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# Neural-Network-Based Dynamic Distribution Model of Parking Space Under Sharing and Non-Sharing Modes

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**Abstract:** In recent years, with the rapid development of China's automobile industry, the number of vehicles in China has been increasing steadily. Vehicles represent a convenient mode of travel, but the growth rate of the number of urban motor vehicles far exceeds the construction rate of parking facilities. The continuous improvement of parking allocation methods has always been key for ensuring sustainable city management. Thus, developing an efficient and dynamic parking distribution algorithm will be an important breakthrough to alleviate the urban parking shortage problem. However, the existing parking distribution models do not adequately consider the influence of real-time changes in parking demand and supply on parking space assignment. Therefore, this study proposed a method for dynamic parking allocation using parking demand predictions and a predictive control method. A neural-network-based dynamic parking distribution model was developed considering seven influencing factors: driving duration, walking distance, parking fee, traffic congestion, possibility of finding a parking space in the target parking lot and adjacent parking lot, and parking satisfaction degree. Considering whether the parking spaces in the targeted parking lots are shared or not, two allocation modes—sharing mode and non-sharing mode—were proposed and embedded into the model. At the experimental stage, a simulation case and a real-time case were performed to evaluate the developed models. The experimental results show that the dynamic parking distribution model based on neural networks can not only allocate parking spaces in real time but also improve the utilisation rate of different types of parking spaces. The performance score of the dynamic parking distribution model for a time interval of 2–20 min was maintained above 80%. In addition, the distribution performance of the sharing mode was better than that of the non-sharing mode and contributed to a better overall effectiveness. This model can effectively improve the utilisation rate of resources and the uniformity of distribution and can reduce the failure rate of parking; thus, it significantly contributes to more smart and sustainable urban parking management.

**Keywords:** dynamic distribution model; neural network; parking demand prediction; predictive control; utility function; sharing distribution mode

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## 1. Introduction

Given the economic development and technical progress, the number of private cars owned by residents has been increasing and the proportion of cars in motorized transportation has risen year

by year. Parking-related problems have been increasing as finding parking spaces has become challenging because of the lack of available parking spaces in the city, especially within the core areas of the city [1]. According to the survey data of China's security exhibition network, from 2015 to 2020, in China, the growth rate of car ownership has become greater than that of parking spaces; thus, the gap between the number of cars and parking spaces continues to expand. Drivers spend more time in searching for available parking facilities, which causes traffic congestion, additional fuel consumption, and pollution. Davis [2] showed that 30% of traffic congestion in road networks is caused when people are cruising to find a parking spot. Yedlin found [3] cruising for curb parking created about 8% of the total vehicle miles travelled in west midtown. Shao [4] highlighted that finding a parking spot and walking to work often constitutes an appreciable fraction of the total travel time. Approximately 30% of vehicles in the urban areas of major cities worldwide take an average of 7.8 min to find vacant parking spaces [5].

To reduce the wastage of money and time in parking search, various smart parking systems have been developed to make efficient utilization of the limited parking resources through optimal allocation of parking requests. A considerable number of studies have focused on the traffic distribution model. According to the characteristics of the research problem and different assumptions, the traffic distribution model can be classified in various ways. Considering whether the traffic demand changes over time, it can be divided into static and dynamic traffic distribution models; considering whether the traffic network abstraction is discrete or continuous, it can be divided into discrete and continuous traffic distribution models; and considering whether traffic demand changes with traffic congestion, it can be divided into fixed demand and elastic demand traffic distribution models. Caicedo [6] developed a demand distribution model to reduce the time and distances involved in locating car parking. Zhao [7] used a novel fuzzy logic controller to create an automatic parallel parking algorithm for parking in tight spaces. Davis et al. [2] evaluated the space allocation of parking lots to estimate the availability of parking spaces according to future demand. Using a decision-making method based on fuzzy knowledge, Leephakpreeda [8] provided advice on car parking. Arnott and Rowse [9] created an integrated model to manage curb-side parking and congestion in a downtown area. Shoup [10] presented a diagram of how drivers choose to ride along the curb-side of car parks or paid off-street car parks. Teodorovic [11] suggested a network of intelligent inventory of parking spaces. Feng et al. [12] developed a hybrid trip network focused on logic for congested road-use pricing and parking. Using a utility feature that combined travel time, search time, waiting time, access time, and parking price, Mei et al. [13] formulated a profit-based parking price to curb parking prices. Chou et al. [14] implemented an intelligent agent program with negotiable parking rates to find the optimal car parking space for a driver. Martin [15] proposed a multinomial logit model that considered systematic and random variations in parking preferences to obtain better adjustments.

With respect to parking behaviour, which considerably affects parking lot selection, Kelly and Clinch [16] proposed that wide discrepancies exist in the sensitivity of the two groups—business travellers and non-commercial travellers. Waerden [17] summarized the effects of existing parking behaviour choices on parking fees. Using a multinomial logistic model, the main factors that influence parking time, such as parking purposes, parking prices, and parking fee payers, were identified and confirmed by Jin-Mei [18]. Simicevic [19] built a model to analyse the elasticity of parking prices, indicating that increasing parking cost will reduce the parking demand. Lam et al. [20] examined the impact of parking rates on the overall network congestion. Keane [21] designed models to predict the decision of where the user was going to park, considering the parking fee, distance from final destination, and the availability of public transport.

With respect to modelling parking in the dynamic sense, Arnott [22] was among the first to study the dynamic equilibrium of users with respect to parking choices. A special network is required in which parking spaces are distributed continuously along the freeway leading to the destination. The dynamics of the framework were embedded in the bottleneck model [23], a single-origin single-destination network, to demonstrate that reducing parking fees will help in social welfare. Zhang et al. [24] further expanded this system to consider evening commutes.

However, the existing models hardly considered the impact of real-time changes of parking demand and supply on the results of parking assignment. Parking demand prediction and predictive control can be helpful in improving the existing models [11]. Parking demand prediction involves using historical data to predict the future parking demand using pre-trained models. The predictive control method is a dynamic control technique that uses the predicted system output to select the best input to learn the optimal solution for the future output. Recently, the development of machine learning and deep learning methods has provided a new method for traffic big data analysis. Dynamic parking models integrate parking demand prediction and predictive control methods to obtain better prediction result and fast responses, which is suitable for the dynamic traffic environment. Sharing parking lots are an effective method to better utilise parking spaces and solve parking problems, especially for the downtown area [4]. However, the performance of the parking lot sharing method is influenced by the parking demand and supply. A dynamic parking model with parking demand prediction and predictive control can be used to determine when to apply the parking lot sharing strategy; this may lead to better overall parking assignment.

Therefore, this study aims to propose a method to perform dynamic parking allocation using parking demand prediction and predictive control methods. This research has the following contributions:

(1) A dynamic parking distribution model was developed based on neural networks. The factors influencing the parking selection, such as driving duration, walking distance, parking fee, congestion, the possibility fail to find vacant parking spaces, and the delight degree of parking, were considered. Real-time parking space allocation was realized by predicting the variable demand and supply status of the parking spaces, based on neural networks.

(2) Two parking distribution modes—the sharing mode and the non-sharing mode—were proposed and embedded into the dynamic parking model.

(3) A real-world case study was conducted to verify the effectiveness of the dynamic parking distribution model and to compare the performances of the sharing and non-sharing modes.

The remainder of this paper is divided into the following parts. Section 2 describes the design of the utility function, which is used to access the available parking lots and select the best parking lot. Both the sharing mode and non-sharing mode and the performance evaluation indexes of the parking distribution algorithm are also introduced in this section. Section 3 introduces the design of the dynamic parking distribution algorithm. Section 4 introduces the traffic environment and results regarding the real case study. The results and analyses of the dynamic parking distribution algorithm and its comparison with the static parking distribution algorithms are explained. Section 5 concludes the study, along with a discussion regarding future studies.

## 2. Utility Function and Distribution Modes

### 2.1. Utility Functions for Parking Lot Assessment

Parking lots are assessed using a utility function, which was established according to discrete choice theory. From the perspective of both drivers and city management, the utility function of parking alternatives  $V$  was established and was based on seven parameters: driving duration from the current location of a car to the assigned parking lot, walking distance from the assigned parking lot to the driver's destination, parking fee, number of congested cars at each parking lot, the probability of failure to find vacant parking space, the level of satisfaction when driver arrives at the assigned parking lot, and the probability of failure to find parking spaces near the assigned but already full parking lots. All seven parameters were normalized via max–min normalization. A max–min normalization is typically done via the following equation:

$$Z = \frac{x - \min(x)}{\max(x) - \min(x)} \quad (1)$$

where  $z$  is the normalized value of  $x$  and  $\min$  and  $\max$  are the minimum and maximum values in  $x$ .

Because the reservation system is not considered in the model, it is necessary to add parameters to ensure that there are available parking spaces when the car reaches the parking lot. Therefore, the probability of finding vacant parking spaces and that of finding vacant parking spaces near the assigned but already full parking lot were also included. Note that the allocation system provides the best parking space to each car for the sake of overall optimization of the whole parking system rather than particular individuals. Hence, drivers do not have a preference, because parking selection is performed and controlled by the central allocation system.

$$V(T_d, D_w, C_p, N_c, A, P_c, S_p) = \beta_1 T_d + \beta_2 D_w + \beta_3 C_p + \beta_4 N_c + \beta_5 A + \beta_6 P_c + \beta_7 S_p \quad (2)$$

where

$T_d$  is the driving duration from the current location of a car to assigned parking lot;

$D_w$  is the walking distance from the assigned parking lot to the driver's destination;

$C_p$  is the parking fee;

$N_c$  is the number of congested cars heading to the targeted parking lot;

$A$  is the probability of failing to find a vacant parking space;

$$A = \frac{\left( \frac{\text{driving duration from current location to parking lot}}{\text{mean time between arrivals at parking lot}} \right)}{\text{currently available number of parking spaces}} \quad (3)$$

$P_c$  is the level of satisfaction when a driver arrives at the assigned parking lot;

$$P_c = \frac{\text{the number of cars that assigned to it at last period}}{\text{the number of cars that request for parking at last period}} * \frac{\text{the number of cars that request for parking}}{\text{its currently available number of parking spaces}} \quad (4)$$

$S_p$  is the probability of failure of finding parking spaces near the assigned but already full parking lots;

$$S_p = \sum \text{distance from target parking lot to near parking lot} * A \quad (5)$$

$\beta_i$  is the coefficient of each parameter in the utility function,  $i = 1, 2, 3, 4, 5, 6,$  and  $7$ .

## 2.2. Coefficient Configuration

Different drivers and the city management attach different importance to the influencing factors of the utility function. For instance, some drivers prefer a shorter driving duration, some prefer shorter walking distances, while some others prefer lower parking fees. The preferences of the participants are represented by coefficient values. For simplicity, this study set three different coefficient values: neutral (1), important (2), and very important (3).

Different configurations of coefficient values were used to represent the different parking preferences. The utility function has seven parameters; thus, there are  $3^7$  different configurations.

## 2.3. Distribution Modes

Consider a situation in which a car is directed to the optimal parking lot, but no parking space is available when the car arrives. Therefore, in this study, two parking distribution modes were proposed. According to whether the parking spaces in the nearby parking lots are shared when the optimal parking lot is full, the sharing or non-sharing allocation mode was selected. Studies on parking space sharing show that different types of parking lots have different usage times and characteristics; thus, they can complement and share parking spaces with each other in time and space [25], which supports the establishment of a sharing mode and non-sharing mode theoretically.

There are three conditions for two or more adjacent buildings to realize parking sharing: time complementarity, close parking locations, and mutual accommodation of parking spaces. A previous study [26] showed that the maximum walking distance after parking acceptable to 95% of the users

is 350 m. Here, we assumed that the parking lots should be open to the outside world or allow outside vehicles to park conditionally.

### 2.3.1. Sharing Distribution Mode

The sharing mode involves overall planning of parking resources in the entire region to improve resource utilisation. When the target parking lot is full, the vehicles that should be assigned to the best parking lot can be directed to the adjacent parking lot, which is described in Figure.1. Suppose there are 35 parking requests, for which the optimal parking lot is A. For each request, the utility function of each parking lot is calculated, and the alternative parking lots are ranked according to the values of their utility functions. If the optimal parking lot is A and parking lot A has vacancies—that is, there are vacancies in the target parking lot that are higher than the number of cars allocated to A at this time unit plus considering the net outflow of vehicles from the target parking lot—this car is assigned to parking lot A. Otherwise, according to the value of the utility function, a sub-optimal parking lot and an inferior parking lot are found and the car is allocated to them. If neither parking lot meets the parking needs, the parking request is refused.

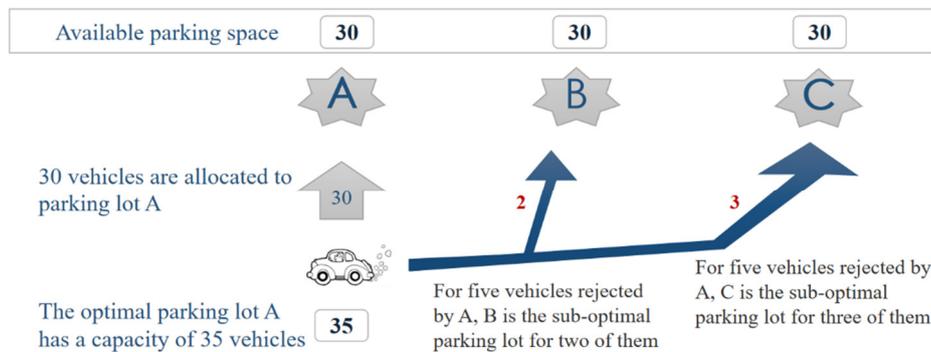


Figure 1. Schematic diagram of the sharing mode.

### 2.3.2. Non-Sharing Distribution Mode

The non-sharing model considers each parking lot as an independent individual, and the parking spaces are not shared with each other, which is described in Figure.2. When a parking lot is full, vehicles assigned to the parking lot are rejected even if parking spaces are available in the adjacent parking lots. The non-sharing model emphasizes that the allocated parking lot is the best choice. For example, there are 35 parking requests allocated to parking lot A according to the utility function. However, there are only 30 parking spaces in parking lot A, then 30 cars are allowed to enter parking lot A according to the sequence of requests, and the last 5 cars are rejected, irrespective of whether there are vacant seats in the adjacent parking lots B and C.

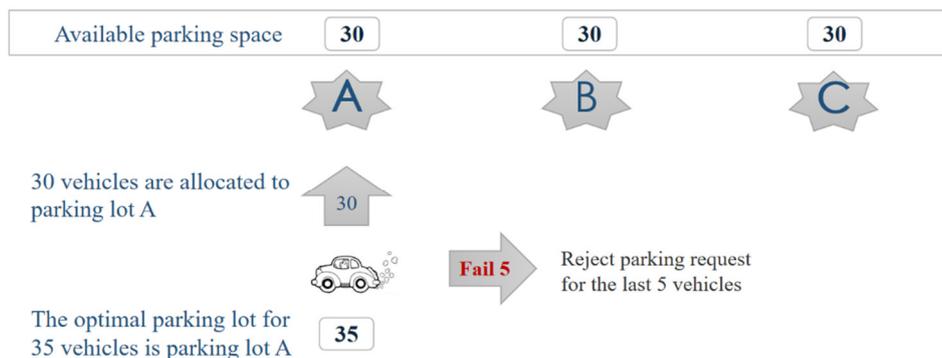


Figure 2. Schematic diagram of the non-sharing mode.

## 2.4. Performance Evaluation

### 2.4.1. Evaluation Indexes

The performance of the parking distribution models can be evaluated using multiple measures from the perspectives of both the private driver and public manager. From the perspective of the driver, shorter driving and walking distances may be preferred. However, from the perspective of the public manager, it is desirable to maximize the utilisation of parking resources with less traffic congestion even at the cost of drivers' convenience. If we only concentrate on one particular evaluation index, the evaluation results would be not comprehensive enough. To incorporate these perspectives into a parking distribution system, the following indicators are considered as performance measures:

(1) Average driving distance/duration: This measure indicates how far drivers drive from the location of the requested parking lot to the assigned parking lot. To calculate it, the total driving distance of all the guided cars is divided by the number of cars. From the personal and public viewpoints, the lower the value, the better.

(2) Average walking distance: Similar to the average driving distance measure, this measure shows how far drivers have to walk to reach their destination from the assigned parking lots. To calculate this, the total walking distance of all drivers is divided by the number of drivers. This measure tends to decrease, while the average driving distance increases.

(3) Average degree of congestion: For the benefit of the public, traffic congestion should be avoided. To this end, it is necessary to monitor the degree of traffic congestion. This measure indicates how severely traffic will be affected by the parking guidance itself. For simplicity, the degree of traffic congestion is estimated using the number of cars heading to the same parking lot on the road at a certain time, because too many cars being guided to a specific parking lot may cause heavy traffic congestion. The variance of the number of cars heading to each parking lot is calculated, and its average during the simulation is used as a measure to estimate the degree of traffic congestion. A lower value indicates that the guided cars are being directed to different parking facilities.

(4) Parking fail rate: There is the possibility of the driver failing to find a free parking space when the driver arrives at the assigned parking facility because the free parking lots became occupied by other cars while the driver was heading to the assigned parking facility. Parking failure leads to driver irritation and to the drivers not trusting the parking guidance system. Thus, it is important to monitor this measure and attempt to reduce it. To calculate the parking fail rate, the number of rejected cars at each parking facility is summed and divided by total number of parking requests during that time.

(5) Parking distribution ratio: Cars need to be assigned evenly to different parking lots. Thus, the situation where one parking lot is full but an adjacent parking lot is nearly empty should be avoided. To assess this objective, we calculated the standard deviation on the occupation ratio of all parking lots.

(6) Average utilisation: To maximize the efficiency of spatial resources in a city, the occupancy rate of parking lots should ideally be increased. To measure the occupancy rate of parking lots, in this study, the average utilisation measure was defined. It is calculated by dividing the occupied time of all parking spaces by the available time of all parking spaces. The average utilisation indicates the extent to which parking lots are occupied on an average. A higher value of this measure is beneficial for parking management of a city. It is not very relevant from the driver's perspective, but it is important from the public service and operational viewpoints.

### 2.4.2. Variation Coefficient Method

For the same allocation results, different evaluation indicators show different trends. Some evaluation indexes are high, while others are low. Therefore, we cannot choose the best result based on the six evaluation indexes separately. The variation coefficient method was adopted to perform a comprehensive evaluation. The final score obtained using the variation coefficient method was

defined as the performance score. First, the variation coefficient of each index is calculated with the following equation:

$$V_j = \frac{\sigma_j}{\bar{I}_j} \quad (6)$$

where  $V_j$  refers to the coefficient of variance of the  $j^{\text{th}}$  variable,  $\sigma_j$  refers to standard deviation of the  $j^{\text{th}}$  variable, and  $\bar{I}_j$  refers to the mean value of the  $j^{\text{th}}$  variable.

Then we get the weight of each index:

$$W_j = \frac{V_j}{\sum_{j=1}^n V} \quad (7)$$

Finally, we calculate the comprehensive performance score:

$$\text{Performance Score} = \sum_{j=1}^n W_j I_j \quad (8)$$

### 2.4.3. Threshold Setting

According to previous studies, the walking distance and walking time are important factors that influence people regarding the choice of a parking lot. Researchers [27] used the term “the 5-min walk” or “pedestrian shed” to describe the distance people are willing to walk before opting to drive. Based on the average walking speed, a 5-min walk is represented by a radius measuring 1/4 of a mile or approximately 400 m [28]. Based on the average walking speed, the time threshold of drivers to reach their destination from the parking lot was set to be 5 min, which is the most commonly cited value of previous studies [27].

In urban areas, searching for parking spaces is a significant problem. In New York City, the searching time for parking spaces is the highest: 15 min for on-street parking and 13 min for off-street parking. In San Francisco and Los Angeles, the average parking searching time is 12 min. The latest study conducted by INRIX [29], a Kirkland-based traffic data and auto-technology company, on parking search times revealed that drivers spend 9 min on average per trip searching for parking. In this study, we used 9 min as the tolerant threshold driving time to the parking lot. If there is no special explanation, our experimental results refer to the score based on the tolerance threshold.

Accordingly, if the walking time is less than 5 min and the driving time is less than 9 min, we regard the walking time and driving time to be acceptable by drivers. As we need to pay more attention to the perspective of the city manager, the walking time and driving time are excluded in the final evaluation score by setting their values to 0. When the driving duration and walking distance exceed 9 min/5 min, they will be included in the final evaluation score.

In Table 1,  $D_{all}$  refers to the total driving distance of all guided cars; Num refers to the number of all cars/number of drivers;  $W_{all}$  refers to the total walking distance of all drivers;  $\sum VarN$  refers to the variance of the number of cars heading to each parking lot;  $T_{all}$  refers to the number of simulation time intervals;  $F_i$  refers to the number of cars which are rejected by guided parking lot  $i$  because the parking lot is already full when they arrived;  $O_i$  refers to occupation ratio of parking lot  $i$ ;  $\bar{O}_i$  refers to the average occupation ratio of all parking lots; and  $A_i$  refers to the available time of parking lot  $i$ .

**Table 1.** Formula of the evaluation index and performance score.

	J	$I_j$	Without threshold	With threshold
Evaluation Index	1	Average driving distance/duration	$D_{all}/Num$	$\begin{cases} 0, & D_{all}/Num \leq 9 \\ D_{all}/Num^{-9}, & D_{all}/Num > 9 \end{cases}$
	22	Average walking distance	$W_{all}/Num$	$\begin{cases} 0, & W_{all}/Num \leq 5 \\ W_{all}/Num^{-5}, & W_{all}/Num > 5 \end{cases}$
	33	Average degree of congestion	$\sum VarN / T_{all}$	$\sum VarN / T_{all}$
	44	Parking fail rate	$\frac{\sum_{i=1}^3 F_i}{Num}$	$\frac{\sum_{i=1}^3 F_i}{Num}$
	55	Parking distribution ratio	$\sqrt{\frac{1}{3} \sum_{i=1}^3 (O_i - \bar{O}_i)}$	$\sqrt{\frac{1}{3} \sum_{i=1}^3 (O_i - \bar{O}_i)}$
Performance Score	66	Average utilisation	$\frac{\sum_{i=1}^3 O_i}{\sum_{i=1}^3 A_i}$	$\frac{\sum_{i=1}^3 O_i}{\sum_{i=1}^3 A_i}$
			$\sum_{j=1}^n W_j I_j$	$\sum_{j=1}^n W_j I_j$

### 3. Neural-Network-Based Dynamic Parking Distribution Model

The neural-network-based dynamic parking distribution model was designed to adapt to the changes in parking demand and in the real traffic situation. The parameter coefficients of the utility function were adjusted in real time to realize the dynamic berths allocation. Specifically, the neural-network model was trained first; this way we can get the prediction results of the performance score. Thus, the coefficient configuration providing the best improvement rates of the performance score compared to the current performance score was selected for the next operation.

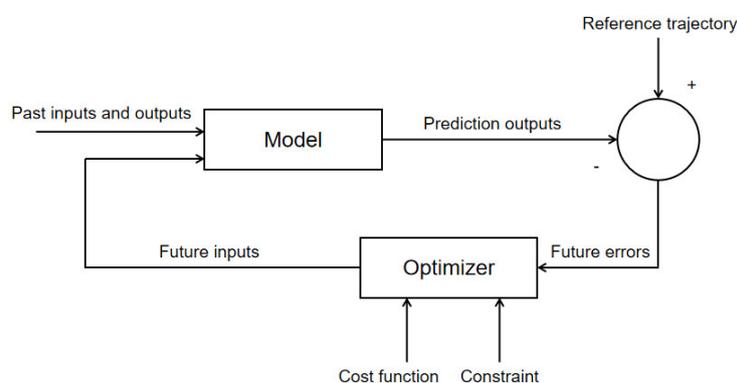
In the dynamic parking distribution model, the demand for parking and the availability of parking spaces can be predicted in advance and the distribution scheme of the parking spaces can be made accordingly. The allocation system works in a discrete time domain expressed as time interval T (measured in minutes) and the time interval T should be as small as possible to achieve the purpose of dynamic allocation. The allocation system only accepts and deals with parking requests at the beginning of each time interval. The time interval T is adjustable and different kinds of time intervals (i.e., 4 min, 6 min, 8 min, 10 min, etc.) were attempted in the experiment.

The expected contributions of the proposed dynamic parking distribution model include the following: (1) the dynamic parking distribution was realized by adjusting the coefficient configuration of the utility function in response to the changing of traffic situation; (2) time interval T can be adjusted within an optimal interval to attain the best allocation performance; and (3) the

allocation performance of the dynamic parking distribution model is better than that of the static parking distribution model with a fixed coefficient configuration of the utility function.

### 3.1. MIMO System

The dynamic parking distribution system can be modelled as a multiple input and multiple output (MIMO) system. Modern dynamic control techniques, such as fuzzy, neural, adaptive, and predictive control [30], can be applied to dynamically modify the coefficient configuration. Among them, predictive control is used in several applications owing to its fast response, which can predict the system output and select the most appropriate input to achieve better future outputs. Especially, model predictive control (MPC) (refer to Figure 3) is useful for predicting the future output of a system. The general MPC is composed of four elements: (1) the input variables to be controlled; (2) the system model to predict the future outputs; (3) an optimizer; and (4) the output variables of the system.



**Figure 3.** Model predictive control flow.

The problem encountered in applying MPC is that it is difficult to define the system model using an accurate description of the dynamic parking distribution system. To resolve this problem, we adopted a neural-network model. By minimizing the error of the system output, the best input variables of the neural-network model can be selected. The learning time of the neural network is also reasonably short so that it can be applied to real life. Therefore, a neural-network approach is considered to be suitable for an MPC model in a dynamic environment.

In the MIMO system, the relative information of all parking lots and the coefficient combination of the utility function act as input variables. The performance measures are both input variables and output variables.

### 3.2. Neural-Network-Based Model Development

Three matrixes were defined. The  $x$  matrix represents the basic information of all parking lots in the area, the  $y$  matrix represents the evaluation index of the distribution model proposed in Section 2.4, and the  $u$  matrix represents the coefficient configuration of the utility equation parameters proposed in Sections 2.1 and 2.2. The historical data of  $x$ ,  $y$ , and  $u$  was used to predict the value of  $y$  in the future moment of time with the neural network. Figure 4 depicts the overall scheme of the dynamic parking distribution model.

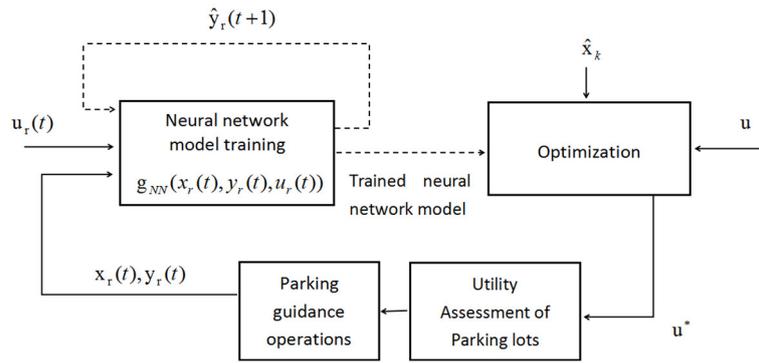


Figure 4. Overall scheme of the dynamic parking distribution model.

The inputs  $(x_r(t), y_r(t), \text{ and } u_r(t))$  of the neural-network model represent the historical data before the current simulation time  $t$ . Note that the parameter  $r$  indicates the past time horizon that is to be used to train the neural-network model.  $x_r(t)$  is a vector indicating the parking environment recorded at the previous  $r$  steps from the current simulation time  $t$ . It represents the historical status of parking demand and supply, which is composed of the number of parking lots available, the number of occupied parking spaces, and the number of cars requesting parking guidance.  $y_r(t)$  indicates the performance value vector of the dynamic parking distribution model at the previous  $r$  steps from the current simulation time  $t$ , which is composed of the six performance measures proposed in Section 2.4.  $u_r(t)$  is a vector composed of the seven coefficients of utility function that are described in Sections 2.1 and 2.2 at the previous  $r$  steps from the current simulation time  $t$ .

$y$  is both the input variables and output variables. The neural-network model is trained using the input variables to reduce the gap between the predicted values of the output variables  $\hat{y}(t + 1)$  and the operational results of the output variables  $y(t + 1)$ . The neural-network model  $g_{NN}$  is formulated as a discrete time nonlinear system described as follows:

$$\hat{y}(t + 1) = g_{NN}(x_r(t), y_r(t), u_r(t)) \quad (9)$$

$$x_r(t) = [x_t, x_{t-1}, \dots, x_{t-(r-1)}], \text{ where } x_t = [\beta_t, \gamma_t, \eta_t] \quad (10)$$

$$y_r(t) = [y_t, y_{t-1}, \dots, y_{t-(r-1)}], \text{ where } y_t = [P_1(t), P_2(t), P_3(t), P_4(t), P_5(t), P_6(t)] \quad (11)$$

$$u_r(t) = [u_t, u_{t-1}, \dots, u_{t-(r-1)}], \text{ where } u_t = [\alpha_1(t), \alpha_2(t), \alpha_3(t), \alpha_4(t), \alpha_5(t), \alpha_6(t), \alpha_7(t)] \quad (12)$$

where

$r$  Past time horizon

$\alpha_i(t)$  Coefficient of  $i^{th}$  parameter in the utility function at time  $t, i = 1, 2, 3, 4, 5, 6, 7$

$\beta(t)$  The available number of parking lots at simulation time  $t$

$\gamma(t)$  The occupied number of parking lots at simulation time  $t$

$\eta(t)$  The number of cars requesting parking at simulation time  $t$

$P_i(t)$  The value of the performance measure  $i$  predicted by the neural-network model under the given input  $x_t = [\beta_t, \gamma_t, \eta_t]$  at simulation time  $t, i = 1, 2, 3, 4, 5, 6$

We built a three-layer neural network to predict the one-step-ahead values of the performance measures of  $y_r(t)$ . The input layer had 16 units, and the output layer had six units. The other network parameters, such as the training cycle, neurons in the hidden layer, learning rate, and momentum term, were selected by performing simulations based on a trial-and-error approach. The model was trained using different configurations of these parameters to achieve the maximum predictability of the network for the test data by analysing the root mean square error (RMSE). This was achieved by keeping the selected parameters constant and slowly moving the other parameters over a wide range of values, as suggested in previous studies [31].

We built support vector machine (SVM) models to predict the values of  $x_r(t)$ . Previous studies [32] have demonstrated that SVM performs better than other models in predicting parking demand and supply. The radial basis function (RBF) was used as a kernel function, and the support vector regression (SVR) was selected as an SVM model. The penalty parameter,  $C$ , and kernel parameter,  $\gamma$ , were selected based on a grid search with cross-validation. In the MPC, the best control move of input variables was decided by optimizing a cost function. For unit  $t$ , we used the value of  $\hat{y}(t+1)$  predicted by the neural network and the value of  $\hat{x}_k$  predicted by the SVM. For each coefficient configuration, the future values of each performance measure  $y(t+2)$  were predicted using the trained neural network. Using the predicted future performance, the sum of the improvement rates of all performance measures from the current ones,  $y(t)$ , was calculated and was used as the value of the cost function. By comparing the future costs according to the coefficient configuration, the configuration of coefficients providing the highest performance improvement (in other words, giving the lowest value of the cost function) was selected as the best coefficient configuration and was used as the new coefficient configuration for the next parking guidance operation, unit  $t+1$ .

$$J = \sum_{i=1}^5 \sum_{k=1}^n \frac{\hat{y}_{ik} - y_i}{y_i} - \sum_{k=1}^n \frac{\hat{y}_{6k} - y_6}{y_6} \quad (13)$$

where

$i$  Indicator for performance measure,  $i = 1, 2, 3, 4, 5, 6$

$k$  Indicator for prediction horizon,  $k = 1, 2, \dots, n$

$n$  Prediction horizon

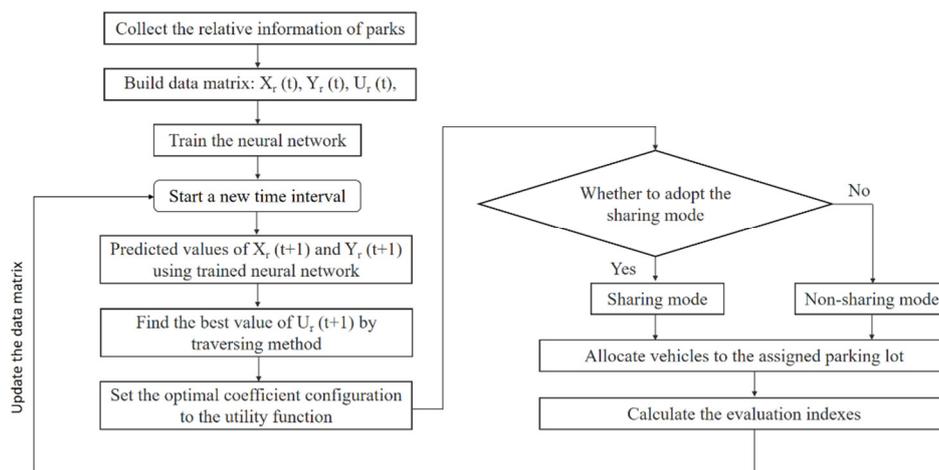
$y_{ik}$  Predicted value of performance measure  $i$  at  $k$ th prediction horizon step

$y_i$  Current value of the performance measure  $i$

$J$  Value of a cost function

### 3.3. Procedure of Dynamic Parking Distribution

The detailed description of the proposed parking space assigning procedure is provided as follows. Figure 5 depicts the overall procedure of the dynamic parking distribution model at each simulation time.



**Figure 5.** Overall procedure of the dynamic parking distribution model.

#### Step 1. Keep track of the current status of parking lots

In this study, the current status of each parking lot is tracked, and the database that records all relevant indicators of parking lots is updated. In addition to the number of available parking spaces, other data related to parking facilities, such as the number of occupied parking spaces, total number

of parking spaces, location, and parking fee, are also stored in the database. The stored data are updated whenever changes occur in parking facilities.

*Step 2. Train the neural network*

First, the neural network is trained to predict the one-step-ahead values of performance measures using the data recorded during parking guidance operations. The inputs ( $x_r(t)$ ,  $y_r(t)$ , and  $u_r(t)$ ) of the neural-network model represent the historical data before the current simulation time  $t$ . With the input variables, the neural-network model is trained to reduce the gap between the predicted values of the output variables  $\hat{y}(t + 1)$  and the operation results of the output variables  $y(t + 1)$ .

*Step 3. Find the best configuration of coefficients at the start of each unit time*

Use a trained neural network to predict the one-step-ahead values of the six performance measures and use SVM to predict the one-step-ahead value of the parking environment. Calculate the cost function value of every coefficient configuration and select the coefficient configuration giving the highest performance improvement as the new coefficient configuration for this parking guidance operation.

*Step 4. For each parking facility, calculate the value of the parking utility function*

Calculate the value of the parking utility function based on the optimal coefficient configuration in Step 3. Among all parking facilities, the parking facility with the lowest value of the parking utility function is suggested to the driver, and a detailed direction guide to the suggested parking facility is provided.

*Step 5. Assign the car to dynamically select the shared mode or non-sharing mode*

According to the judgment regarding the relative merits of the sharing and non-sharing modes, the dynamic parking distribution model determines whether to use the sharing mode or the non-sharing mode.

*Step 6. Repeat Steps 2–5 for all parking requests at a certain time quantum*

When a new time unit starts, go to Step 2.

### 3.4. Assumptions of the Model

The model was developed and implemented under the following assumptions:

(1) Once a driver with a parking request is assigned to a parking lot, the driver will proceed to the park without changing his destination or the assigned park.

(2) If there is no parking space when the car arrives at the suggested parking facility, the driver will leave the parking lot.

(3) The estimated driving duration and walking time are the same as the actual occurring values.

(4) The impact on the increase of driving duration caused by traffic congestion is not considered in this study.

(5) If two parking lots have the same utility value for a vehicle, it will be allocated to the parking lot with a shorter driving distance. If more than one vehicle is allocated to a parking lot at the same time, but there is not enough available parking space in the parking lot, the vehicles that arrive at the parking lot first have has priority to use the parking space.

### 3.5. Development of a Static Distribution Model for Comparison

Given the utility function, a static parking distribution model was developed for comparison. By assigning coefficients to the seven parameters of the utility equation, the utility values of the different parking lots were calculated and compared, and the parking lots with lower utility values were returned to the drivers as results. In the static parking distribution model, the coefficients are fixed over a period of time. The allocation algorithm does not change because of changes in the parking supply and demand and traffic environment, which is not the case with the dynamic distribution model. Figure 6 depicts the overall procedure of the static parking distribution model at each assigning time unit. To verify the effectiveness and superiority of the dynamic allocation model, the results of the dynamic allocation model are compared with those of the static allocation model in the following case study.

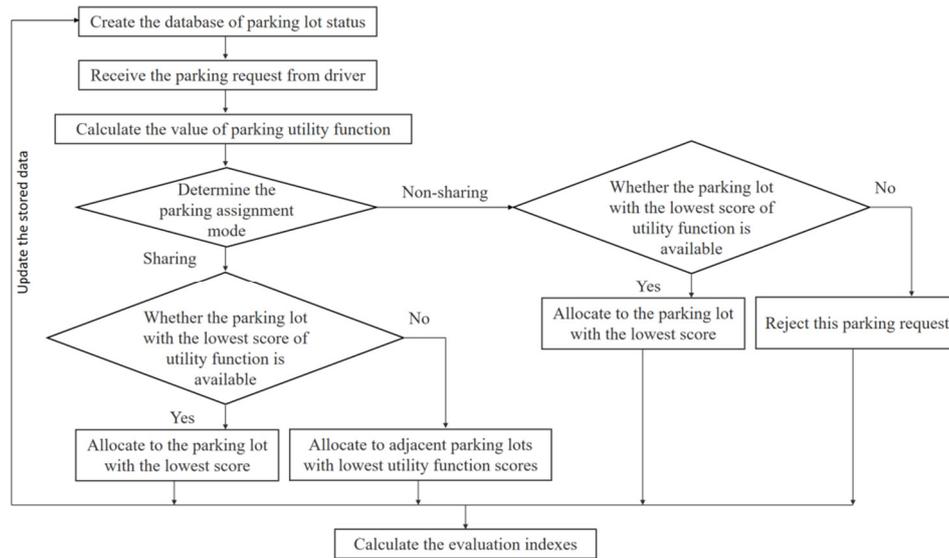


Figure 6. Overall procedure for the static parking distribution model.

#### 4. Case Study

##### 4.1. Overall Scheme

The overall scheme of the case study was shown in Figure 7. First, the parking data were collected. Three adjacent parking lots located in Shenzhen were selected to obtain the records, and the sharing scheme was verified from three aspects. Next, a utility function was comprehensively established considering seven factors. Both the sharing and non-sharing modes and performance evaluation indexes were defined, based on which a dynamic parking distribution model based on neural networks was developed and trained. Finally, the performance of the dynamic parking distribution model was analysed with the case study.

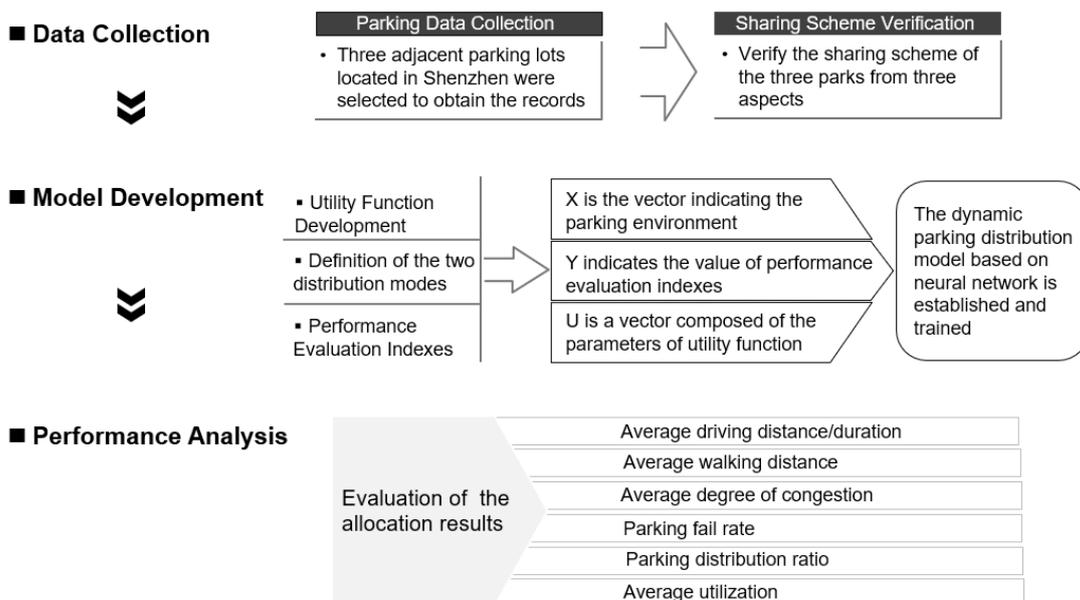


Figure 7. Schematic diagram of the case study.

#### 4.2. Parking Data Collection

Part of the data used in the case were collected through field research, and some data that could not be directly obtained were generated via simulation. The real data and partial simulation data in the case study are summarized in Table 2.

**Table 2.** Data used in the case study.

Data Type	Data Used in The Case	Acquisition Method
Real World Data	Latitude and longitude of parking lot	Investigation and collection
	Vehicle entry and exit rate in parking lots	
	Capacity of each parking lot	
	Parking fee	
Simulation Data	Vehicles' original location and destination	Random distribution
	Requesting parking number	Uniform distribution
	Duration of stay	Random distribution

The real case study was conducted in Shenzhen, in the Luohu district. Three adjacent parking lots located in Shenzhen were selected, and records of entries and exits for seven consecutive weeks were obtained. The information regarding the three parking lots is listed in Table 3. The parking data for 55 days, from June 2, 2018, to July 27, 2018, of the three parking lots were collected. During the analysis, there were 606,959 records of the times at which cars entered and left the parking lot, and the number of cars entering and leaving the parking lot per unit time was calculated.

**Table 3.** Basic information of the three parking lots.

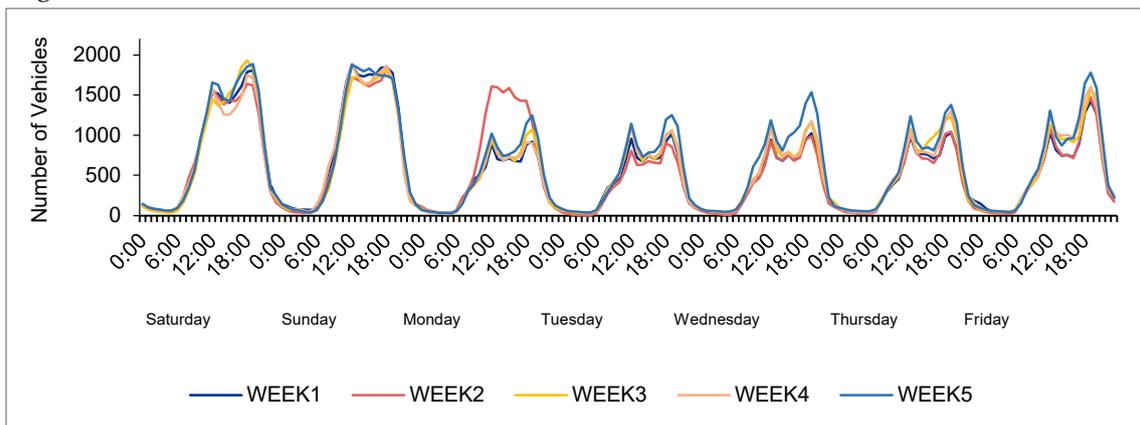
Name	Haiya	Zi'an	Jingfa
Abbreviation	PL1	PL2	PL3
City	Shenzhen	Shenzhen	Shenzhen
Scale	2000	170	660
Initial Vehicles	64	16	0
Type	Commercial parking	Office parking	Office parking
Address	No. 99, Jian'an 1st Road, Bao'an District, Shenzhen	Zi'an business building, No. 71, Longjiang second lane, Bao'an District, Shenzhen	Jingfa building, No. 46, Baoqian lane, Bao'an District, Shenzhen

As shown in Figure 8, the bigger rectangle is the research region and the smaller one is the central region. The area of destination generation was narrowed to the central city region because the city centre has a higher parking demand. Google Maps was used to estimate the driving duration for each car from the starting point to the alternative parking lots and the walking distance from the assigned parking lots to the destination. The global positioning system (GPS) data of each parking facility, including latitude, longitude, and distance between the landmarks, were acquired from the geographical information system (GIS) of Google Maps. The capacity, parking fee, and vehicle entry and exit records of each parking facility were determined based on offline investigations.

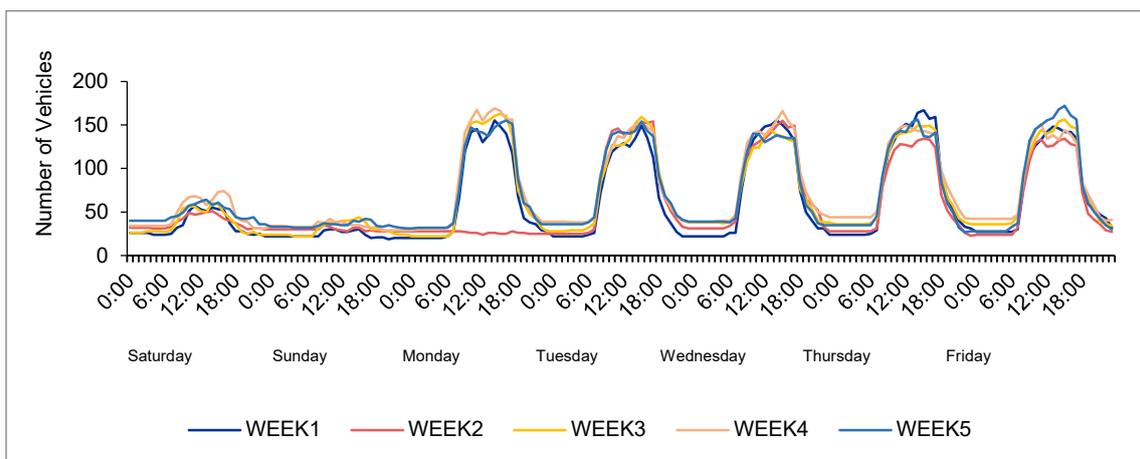


Figure 8. Research region and the core area for research.

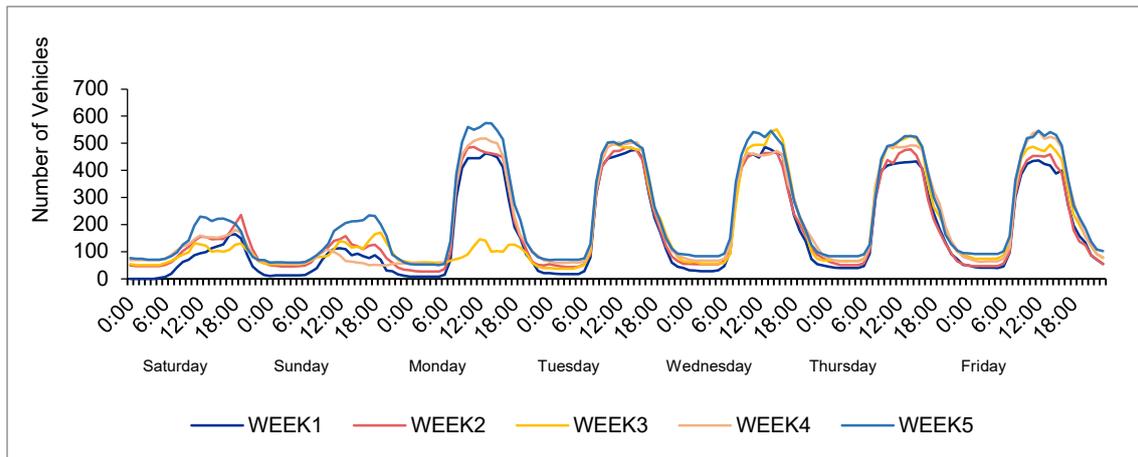
According to the entry and exit records of the three parking lots, we calculated the number of vehicles parked in the parking lots per hour. The initial number of vehicles for each parking lot were obtained through inquiry. The initial number of vehicles for PL1, PL2, and PL3 were 64, 16, and 0, respectively. The parking curves of the three parking lots for the five consecutive weeks are shown in Figure 9.



a)



b)



c)

**Figure 9.** Parking curves of parking lots 1, 2, and 3. (a) Parking curve of parking lot 1; (b) parking curve of parking lot 2; and (c) parking curve of parking lot 3.

There are three conditions for parking space sharing: complementary time, close parking location, and mutual accommodation of parking spaces [25]. Based on the parking curves of the three parking lots, it can be concluded that the parking demand at commercial parking lot PL 1 is higher during the weekends, as evidenced by the peak positions of the parking curve. In contrast, PL 2 and PL 3 are office parking lots, and the parking demands are larger on weekdays. The parking curves for PL 2 and PL 3 peak on the weekdays. Therefore, the three parking lots are complementary in time. Because the total parking demand of any day does not exceed the total parking space capacity of the three parking lots, the parking spaces of three parking lots are mutually compatible. In addition, the three parking lots are closely located, so they have the potential for sharing, and the parking distribution algorithms under the sharing mode can be implemented.

#### 4.3. Parameter Setting

We used 80% of the data as the training dataset, and 20% of the data as the test dataset. We built a three-layered neural network. The input layer has 16 neurons and the output layer has 6 neurons. The number of neurons in the hidden layer is determined by the following experience formula:

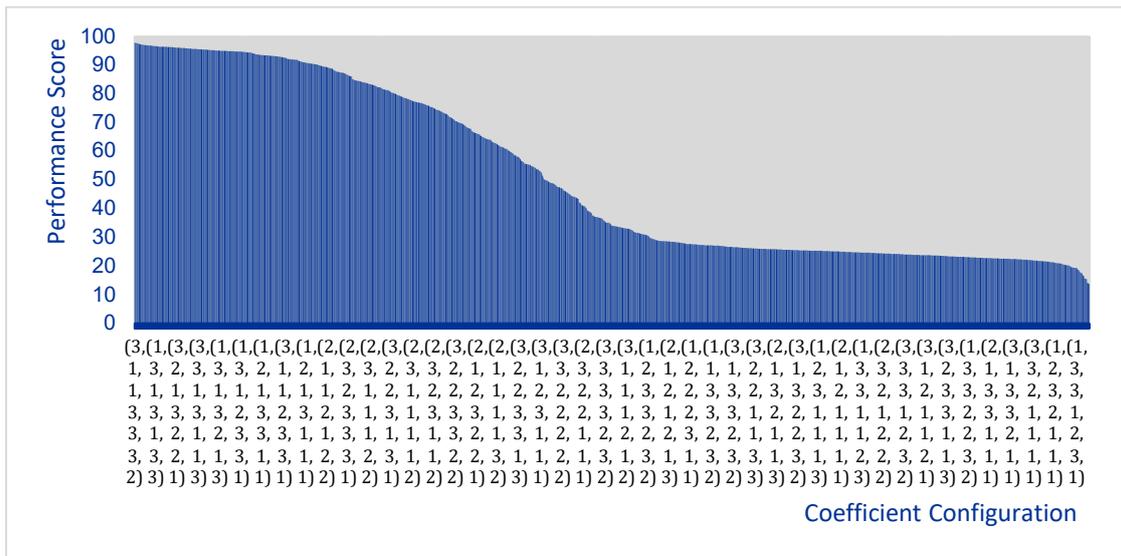
$$h = \sqrt{m + n} + a \quad (14)$$

where  $h$  is the number of neurons in the hidden layer,  $m$  is the number of neurons in the input layer,  $n$  is the number of neurons in the output layer, and  $a$  is a regulation constant between 1 to 10. ReLU is used as the activation function for hidden layer.

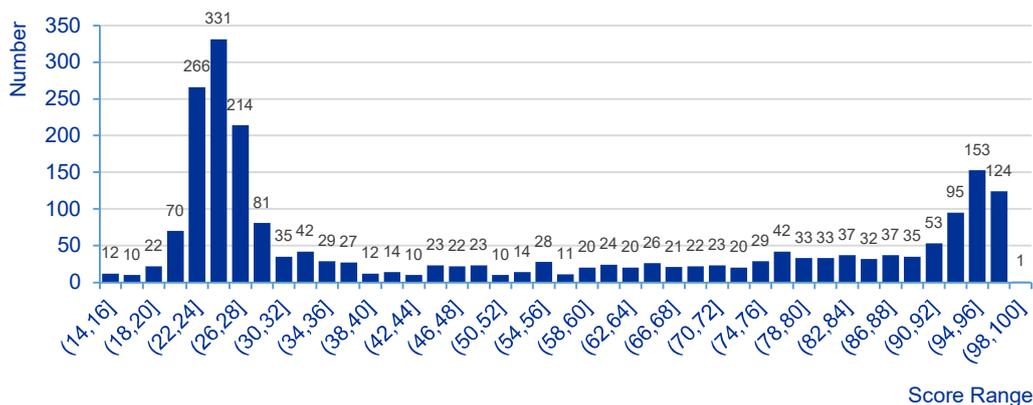
The network parameters, including the training cycle, learning rate, and momentum term, were selected by performing simulations using a trial-and-error approach. The model was trained using different combinations of these parameters to achieve the maximum predictability of the network for the test data by analysing the RMSE. This was achieved by keeping the selected parameters constant and slowly varying the other rest parameters over a wide range of values.

#### 4.4. Results of the Static Parking Distribution Model

As described in Section 2.1, there are  $3^7$  types of coefficient configurations for the utility function. To comprehensively test the allocation performance of the static parking distribution model, the  $3^7$  coefficient configurations were used as the allocation benchmark to assign the parking request within 24 h under the sharing mode because the performance of the sharing mode is better than that of the non-sharing mode, which was tested during the preparation work for this study. The statistical performance scores of the  $3^7$  coefficient configurations are shown in Figures 10 and 11.



**Figure 10.** Scatter diagram of the performance scores of  $3^7$  coefficient configurations for the static parking distribution model (T = 6 min).



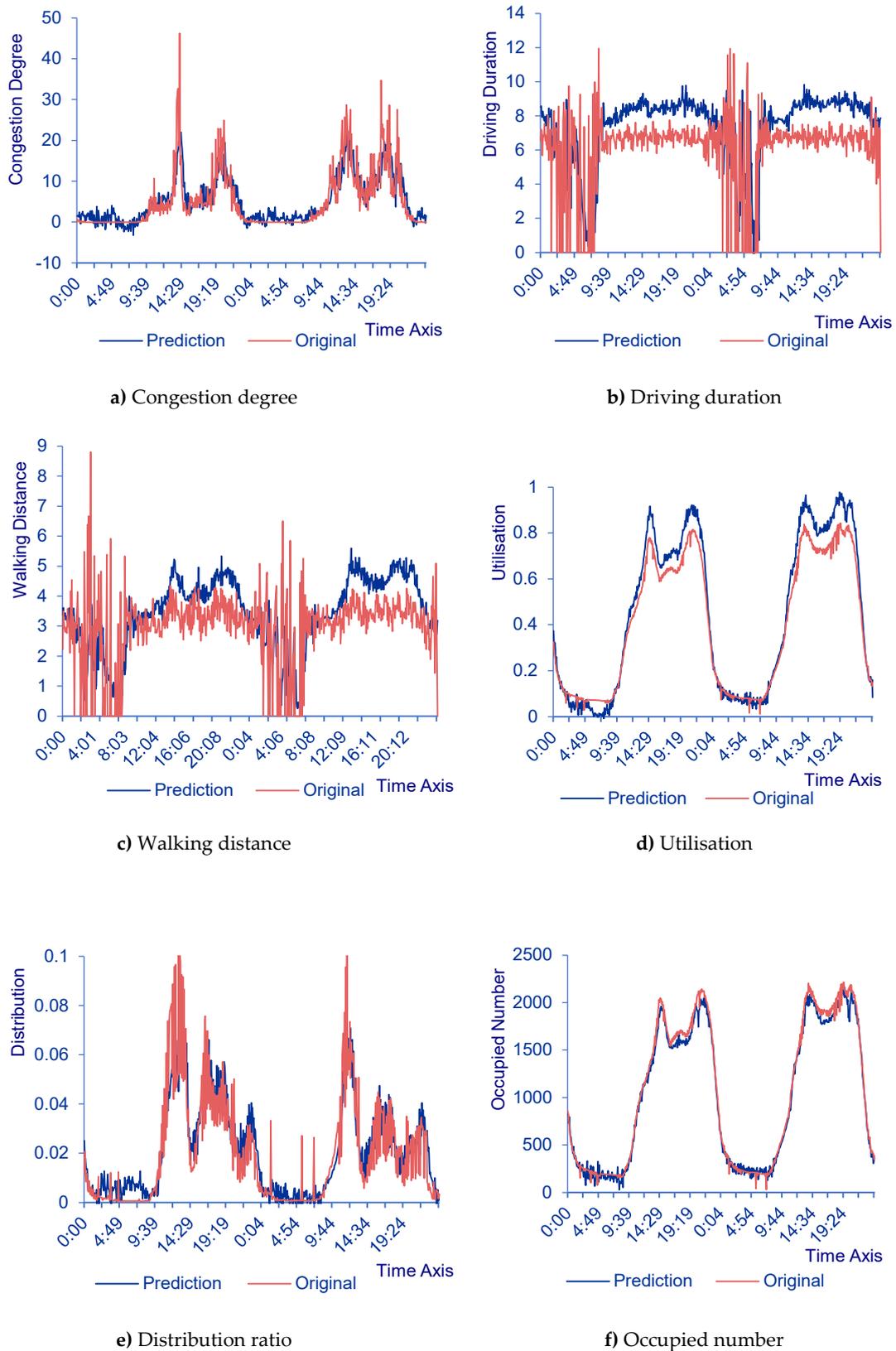
**Figure 11.** Statistical performance scores of the  $3^7$  coefficient configurations.

The highest allocation score of the static parking distribution model was 97.8, and the lowest score was 13.7. A total of 426 coefficient configurations scored more than 90 points. Taking T = 2 min as the interval step length, the scores of the static parking distribution model in the case were statistically plotted, and the score distribution presented a double-hump shape; the humps were larger in the low-scoring range.

#### 4.5. Results of the Dynamic Parking Distribution Model

##### 4.5.1. Effectiveness of the Neural-Network-Based Dynamic Model

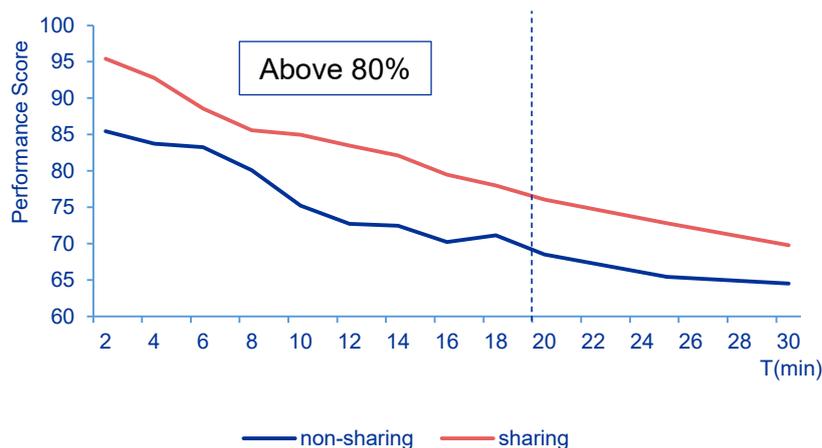
Before using a neural-network model to predict and allocate parking spaces, first, the neural network needs to be trained to ensure its validity and accuracy. After training the neural network, the values of the loss function and mean square error (MSE) decreased and tended to become stable. The X-matrix and Y-matrix were tested by the neural-network model using the test dataset. Through a comparison between the predicted and original values, the model was found to be good enough to accurately capture the trend of most of the evaluation indexes. The model gave excellent prediction in terms of congestion degree, utilisation, distribution ratio, and occupied number. The results were shown in Fig.12.



**Figure 12.** The predicted values and the original values of the X-matrix and Y-matrix used in the neural-network model. (a) Congestion degree; (b) driving duration; (c) walking distance; (d) utilisation; (e) distribution ratio; (f) occupied number.

#### 4.5.2. Determination of the Optimal Time Interval T

In this part, the impact of time interval T on the allocation results of the dynamic parking distribution model was explored. According to the statistical analysis of the dataset, the number of parking requests per hour N was found to be no more than 1100. To make the results of the sharing and non-sharing modes significantly different, it is assumed that three quarters of the parking spaces in PL1 and one half of the parking spaces in PL3 are unusable due to some objective factors like parking facility maintenance. By setting  $T = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 25,$  and 30 min, the performance scores of the dynamic parking distribution model considering the waiting factor were calculated; the results are as follows.



**Figure 13.** Performance scores of the changes in the dynamic parking distribution model with the changes of T in the sharing mode and non-sharing mode.

As can be seen from the results in Figure. 13, the performance score slopes downward with an increasing T, and the optimal range of assigning time T is considerably broad. Compared with the best performance score, the performance scores when T is between 2 min and 20 min were all above 80% of the highest scores. Therefore, rational distribution results can be obtained when T is kept within 20 min. Within the capacity of the computing system, the smaller the T value, the better the performance. Moreover, the performance of the dynamic parking distribution model in the sharing mode was found to be better than that in the non-sharing mode, with a score improvement rate of 6–15%.

To analyse the reasons for the excellent performance of the sharing mode over the non-sharing mode for the dynamic parking distribution model, six evaluation indexes with a large score difference of  $T = 4$  min were selected for comparison, and the results are shown in Table 4.

**Table 4.** Comparison of the evaluation indexes of the sharing mode and non-sharing mode for the dynamic parking distribution model without a tolerance threshold ( $T = 4$  min).

Distribution mode	Non-Sharing	Sharing
Congestion	1.9	1.5
Driving duration	6.7	6.7
Utilization	0.51	0.51
Walking distance	3.2	3.2
Fail rate	$3.1 \times 10^{-2}$	$1.7 \times 10^{-2}$
Distribution	$8.0 \times 10^{-3}$	$7.0 \times 10^{-3}$

For the dynamic parking distribution model, the performance of the sharing mode was better than that of the non-sharing mode mainly because of less traffic congestion and the lower parking

rejection rate. The driving duration and walking distance in the shared mode were slightly worse than those in the non-sharing mode. There is not much difference between the utilisation and the spatial distribution.

#### 4.5.3. Comparison between the Adjusting Coefficient Configuration and Fixed Coefficient Configuration

Because the dynamic parking distribution model can optimize the allocation results at every time interval, the overall allocation performance is theoretically better than the static parking distribution model. To verify this reasonable conjecture, the top ten results of the fixed coefficient configuration in the static model were compared with those of the dynamic parking distribution model. The result was shown in Table 5.

**Table 5.** Allocation performance of the dynamic parking distribution model compared with the top ten alternatives of the static parking distribution model with all parking spaces open.

T = 2 min				T = 4 min					
Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score		
Dynamic	Weight adjusting	65.4		Dynamic	Weight adjusting	84.6			
	(3, 1, 1, 3, 2, 1, 2)	64.6	$1.0 \times 10^{-2}$		(3, 1, 1, 3, 3, 2, 1)	77.5	$9.0 \times 10^{-2}$		
	(3, 1, 1, 3, 3, 1, 2)	64.3	$2.0 \times 10^{-2}$		(3, 1, 1, 3, 2, 2, 1)	77.4	$9.0 \times 10^{-2}$		
	(3, 1, 1, 3, 3, 2, 1)	35.4	0.85		(3, 1, 1, 3, 1, 1, 1)	71.6	0.18		
	(3, 1, 1, 3, 2, 2, 1)	34.3	0.91		(3, 1, 1, 3, 1, 2, 1)	71.2	0.19		
	Static	(3, 1, 1, 3, 1, 2, 1)	30.8		1.1	Static	(3, 1, 1, 3, 2, 1, 1)	69.0	0.23
		(3, 1, 1, 3, 2, 1, 1)	28.3		1.3		(3, 1, 1, 3, 3, 1, 1)	67.6	0.25
		(3, 1, 1, 3, 1, 1, 1)	27.8		1.4		(3, 1, 1, 3, 2, 2, 2)	33.6	1.5
		(3, 1, 1, 3, 3, 1, 1)	26.8		1.4		(3, 1, 1, 3, 2, 1, 2)	30.7	1.8
(3, 1, 1, 3, 2, 2, 2)		21.9	2.0	(3, 1, 1, 3, 1, 1, 2)	29.8		1.8		
(3, 1, 1, 3, 1, 1, 2)	14.1	3.6	(3, 1, 1, 3, 3, 1, 2)	19.1	3.4				
T = 6 min				T = 8 min					
Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score		
Dynamic	Weight adjusting	79.3		Dynamic	Weight adjusting	79.1			
	(3, 1, 1, 3, 2, 2, 1)	79.1	$2.0 \times 10^{-3}$		(3, 1, 1, 3, 3, 1, 1)	68.8	0.15		
	(3, 1, 1, 3, 3, 2, 1)	74.9	$5.8 \times 10^{-2}$		(3, 1, 1, 3, 2, 1, 1)	66.9	0.18		
	(3, 1, 1, 3, 1, 2, 1)	74.6	$6.3 \times 10^{-2}$		(3, 1, 1, 3, 1, 1, 1)	66.6	0.19		
	(3, 1, 1, 3, 3, 1, 1)	47.0	0.69		(3, 1, 1, 3, 1, 2, 1)	54.9	0.44		
	Static	(3, 1, 1, 3, 2, 1, 1)	43.2		0.84	Static	(3, 1, 1, 3, 2, 2, 1)	51.7	0.53
		(3, 1, 1, 3, 1, 1, 1)	42.3		0.88		(3, 1, 1, 3, 1, 1, 2)	49.7	0.59
		(3, 1, 1, 3, 1, 1, 2)	29.0		1.7		(3, 1, 1, 3, 2, 1, 2)	48.2	0.64
		(3, 1, 1, 3, 2, 2, 2)	27.8		1.9		(3, 1, 1, 3, 3, 1, 2)	47.8	0.66
(3, 1, 1, 3, 3, 1, 2)		27.5	1.9	(3, 1, 1, 3, 3, 2, 1)	41.6		0.90		

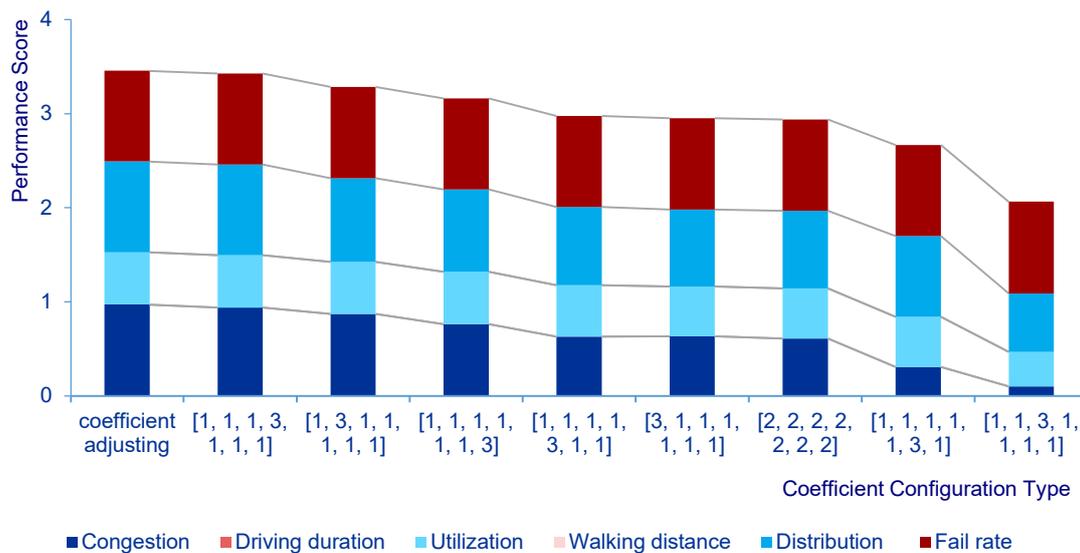
T = 10 min				T = 12 min			
Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score
	(3, 1, 1, 3, 2, 1, 2)	27.2	1.9		(3, 1, 1, 3, 2, 2, 2)	25.0	2.2
Dynamic	Weight adjusting	77.8		Dynamic	Weight adjusting	78.9	
	(3, 1, 1, 3, 3, 2, 1)	65.1	0.15		(3, 1, 1, 3, 2, 1, 1)	61.7	0.28
	(3, 1, 1, 3, 2, 1, 1)	63.7	0.18		(3, 1, 1, 3, 3, 1, 1)	59.6	0.32
	(3, 1, 1, 3, 1, 2, 1)	63.3	0.19		(3, 1, 1, 3, 2, 2, 1)	59.5	0.33
	(3, 1, 1, 3, 3, 1, 1)	62.4	0.44		(3, 1, 1, 3, 1, 2, 1)	57.6	0.37
Static	(3, 1, 1, 3, 1, 1, 1)	61.6	0.53	Static	(3, 1, 1, 3, 1, 1, 1)	55.4	0.43
	(3, 1, 1, 3, 2, 2, 1)	60.6	0.59		(3, 1, 1, 3, 3, 2, 1)	49.6	0.59
	(3, 1, 1, 3, 1, 1, 2)	48.9	0.64		(3, 1, 1, 3, 2, 2, 2)	38.5	1.1
	(3, 1, 1, 3, 3, 1, 2)	48.5	0.66		(3, 1, 1, 3, 1, 1, 2)	35.2	1.2
	(3, 1, 1, 3, 2, 1, 2)	47.8	0.90		(3, 1, 1, 3, 3, 1, 2)	34.3	1.3
	(3, 1, 1, 3, 2, 2, 2)	36.1	2.2		(3, 1, 1, 3, 2, 1, 2)	32.3	1.4
T = 14 min				T = 16 min			
Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score
Dynamic	Weight adjusting	87.3		Dynamic	Weight adjusting	66.8	
	(3, 1, 1, 3, 1, 1, 1)	57.7	0.51		(3, 1, 1, 3, 1, 1, 1)	62.3	$7.2 \times 10^{-2}$
	(3, 1, 1, 3, 2, 1, 1)	57.6	0.52		(3, 1, 1, 3, 3, 1, 1)	60.9	0.10
	(3, 1, 1, 3, 1, 2, 1)	55.9	0.56		(3, 1, 1, 3, 2, 1, 1)	57.2	0.17
	(3, 1, 1, 3, 3, 1, 1)	55.8	0.57		(3, 1, 1, 3, 1, 2, 1)	55.0	0.21
Static	(3, 1, 1, 3, 2, 2, 1)	54.2	0.61	Static	(3, 1, 1, 3, 3, 2, 1)	54.7	0.22
	(3, 1, 1, 3, 3, 2, 1)	52.4	0.67		(3, 1, 1, 3, 2, 2, 1)	53.2	0.26
	(3, 1, 1, 3, 2, 2, 2)	46.2	0.89		(3, 1, 1, 3, 2, 2, 2)	43.6	0.53
	(3, 1, 1, 3, 1, 1, 2)	40.4	1.2		(3, 1, 1, 3, 1, 1, 2)	40.8	0.64
	(3, 1, 1, 3, 3, 1, 2)	37.7	1.3		(3, 1, 1, 3, 2, 1, 2)	37.9	0.76
	(3, 1, 1, 3, 2, 1, 2)	35.2	1.5		(3, 1, 1, 3, 3, 1, 2)	37.8	0.77
T = 18 min				T = 20 min			

Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score
Dynamic	Weight adjusting	65.1		Dynamic	Weight adjusting	75.5	
	(3, 1, 1, 3, 1, 1, 1)	63.2	$3.0 \times 10^{-2}$		(3, 1, 1, 3, 3, 1, 1)	62.6	0.21
	(3, 1, 1, 3, 1, 2, 1)	61.3	$6.2 \times 10^{-2}$		(3, 1, 1, 3, 1, 1, 1)	62.0	0.22
	(3, 1, 1, 3, 2, 1, 1)	60.7	$7.3 \times 10^{-2}$		(3, 1, 1, 3, 2, 1, 1)	58.7	0.29
Static	(3, 1, 1, 3, 3, 1, 1)	60.0	$8.6 \times 10^{-2}$	Static	(3, 1, 1, 3, 2, 2, 1)	47.2	0.60
	(3, 1, 1, 3, 3, 2, 1)	48.8	0.33		(3, 1, 1, 3, 3, 2, 1)	43.9	0.72
	(3, 1, 1, 3, 2, 2, 1)	42.5	0.53		(3, 1, 1, 3, 1, 2, 1)	43.8	0.72
	(3, 1, 1, 3, 3, 1, 2)	42.0	0.55		(3, 1, 1, 3, 1, 1, 2)	37.3	1.0
	(3, 1, 1, 3, 2, 2, 2)	41.5	0.57		(3, 1, 1, 3, 2, 2, 2)	36.8	1.1
	(3, 1, 1, 3, 2, 1, 2)	39.4	0.65		(3, 1, 1, 3, 2, 1, 2)	33.3	1.3
	(3, 1, 1, 3, 1, 1, 2)	39.4	0.65		(3, 1, 1, 3, 3, 1, 2)	29.1	1.6
	T = 25 min				T = 30 min		
Model	Coefficient configuration	Score	Increase rate of performance score	Model	Coefficient configuration	Score	Increase rate of performance score
Dynamic	Weight adjusting	80.6		Dynamic	Weight adjusting	96.7	
	(3, 1, 1, 3, 3, 1, 1)	76.9	$4.8 \times 10^{-2}$		(3, 1, 1, 3, 2, 1, 2)	91.7	$5.5 \times 10^{-2}$
	(3, 1, 1, 3, 2, 1, 1)	75.7	$6.4 \times 10^{-2}$		(3, 1, 1, 3, 3, 1, 2)	91.3	$6.0 \times 10^{-2}$
	(3, 1, 1, 3, 1, 1, 1)	72.1	0.12		(3, 1, 1, 3, 1, 1, 2)	90.4	$7.0 \times 10^{-2}$
Static	(3, 1, 1, 3, 3, 1, 2)	71.7	0.12	Static	(3, 1, 1, 3, 3, 1, 1)	86.6	0.12
	(3, 1, 1, 3, 1, 1, 2)	70.1	0.15		(3, 1, 1, 3, 2, 1, 1)	86.1	0.12
	(3, 1, 1, 3, 2, 1, 2)	68.1	0.18		(3, 1, 1, 3, 1, 1, 1)	86.0	0.13
	(3, 1, 1, 3, 2, 2, 2)	46.2	0.74		(3, 1, 1, 3, 2, 2, 2)	45.8	1.1
	(3, 1, 1, 3, 1, 2, 1)	10.0	7.0		(3, 1, 1, 3, 3, 2, 1)	17.2	4.6
	(3, 1, 1, 3, 3, 2, 1)	6.1	12		(3, 1, 1, 3, 1, 2, 1)	5.4	17
	(3, 1, 1, 3, 2, 2, 1)	5.4	$1.4 \times 10^1$		(3, 1, 1, 3, 2, 2, 1)	0.6	$1.6 \times 10^2$

Note: The increase rate of the performance score indicates the increase percentage of the performance score of the dynamic model compared with that of the static distribution model.

It can be seen from the experimental results that under different assigning time intervals  $T$ , the distribution results of the dynamic parking distribution model are better than those of the top ten alternatives of the fixed coefficient configuration in the static model. The allocation scores show that, compared with the ten alternatives of the static models, the improvement degrees of the dynamic model are different: the lowest increase rate is 0.2% and the highest increase rate is nearly 158. The dynamic parking distribution model has better applicability in the real case.

To analyse the reasons for the excellent performance of the adjusting coefficient configuration over the fixed coefficient configurations, six evaluation indexes when  $T = 4$  min were selected for comparison, and the results are shown as Figure 14. In addition, to highlight the final performance score more clearly, the representative eight coefficient configurations were selected for comparison because the scores of the adjusting coefficient configuration and the top ten fixed coefficient configurations were very close.

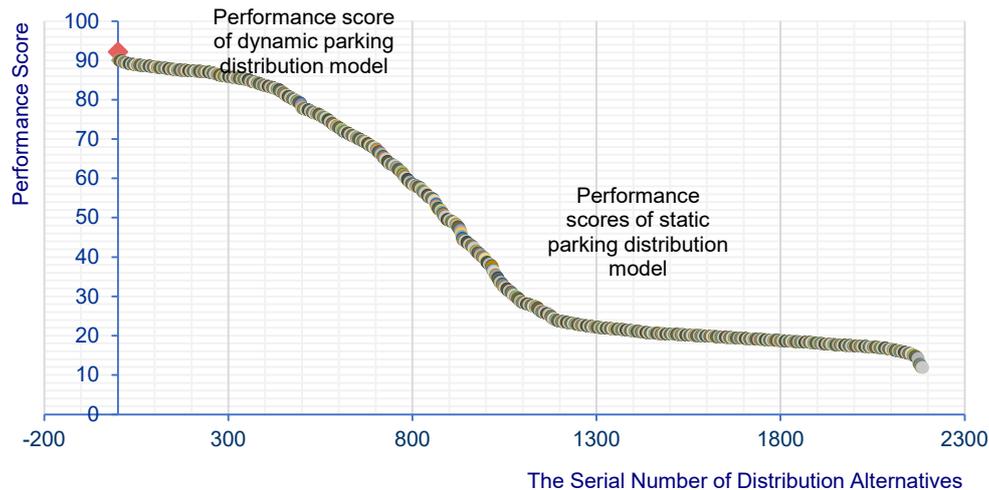


**Figure 14.** Comparison of the adjusting coefficient configuration over the fixed coefficient configurations with a tolerance threshold ( $T = 4$  min).

The performance of the adjusting coefficient configuration was better than that of the fixed coefficient configuration mainly because of less traffic congestion. The utilisation, the spatial distribution, and fail rate of the adjusting coefficient configuration were slightly better than those of the fixed coefficient configurations, but the differences were not significant.

#### 4.5.4. Comparison of the Static Parking Distribution Model and the Dynamic Parking Distribution Model

The best results of the static parking distribution model and those of the dynamic parking distribution model were compared; that is, the distribution results of the dynamic model were compared with those of the static model for  $T = 4$  min. For the case in which  $T$  is kept constant, the results of the static model and dynamic model were compared. Thus, the effectiveness of the dynamic parking distribution model was verified.



**Figure 15.** Evaluation scores of the dynamic parking distribution model when  $T = 4$  min compared with the scores of the static parking distribution model when  $T = 4$  min.

The red square in the graph represents the values of the dynamic parking distribution model, while the results of static parking distribution model are denoted by circular markers. It can be concluded from Figure 15 that when  $T$  is equal to 4 min, the performance of the dynamic parking distribution model is better than that of the static parking distribution model. Compared with the static parking distribution model, the performance score of the dynamic parking distribution model increased by at least 2.3%. Generally, the dynamic parking distribution model proposed here showed remarkable performance when compared with the traditional static parking distribution model and was found to meet the needs of dynamic parking allocation in practical application scenarios.

#### 4.6. Overall Findings

For the evaluation of the proposed dynamic parking distribution model, a comparison between the static parking distribution model and dynamic parking distribution model was conducted in a real case study. We found that the allocation performance of the dynamic distribution model was improved by 6%-15% in the sharing mode compared with the non-sharing mode. The advantages of the sharing mode are mainly reflected in less traffic congestion and a lower rejection rate of parking. Therefore, the sharing mode is recommended to be widely adopted in parking allocation systems. As for the time interval  $T$ , the smaller the  $T$  is, the better the allocation performance of the dynamic distribution model will be. The performance score of the dynamic parking distribution model when  $T$  is between 2 min and 20 min were all kept above 80% of the highest scores. Moreover, the smaller the  $T$  is, the more real-time feedback on parking requests the dynamic distribution model can provide. It was also found that, compared with the static parking distribution model, the performance score of the dynamic parking distribution model increased by at least 2.3%. Since the dynamic parking distribution model can effectively capture the real time change in parking supply and demand, a smart and sustainable parking space distribution can be achieved for the benefit of both public managers and individual drivers, and can be considered to have better applicability in real life.

## 5. Conclusions

In this study, we proposed a method to perform dynamic parking allocation using parking demand prediction and predictive control methods. A utility function that assess parking spaces was designed based on seven parameters: driving duration, walking distance, parking fee, traffic congestion, possibility of finding parking space in the target parking lot and adjacent parking lot, and parking satisfaction degree. In addition, two allocation modes—sharing and non-sharing modes—

were proposed for comparison. Integrating these parts, a neural-network-based dynamic parking allocation model was designed; this model considers a balanced allocation of parking resources and maximum utilisation of parking spaces in the entire city. The performance of this model was verified and compared with that of a static model through a case study.

The results of the case study of the static parking distribution model showed that the highest allocation score of the static parking distribution model in the real case was 97.8, and the lowest score was 13.7. A total of 426 coefficient configurations scored more than 90 points, and the score distribution presented a double-hump shape. According to the results of the real case study of the neural-network-based dynamic parking distribution model, it can be concluded that i) the results of the dynamic parking distribution model are better than those of the static parking distribution model in the real case; ii) the performance score of the dynamic parking distribution model when T is between 2 and 20 min was above 80% of the highest scores; and iii) the performance of the dynamic parking distribution model in the sharing mode was better than that in the non-sharing mode, with a score improvement rate of 6%–15%.

The neural-network-based dynamic parking distribution model can effectively improve the utilisation rate of resources and the uniformity of distribution and can reduce the failure rate of parking, thereby improving the assignment process of parking spaces. Generally, the dynamic parking distribution model proposed in this study performed remarkably well and can satisfy the requirements of dynamic parking allocation in urban parking management. Based on the limitation of data availability and the restriction of some objective factors, there are still some suggestions for future research. Due to the limited amount of survey data on the occupancy of parking spaces in urban classified parking lots, it is difficult to expand the model to a large span in limited time. It will be better to conduct the research based on the amount of data in different seasons and regions in later stages. In addition, the hourly parking fee of parking lots was fixed in this study according to the local parking policy; however, it will be of interest in future research to adjust the fees of the parking lot flexibly according to the supply and demand.

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