Abstract: The low-carbon economy has become the focus of global attention and scientific measurement standards with the concepts of low energy consumption, low pollution, and sustainable development. More and more attentions are paid to the research of low-carbon supply chains. Based on a two-level low-carbon supply chain in the context of carbon trading, a Stackelberg game model was established for government subsidies to determine a coordinated and balanced solution for supply chains in situations dominated by manufacturers. The optimal strategies for low-carbon technology innovation are analyzed within the context of governmental subsidies. This study’s conclusions are as follows: (1) When government subsidies are in place, regardless of who the government subsidies are meant for, manufacturers and retailers that do not generate carbon emissions will transfer the subsidies to the companies that generate carbon emissions by adjusting wholesale prices and retail prices to maximize their own profits. (2) When consumer prices are sensitive, the government’s optimal subsidy intensity increases as consumers’ low-carbon preferences increase. When consumer prices are not sensitive, the government should not provide any subsidies. (3) When consumers’ low-carbon preferences are weak, the retail price of products will decrease with the increase in subsidies; when consumers’ low-carbon preferences are strong, the opposite dynamic occurs.

Keywords: carbon trading policy; low-carbon subsidy policy; carbon reduction technology; Stackelberg game

1. Introduction

The ‘low-carbon economy’ was first mentioned by officials in the 2003 British Energy White Paper ‘The Future of Our Energy: Creating a Low-Carbon Economy.’ As a pioneer of the first industrial revolution, Britain fully recognized the negative effects of climate change. In this regard, the transition to a low-carbon economy has become a major trend in the development of the world economy.

In this context, a series of new concepts and policies related to low-carbon emissions have been successively produced worldwide. At the same time, it has also promoted the research, development, and use of new energy [1]. These developments may provide new ways of thinking in modern economical interactions and energy applications, i.e., abandoning traditional concepts, directly applying new technologies, studying the low-carbon economy, implementing low-carbon lifestyles, and ultimately achieving sustainable development.

Several countries have already taken leadership roles in restricting carbon emissions, and have formed more mature carbon trading markets [2–4]. In some countries, carbon trading policies are only in the pilot stage in some regions, and carbon tax policies are still in the exploratory stage [5–7]. Companies that develop low-carbon technologies and low-carbon economies cannot only shoulder their own social responsibility for environmental protection, but complete national energy conservation
and consumption reduction targets; additionally, they must adjust industrial structures, improve energy efficiency, and build a green ecological civilization.

Spurred by relevant low-carbon policies, many companies have researched low-carbon technology innovations to maintain a competitive advantage and obtain greater profits [8,9]. To motivate enterprises to improve technological innovation and reduce carbon emissions, as well as cultivate consumers’ green awareness and enhance low-carbon preferences, relevant government departments can also implement low-carbon subsidies for enterprises and consumers. This study is based on a secondary supply chain consisting of a manufacturer and retailer. It is determined by the research of other scholars and the relationship between the government and enterprises. This article uses a combination of qualitative and quantitative research. We constructed a two-stage game model, and it aims to solve the following problems by studying the strategic interactions between governments, enterprises, and consumers within the context of the carbon cap trading mechanism.

1. When the government provides low-carbon subsidies to different entities, will the company’s optimal strategy change? What are the relationships between several different situations?
2. What will happen to the total carbon emissions, consumer utility, and retail price of products when a government offers different subsidies? How do these metrics compare to when there is no government subsidy?

Finally, an example analysis is used to verify the correctness of the conclusion.

The structure of this paper is as follows: Section 2 is a literature review that summarizes the current research status of low-carbon technology innovations under government subsidies and from independent enterprise (i.e., no subsidies). Section 3 uses the Stackelberg game model to analyze the behavior of anarchic subsidies and government subsidies to manufacturers, retailers, and consumers to directly and indirectly encourage manufacturers to adopt optimal supply chain decisions under low-carbon technology innovations. Section 4 puts forward some propositions and inferences, and then proves their significance. Section 5 uses numerical analysis to analyze the changes in retail prices and social welfare of goods under different circumstances. Section 6 provides the discussion.

2. Literature Review

Consumers are willing to pay higher prices for greener and low-carbon products, and thereby encourage manufacturers to produce low-carbon products to gain a competitive advantage. At the same time, the production of low-carbon products is conducive to sustainable development, and also helps to improve the social image of enterprises. Therefore, some scholars have researched issues related to the independent technological innovations of enterprises. Benjaafar et al. considered carbon emission factors in corporate decision-making models and reached many management-oriented conclusions that were groundbreaking in carbon emissions research [10]. A low-carbon technology innovation does not necessarily represent an improvement in production technology, instead, it represents all links that can reduce carbon emissions. Peng et al. posited that a low-carbon technology innovation includes all production processes, products, and services that are conducive to energy Conservation and emission reduction [11]. The current research on enterprises’ low-carbon technology innovation is mainly carried out from the perspective of improving supply chain processes and low-carbon technology investment. The key is to balance carbon emissions, cost, and profit [12].

2.1. Research on Independent Low-Carbon Technology Innovations

Ghosh, D and Shah, J studied the strategies and supply chain structure of various companies in a green clothing supply chain of products, and showed how the level of greening, prices, and profits are affected by the channel [13]. El Ouardighi, F et al. attempted to determine how the double marginalization phenomenon affects the trade-off between pollution emissions and pollution accumulation-related emission reduction activities in a supply chain consisting of a manufacturer and a retailer [14]. At the same time, more and more companies have begun to pay attention to the
carbon footprint in addition to production emissions. For example, Hoen, KMR et al. have studied the impact of carbon emission regulations on corporate transportation modes under random demand [15]. Manufacturers and retailers can often jointly reduce emissions. For example, Zhou et al. discussed the cooperation between retailers and manufacturers in advertising and emission reductions with the assumption that retailers have fair concerns and consumers have low-carbon preferences [16]. Xie et al. comprehensively used game theory to discuss the emission reduction efficiencies and profits of enterprises in the supply chain under different cooperation models; their findings provided a certain reference for manufacturers in making emission reduction decisions [17]. Ye studied a joint supply chain emission reduction strategy for a secondary supply chain by considering the effects of referenced low-carbon levels and consumers’ low-carbon preferences [18]. Pu, XJ et al. Studied the impact of consumers’ low-carbon preferences on the market equilibrium of supply chain product selection strategies when manufacturers and retailers collaborated on low-carbon production based on game theory models [19]. Jorgensen, S and Zaccou, G hoped that two neighboring countries could work together to control carbon emissions. They used a differential game model to use carbon emissions and investment in low-carbon technology innovation as control variables, and reached a solution that maximized common benefits [20]. Takashima, N considers international cooperation. Through research and investment in technological innovation, research and development recycling and licensing, an international environmental agreement can be reached to reduce carbon emissions [21].

2.2. Impact of Government Carbon Policy on Low-carbon Technology Innovation

With the introduction of policies such as carbon quotas and carbon taxes, many enterprises have also promoted low-carbon technological innovations that can adapt to the new competitive environment. Brauneis et al. found that stricter carbon emission policies can guide companies to adopt innovations in carbon emission reduction technologies [22]. Wang et al. found that when retailers dominate, product demand will be greatly affected by the manufacturer’s low-carbon reputation and retail promotion methods, and active emission reduction strategies can effectively enhance the low-carbon reputation; this reputation is optimal when working with retailers to reach a contract [23]. In the context of carbon trading, Ji considered the secondary supply chain pricing and emission reduction strategies of consumers’ low-carbon preferences and channel preferences. They concluded that joint emission reduction can increase the profits of both parties, and that consumers’ low-carbon preferences affect the development of both parties [24]. Luo et al. found that companies introduced capital investment in carbon emission reduction technological research and development into the supply chain game model based on the carbon trading mechanism [25]. Deng et al. discussed the optimal strategies of governments and enterprises for fostering low-carbon technology innovations from the perspective of political competition [26]. Subramanian, R et al. considered the situation in which regulators auctioned a fixed number of emissions permits to companies, and the study found that changing the number of available permits has a lesser impact on cleaner industries than cleaner industries. As the level of industrial pollution increases, the level of emission reductions gradually decreases [27]. Konur, D and Schaefer, B respectively studied the issue of integrated inventory control and transportation planning under carbon caps, caps and trades, caps and offsets, and carbon tax policies [28]. Toptal, A et al. analyzed the joint decisions of retailers on stock replenishment and investment in low-carbon technology innovation under the three carbon emission control policies of carbon caps, carbon taxes, caps and trades, and then compared different carbon emission control policies in terms of cost and emissions [29]. Jaber, MY et al. proposed a two-level supply chain model with a coordination mechanism based on EU-ETS. At the same time, they considered the manufacturer’s carbon emissions, and proposed several different carbon trading schemes and possible combinations between these schemes [30].
2.3. Impact of Government Subsidies on Low-Carbon Technology Innovation

Enterprises’ independent low-carbon technology innovations sometimes face certain risks and high costs. In addition to certain external environmental factors, to achieve carbon emission reductions, the government will provide low-carbon subsidies to enterprises. Studies such as Zhang found that companies actively developing carbon emission reduction technologies that have certain externalities, and the investment costs of technology research and development, as well as their associated risks, are both high. Government subsidies are needed to motivate risk-averse enterprises [31]. Zhao et al. studied the decision-making processes in manufacturers’ emission reduction strategies within the context of government regulation [32]. Li et al. constructed a game model between manufacturers and retailers to explore and analyze the impact of the government’s low-carbon subsidy policy on low-carbon R&D investment in the supply chain [33]. Montero found that governmental carbon reduction subsidies to carbon-producing companies can improve these companies’ negative externalities to the environment, as well as encourage them to increase investment in carbon reduction research and development, and ultimately reduce carbon emissions in the supply chain [34]. Lou et al. studied supply chains with information asymmetry in the context of carbon trading, and concluded that controlling carbon trading prices and providing technical subsidies can effectively stimulate manufacturers’ technological innovation [35]. Xie et al. studied the driving factors of low-carbon technology innovations from the perspective of government incentives, and explained the benefits of low-carbon subsidies through these innovations; these benefits may guide manufacturing companies in eliminating bottlenecks when implementing low-carbon innovations [36]. In Chen et al., the evolutionary game theory was applied to examine the behavioral strategies of manufacturers in response to various combinations of carbon taxes and subsidies, and the authors analyzed strategies that could more effectively encourage manufacturers to adopt low-carbon technology innovations [37]. Cao et al. compared carbon trading policies with low-carbon subsidy policies, and found that when the degree of environmental damage is low, low-carbon subsidy policies are more beneficial to society. When the degree of environmental damage is greater, carbon trading policies with total control are more beneficial to society [38]. Hou et al. studied a dynamic emission reduction investment decision-making problem between secondary supply chains by comparing profits and social benefits under different circumstances; this approach guided the supply chain to make low-carbon technology innovation decisions and governmental departments to determine low-carbon subsidy costs [39]. Guo et al. divided low-carbon technology innovations into low-carbon product innovations and process innovations, and measured the emission reduction levels of these innovations [40]. Wang et al. discussed the impact of low-carbon insurance subsidies and government subsidies on low-carbon technology innovations from enterprises, and found that low-carbon insurance subsidies cannot improve the level of innovation and corporate profits; however, they can reduce the risk of corporate innovation [41]. In addition to providing low-carbon subsidies to enterprises, the government will subsidize consumer purchases. For example, Cohen considered the uncertainty of the early demand for low-carbon products and studied how it affects government subsidy strategies for consumers [42]. Bi, GB considers a perfectly competitive market where consumers have low-carbon preferences and price preferences, how government subsidies encourage companies to innovate in low-carbon technologies, and government subsidy strategies. Studies show that green technology levels, environmental improvement factors, and unit costs increase coefficients play an important role in government subsidy strategies [43]. Based on game theory, Nielsen, IE et al. studied the impact of government subsidy strategies on closed-loop supply chain decisions, and studied the government’s subsidies to consumers and manufacturers to maximize profits, social welfare, and the impact of carbon emissions [44]. In a centralized and decentralized environment, Saha, S et al. studied the situation when the government subsidized consumers and manufacturers through a three-stage game. Both contract mechanisms can produce a scenario of Pareto efficiency and a higher government society welfare [45]. Hussain, J et al. studied the profit maximization behavior of a company, and considered whether to accept low-carbon subsidies to choose low-carbon technology innovation. The study found that subsidies enable enterprises to maximize their profits while ensuring
the implementation of low-carbon technologies, however, in some cases, companies will not adopt low-carbon technologies [46]. Hutchinson et al. found that subsidies for the production of low-carbon energy may have harmful effects on emissions [47]. Chen, JY et al. proposed a three-stage game to study the impact of government subsidies on a secondary supply chain composed of manufacturers and retailers, and classified the government’s optimal low-carbon subsidy strategy [48].

In summary, many scholars have researched low-carbon technology innovations in enterprises. However, a large amount of research has focused on the optimal emission reduction strategies of independent low-carbon technology innovations from enterprises, the impacts of carbon policies on low-carbon technology innovations of enterprises, low-carbon technology innovations produced only under government subsidies, or comparisons of the pros and cons of different strategies. Few studies have considered optimal strategies for low-carbon technology innovations under the combined influence of carbon policies and government subsidies. Therefore, based on the carbon trading mechanism, this study focuses on optimal supply chain strategies when the government subsidizes different entities, analyzes the impacts of government subsidies and the relationships between different subsidy types, further analyzes changes in existing models, and discusses the obtained findings.

3. Model

3.1. Problem Description, Basic Assumptions, and Parameters

Consider a supply chain system consisting of a single retailer and a single manufacturer with government involvement. The government can choose to subsidize manufacturers, retailers, and consumers to directly or indirectly encourage manufacturers to carry out low-carbon technological innovation. Of course, when there is no government subsidy, manufacturers may also choose independent low-carbon technology innovation. Manufacturers will be subject to carbon quotas allocated by the government when conducting low-carbon technological innovations, and will also participate in carbon trading. The market demand is determined by the unit product price and the unit product low-carbon technological innovation level.

Combined with the actual situation, make the following assumptions:
- Both government and supply chain members seek to maximize their profits.

1. Assume that only manufacturers in the entire supply chain will produce carbon emissions when producing products.
2. The information between the government and the enterprise and the supply chain members is completely symmetrical, and there is no deception

Other assumptions related to modeling will be mentioned in corresponding situations.

Table 1 provides the main parameter symbols involved in this article:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_G$</td>
<td>Government’s free carbon quota for enterprises</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Emission reduction technology innovation level</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Manufacturer unit cost</td>
</tr>
<tr>
<td>$w$</td>
<td>Product wholesale price</td>
</tr>
<tr>
<td>$p$</td>
<td>Product retail price</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Retailer unit cost of sales</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Carbon trading price</td>
</tr>
<tr>
<td>$e$</td>
<td>Carbon emissions per unit product without technological innovation</td>
</tr>
<tr>
<td>$\Delta e$</td>
<td>Carbon emission reduction per unit product after technological innovation</td>
</tr>
<tr>
<td>$D(p, \theta)$</td>
<td>Demand</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Market capacity</td>
</tr>
</tbody>
</table>
3.2. Decision Analysis Based on a Manufacturer’s Independent Technological Innovation

When there is no government subsidy, as shown in Figure 1, if a company has a certain free carbon quota $C_g$, manufacturers with carbon emission reduction technology level $\theta$ produce low-carbon products at unit cost $C_m$ and wholesale to retailers at unit wholesale price $w$, and retailers sell to consumers at retail price $p$. The unit’s cost of sale is $C_r$. From the manufacturer’s perspective, the part that exceeds the free carbon quota needs to be individually bought from the carbon trading market, and the remaining part can also be profitable by selling it to other companies on the carbon trading market. Assuming that the price $p_r$ of carbon trading is an exogenous variable. the manufacturer’s carbon reduction technology innovation level is $\theta$, and $\theta = \frac{\Delta e}{e}$, $\theta \in [0, 1)$, where $e$ is the carbon emission amount produced by the unit before the manufacturer implements a technological innovation, and $\Delta e$ is the carbon emission reduction amount of the unit produced after the technological innovation. Consumers have a certain low-carbon preference, so the demand function of the product can be expressed as: $D(p, \theta) = a - \beta p + \mu \theta$, $a$ is the market capacity. $\beta$ is the consumer’s sensitivity coefficient to product prices, and $\mu$ is the consumer’s sensitivity coefficient to carbon emission reduction technology innovations. $a$ and $\beta$ are exogenous variables, and all are positive numbers. Based on related research [49,50], we assume that the manufacturer’s carbon emission reduction technology investment function is $C = \frac{\alpha^2}{2}$, and $\varepsilon > 0$ represents the difficulty factor for developing carbon emission reduction technologies.

![Figure 1. Supply chain system diagram.](image_url)

When the manufacturer develops its own technological innovation, the demand function is $D(p, \theta) = a - \beta p + \mu \theta$. In decentralized decision-making, a Stackelberg game between the manufacturer and the retailer commences, in which the manufacturer is the dominant player, and the inverse solution method is used in the solution.
The retailer’s profit function is:
\[ \pi_r = D(p, \theta)(p - w - C_r) \]  
(1)

The manufacturer’s profit function is:
\[ \pi_m = D(p, \theta)(w - C_m) - \frac{\epsilon \theta^2}{2} - p_c \left[ D(p, \theta)(\epsilon - \Delta e) - C_g \right] \]  
(2)

We can respectively obtain the best wholesale price for manufacturers \( w^* \), the best technological innovation level \( \theta^* \), and the best retail price \( p^* \) for retailers as follows:
\[ w^* = \frac{(2\epsilon - \mu ep_c)\left[\alpha + \beta ep_c + \beta(C_m - C_r)\right] - \beta \epsilon^2 p_c^2(\alpha - \beta C_r) - \mu^2(C_m + ep_c)}{4\beta \epsilon - (\mu + \beta ep_c)^2} \]  
(3)
\[ \theta^* = \frac{(\mu + \beta ep_c)\left[\alpha - \beta ep_c - \beta(C_m + C_r)\right]}{4\beta \epsilon - (\mu + \beta ep_c)^2} \]  
(4)
\[ p^* = \frac{\alpha(2\epsilon - \beta \epsilon^2 p_c^2) - \mu^2(ep_c + C_m + C_r) + (\epsilon - \mu ep_c)\left[\alpha + \beta ep_c + \beta(C_m + C_r)\right]}{4\beta \epsilon - (\mu + \beta ep_c)^2} \]  
(5)

The maximum profits for retailers and manufacturers are as follows:
\[ \pi_r^* = \frac{\beta \epsilon^2\left[\alpha - \beta ep_c - \beta(C_m + C_r)\right]^2}{\left[4\beta \epsilon - (\mu + \beta ep_c)^2\right]^2} \]  
(6)
\[ \pi_m^* = \frac{\epsilon\left[\alpha - \beta ep_c - \beta(C_m + C_r)\right]^2}{2\left[4\beta \epsilon - (\mu + \beta ep_c)^2\right]} + p_c C_g \]  
(7)

From equation (7), when \( 4\beta \epsilon - (\mu + \beta ep_c)^2 > 0 \), \( \pi_m^* > p_c C_g \), the manufacturer can then make a profit by producing the product; otherwise, the manufacturer’s optimal decision is to stop producing any products and only rely on selling carbon allowances for profit. This constraint is valid for all formulas in the full text.

Social welfare can be considered next. Because carbon trading policies directly limit total carbon emissions, carbon emissions will not be considered when calculating social welfare. Social welfare = total profit of the supply chain + consumer surplus, where the consumer surplus is the consumer’s utility value to the product minus the actual value of the product. Suppose the consumer’s utility function is \( U = u + \mu \theta \), where \( u \) represents the consumer’s utility value for a common product. Assuming that \( u > ep_c + C_m + C_r \), the company will have the motivation to produce and sell products. Thus, the social welfare expression is as follows:
\[ SW^* = \pi_m^* + \pi_r^* + (u + \mu \theta^* - p^*)D(p^*, \theta^*) \]  
(8)

Social welfare can be quantified by substituting formulas (4), (5), (6), and (7) into formula (8).

3.3. Direct Governmental Subsidies for Technological Innovation to Manufacturers

3.3.1. Problem Description

To expand their competitive advantages and achieve higher profits, manufacturers can choose to innovate low-carbon technologies independently [51]. Thus, to stimulate manufacturers to innovate low-carbon technologies or stimulate consumers to purchase low-carbon products, the government can implement research and development measures such as subsidies or consumer subsidies as
shown in Figure 2, a government subsidy for manufacturers to produce low-carbon products has been added to the basic model. Based on a certain free carbon quota allocated to manufacturers, the government subsidizes manufacturers by subsidizing the production of low-carbon products to stimulate manufacturers’ independent technological innovations for reducing carbon emissions. These subsidies are only intended for low-carbon products produced by manufacturers using carbon emission reduction technologies, thereby guiding manufacturers to conduct relevant innovations through research and development.

![Figure 2. Structure diagram when the government subsidizes the manufacturer.](image)

Assume that the government’s unit product subsidy for manufacturers’ carbon emission reduction innovations is \( k_1 \), and the retailer and consumer subsidies are 0. The demand function is thus \( D(p^{G1}, G^{G1}) = \alpha - \beta p^{G1} + \mu G^{G1} \). The superscript, defined as \( G1 \) here, indicates that the manufacturer chooses to carry out a technological innovation when there is a government subsidy.

3.3.2. Model Building

Throughout the entire process, the goals of companies in the supply chain are to pursue profit maximization, and these have an outsized influence on corporate decision-making. In decentralized decision-making, the game between the manufacturer and the retailer is a Stackelberg game, in which the manufacturer is the dominant player and the inverse solution method is used in the solution.

The retailer’s profit function is:

\[
\pi_r^{G1} = D(p^{G1}, G^{G1})[p^{G1} - w^{G1} - C_r]
\]  
(9)

The manufacturer’s profit function is:

\[
\pi_m^{G1} = D(p^{G1}, G^{G1})[w^{G1} - C_m + k_1 G^{G1} - p_c(e - \Delta e)] - \frac{c(G^{G1})^2}{2} + p_c C_g
\]  
(10)

We can obtain the best wholesale price for manufacturers \( w^{G1*} \), the best technological innovation level \( G^{G1*} \), and the best retail price for retailers \( p^{G1*} \), respectively, as follows:

\[
w^{G1*} = \frac{(2 \epsilon - \mu k - \mu e_p) [\alpha + \beta e_p + \beta (C_m - C_r)] - \beta (e_p + k_1)^2 (\alpha - \beta C_r) - \mu^2 (C_m + e_p)}{4 \beta e - (\mu + k_1 \beta + \beta e_p)^2}
\]  
(11)

\[
G^{G1*} = \frac{(\mu + k_1 \beta + \beta e_p) [\alpha - \beta e_p - \beta (C_m + C_r)]}{4 \beta e - (\mu + k_1 \beta + \beta e_p)^2}
\]  
(12)
The maximum profits for retailers and manufacturers are as follows:

\[
\pi^G_{p} = \frac{\beta \varepsilon [\alpha - \beta \varepsilon p_c - \beta (C_m + C_r)]^2}{4 \beta \varepsilon - (\mu + k_1 \beta + \beta p_c)^2} \\
\pi^G_{m} = \frac{\varepsilon [\alpha - \beta \varepsilon p_c - \beta (C_m + C_r)]^2}{2 [4 \beta \varepsilon - (\mu + k_1 \beta + \beta p_c)^2]} + p_c C_g
\]

According to formula (14), when \(4 \beta \varepsilon - (\mu + k_1 \beta + \beta p_c)^2 > 0\), \(i = 1, 2, 3\) and \(\pi^G_{m} > p_c C_g\) are established, and the manufacturer can make profits by producing products; otherwise, the manufacturer’s optimal decision is to stop producing any products and only rely on selling carbon allowances to obtain profits. This constraint is valid for all formulas in the full text.

Social benefits under government subsidies can then be obtained. Social welfare = total profit of the supply chain – government subsidy expenditure + consumer surplus. Therefore, the expression of social welfare when the government subsidizes manufacturers is as follows:

\[
SW^G = \pi^G_{m} + \pi^G_{r} - k_1 \theta^G D\left(p^G, \theta^G\right) + \left(u + \mu \theta^G - p^G\right) D\left(p^G, \theta^G\right)
\]

Substituting formulas (12), (13), (14), and (15) into formula (16), we can obtain social welfare.

3.4. Government Subsidies for Retailers to Stimulate Manufacturers’ Technological Innovations

3.4.1. Problem Description

As shown in Figure 3, the government subsidizes retailers’ sales of low-carbon products, reduces the price of products, indirectly subsidizes consumers, and influences consumers’ carbon preferences to increase their desire to purchase low-carbon products. At the same time, subsidies to retailers make these companies more inclined to cooperate with manufacturers with higher levels of carbon reduction technology innovations; accordingly, manufacturers will choose carbon emission reduction technology innovations to more closely cooperate with retailers and protect their own interests. Suppose that the government’s unit subsidy for the sale of low-carbon products by retailers is \(k_2 \theta\), and the subsidy for manufacturers and consumers is 0, then the demand function is: \(D(p^{G2}, \theta^{G2}) = \alpha - \beta p^{G2} + \mu \theta^{G2}\).

The purpose of the government’s subsidy to retailers is to allow retailers to choose manufacturers with advanced carbon emission reduction technology in order to obtain maximum benefit, thereby increasing the carbon emission reduction of the entire supply chain and encouraging manufacturers to adapt to the market. To promote emission reductions, the government provides a subsidy mechanism with an intensity of \(k_2 \theta\) per unit of retail sales of low-carbon products. When the products sold by retailers are not low-carbon products, the subsidy is zero. The superscript G2 means that when the government subsidizes the retailer, the manufacturer that cooperates with the retailer chooses to conduct carbon technology research and development.
3.4.2. Model Building

The game between the manufacturer and the retailer is a Stackelberg game, where the manufacturer is the dominant player, and the inverse solution method is used in the solution.

The retailer’s profit function is:

\[ \pi_r^{G2} = D \left( p^{G2}, \theta^{G2} \right) \left( p^{G2} + k_2 \theta^{G2} - w^{G2} - C_r \right) \]  \hfill (17)

The manufacturer’s profit function is:

\[ \pi_m^{G2} = D \left( p^{G2}, \theta^{G2} \right) \left[ w^{G2} - C_m - p_c (e - \Delta c) \right] - \frac{\epsilon \theta^{G2}}{2} + p_c C_g \]  \hfill (18)

We can respectively obtain the best wholesale price for manufacturers \( w^{G2*} \), the best technological innovation level \( \theta^{G2*} \), and the best retail price for retailers \( p^{G2*} \), as follows:

\[ w^{G2*} = \frac{(2e - k_2 \beta p_c - \mu p_c)[\alpha + \beta p_c + \beta(C_m + C_r)] - \beta \epsilon p_c^2(\alpha - \beta C_r) - (k_2 \beta + \mu)^2(C_m + p_c)}{4\beta - (\mu + k_2 \beta + \beta p_c)^2} \]  \hfill (19)

\[ \theta^{G2*} = \frac{(\alpha + k_2 \beta + \beta p_c)[\alpha - \beta p_c - \beta(C_m + C_r)]}{4\beta - (\mu + k_2 \beta + \beta p_c)^2} \]  \hfill (20)

\[ p^{G2*} = \frac{\alpha[2e - \beta(k_2 + ep_c)^2] - \mu^2(ep_c + C_m + C_r) + (e - \mu ep_c - k_2C_r)[\alpha + \beta p_c + \beta(C_m + C_r)]}{4\beta - (\mu + k_2 \beta + \beta p_c)^2} \]  \hfill (21)

The maximum profits for retailers and manufacturers, respectively, are as follows:

\[ \pi_r^{G2*} = \frac{\beta \epsilon^2 [\alpha - \beta C_r - \beta(C_m + C_r)]^2}{4\beta - (\mu + k_2 \beta + \beta p_c)^2} \]  \hfill (22)

\[ \pi_m^{G2*} = \frac{\epsilon [\alpha - \beta p_c - \beta(C_m + C_r)]^2}{2[4\beta - (\mu + k_2 \beta + \beta p_c)^2]} + p_c C_g \]  \hfill (23)

The expression of social welfare when the government subsidizes retailers is as follows:

\[ SW^{G2*} = \pi_m^{G2*} + \pi_r^{G2*} + (u + \mu \theta^{G2*} - p^{G2*} - k_2 \theta^{G2*})D \left( p^{G2*}, \theta^{G2*} \right) \]  \hfill (24)

Figure 3. Schematic diagram of government subsidies to retailers.
The social welfare value can be obtained by substituting formulas (20), (21), (22), and (23) into Equation (24).

3.5. Government Subsidies to Consumers to Stimulate Technological Innovations from Manufacturers

3.5.1. Problem Description

As shown in Figure 4, the government only subsidizes consumers’ purchases of low-carbon products, thus increasing consumers’ desire to buy such products. To maximize profits, manufacturers will upgrade their technology and produce low-carbon products. Assume that the government’s unit subsidy for consumers to purchase low-carbon products is \(k_3\theta\), the subsidy for manufacturers and retailers is 0, and the demand function is: 

\[
D(p^G, \theta^G) = a - \beta(p^G - k_3\theta^G) + \mu\theta^G.
\]

Thus, the purpose of government subsidies to consumers is to allow manufacturers to choose technological innovations that can produce low-carbon products, thereby increasing reductions in carbon emissions from the entire supply chain and motivating consumers to purchase low-carbon products. To promote emission reductions, the government provides consumers with a subsidy mechanism with an intensity of \(k_3\theta\) per unit of low-carbon products purchased. When the products purchased by consumers are not low-carbon products, the subsidy is zero. The definition of superscript G3 means that when the government subsidizes consumers, manufacturers choose to generate low-carbon technological innovations through research and development.

![Figure 4. Schematic diagram of government subsidized consumer structure.](image)

3.5.2. Model Building

The game between the manufacturer and the retailer is a Stackelberg game, where the manufacturer is the dominant player, and the inverse solution method is used in the solution.

The retailer’s profit function is as follows:

\[
\pi^G_r = D(p^G, \theta^G)(p^G - w^G - C_r)
\]

(25)

The manufacturer’s profit function is as follows:

\[
\pi^G_m = D(p^G, \theta^G)
\left[ w^G - C_m - p_c(e - \Delta) \right] - \frac{\epsilon\theta^G}{2} - p_c C_g
\]

(26)

We can respectively obtain the best wholesale price for manufacturers \(w^{G^3}\), the best technological innovation level \(\theta^{G^3}\), and the best retail price for retailers \(p^{G^3}\), as follows:

\[
w^{G^3} = \frac{(2\epsilon - k_3\beta p_c - \mu p_c)(\alpha + \beta p_c + \beta(C_m - C_r)) - \beta\epsilon p_c^2(\alpha - \beta C_r) - (k_3\beta + \mu)^2(C_m + ep_c)}{4\beta\epsilon - (\mu + k_3\beta + \beta p_c)^2}
\]

(27)
\[ \theta^{G3} = \frac{(\mu + k_3 \beta + \beta \varepsilon p_c) \alpha - \beta \varepsilon p_c - \beta (C_m + C_r)}{4 \beta \varepsilon - (\mu + k_3 \beta + \beta \varepsilon p_c)^2} \] (28)

\[ p^{G3} = \frac{\alpha (2 \varepsilon - \beta \varepsilon p_c + \mu k_3 + k_3^2 \beta^2) - \beta \varepsilon p_c + \beta (C_m + C_r) + (\varepsilon - \mu \varepsilon p_c - 2 \mu k_3 - k_3^2 \beta)(\alpha - \beta \varepsilon p_c - \beta (C_m + C_r))}{4 \beta \varepsilon - (\mu + k_3 \beta + \beta \varepsilon p_c)^2} \] (29)

The expression of social welfare when the government subsidizes consumers is as follows:

\[ SW^{G3} = \pi^{G3}_m + \pi^{G3}_r + (u + \mu \theta^{G3} - p^{G3})D(p^{G3}, \theta^{G3}) \] (32)

The social welfare value can be obtained by substituting formulas (28), (29), (30), and (31) into formula (32).

4. Results

According to the equilibrium results calculated by the model, the following propositions can be obtained:

**Proposition 1.** When government subsidies and market capacity \( \alpha \) meets:

\[ \beta \varepsilon p_c + \beta (C_m + C_r) < \alpha < \frac{4 \beta \varepsilon - (\mu + k_3 \beta + \beta \varepsilon p_c)^2}{\mu + k_3 \beta + \beta \varepsilon p_c} \]

The manufacturer will choose a low-carbon technology innovation. Otherwise, it will continue to produce ordinary products.

**Proof.** Demand after innovation can be calculated by formula (12) and formula (13) as: \( D(p^{G1*}, \theta^{G1*}) = \frac{\beta \varepsilon \alpha - \beta \varepsilon p_c - \beta (C_m + C_r)}{\mu + k_3 \beta + \beta \varepsilon p_c} \). We can even further prove that the demand in the three cases is equal, namely: \( D(p^{G1*}, \theta^{G1*}) = D(p^{G2*}, \theta^{G2*}) = D(p^{G3*}, \theta^{G3*}) \). If \( \alpha \leq \beta \varepsilon p_c + \beta (C_m + C_r) \), demand is negative. Because the low carbon technology innovation level \( \theta = \frac{k_3}{\beta} \) is a real number from 0 to 1, let \( 0 < \theta < 1 \). Proposition 1 is thus proven. \( \square \)

The proposition states that manufacturers will choose technological innovations only when the market capacity is within a certain range. If the market capacity is overly small, companies cannot rely on the production of products to make profits. If the market capacity is too large, companies do not need to spend extra money on technological innovations to obtain high profits.

**Corollary 1.** When the government does not subsidize and the market capacity \( \alpha \) satisfies: \( \beta \varepsilon p_c + \beta (C_m + C_r) < \alpha < \frac{4 \beta \varepsilon - (\mu + k_3 \beta + \beta \varepsilon p_c)^2}{\mu + k_3 \beta + \beta \varepsilon p_c} \). The manufacturer will choose low-carbon technology innovations. Otherwise, it will continue to produce ordinary products.

**Proposition 2.** When government subsidies occur, \( \theta^{G1*}, \pi^{G1*}_r \), and \( \pi^{G1*}_m \) (\( i = 1, 2, 3 \)) are positively correlated with \( k \).

**Proof.** The conclusion of the proposition can be easily obtained by seeking the first derivative of \( k \) for the three cases of government subsidy, i.e., \( \theta^{G*}, \pi^{G*}_r \), and \( \pi^{G*}_m \), respectively. \( \square \)
Proposition 2 indicates that when government subsidies exist, the level of low-carbon technological innovation, retailer profits, and manufacturer profits in all three scenarios are positively correlated with the subsidy intensity $k$.

**Corollary 2.** It can be proven that the independent technological innovation of the manufacturer, the low-carbon technological innovation level, the profit of the retailer, and the profit of the manufacturer are all related to the low-carbon preference $\mu$ of consumers.

**Proposition 3.** There are $\pi_m^{G^1*} = \pi_m^{G^2*} = \pi_m^{G^3*} > \pi_m^*$, $\pi_r^{G^1*} = \pi_r^{G^2*} = \pi_r^{G^3*} > \pi_r^*$, $\theta^{G^1*} = \theta^{G^2*} = \theta^{G^3*} > \theta^*$ when $k_1 = k_2 = k_3 > 0$.

**Proof.** Comparing formulas (11), (18), and (25), the expressions of $\theta$ are the same in the three cases where government subsidies are obtained. The expressions of $\pi_r^*$ and $\pi_m^*$ are also the same in the three cases. In formulas (4), (6), and (7), $\theta^*$, $\pi_r^*$, and $\pi_m^*$ are compared with government subsidies. When $k \neq 0$, $4\beta \leq \beta - (k\beta + \mu + \beta\mu)^2 < 4\beta - (\mu + \beta\mu)^2$, and the rest of the expression is the same. Thus, $\pi_m^{G^i*} > \pi_m^*, \pi_r^{G^i*} > \pi_r^*, \theta^{G^i*} > \theta^* (i = 1, 2, 3)$, thereby proving this proposition. $\square$

Proposition 3 states that as long as the government subsidies are fixed, regardless of who the subsidy is intended for, the profits of manufacturers who choose to develop technological innovations will have the same increase, and the optimal technological innovation level of manufacturers will be improved by the same degree as without government intervention. Owing to the “free-rider” behavior of retailers, profits will also increase at the same rate in all cases. Several subsidy strategies can be transformed into each other without any absolute advantages or disadvantages.

**Proposition 4.** There are $w^{G^3*} = w^{G^2*} = w^{G^1*} + k_3\theta^{G^2*}$, $p^{G^3*} - k_3\theta^{G^3*} = p^{G^2*} = p^{G^1*}$ when $k_1 = k_2 = k_3$.

**Proof.** Comparing equations (17) and (24), we can determine that when $k_2 = k_3$, $w^{G^3*} = w^{G^2*}$. Equation (10) is subtracted from Equation (17) to obtain $w^{G^2*} - w^{G^1*} = k_2\theta^{G^2*}$. Similarly, we find that $\pi^{G^i*} = \pi^*, \pi_r^{G^i*} = \pi_r^*, \theta^{G^i*} > \theta^* (i = 1, 2, 3)$, thereby proving this proposition. $\square$

Proposition 4 explains why Proposition 3 occurs. When other conditions remain the same, regardless of the main body of government subsidies, the decentralized decisions of the manufacturers and retailers are based on adjustments to wholesale and retail prices; thus, this part of the subsidized funds will be transferred to the carbon emissions manufacturers through decisions that maximize the profits of both parties.

**Proposition 5.** When $0 < \beta \leq 1$, $\frac{\partial SW}{\partial \epsilon_i} < 0$. When $1 < \beta < \frac{\alpha}{\epsilon_i + C_m + C_r}$, there is at least one real solution, so that $\frac{\partial SW}{\partial \epsilon_i} = 0 (i = 1, 2, 3)$.

**Proof.** Let $M = \alpha - \beta\epsilon_i - \beta(C_m + C_r), t = \mu + \beta k + \beta\epsilon_i$. We must prove that $\frac{\partial SW}{\partial \epsilon_i} < 0$. Finding the derivative of $i$ for social welfare $SW$ provides the following equation: $\frac{\partial SW}{\partial \epsilon_i} = 2M\beta \epsilon \left(\alpha - u\beta - 2M\right) + M^2\beta \epsilon \left(3\mu\beta + \mu + \beta\mu\epsilon_i\right)^2 + 8M\beta \epsilon^2 \left(M - \alpha - u\beta\right)t + 4M^2\epsilon^2 \left(\beta - 1\right)$ when $0 < \beta \leq 1$ because: $\alpha - u\beta - 2M = -M - \beta \left(u - \epsilon_i - C_m - C_r\right)$ and $u > \epsilon_i + C_m + C_r$, the first three coefficients of the polynomial are negative, positive, and negative, respectively. We may deduce from Proposition 1 that $\epsilon > \frac{Mt + t^2}{4\beta}$.

Substituting in the original formula gives the following equation:

$$\frac{\partial SW}{\partial \epsilon_i} < M\beta \epsilon \left[-2(M + 2u\beta)t^2 + M(2M - 2\alpha - 2u\beta + \mu + \epsilon_i) + \mu M^2(\beta - 1)\right] \leq 0$$

Specifically, when $0 < \beta \leq 1$, $\frac{\partial SW}{\partial \epsilon_i} < 0$.

When $1 < \beta < \frac{\alpha}{\epsilon_i + C_m + C_r}$, $\frac{\partial SW}{\partial \epsilon_i} \bigg|_{\epsilon_i = 0} = 4M^2\epsilon^2 \left(\beta - 1\right) > 0$. Then, we must find the limit of $\frac{\partial SW}{\partial \epsilon_i}$ when $t$ tends to positive infinity, and the limit value is $-\infty$. There is at least one real solution greater than 0 so that $\frac{\partial SW}{\partial \epsilon_i} = 0$, thereby proving this proposition. $\square$
Proposition 5 states that when consumer prices are not sensitive, the government should not provide any subsidies; when consumer prices are sensitive, there should be an optimal government subsidy to maximize social welfare.

5. Analysis Example

5.1. Product Retail Price Analyses

Through analyses of the above examples, the influencing factors and changing trends of retail prices of products developed through low-carbon technology innovations from independent manufacturers and government subsidies are obtained. Referencing the Benjaafar [10] hypothesis-related parameters, we assume that: $p_c = 3$, $C_m = 30$, $C_r = 2$, $e = 100000$, $e = 0.9$, $\beta = 1.2$, and $\alpha = 100$. The simulation results are shown in Figures 5–8.

Figure 5. Trend chart of retail prices during independent technological innovation.

Figure 6. Trend chart of retail prices when the government subsidized companies.

Figure 7. Trend chart of retail prices when the government subsidizes consumers.

Figure 8. Comparison of retail prices with and without government subsidies.
As shown in Figure 5, when manufacturers innovate independently, and as consumers’ low-carbon preferences increase, the retail prices of goods will continue to increase. This is because consumers’ low-carbon preferences directly affect the manufacturer’s optimal low-carbon technology level. The stronger the consumers’ low-carbon preferences, the more willing the manufacturer is to innovate their technology, and the more low-carbon conscious consumers are willing to spend on buying low-carbon products, so retail prices will increase.

As shown in Figure 6, the retail price of goods increases as consumers’ low-carbon preferences increase. In the case of government subsidies to enterprises, when consumers’ low-carbon preferences are weak, the retail prices of goods decrease as the subsidy increases. When these preferences are strong, the retail prices of goods increase as the subsidy increases.

As shown in Figure 7, when the government subsidizes consumers, the retail prices of goods are positively correlated with the intensity of subsidies and consumers’ low-carbon preferences. From Proposition 4, when the government subsidizes consumers and enterprises, the same consumer surplus occurs. Therefore, it is only necessary to compare the retail prices of goods in the two cases of technological innovations from independent manufacturers and government-subsidized enterprises.

As shown in Figure 8, when consumers’ low-carbon preferences are weak and government subsidies are high, the retail prices of products under subsidies will be lower than the retail prices of products under independent innovation. At stronger levels of preferences, the opposite dynamic is true.
5.2. Analysis of Social Welfare Influencing Factors

To verify the impact of governmental subsidies and consumers’ low-carbon preferences on social welfare among consumers with different price sensitivity coefficients, an analysis example is performed. Make $\beta^{H} > 1$ price sensitive and $\beta^{L} \leq 1$ price insensitive. The relevant parameters are set as follows: $p_c = 3$, $C_m = 30$, $C_r = 2$, $\epsilon = 100000$, $\beta^H = 1.2$, $\beta^L = 0.8$, $\alpha = 100$, $u = 72$, and $C_g = 200$. The simulation results are shown in Figures 9 and 10.

Figure 9. Trend of social welfare changes when consumers are price sensitive.

Figure 10. Trend of social welfare changes when consumer prices are not sensitive.

Figure 9 shows that when consumer prices are sensitive, social welfare tends to increase first, and then decrease with the increase in government subsidies. Therefore, there is an optimal government subsidy that can maximize social welfare. The intensity of subsidies is positively related to consumers’ low-carbon preferences. Specifically, when consumers’ low-carbon preference is zero, the government should not provide any subsidies.

Figure 10 shows that when consumer prices are not sensitive, social welfare and government subsidies are always negatively correlated. At this time, the government should not provide any subsidies.

6. Discussion

6.1. Conclusions

This article reveals the impact of government subsidies, consumer low-carbon preferences, and consumer price preference coefficients on supply chain enterprise profits and social welfare under low-carbon policies, and studied the relationship between them. Based on a two-level low-carbon supply chain dominated by manufacturers, this study establishes game models for manufacturers’ independent low-carbon technology innovations, and government-subsidized manufacturer, retailer,
and consumer policies. By comparing the equilibrium solutions in these cases, the interaction rules between them are obtained. The research indicates that:

1. When governmental subsidies are implemented and market capacity is in the proper range, enterprises can obtain greater profits, and manufacturers’ low-carbon technological innovation levels will increase based on the intensity of the subsidies. Regardless of who the subsidy is intended for, as long as other conditions remain unchanged, a same subsidy effect can be implemented for any other object by adjusting the wholesale and retail prices. In a secondary supply chain, where only manufacturers produce carbon emissions, the three categories of subsidy policies can be interchangeable without absolute advantages or disadvantages.

2. The retail prices of goods are always positively related to consumers’ low-carbon preferences. With manufacturers’ independent technological innovations and government subsidies to consumers, the retail prices of goods are positively correlated with the intensity of subsidies. However, with government subsidies to enterprises, retail prices are positively correlated with the subsidies only when consumers have strong low-carbon preferences, and retail prices are negatively correlated with the subsidies when consumers have weak low-carbon preferences. Therefore, with high subsidies and strong preferences, the retail price exceeds the product price when there is no government subsidy, and at high subsidies and weak preferences, the retail price is lower than the product price when there is no government subsidy.

3. Social welfare does not always increase with increasing government subsidies. In the price-sensitive consumer group, once the subsidy intensity exceeds a certain level, the social benefits provided by technological innovations are not proportional to the government’s input, so social welfare begins to decline rapidly. At this time, the government’s optimal subsidy intensity positively correlates with consumers’ low-carbon preferences. In price-insensitive consumer groups, the government should not provide any subsidies.

6.2. Managerial Implications

It has a certain guiding significance for supply chain enterprises to obtain their own maximum benefits while reducing carbon emissions. At the same time, it also has a certain reference role for the government’s low-carbon subsidy decision.

For supply chain companies: There is a range of product market capacities that manufacturers can refer to, to decide whether to carry out low-carbon technological innovation. When the market capacity is within this range, manufacturers should increase their level of low-carbon technological innovation as government subsidies increase to obtain their own maximum profits. Consumers’ low-carbon preference should be taken seriously. The higher the consumer’s low-carbon preference level, the higher the profitability of the company. Therefore, companies should take actions to increase consumers’ low-carbon preference, such as low-carbon campaigns.

For the government: Consumers’ price preference determines whether they should receive low-carbon subsidies. When low-carbon subsidies are beneficial to social welfare, it is not that the greater the subsidy, the better, but that there is an optimal level of subsidies. When the government conducts low-carbon subsidies, it is the most efficient way to directly apply the subsidies to the links that generate large amounts of carbon emissions. In order to make manufacturers willing to carry out low-carbon technological innovation activities, the government should also actively guide market demand and control it within an appropriate range. In addition, the low carbon preference of consumers also directly affects social welfare, and the government should strive to cultivate consumers’ environmental awareness.

6.3. Suggestions for Future Research

There are also some shortcomings in this study. This article assumes that the information flowing between the government and the enterprises is completely symmetrical. Deceptive behavior that
occurs in reality requires further analysis. In addition, assuming that the retailer does not generate any carbon emissions, and there is no contract between the retailer and the manufacturer, the retailer will display a “free-rider” behavior.

Future research may focus on how the retailer shares responsibility and costs with the manufacturer, and the best supply chain strategies when retailers also generate carbon emissions and are also affected by carbon cap policies.

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