\[ C_2 \]

\[ AVRE_5 \]
5.1. Comparison with Method For Selecting Service Activities Before Resource Allocation

To validate the effectiveness of combining service activities selection and resource allocation, the same case was solved with another method as a contrast. In summary, the details were as follows:

Firstly, all paths from triggered ss to \( s_0 \) were searched by SL-DFT and all service activity compositions were obtained. Next, all compositions were filtered by objective functions as executing time, cost, and reliability to obtain Pareto optimal service activity compositions. Then, Pareto optimal solutions of resource allocation for each composition were acquired using the improved Pareto-IMODCS with same parameters as last time. Finally, all solutions were filtered according to Pareto optimality. As a result, six non-dominated solutions were recorded as set B (Table 3).

<table>
<thead>
<tr>
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<th>5</th>
<th>6</th>
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<tr>
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<td>8(21,25,42,83,90)</td>
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<td>22(29,43,45,100)</td>
<td>22(32,45,47,100)</td>
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</tr>
</tbody>
</table>

Table 3. Solutions in set B.

By analyzing solutions in Tables 2 and 3, it can be found that set A and set B share four common solutions. Nevertheless, set A also contains other solutions in which service activity compositions are different from solutions in set B. The other two solutions in set B (solutions B5 and B6) are dominated by solution A3. Obviously, combining the service activities selection and resource allocation effectively enriches the diversity of the optimal solution set by retaining more non-dominated solutions. The last ones might have been hindered by using the method where service activities selection finished individually before resource allocation.

5.2. Comparison with the Method Using Different Objective Functions

By calculating the importance degrees of service activities related to triggered \( s_{81} \) and \( s_{82} \), it was found that the importance degree of \( s_{9} \) is significantly higher than other service activities. Reverting to the actual setting of the case, \( s_{9} \) was frame maintenance in the running system maintenance. The frame is the base of the whole automobile assembly, which makes frame maintenance more important compared to other activities in the maintenance of the running system and the steering system.

To validate the maximization of \( AVRE_S \), an objective function which improves the executing reliability of service activities with higher importance degrees, the same case was solved in another way, as follows.

The value 1 was given to the weights of all \( exre_j \) in the objective function of executing reliability, regardless of the importance degrees of the service activities. The altered objective function of the executing reliability is shown in Formula (23), where \( EXRE_S \) denotes the overall executing reliability of all selected activities. As a result, 13 Pareto optimal solutions were obtained, which were recorded as set C.

\[
\max EXRE_S = \sum_{j=1}^{\delta} y_j \cdot exre_j. \quad (23)
\]
The executing reliabilities of \( sa_9 \) (\( exre_9 \)) in different solutions with set A and set C are shown in Figure 10. The values of \( exre_9 \) in set A are all higher than those in set C. The values of \( exre_9 \) in solution A3 and C11 are the highest in set A and set C, respectively. Service activity compositions in solution A3 and C11 are the same. Comparing and analyzing these two solutions shows why maximizing \( AVRE_s \), as one of the objective functions, could facilitate more beneficial solutions, in which service activities with higher importance degrees have higher executing reliabilities.

![Figure 10. \( exre_9 \) in different solutions in set A and set C.](image)

The Gantt chart of selected service activity nodes in solution A3 and C11 is depicted in Figure 11. From the perspective of the service time interval, \( sa_9 \) overlaps with \( sa_{19} \) and \( sa_{22} \), \( sa_{19} \) overlaps with \( sa_8 \), \( sa_8 \) overlaps with other nodes. Situations of service time interval overlap between service activities are intricate. Consequently, resource allocation of one activity might influence another one and indirectly impact other activities.

![Figure 11. Gantt chart of selected service activity nodes.](image)

The types and quantities of resources demanded by \( sa_9, sa_9, sa_{19}, \) and \( sa_{22} \) are shown in Table 4. The actual situation of resource allocation about the \( kr \) related to \( sa_9 \) in solution A3 and C11 is illustrated in Table 5. \( w_{ij}^{RS} \) on the edges that connect \( sa_j \) and \( r_i \) (all \( pr_i \) and \( cr_j \) that are related to \( kr_{124}, kr_{126}, kr_{134}, \) and \( kr_{136} \)) are detailed in Figures 12 and 13.

<table>
<thead>
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<th>Table 4. Resource quantity demands of ( sa )</th>
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<tr>
<td>( sa_9 )</td>
</tr>
<tr>
<td>( sa_8 )</td>
</tr>
<tr>
<td>( sa_{19} )</td>
</tr>
<tr>
<td>( sa_{22} )</td>
</tr>
</tbody>
</table>
Table 5. $r_i$ that are allocated to $s_{8i}$, $s_{9i}$, $s_{19i}$, and $s_{22i}$.

<table>
<thead>
<tr>
<th></th>
<th>$s_{8i}$</th>
<th>$s_{9i}$</th>
<th>$s_{19i}$</th>
<th>$s_{22i}$</th>
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</thead>
<tbody>
<tr>
<td>A3</td>
<td>$pr_{71}$</td>
<td>$pr_{46}$</td>
<td>$cr_{66}$</td>
<td>$pr_{28}$</td>
</tr>
<tr>
<td>C11</td>
<td>$pr_{30}$</td>
<td>$pr_{45}$</td>
<td>$cr_{66}$</td>
<td>$pr_{31}$</td>
</tr>
</tbody>
</table>

Figure 12. $w_{ij}^{RS}$ on edges that connect $pr_i$ to $s_{8i}$, $s_{9i}$, $s_{19i}$, and $s_{22i}$.

Figure 13. $w_{ij}^{RS}$ on edges that connect $cr_i$ to all $si$.

The decision process in multi-objective optimization is complicated. In solution C11, $r_i$ with the highest $w_{ij}^{RS}$ is not allocated to $s_{8i}$ in the process of overall executing reliability maximization. In solution A3, $pr_{28i}$, $pr_{46i}$, $cr_{66i}$, and $cr_{98i}$ are all allocated to $s_{9i}$. $w_{ij}^{RS}$ of these four resources is the highest among the same type of resources. In turn, the $cr_{98i}$ in solution A3 is higher than solution C11. For the remaining three $si$, for whom the importance degrees are not conspicuously higher than each other, the resource allocation is not only based on $w_{ij}^{RS}$, but also on the cost (as shown in Figure 9). By including the importance degree of $sa_j$ with the objective function of executing reliability as weight, it enables more important $sa_j$ to have higher executing reliabilities, by allocating $r_i$ with the highest $w_{ij}^{RS}$ to this $sa_j$. If the importance degree of $sa_j$ is significantly higher, the impact on the overall executing reliability and executing cost will be tolerable.
6. Discussion

As an imperative part of PSS design, PSS configuration is usually studied in a predefined level of module granularity \([21,22]\). The service modules in existing studies of PSS \([23,34]\) are essentially whole service flows, which can be decomposed into service activities. In the studies of service activity composition \([42,45]\), the impact of resource allocation on service activity is not considered during the service activities selection. This paper studied the PSS configuration in a finer granularity level to inspire the final configuration’s optimization. By combining service activities selection and resource allocation, the proposed approach enriched the diversity of the optimal solution set by retaining more non-dominated solutions. These solutions might be eliminated in advance within the process of service activities selection, when service activities selection and resource allocation are divided into two independent processes in the PSS configuration optimization. Furthermore, the above-mentioned existing studies highlight the multi-objective optimization of the overall solution. The proposed approach improves the executing reliabilities of service activities with a higher importance degree, by allocating resources with higher reliabilities to them. In terms of modeling, a multilayer network model can provide a better description of relationships among three types of elements (product parts, service activities, and resources), compared to an ontology-based model \([43]\), structure tree model \([44]\), and Petri net model \([45,46]\). The mathematical characteristics of multilayer networks are useful for analyzing the importance degree of product parts and service activities. The sparse matrices of edges in multilayer networks are applicable as the input of the proposed optimization algorithm.

This paper focused on product-oriented PSS. Adding services to traditional products is used in product-oriented PSS as an approach to reach sustainability \([66]\). The ultimate purpose of the proposed approach is to achieve PSS configuration optimization. Even if the purpose of sustainability is not directly mentioned, the PSS configuration optimization is still related to achieving sustainability. Optimization of the executing time means a reduction in energy consumption and decreased work time of workers and equipment. The reduced resource utilization can lead to the concept of sustainability in PSS \([5]\). The optimization of executing costs is an economic benefit. The optimization of executing reliability can also be regarded as a guarantee of the optimization of executing time and cost. Furthermore, one goal of the proposed multi-objective optimization is to improve customer satisfaction. Economic perspective and customer satisfaction are also important measurement indicators of sustainability \([9]\).

7. Conclusions

PSS enables manufacturing companies to provide customers with solutions by integrating preferable product modules and service modules. How to optimize PSS configuration in a fine-grained perspective when modules are confirmed represents an important issue for PSS management. A PSS multilayer network-based method combining service activities selection and resource allocation was developed to obtain optimal PSS solutions. Product parts, services, and resources were mapped as different layers in a PSS multilayer network. In particular, the mapping rules from logical relationships among service activities to edges among nodes were developed to model service layers appropriately. Service activities selection and resource allocation were combined to evaluate the objective functions of solutions systematically and precisely. The importance degree of a service activity was proposed to ascertain the priority of service activities in resource allocation. The advantages of the proposed method are as follows:

1. The impact of resources on the cost and reliability of service activities is thoroughly considered, which can affect the PSS configuration optimization equally with service activities. A combination of service activities selection and resource allocation can make the calculation of optimizing indexes closer to the actual implementation;
of the PSS and build optimization functions easily;
3. Cross layer importance degree can sort the service activities based on the importance of the product parts and the objects of service activities.

Although the approach optimizes PSS configuration in a more fine-grained perspective, it also has some limitations. The combination of service activities selection and resource allocation leads to more complexity within the calculation. The optimization is based on a given product in a situation where customers have explicit targets on product module selection. Further work would consider improving the algorithm to shorten the calculation’s response time. Besides, the multilayer network-based modeling method might be extended to the design of use-oriented PSS and result-oriented PSS to satisfy demands for different types of PSS.

Author Contributions: Conceptualization, Z.Z. and D.X.; methodology, Z.Z. and D.X.; software, D.X.; validation, Z.Z., D.X., E.O. and H.C.; formal analysis, D.X.; investigation, D.X.; resources, Z.Z.; data curation, D.X.; methodology, Z.Z. and D.X.; software, D.X.; validation, Z.Z., D.X., E.O. and H.C.; supervision, Z.Z.; project administration, Z.Z.; funding acquisition, Z.Z. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

\[
\min T_S = \sum_{j=1}^{\delta} y_j \cdot T_j 
\]

\[
\min C_S = \sum_{j=1}^{\delta} y_j \cdot (C_j + \sum_{i=1}^{\alpha+\beta} x_{ji} \cdot c_i \cdot T_j) 
\]

\[
\max AVRE_S = \frac{1}{\sum_{j=1}^{\delta} y_j \cdot D_j} \cdot \sum_{j=1}^{\delta} \left( y_j \cdot D_j \cdot \prod_{i=1}^{\alpha+\beta} z_i \right) 
\]

\[
y_j \cdot \left( \sum_{\varphi=\delta+1}^{\delta+\gamma} e_{\varphi}^\delta \cdot u_{\varphi} \right) = y_j, j = 1, 2, \ldots, \delta 
\]

\[
\sum_{j=1}^{\delta} y_j \cdot e_{\varphi j}^\delta \cdot w_{\varphi j}^\delta = u_{\varphi}, \varphi = \delta + 1, \ldots, \delta + \gamma 
\]

\[
\sum_{j=1}^{\delta} y_j \cdot e_{\varphi j}^\delta \cdot w_{\varphi j}^\delta = \sigma 
\]

\[
\left[ \left( \sum_{j=1}^{\delta} y_j \cdot e_{ij}^\delta \cdot w_{ij}^\delta \right) + e_{\varphi}^\delta \cdot w_{\varphi}^\delta \right] \cdot y_i = y_i \cdot \left[ \left( \sum_{\varphi=\delta+1}^{\delta+\gamma} e_{\varphi}^\delta \cdot w_{\varphi}^\delta \right) + \left( \sum_{j=1}^{\delta} y_j \cdot e_{ij}^\delta \cdot w_{ij}^\delta \right) \right], i = 1, 2, \ldots, \delta 
\]

\[
y_j \cdot e_{\varphi j}^\delta \cdot f_{lj} \leq u_{\varphi} \cdot \text{ddl}_{lj}, j = 1, 2, \ldots, \delta; \varphi = \delta + 1, \ldots, \delta + \gamma 
\]

\[
\sum_{i=1}^{\alpha+\beta} e_{hi}^\beta \cdot x_{ji} = y_j \cdot e_{Sj}^\beta \cdot w_{Sj}^\beta, j = 1, 2, \ldots, \delta; h = \alpha + \beta + 1, \ldots, \alpha + \beta + b 
\]

\[
\sum_{i=1}^{\alpha+j} \sum_{j=1}^{\delta} \sum_{h=1}^{\delta} \left( x_{ji} \cdot x_{hi} \cdot o_{lj} \right) = 0, j \neq h 
\]

\[
o_{lj} = \begin{cases} 1, & (bt_j, f_t) \cap (bt_h, f_t) \neq \emptyset \\ 0, & (bt_j, f_t) \cap (bt_h, f_t) = \emptyset, j \neq h \end{cases} 
\]

\[
z_{ji} = \begin{cases} w_{ij}^R, & x_{ji} = 1 \\ 1, & x_{ji} = 0 \end{cases} 
\]
\[ x_{ji} = ax_{ji} \cdot e_{ji}^{RS} \cdot y_j \quad (A13) \]
\[ ax_{ji} \cdot (ax_{ji} - 1) = 0 \quad (A14) \]
\[ y_j \cdot (y_j - 1) = 0 \quad (A15) \]

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