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A Multi-Item Replenishment Problem with Carbon Cap-And-Trade under Uncertainty

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Received: 26 May 2020; Accepted: 12 June 2020; Published: 15 June 2020

Abstract: Recently, as global warming has become a major issue, many companies have increased their efforts to control carbon emissions in green supply chain management (GSCM) activities. This paper deals with the multi-item replenishment problem in GSCM, from both economic and environmental perspectives. A single buyer orders multiple items from a single supplier, and simultaneously considers carbon cap-and-trade under limited storage capacity and limited budget. In this case we can apply a can-order policy, which is a well-known multi-item replenishment policy. Depending on the market characteristics, we develop two mixed-integer programming (MIP) models based on the can-order policy. The deterministic model considers a monopoly market in which a company fully knows the market information, such that both storage capacity and budget are already determined. In contrast, the fuzzy model considers a competitive or a new market, in which case both of those resources are considered as fuzzy numbers. We performed numerical experiments to validate and assess the efficiency of the developed models. The results of the experiments showed that the proposed can-order policy performed far better than the traditional can-order policy in GSCM. In addition, we verified that the fuzzy model can cope with uncertainties better than the deterministic model in terms of total expected costs.

Keywords: green supply chain management; carbon tax and cap; can-order policy; mixed-integer programming; fuzzy constraints

1. Introduction

Since the 1997 Kyoto protocol, many countries and organizations have presented legislation or policies about managing carbon emissions as global warming destroys the Earth's ecosystem. Accordingly, any company can concentrate on reducing and managing its carbon emissions in a variety of areas such as supply chain management (SCM), production, contracts, inventory, and replenishment, with the result that green supply chain management (GSCM) is quickly spreading [1]. The main purpose of GSCM is to minimize supply chain costs and simultaneously reduce carbon emissions. In line with this goal, many governments have implemented carbon cap-and-trade regulation, which is well known as an effective economic-based mechanism [2]. Under such regulation, each company is allocated limited carbon emissions credits from its government, and it can buy or sell rights to emit carbon emissions with other companies in the carbon trading market [3,4]. In the European Union, the European Union Emissions Trading Scheme, which is the largest carbon trading market, has covered almost 50% of total carbon emissions [5]. Therefore, it becomes important to consider GSCM in the context of carbon cap-and-trade regulation.

Beyond governments' efforts to reduce carbon emissions, some companies have tried to manage their own carbon emissions through their supply chains. For example, the retailers Asda, Tesco, Wal-

Mart, and H&M require their suppliers to reduce carbon emissions during multi-item replenishment activities [6,7]. In this way, a company considers carbon emissions simultaneously with the multi-item replenishment problem under limited resources, such as storage capacity and budget [8]. However, a company could face two realistic situations based on either known or unknown market information [9]. When a company is in a monopoly market, it knows all of the relevant market information so it can easily decide on storage capacity and budgets. In contrast, when a company is in a competitive or a new market, the needed market information is difficult to grasp. In this case, because of weak market information, neither proper storage capacity nor budgets can be estimated in a stochastic sense. It is difficult to predefine them [10]. Therefore, this paper focuses on the multi-item replenishment problem with limited resources under two market information cases, the certain and the uncertain.

Given current real-world practices, this paper considers the multi-item replenishment problem with carbon cap-and-trade under limited storage capacity and budget. This work (1) develops two mixed-integer problem (MIP) models based on a periodic can-order policy, which is a well-known multi-item replenishment policy; (2) includes carbon cap-and-trade for GSCM; and (3) covers two market information cases: certain and uncertain information. This paper makes the following contributions. First, we develop a deterministic model with carbon cap-and-trade for GSCM with certain (known) market information. In this model, both storage capacity and budget are already predefined. The deterministic model can be applied in a monopoly market case. Based on this model, we develop a fuzzy model applying fuzzy constraints. Because of uncertain market information, both storage capacity and budget are considered as fuzzy numbers. The fuzzy model can be applied in a competitive market case. Second, we suggest both MIP models based on the periodic can-order policy. Thus, each model can obtain optimal results under replenishment planning and inventory control.

The structure of this paper is as follows. A literature review is presented in Section 2. Notation, assumptions, and problem definitions are introduced in Section 3. The deterministic model and the fuzzy model are developed in Sections 4 and Section 5, respectively. We present numerical experiments in Section 6. Academic, managerial, and environmental insights are presented in Section 7. Finally, conclusions are presented in Section 8.

2. Literature Review

This paper is related to three elements of the relevant literature: a multi-item replenishment problem for GSCM with carbon emissions, the can-order policy, and GSCM with fuzzy constraints. The research on multi-item replenishment for GSCM with carbon emissions has been approached in various ways. Konur [11] suggested an integrated inventory-transportation model which considers a carbon cap and the emissions characteristics of trucks during transportation. Nia, Far, and Niaki [8] considered an economic order quantity model under the green vendor-managed inventory (VMI) policy, which includes limited warehouse capacity, pallets, deliveries, and greenhouse-gas emissions. Mokhtari and Rezvan [12] applied VMI policy to solve a multi-item replenishment problem in GSCM. In their model, each retailer decides on a replenishment plan based on a limited amount of total GHG emissions. Noh and Kim [1] considered a single-setup/multiple-delivery policy for a green supply chain contract under an uncertain demand situation. They proved that the cooperative contract is useful to improve the performance of GSCM. Cui et al. [13] focused on a business-to-consumer (B2C) e-business company with distribution centers, and they utilized the strategy of multi-item joint replenishment-distribution.

The theory of the can-order policy was first established by Balintfy [14]. Based on that study, Silver [15] focused on an inventory replenishment problem with Poisson demand and non-zero lead time. He established that a can-order policy performed better than an independent order policy. Liu and Yuan [16] developed a Markov model for a two-item inventory system with coordinated replenishment and a heuristic method for solving the problem. Kayaş et al. [17] presented a continuous can-order policy model with two items with Poisson demand under a semi-Markov decision process, and developed a simple enumeration algorithm to solve the problem. Tsai et al. [18]

developed an association clustering method that gathers items with similar demand in a hierarchal way to evaluate the correlated demand for handling a large number of items. Kouki et al. [19] considered a continuous review can-order policy, developed as a Markov process with perishable items and zero lead time.

According to the basic theory of the can-order policy, the inventory system should be continuously reviewed. However, because the supplier has limited replenishment opportunities, Johansen and Melchior [20] suggested a can-order policy model with a periodic review system. To simulate their idea, they suggested a new method based on Markov decision theory to obtain a near-optimal solution. Nagasawa et al. [21] presented a periodic can-order policy model that uses multi-objective programming to obtain the optimal can-order level. Most previous studies dealt with the continuous review system and focused on deriving the reorder level, can-order level, and order-up-to level based on various algorithms. In the real world, a supplier might ship items once or twice a day, so a company might not receive items as frequently as desired [20]. Although those researchers assumed that those levels cannot be fixed to certain values, a decision maker usually sets those levels according to inventory strategy and service level, respectively. Besides, they did not consider carbon cap-and-trade and limited resources, which are storage capacity and budget in the uncertain market information case.

In a competitive or a new market, it is impossible to describe the market information as a specific stochastic distribution because of this uncertainty. To handle this problem, many researchers apply the fuzzy method, a common way to handle uncertainty and non-stochastic situations [22]. Some researchers have tried to apply the fuzzy method to replenishment models. Sadeghi et al. [23] considered the economic production quantity policy under the consignment stock policy in a fuzzy demand situation, and they applied particle swarm optimization to solve their model. Nia et al. [24] presented a fuzzy resource nonlinear integer programming model that regards customer demand, storage capacity, and the budget as fuzzy numbers. Sadeghi et al. [25] focused on a nonlinear integer programming model with a trapezoidal fuzzy number for demand. However, for handling uncertain resource constraints, no previous studies have applied the fuzzy method to GSCM which focuses on a multi-item replenishment problem.

3. Notation, Assumptions, and Problem Definitions

3.1. Notation

Index:

i item, $i = 1, \dots, I$

t period, $t = 1, \dots, T$

Decision variables:

y_t binary variable indicating the order during period t

l_t^i inventory level of item i at the end of period t

l_t^{i+} on-hand inventory of item i at the end of period t

l_t^{i-} backorder level of item i at the end of period t

x_t^i order amount for item i in period t

S_t^i the order-up-to level of item i in period t

α_t^i if l_t^i drops below s^i , then $\alpha_t^i = 1$, otherwise $\alpha_t^i = 0$

β_t^i if l_t^i drops below c^i , then $\beta_t^i = 1$, otherwise $\beta_t^i = 0$

γ_t^i if l_t^i drops below c^i and at least one item is ordered in period t , then $\gamma_t^i = 1$, otherwise $\gamma_t^i = 0$

z_t^i if minor setup is done for item i during period t , then $z_t^i = 1$, otherwise $z_t^i = 0$

e_t^+ amount of buying carbon credit in period t

e_t^- amount of selling carbon credit in period t

Parameters:

u_t major ordering cost in period t (\$/order)

v_t^i	minor ordering cost of item i in period t (\$/order)
b_t^i	per unit backorder cost of item i in period t (\$/unit)
h_t^i	per period holding cost of item i in period t (\$/unit)
p	carbon tax (\$/ton)
CE	carbon cap for entire planning horizon
\widehat{h}_t^i	amount of carbon emissions when a buyer holds inventory of item i in period t
\widehat{v}_t^i	amount of carbon emissions when a buyer orders inventory of item i in period t
d_t^i	demand for item i during period t
o^i	volume of item i
w_t	storage capacity during period t
g^i	purchase price of item i
P_t	amount of budget during period t
c^i	can-order level of item i
s^i	reorder level of item i
M	big M, very big number

3.2. Assumptions

1. A single buyer orders multiple items from a single supplier and simultaneously considers carbon cap-and-trade under limited storage capacity and budget.
2. The system considers a periodic review can-order policy to obtain the order-up to level. The supplier can utilize limited transportation, so the review period is dependent on the contract period between the buyer and the supplier.
3. Both the buyer and the supplier share the demand information of the items in real time. Thus, the supplier can deliver multiple items with no lead time. Also, the demand for each item is known.
4. The reorder level and the can-order level are assumed as constant.
5. The storage capacity and budget are assumed as constant in the deterministic model and fuzzy numbers in the fuzzy model.
6. The buyer's carbon emissions occur throughout the ordering item and holding inventory. A buyer has a carbon cap and could buy or sell its own carbon credit to other company depending on the carbon emissions.

3.3. Problem Definition

In this GSCM, a single buyer orders multiple items from a single supplier, considering storage capacity, budget, and carbon emissions. Because of limited market information, both the storage capacity and the budget could have some level of uncertainty. For developing multi-item replenishment, we apply a can-order policy to GSCM. Figure 1 compares the inventory patterns with the traditional can-order policy and the proposed can-order policy for two items. By the proposed can-order policy, the order-up-to level S_t^i , where $x_t^i = S_t^i - l_t^i$, is decided individually through the planning horizon. The inventory level of both items in period t_0 explains the initial inventory level. In period t_1 , only the inventory level of item 1 is lower than reorder level s^1 , so item 1 is replenished up to S_1^1 . In period t_2 , the inventory levels of items 1 and 2 are lower than the reorder level s^1 and can-order level c^2 , respectively, so both items are replenished up to S_2^1 and S_2^2 . In period t_3 , both items are replenished because both of their inventory levels are lower than the reorder level. In contrast, in the traditional can-order policy, when the inventory level of an item drops to or below the reorder level s^i , an order is placed and the order amount x_t^i can make the inventory level up to S^i . Thus, the order-up-to level S^i , where $x_t^i = S^i - l_t^i$, is always the same through the planning horizon.

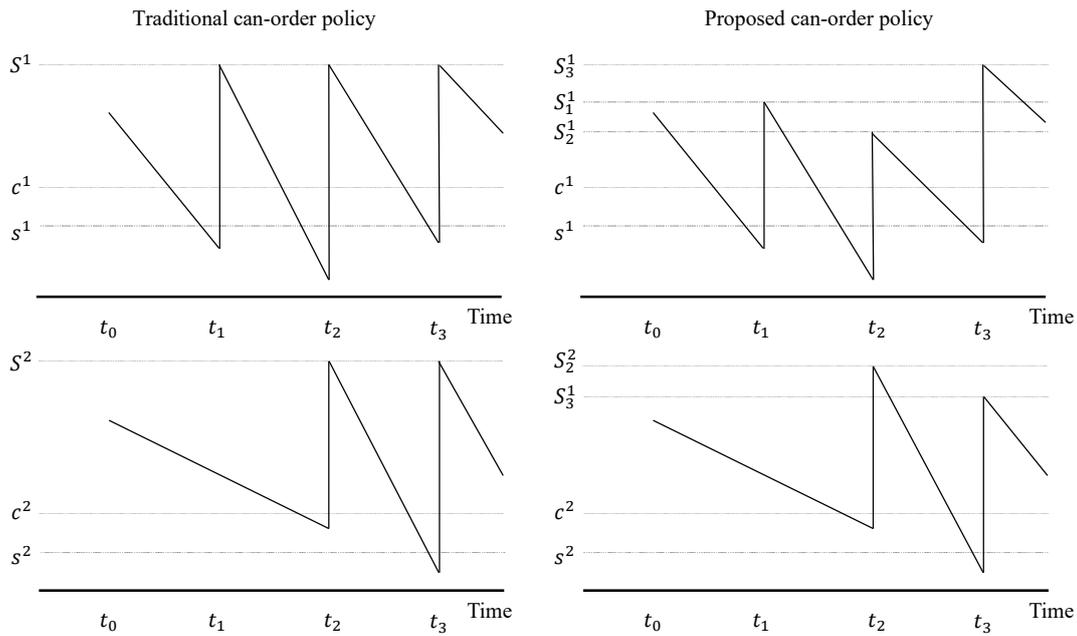


Figure 1. Inventory patterns with can-order policies for two items.

4. Deterministic Model

In this section we use MIP to develop a deterministic model for GSCM. The total cost consists of the major ordering cost, minor ordering cost, backorder cost, inventory holding cost, and carbon tax cost. The major ordering cost, incurred when an order is placed during period t , is given by:

$$\sum_{t=1}^T u_t y_t \tag{1}$$

where

$$y_t = 0 \text{ or } 1, \quad t = 1, 2, \dots, T.$$

The minor ordering cost of each item occurs when item i is ordered in period t , and it cannot be incurred unless the major ordering cost is also incurred. It is given by:

$$\sum_{t=1}^T \sum_{i=1}^I v_t^i z_t^i \tag{2}$$

The holding cost can be derived as the sum of the order-up-to level and the on-hand inventory at the end of each period. At the beginning of each period, the order-up-to level is delivered, and that amount is decreed by the amount of on-hand inventory at the end of the preceding period. Thus, the expected inventory of each item in each period is approximated as the average of those amounts.

$$\sum_{t=1}^T \sum_{i=1}^I h_t^i \left(\frac{S_t^i + I_t^{i+}}{2} \right) \tag{3}$$

The backorder cost of item i at the end of period t is:

$$\sum_{t=1}^T \sum_{i=1}^I b_t^i I_t^{i-} \tag{4}$$

A buyer has its own carbon cap. If the buyer emits less carbon than the given carbon cap, it would get rewarded by receiving money. Otherwise, the buyer pays penalties in the form of a carbon tax [26]. The buyer's carbon cap cost at period t is:

$$\sum_{t=1}^T p(e_t^+ - e_t^-) \quad (5)$$

Based on Equations (1)–(5), the following mathematical model can be developed:

$$\text{Min} \sum_{t=1}^T \sum_{i=1}^I \left(h_t^i \left(\frac{S_t^i + l_t^{i+}}{2} \right) + b_t^i l_t^{i-} + v_t^i z_t^i \right) + \sum_{t=1}^T (u_t y_t + p(e_t^+ - e_t^-)), \quad (6)$$

subject to

$$S_t^i - l_t^i = d_t^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (7)$$

$$l_t^i = l_t^{i+} - l_t^{i-}, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (8)$$

$$x_t^i \leq M y_t, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (9)$$

$$l_{t-1}^i + M \alpha_t^i \geq s^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (10)$$

$$l_{t-1}^i - M(1 - \alpha_t^i) \leq s^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (11)$$

$$S_t^i \leq l_{t-1}^i + x_t^i + M(1 - \alpha_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (12)$$

$$S_t^i \geq l_{t-1}^i + x_t^i - M(1 - \alpha_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (13)$$

$$l_{t-1}^i + M \beta_t^i \geq c^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (14)$$

$$l_{t-1}^i - M(1 - \beta_t^i) \leq c^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (15)$$

$$\gamma_t^i \leq \beta_t^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (16)$$

$$\sum_{i=1}^I \alpha_t^i \leq M \delta_t, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (17)$$

$$\delta_t \leq \sum_{i=1}^I \alpha_t^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (18)$$

$$\gamma_{it} \leq \delta_t, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (19)$$

$$\gamma_{it} \geq \delta_t + \beta_t^i - 1, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (20)$$

$$S_t^i \leq l_{t-1}^i + x_t^i + M(1 - \gamma_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (21)$$

$$S_t^i \geq l_{t-1}^i + x_t^i - M(1 - \gamma_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (22)$$

$$x_t^i \leq M(\alpha_t^i + \gamma_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (23)$$

$$S_t^i \leq l_{t-1}^i + M(\alpha_t^i + \gamma_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (24)$$

$$S_t^i \geq l_{t-1}^i - M(\alpha_t^i + \gamma_t^i), \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (25)$$

$$\alpha_t^i + \gamma_t^i \leq M z_t^i, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T, \quad (26)$$

$$\sum_{t=1}^T \sum_{i=1}^I \left(\widehat{h}_t^i \left(\frac{S_t^i + l_t^{i+}}{2} \right) + \widehat{v}_t^i z_t^i \right) + \sum_{t=1}^T e_t^- \leq CE + \sum_{t=1}^T e_t^+, \quad (27)$$

$$\sum_{i=1}^I o^i S_t^i \leq W_t, \quad t = 1, 2, \dots, T, \quad (28)$$

$$\sum_{i=1}^I g^i x_t^i \leq P_t, \quad t = 1, 2, \dots, T, \quad (29)$$

$$y_t, \alpha_t^i, \beta_t^i, \gamma_t^i, z_t^i \in \{0, 1\}, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T,$$

$$S_t^i, x_t^i, l_t^{i+}, l_t^{i-} \in \mathbb{Z}_+ + \{0\}, \quad l_t^i \in \mathbb{Z}, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T,$$

$$e_t^+, e_t^- \in \mathbb{Z}_+ + \{0\}, \quad t = 1, 2, \dots, T,$$

Equation (6) presents the objective function that minimizes the total inventory holding cost, backorder cost, major ordering cost, and minor ordering cost. Equation (7) ensures that the inventory position at the end of period t is equal to the difference between the order-up-to level and demand. Equation (8) ensures that the inventory at the end of period t is equal to the difference between the on-hand inventory and the backorder level. Equation (9) presents the constraint of incurring the major ordering cost whenever at least one item is ordered. Equations (10) and (11) regulate ordering when the inventory position falls below the reorder level. If l_{t-1}^i , the initial inventory level at t , drops below s^i , then $\alpha_t^i = 1$, otherwise $\alpha_t^i = 0$. According to Equations (12) and (13), if the inventory position at the end of the previous period is below the reorder level, the item is ordered in the amount equal to the difference between the order-up-to level and the inventory position at the end of the previous period. Equations (14) and (15) regulate whether the inventory position is below the can-order level. If l_{t-1}^i , the initial inventory level at t , drops below c^i , then $\beta_t^i = 1$, otherwise $\beta_t^i = 0$. According to Equations (16)–(20), if the inventory position of at least one item is below its reorder point, the following order includes other items whose inventory positions are below their can-order level. According to Equations (21) and (22), if the inventory position at the end of the previous period is below the can-order level, that item is ordered in the amount equal to the difference between its order-up-to level and its inventory position at the end of the previous period. Equation (23) regulates the ordering of items whose inventory position is below the reorder or can-order level. Equations (24) and (25) indicate that the order-up-to level is set based on the inventory position, which is below the reorder level or the can-order level. Equation (26) indicates that any item below its reorder level or can-order level incurs the minor ordering cost. Equation (27) ensures the carbon cap-and-trade constraint. The carbon emissions are incurred from the ordering and holding activities. Based on the amount of emissions, the buyer could buy or sell the carbon credit from the participating companies [26]. Equation (28) ensures that the sum of the inventory position for each item at the beginning of each period and the order-up-to level will not exceed the storage capacity. Equation (29) ensures that the buyer cannot order items over its budget.

5. Fuzzy Model

We here develop a fuzzy model based on the deterministic model. A company that is in a competitive market or moves into a new market usually has less information about that market. In this case, to fit the information as a specific stochastic distribution is usually impossible. It is difficult to predetermine the amount of resources, such as limited storage capacity and/or budget. This uncertain market information can be handled by using the fuzzy method.

To apply the fuzzy method to our deterministic model, which is also called a crisp model, the model must first be transformed into a fuzzy model. In order to obtain a crisp value from the fuzzy model, the fuzzy model should be converted to a new crisp model through a defuzzification process.

Although many researchers have developed various defuzzification methods, we used the symmetry method introduced by Zimmermann [22]. The crisp objective function with fuzzy constraints, where $A_i x \leq \tilde{b}_i$ and $D_i x \leq C_i$ are a set of fuzzy and crisp constraints, respectively, is formulated as:

$$\text{Min } f(x) = c^T x, \quad (30)$$

subject to

$$A_i x \leq \tilde{b}_i, \quad i = 1, 2, \dots, m,$$

$$D_i x \leq C_i, \quad i = 1, 2, \dots, m,$$

$$x \geq 0.$$

Now, using a membership function for a fuzzy set, an element x of X is mapped to a value between 0 and 1. The membership function for the fuzzy sets that represent the fuzzy constraints can be defined as:

$$\mu_i(x) = \begin{cases} 1 & \text{if } A_i x \leq b_i \\ \frac{b_i + p_i - A_i x}{p_i} & \text{if } b_i < A_i x \leq b_i + p_i \\ 0 & \text{if } A_i x > b_i + p_i \end{cases}, \quad i = 1, 2, \dots, m + 1. \quad (31)$$

In Equation (31), $\mu_i(x)$ could be 1 when the constraints are well satisfied, otherwise 1. The p_i is the tolerance interval which is assumed a constraint to be linearly increasing. Defining the membership function of the objective function requires solving the following two problems. The first problem is the original crisp model, and the optimal result is set as $\text{sup}_{R_1} f = (c^T x)_{\text{opt}} = f_1$.

$$\text{Min } f(x) = c^T x, \quad (32)$$

subject to

$$A_i x \leq b_i, \quad i = 1, 2, \dots, m,$$

$$D_i x \leq C_i, \quad i = 1, 2, \dots, m,$$

$$x \geq 0$$

In the second problem, the fuzzy constraints in Equation (32) are changed to the crisp constraints with tolerances, and the optimal result is set as $\text{sup}_{S(\tilde{R})} f = (c^T x)_{\text{opt}} = f_0$.

$$\text{Min } f(x) = c^T x, \quad (33)$$

subject to

$$A_i x \leq b_i + p, \quad i = 1, 2, \dots, m,$$

$$D_i x \leq C_i, \quad i = 1, 2, \dots, m,$$

$$x \geq 0.$$

In short, f_0 and f_1 are the minimum total cost with and without tolerances for resources, respectively. Based on that, the membership function of the objective function is obtained as:

$$\mu_{\tilde{f}}(x) = \begin{cases} 1 & \text{if } f_1 < c^T x \\ \frac{c^T x - f_0}{f_1 - f_0} & \text{if } f_0 \leq c^T x \leq f_1 \\ 0 & \text{if } c^T x < f_0 \end{cases}. \quad (34)$$

Based on Equations (30)–(34), both the objective function and the constraints have ‘symmetry’ such that the crisp model by the defuzzification process is transformed.

$$\mathbf{Max} \lambda, \quad (35)$$

subject to

$$\mathbf{c}^T \mathbf{x} \leq \lambda(\mathbf{f}_0 - \mathbf{f}_1) + \mathbf{f}_1,$$

$$\mathbf{A}_i \mathbf{x} \leq \mathbf{b}_i + (1 - \lambda)\mathbf{p}_i, \quad i = 1, 2, \dots, m,$$

$$\mathbf{D}_i \mathbf{x} \leq \mathbf{C}_i, \quad i = 1, 2, \dots, m,$$

$$\mathbf{x} \geq \mathbf{0},$$

$$0 \leq \lambda \leq 1.$$

As mentioned earlier, we considered the storage capacity and the budget as fuzzy numbers. The following table methodically shows the fuzzy constraints of Equations (28) and (29).

$$\sum_{i=1}^I o^i S_t^i \leq \widetilde{W}_t, \quad t = 1, 2, \dots, T,$$

$$\sum_{i=1}^I g^i x_t^i \leq \widetilde{P}_t, \quad t = 1, 2, \dots, T.$$

Next, those constraints are converted to the defuzzification of the storage capacity and budget, where $p_{1,t}$ and $p_{2,t}$ are their respective tolerances:

$$\sum_{i=1}^I o^i S_t^i \leq W_t + (1 - \lambda)p_{1,t}, \quad t = 1, 2, \dots, T, \quad (36)$$

$$\sum_{i=1}^I g^i x_t^i \leq P_t + (1 - \lambda)p_{2,t}, \quad t = 1, 2, \dots, T. \quad (37)$$

Finally, the equivalent crisp model, transformed from the fuzzy model using the method of Zimmermann [22], is obtained:

$$\mathbf{Max} \lambda, \quad (38)$$

subject to

$$\sum_{t=1}^T \sum_{i=1}^I \left(h_t^i \left(\frac{S_t^i + l_t^{i+}}{2} \right) + b_t^i l_t^{i-} + v_t^i z_t^i + G^i e^i x_t^i \right) + \sum_{t=1}^T u_t y_t \leq \lambda(\mathbf{f}_0 - \mathbf{f}_1) + \mathbf{f}_1,$$

$$\sum_{i=1}^I o^i S_t^i \leq W_t + (1 - \lambda)p_{1,t}, \quad t = 1, 2, \dots, T,$$

$$\sum_{i=1}^I g^i x_t^i \leq P_t + (1 - \lambda)p_{2,t}, \quad t = 1, 2, \dots, T,$$

with Equations (7)–(27)

$$y_t, \alpha_t^i, \beta_t^i, \gamma_t^i, z_t^i \in \{0, 1\}, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T,$$

$$S_t^i, x_t^i, l_t^{i+}, l_t^{i-} \in \mathbb{Z}_+ + \{0\}, l_t^i \in \mathbb{Z}, \quad i = 1, 2, \dots, I, \quad t = 1, 2, \dots, T,$$

$$e_t^+, e_t^- \in \mathbb{Z}_+ + \{0\}, \quad t = 1, 2, \dots, T,$$

$$0 \leq \lambda \leq 1.$$

We denote $p_{1,t}$ and $p_{2,t}$ as the ‘tolerance intervals’ of W_t , and P_t , respectively.

6. Numerical Experiments

To validate our proposed two MIP models, we tested three kinds of numerical experiments. First, we conducted an efficiency test for the deterministic model by comparing it to the traditional can-order policy with predetermined values, (s^i, c^i, S^i) . Second, we compared the traditional can-order policy with the proposed can-order policy. In this experiment, we determined which the policy best fit with GSCM. Third, we tested the fuzzy model with various tolerances. In all the experiments we set the very large number M at 1,000,000. To solve the MIP models, we used LINGO 17.0 software.

6.1. Efficiency Test

To show the efficiency of the proposed can-order policy, we compared it to the traditional can-order policy with predetermined values. In the traditional can-order policy, a decision maker sets the values for (s^i, c^i, S^i) based on previous data or their experience. Thus, this comparison is necessary to show whether the proposed deterministic model can reduce the total cost compared to the traditional can-order policy with predetermined values.

Table 1 presents the basic input parameters for three items in 12 periods. The major ordering cost (u_t) is \$1000/order. It is difficult to apply constraints, such as storage capacity, budget, or carbon cap-and-trade, to it in the traditional can-order policy with predetermined values, so we did not consider those constraints in this test.

Table 1. Input parameters for the basic test.

Item	1	2	3
v_t^i	$U(38, 46)$	$U(89, 91)$	$U(25, 30)$
b_t^i	$U(137, 140)$	$U(239, 288)$	$U(83, 85)$
h_t^i	$U(68, 75)$	$U(119, 144)$	$U(51, 64)$
d_t^i	$N(600, 50^2)$	$N(500, 30^2)$	$N(100, 5^2)$

Notes: $U(*,*)$: uniform distribution; $N(*,*)$: normal distribution

Table 2 shows the initial inventory position, the reorder level, and the can-order level that we used during the test of the traditional can-order policy with predetermined values. The order-up-to level for the traditional can-order policy with predetermined values was assumed to be $S_1 = 697$, $S_2 = 590$, and $S_3 = 184$, the highest demand for each item during the 12 periods. We used Excel 2017 for calculating the traditional can-order policy with predetermined values.

Table 2. Initial inventory position, can-order level, and reorder level.

Item	l_0^i	s^i	c^i
1	0	50	80
2	0	70	100
3	0	10	20

Table 3 shows that the proposed model led to a total cost of \$757,120.50. To see the precise efficiency of the proposed model, we tested it against the traditional can-order policy with predetermined values. Otherwise, it is difficult to discover whether the order-up-to level is biased in

predetermined values. We set the order-up-to level from 95% to 75% of the highest demand in 5% decrements. Table 4 shows the results of this test.

Table 3. The results of the basic test.

Traditional Can-Order Policy with Predetermined Values (\$)	Proposed Model (\$)
1,107,962.00	757,120.50

Table 4. The effect of the order-up-to level.

Test Number	Percent of the Highest Demand	Order-Up-to Level			Total Cost of The Traditional Can-Order Policy with Predetermined Values (\$)
		S_1	S_2	S_3	
1	95%	662	561	175	928,417.00
2	90%	627	531	166	935,320.50
3	85%	592	502	157	940,683.00
4	80%	558	472	148	1,045,999.00
5	75%	523	443	138	1,155,508.00

As shown in Table 4, the set of order-up-to level $S_1 = 662$, $S_2 = 561$, and $S_3 = 175$ obtains the best result, \$928,417.00. The total cost is increased with a lower order-up-to-level because the company cannot cope with the demand. The proposed model strongly outperforms the traditional can-order policy with predetermined values. This result shows that predetermined values, which are biased by a decision maker, rarely set the order-up-to level properly, which increases the total cost.

6.2. Comparison Test between the Proposed can-order Policy and the Traditional can-order Policy

To determine the better policy, we compared the proposed can-order policy to the traditional can-order policy. Based on the deterministic model, we developed a new model about the traditional can-order policy, which can obtain the same order-up-to level through the planning horizon. We conducted this test using an experimental design in which we varied the number of items and periods from 6 to 10 and 10 to 20, respectively. The major ordering cost was \$1000/order and the initial inventory position was zero. The amount of carbon emissions for holding and ordering were assumed as 10% of each cost. Both carbon tax and carbon cap were assumed as \$2 and \$80,000, respectively. The storage capacity was set as $4,000m^2$ and the amount of budget was set as \$80,000. Table 5 presents each item's input data. The demands for each item were generated using a normal distribution. The reorder level was set as 99% of the safety stock, and the can-order level was assumed as 20% above the reorder level.

Table 5. Input parameters for the comparison test.

Item Number	v_t^i (\$)	b_t^i (\$)	h_t^i (\$)	d_t^i	o^i	g^i
1	1	13	1	$N(200, 10^2)$	0.1	1
2	2	14	2	$N(400, 20^2)$	0.5	0.1
3	1	10	1	$N(300, 15^2)$	1	0.2
4	1	15	3	$N(200, 20^2)$	0.8	1
5	2	8	1	$N(500, 10^2)$	1.1	2
6	2	9	2	$N(100, 20^2)$	2	0.2
7	1	6	1	$N(60, 5^2)$	1	0.1
8	1	10	1	$N(1000, 50^2)$	0.9	0.3
9	2	11	2	$N(200, 15^2)$	1.5	2
10	1	13	1	$N(100, 5^2)$	1.3	0.1

Table 6 shows the results of the comparison test. In all cases, the proposed can-order policy outperformed the traditional can-order policy because the proposed policy produced less total cost than the traditional policy. In some of the five-period tests the total costs are negative values, which is an interesting point. Those cases illustrate that a company sells carbon caps to other companies so that it receives profits. The result proves that the proposed policy benefits the company that replenishes multi-items under carbon cap-and-trade. The proposed policy could be efficacious in the large-scale multi-item replenishment problem in GSCM. Thus, based on these discussions, the proposed model is promising for practical applications.

Table 6. The results of the comparison test.

Number of Items	Periods	Total Cost (\$)	
		Traditional Can-Order Policy	Proposed Can-Order Policy
6	5	-5599.80	-7264.20
	10	8800.40	5471.60
	15	23,200.60	18,207.40
	20	37,600.80	30,943.20
8	5	-895.60	-4021.20
	10	18,208.80	11,957.60
	15	37,313.20	27,936.40
	20	56,417.60	43,915.20
10	5	987.80	-2448.60
	10	21,975.60	15,102.80
	15	42,963.40	32,654.20
	20	63,951.20	50,205.60

6.3. Fuzzy Model Test

We also tested the fuzzy model to demonstrate the effects of changing the tolerances. To initialize and solve the model in Equation (38), we first solved the two models in Equations (33) and (35) such that we could obtain the values of f_0 and f_1 . We considered three items during 12 periods and used the parameters in Section 6.2. Table 7 shows the tolerances for the parameters of the storage capacity and the budget. The base values of the storage capacity and the budget were 1000 m² and \$8000, respectively. Table 8 also shows the value of f_1 which is the total cost of the deterministic model, and the result of the fuzzy model.

Table 7. The various tolerances for the parameters.

Test Number	p_1	p_2
1	300	3000
2	300	4000
3	400	2000
4	400	4500
5	450	2500

Table 8. The results of the fuzzy model test.

Test Number	f_0 (\$)	Fuzzy Total Cost (\$)	λ	$(1 - \lambda)p_1$	$(1 - \lambda)p_2$	f_1 (\$)
1	907,298.50	1,039,618.0	0.73	1081	8810	
2	884,448.50	1,011,266.0	0.75	1075	9000	
3	930,148.50	1,078,992.0	0.68	1128	8640	1,400,277.0
4	881,762.50	1,002,462.0	0.77	1092	9035	
5	918,723.50	1,062,622.0	0.72	1126	8700	

Table 8 shows that all the results of the fuzzy model test produced better results than the deterministic model, \$1,400,277. Two factors explain this. First, the uncertain market information on storage capacity and budget might lead to large on-hand inventories to avoid large backorders. Second, the strict constraints of storage capacity and budget in the deterministic model are strongly fixed. Based on the value of λ , the decision maker can obtain the values of the storage capacity and budget, being $(1 - \lambda)p_{1,t}$ and $(1 - \lambda)p_{2,t}$, respectively. This test illustrates that the fuzzy model not only handles the uncertainty but also improves the system performance. Thus, the fuzzy model is a good option for a decision maker when the market information has uncertainty.

7. Academic, Managerial, and Environmental Insights

7.1. Academic Insights

We developed two MIP models dealing with the periodic can-order policy for GSCM with limited storage capacity, limited budget, and carbon cap-and-trade. We also developed a deterministic model under certain (known) market information, based on which we suggested a fuzzy model that considers the fuzzy numbers of storage capacity and budget. This is the first study to develop both deterministic and fuzzy models of the can-order policy under carbon cap-and-trade for GSCM. Thus, our study can be considered initial research which considers multi-item replenishment with carbon cap-and-trade for GSCM.

7.2. Managerial Insights

A company interested in GSCM could benefit from our study. The deterministic and fuzzy models developed here can help a company systematically replenish multi-items with carbon cap-and-trade regulation. The deterministic model suggests practical insights on multi-item replenishment for minimizing the total cost under limited resources and carbon cap-and-trade regulation. It can also help a company in a monopoly market make sound investment decisions. For a company that has uncertain market information, the fuzzy model can support preparation of appropriate storage capacity and budget, and also planning for cost minimization. Thus, the correct implementation of these models will give a company better decisions in managing GSCM and reducing its total cost.

7.3. Environmental Insights

This paper incorporates carbon cap-and-trade regulation into GSCM. Considering carbon cap-and-trade regulation, the companies can increase resource-use efficiently and grow together. This sustainable situation is regarded on the 2030 Agenda and the Sustainable Development Goal 9 (SDG 9), which is industry, innovation, and infrastructure. Thus, using this paper, the company can help protect environment while earning profits.

8. Conclusions

This paper presents two MIP models, a deterministic model and a fuzzy model, which address the multi-item replenishment problem with carbon cap-and-trade for GSCM under limited resources. We developed the two models based on the can-order policy, which is one of the well-known multi-item replenishment policies. Reflecting real-world situations, we considered limited storage capacity, budget, and carbon cap-and-trade regulation. The deterministic model can be used when a decision maker has solid market information, while the fuzzy model can be applied when a decision maker faces uncertain market information in a competitive or a new market. In this model, both limited storage capacity and budget are denoted as fuzzy numbers.

We carried out three experiments to test the efficiency of the two models. First we compared our deterministic model with the traditional can-order policy which already has predetermined values (s^i, c^i, S^i) , and proved that our deterministic model was significantly better because it resulted in lower total costs. In the second experiment, by using our deterministic model, we compared the

proposed can-order policy with the traditional can-order policy. The result showed that the proposed can-order policy outperformed the traditional can-order policy by, again, resulting in lower total costs. Finally, we quantified the effects of the fuzzy model with various tolerances. The results showed that applying fuzzy constraints is useful to make decisions under uncertain situations. We demonstrated the validity and practicality of our models in those experiments, confirming that our models can be useful for multi-item replenishment in GSCM.

There are some research limitations in this study and also some indications for possible future works. First, this paper only considered a single supplier and a single buyer. In the real world there are many suppliers, so it is an important decision to select one among several. Our current models could be applied to extend GSCM with this supplier-selection problem. Also, this paper assumed the deterministic demand. For replenishment planning, forecasting demand theory could be considered.

Author Contributions: Conceptualization, J.S.K.; Funding acquisition, S-J.H.; Methodology, J.N; Project .administration, S-J.H.; Writing – original draft, J.N.; Writing – review & editing, J.S.K. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2019S1A5C2A04083153).

Conflicts of Interest: The authors declare no conflicts of interest.

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