Article

Design of Flexible Vehicle Scheduling Systems for Sustainable Paratransit Services

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Abstract: The aging population has led to an increase in the variety and volume of transportation demands by people facing travel difficulties. Hence, transportation organisations need to provide flexible and sustainable paratransit services to meet these increasing demands. In this study, we investigate the design of flexible vehicle scheduling systems in order for a community organisation to serve more people and achieve higher operational efficiency. We analyse and propose a system design based on user requirements for different types of paratransit types. Further, we identify an integrated service option and process flow for dial-a-ride passengers to ride on a vehicle with schedule route passengers. Because this option involves a complex decision, we formulate the problem as a two-stage decision model. To verify the effectiveness of our proposed design, we perform numerical simulations and conduct a case study by collaborating with a transportation organisation. We found that the proposed system would enable the organisation to serve more people with fewer vehicles but without an increase in the travelling time. These results demonstrate the importance of a flexible vehicle scheduling system for accessible transportation organisations to sustain their service operations.

Keywords: paratransit services; dial-a-ride; schedule route; vehicle routing problem; system design

1. Introduction

The aging population has led to an increase in the number of people with travel difficulty; this increasing demand has resulted in considerable pressure on many paratransit service organisations to sustain their services in many countries. Recently, because of COVID-19, these services have become critical among the elderly to help them maintain social distance while also supporting them with their daily activities with door-to-door accessible transportation services. The elderly and disabled people generally need to move using auxiliary tools such as a cane, crutch, or wheelchair, which could inconvenience them while travelling. Without sufficient ancillary transport facilities and support from the public, people with such travel difficulties can often feel isolated from their communities and face barriers when participating in various physical activities; this can affect their emotional wellbeing and overall quality of life [1]. To facilitate their integration into the community, it is necessary to provide sustainable paratransit services as a means to enhance mobility for people with travel difficulties [2]. Currently, such paratransit service organisations face challenges in providing these services in a sustainable manner as the operation costs of providing accessible transportation services with special wheelchair lifts are generally high. Owing to limited social welfare expenditure, these service organisations find it difficult to meet the increasing demand if subsidies do not increase. Further, paratransit services cannot be substituted by public transportation services, as most public transportation operations are equipped with sufficient wheelchair lifts. The lack of substitutive services...
as well as the increasing demand of paratransit services raise social concerns to identify means to serve people with disabilities and travel inconvenience.

In this study, we aim to design a flexible vehicle scheduling system that can utilise a vehicle for various types of paratransit services. Traditional transportation services with a fixed route and timetable schedule can suffer from issues such as low seat utilisation; however, a flexible vehicle scheduling system can overcome this issue by allowing changes in routing, vehicle assignment, scheduling, and the passengers to be served [3]. Further, such a system can schedule vehicles based on various user requests and effectively manage the paratransit service. Further, instead of planning each type of paratransit service separately, the flexible vehicle scheduling system can integrate various types of paratransit services when planning, and it can enable changes to the scheduling of travel routes based on daily request in a flexible manner so that more people in need can be served.

To illustrate how a flexible vehicle scheduling system can enhance the vehicle utilisation for various paratransit services, we collaborated with the largest community transportation organisation in Hong Kong as a case study. The organisation currently provides four types of paratransit services, including scheduled route (SR), dial-a-ride (DAR), feeder, and pooled DAR. The requirements of those paratransit services and the service characteristics are different based on the purpose of the trip such as going to work, school, the hospital, or for social activities. In terms of the number of people served and the service requested, the two main two services are SR and DAR. Similar to a bus service, SR has fixed pickup and drop-off times and set locations. In contrast, DAR is similar to a taxi service that provides door-to-door transportation in a charter mode or shared dial-a-ride (SDAR) mode. In addition to the SR and DAR services, the service organisation also provides a feeder service, which is considered as an intermediate transportation service for people with disabilities that travel between metro stations and hospitals. If a group of passengers have the same destination or origin on multiple days, they need to apply for the pooled DAR service. The organisation faces the challenge of unfilled demands in variety and volume because of the lack of a flexible vehicle scheduling system, especially for these two services.

In this study, we focus on the design of vehicle scheduling systems for the DAR service that can serve more people with travel inconvenience by flexibly integrating this service with the SR service. First, we analysed the fundamental requirements of a vehicle scheduling system to provide DAR and SR services. Second, we proposed the system design framework of the flexible vehicle scheduling systems. Third, based on the system requirement and design framework, we formulated the key decision model to integrate the SR service with the DAR service. Finally, we conducted experiments to evaluate service performances using the flexible vehicle scheduling approach.

The contributions of this study are twofold. First, with regard to the contributions in the areas of design of vehicle scheduling systems and decision models, we provide an integrated perspective on the investigation of SR and DAR services. Both services are generally investigated separately in the research on vehicle routing problems in the literature, and this study belongs to a pilot study of integrating both services in systems design and a decision model. Besides this theoretical contribution, this study also acts as an operational guideline for practitioners in the accessible transportation community. It is common practice for a community organisation to provide multiple types of paratransit services with a shared pool of resources; however, most organisations face challenges in managing resources efficiently for multiple services because of their different operational characteristics. This paper provides a design framework with system components, process flow, and experiment results as an operations guideline to illustrate how different paratransit services can be offered in a flexible manner, while using the resources optimally. These results can help to unlock the potential of the community organisations in serving more people with travel inconvenience sustainability with fewer vehicles. The sustainability of service organisations affects the social sustainability in terms of economic growth, social inclusion, and environmental protection in the society.

The remainder of the paper is organised as follows. Section 2 presents a literature review on vehicle scheduling methods for transportation services with consistent schedule and DAR paratransit
services. Section 3 describes the design framework of flexible vehicle scheduling systems, identifying the system requirements and architecture for SR and DAR services. In Section 4, we formulate the flexible vehicle scheduling model to integrate SR with DAR services while meeting service requirements of both services. Section 5 illustrates the effect of the integrated service through a numerical illustration and a case study. Section 6 discusses the limitation of the study and the contributions to literature. Finally, Section 7 concludes the findings and future research.

2. Literature Review

2.1. Sustainable Transportation and Paratransit Service

Several previous studies on sustainability focused on economic, environmental, and social effects. The economic and social environment can be improved by utilising a decision support system and information communication technologies in policy making process of transport services [4–6]. A few studies also focussed on the environmental elements during the modelling of route problems [7,8]. Croce et al. [6] proposed a decision-making framework of transport system model to optimise renewable energy saving. To reduce carbon emission, the control strategy of route diversion control was proposed as an alternative method [7]. Using a transportation model while also considering traffic flow density in the city [8], it was shown that the travelling time of vehicles could be improved thereby reducing carbon emission. Hence, a transportation model was generally used to address the carbon emission via the estimators of traffic flow in urban planning [9]. In contrast to the above-mentioned studies, in our study we focus on the design of a vehicle routing system for various paratransit services from a community transportation service provider perspective. An integrated vehicle routing optimisation model is proposed to serve more people with less resource. Our study could be considered as a contribution to micro-sustainability at organization level, instead of focusing on macro-sustainability at national level for public policy making.

Regarding social components in the public transport service, several studies focused on the mobility of older people [10–12]. Musselwhite and Haddad [10] conducted a survey on the importance of mobility in older people in the UK. It was found that the abandonment of driving would reduce utilitarian and social needs. Shergold et al. [11] analysed four scenarios of the elderly living pattern and the use of transport. The ageing population emerged as an important factor in the policy making of transport services [12]. Our study differs from the methodology based on survey and interview to investigate the social factors, as our study aims to achieve sustainable operations of paratransit service providers.

In every city, paratransit services provide crucial means of mobility and connectivity for people. Few recent studies investigated the importance of quality and efficiency improvement in providing paratransit services [13,14]. Plano et al. [15] performed a survey on driver attitude of interventions in evening paratransit service. It was concluded that higher security and a higher fare were critical factors in operating evening paratransit services. Unlike other studies that identify the critical factors in operating paratransit service, our study only focuses on the integration of various types of paratransit services in the design of flexible vehicle routing system.

2.2. Flexible Vehicle Routing System

Paratransit scheduling is an integral service for people with disabilities and the elderly to have healthy and independent lives in the community [16]. A lack of paratransit transportation can cause barriers for such people in community participation [17]. However, because of the high cost and limited resources, i.e., vehicles with special equipment including automatic tail lifts and indicator lights for boarding and alighting, only a small number of qualified users can be served by assigning SR services. Those who cannot be served can only use public transport to travel or must stay at home, which indicates a shortage in the supply of these services. To overcome this problem, a flexible transport service is considered a promising approach, as it allows serving a larger number of people with travel
difficulties. Unlike regular public transport services, a flexible transport service offers greater flexibility to cater to diverse travel needs and match the availability of vehicles. Further, a flexible vehicle routing system in paratransit services can not only increase vehicle utilisation, but also promote social inclusion for people with travel difficulties.

Currently, transportation services for people with travel difficulty are usually provided in the form of a DAR program that offers greater flexibility at a higher cost, or as a fixed-route transit, which serves several people [18,19]. Appropriate route planning for paratransit services based on demand forecasting can assist service providers in allocating resources to increase service rates. Paquette et al. [20] considered the quality of service in a paratransit service, which consists of determining routes and schedules for vehicles that transport users on demand from a pickup point to a destination. Deka and Gonzales [21] identified the generators of paratransit trips for people with disabilities as to optimise the cost of service. Torkjazi and Huynh [22] proposed a dynamic insertion scheduling strategy for paratransit services; this strategy allows for the pickup of unscheduled customers if they are near a prescheduled stop. This thus helps in reducing the low seat utilisation problem. The above-mentioned literature indicates that formulating a flexible paratransit plan in terms of route and vehicle allocation is essential to fulfil the demand-responsive transport needs of people with travel difficulties.

2.3. Transportation Services with Consistent Schedule

A passenger-oriented routing problem is can be employed to improve service quality and satisfy the special requirements of passengers. Among the past studies related to transportation services with consistent schedule, the consistent vehicle routing problem (ConVRP) is one of the main categories of existing and emerging vehicle routing problem variants [23]. ConVRP was first proposed by Groër, Golden, and Wasil [24], who defined ConVRP as a variant of classic capacity VRP that assigned drivers to serve the same customers at roughly the same time in a planning horizon. The heuristics were widely studied for solving ConVRP [25–28]. Further, the ConVRP problem dealing with different types of customers was solved using a template-based tabu search algorithm proposed by Tarantilis, Stavropoulou, and Repoussis [13]. The algorithm involves a two-level decomposition scheme. The master sub-problem is designing the template route of frequent customers, and the slave sub-problem determines the actual daily route for both frequent and non-frequent customers.

A generalised ConVRP was introduced by Kovacas et al. [29], which allowed the customers to be visited by a limited number of drivers. Customers had time windows in AM/PM, and it was possible to change the vehicle departure times. Recently, Goeke, Roberti, and Schneider [30] investigated the first exact methods based on column generation for ConVRP with both driver and arrival time constraints. This method can solve the medium-sized instance with five days and 30 customers. In a broader context, the definitions of consistency constraints range between hard and soft constraints. Hard consistency constraints include same driver consistency [24,30] and arrival time consistency within a certain time limit [31]. Softer consistency constraints allow a set of drivers to be considered [29] or to only consider time consistency in terms of time class [32].

ConVRP has been used in many practical applications such as in the transportation of disabled people [32], DAR service [33], and pharmaceutical distribution [34]. Tellez et al. [21] proposed a route-based MILP formulation of time-consistency DARP, which was a bi-objective problem solved by the set partitioning-based ε-constraint metaheuristic that traces the Pareto front approximation between the two objectives. The model formulation in this study follows a general arc-based VRP modelling instead of route-based modelling; this is because arc-based modelling has a closer representation with a general vehicle routing system, and it can provide information on the travelling time and speed of any road in a digital map.

In this study, we focus on soft time consistency constraints and measure time consistency in terms of time class [32,33], which is different from the time gap between the earliest and latest pickup time in some of the literature [24,30]. In our transportation decision model, SR passengers are considered frequent customers and SDAR passengers are considered as nonfrequent customers.
The time consistency constraints influence only frequent customers; the satisfaction of time consistency constraints is part of service quality. Unlike previous studies, we propose an integrated decision model that can satisfy the time consistency requirement of SR passengers and increase opportunities for serving more SDAR passengers using a limited number of vehicles. Our study extends the applications of ConVRP to a DAR problem for paratransit services.

2.4. Dial-a-Ride Paratransit Services

The dial-a-ride problem (DARP) had early applications in door-to-door transportation services for people with disabilities who had a limited selection of public transportation and more service requirements such as service time for loading and unloading the wheel chair and requirement for special seats [35–37]. The DARP was considered a variant of the pickup and delivery problem with time windows (PDPTW) [38,39]. In these contexts, narrow time windows were often considered, which indicate the time passengers ordered to be picked up and the actual arrival time. Cordeau [39] presented an exact approach for solving static DARP. Recently, researchers formulated a general mixed-integer programming model and presented solutions for the dynamic shared-ride taxi problem [40,41].

In this study, we investigated the integration model for SR and shared DAR services. The design of a paratransit service with SR, DAR, feeder, and pooled DAR has been studied by Mo et al. [42], who proposed a service family framework to represent each type of service. The service family framework is crucial to identifying the commonality and different characteristics of various services for mass customising services. In this study, we extended the study of DAR from a shared ride option to an integrated model with SR services. When compared with the previous study, the planning horizon is different, as the requests of SR passengers are known before the planning horizon, while the requests of SDAR passengers are known at least one day before the transport day. This multiperiod planning horizon affects the design of the flexible vehicle scheduling system, service process flow, and the integrated decision model.

3. Design Framework of Flexible Vehicle Scheduling System

For the design of flexible vehicle scheduling systems for paratransit services, several considerations and solution requirements are identified and compared with vehicle scheduling systems for general logistics management. Such requirements come from different stakeholders such as users, service providers, and the government. For instance, users typically expect more service options, shorter and consistent traveling time with lower price, whereas the service providers are expected to prioritise and satisfy those expectations that are subject to the operation constraints. When compared with generic transportation systems, most requirements are similar in the design of flexible vehicle scheduling system except for the flexibility to integrate SDAR service orders with SR service orders. To facilitate the integration of different transportation requirements, we identify the main functions of the system in module layers, followed by the process of integrated services.

3.1. Layers of Vehicle Scheduling Systems

A vehicle scheduling system is designed to support a planning manager in scheduling vehicles for fulfilling transportation request orders represented in a digital form and stored in a database. These data are captured via various applications of information communication technology such as webpage, location sensors, and digital phone systems. These orders are handled by a set of vehicles identified by the system’s artificial intelligence. The planning manager evaluates the results via visualisation of route plan in a digital map based on the decisions made by the system. The information flows are represented as different module layers of a vehicle scheduling system, which represents order in a digital map, makes resource management decisions and visualizes the results for a planning manager. The entire system integrates a decision support module with different types of information technology applications interacting with a planner manager and users.
Figure 1 shows the three module layers of a vehicle routing system: the network, decision, and user application layers. The network layer is the fundamental layer in the vehicle routing system, and it determines the representation of a transportation request order, road connection, and travelling time and/or distance. These data are represented as objects and stored in a digital network map, such as Google Map and ESRI’s ArcGIS. This information is used to support decision making in the decision layer of the vehicle scheduling system. In the decision layer, the objective is to optimise vehicles resources and fulfil various requirements of the order requests. Most of the requirements of SR services are similar to those of DAR services, except for the consistent requirement of a SR. Owing to different requirements among paratransit services, the managers need to allocate and prioritise resources when the demands are more than the available vehicles. In the sector of community transportation service, it is crucial to serve people with travel difficulties for regular daily activities as they cannot use alternative modes of public transport systems, and the priority of SR services is higher than that of DAR services. Such priority affects the planning process flow, which requires the scheduling of SR services before DAR services.

The integrated decisions of scheduling multiple services for a flexible system are very complex, particularly generating flexible and efficient route plans for the best use of vehicles. The route plan generated from the decision layer is forwarded to the user application layer. The user application layer serves as an interface between decision of the manager and computer. Most functions in this layer support the planning manager to evaluate what-if scenarios and visualise the route plan on a digit map. In the design of flexible vehicle scheduling systems, the function of visualising the optimal route plan is vital, particularly because it can facilitate communication with drivers for any ad-hoc change requests in service.

3.2. Process Flow of Integrated Services

A flexible vehicle scheduling system needs to be aligned, and it should incorporate the corresponding process flow. In general, SR service orders would be planned prior to SDAR service orders, as SR services require early planning because of the time consistency requirement, while DAR services are ad-hoc demands. We focus on the process flow of integrating an SDAR service order with a planned SR service order. The process flow of the integrated services involves two stages: order planning and vehicle routing.

Figure 2 illustrates the process flow of integrating an SDAR service order with a planned SR service. In the first phase of order planning, the DAR service request is retrieved from the database and the corresponding location information is inputted from the module functions to the network layer. Such requests are planned to be handled by the flexible vehicle routing system. If the request is in the charter mode and a vehicle is available, the request would be processed, and the corresponding vehicle
would be reserved. However, when a vehicle is not available, a planning operator will communicate to the user to refine the order request if possible. If all vehicles have been scheduled, the planning operator will check with the user if he/she is willing to ride on a shared vehicle with other SR passengers, while the system advises the user on alternatives of available services. The status of the request would then be changed to the status of confirm, wait, or cancel, subject to the preference of the DAR user. After all requests have been planned prior to the service dates, the service requests are confirmed as the service orders in the second phase of vehicle routing. The service orders would be scheduled by the optimisation module in the decision layer of flexible vehicle routing system to enhance the possibility of serving a higher number of people with different requirements while using the same pool of vehicle resources. The route plan would then be generated as the output for the drivers to pick-up, transport, and drop-off the passengers.

![Figure 2. Process flow of a shared dial-a-ride (SDAR) service order with the merging option of SR service.](image)

### 4. Decision Models of Integrating Services

#### 4.1. Problem Description

Two types of passengers—SR and SDAR passengers—are considered in the integrated decision model. The SR passengers place a routine request and need to be served consistently in time, while they make one-off transportation requests. Therefore, it is assumed that SR service orders would be planned in prior to SDAR service orders. The characteristic of SR service orders includes time consistent requirements with an advance notice of pickup and drop-off points, while the SDAR service order has an unknown pickup and drop-off point until the order is confirmed. A time consistency requirement is defined by the difference in the pickup and drop-off among dates over a planning horizon. An order is handled with the time consistency if the corresponding SR passenger is served with similar pickup or drop-off times on each day in a week. We adopted the measurement of time consistency using a unit of time class, as proposed by Feillet et al. [20]. It is measured by the number of similar pickup or drop-off times over a planning horizon. For example, the number of time class is 2 for an SR passenger served at [7:50, 8, 8:05, 9, and 9:15 a.m.] on [Mon, Tue, Wed, Thu, and Fri] with the
time window of 15 min, as the served times could be considered as two time classes subsets \( [7:50, 8, 8:05 \text{ a.m.}] \) and \( [9, 9:15 \text{ a.m.}] \).

In the first stage of our decision problem, the objective is to identify the minimal number of time classes based on a set of SR service order requests. The expected output of the stage one decision is a set of pickup and drop-off times of those locations, which would provide the minimal number of time class. The set of pickup and drop-off times identified in the first stage of the decision problem affects the chance of integrating SDAR service orders, as the corresponding set becomes part of constraints in the second stage of the decision making problem, whereas the objective of the second stage is to obtain an optimal route plan for both SR and SDAR service orders.

### 4.2. Formulating the Stage-One Decision Model: Identification of Time-Class Consistency

In stage one, the decision model is aimed to handle the time consistency requirement of SR passengers. Two types of passengers are considered: let \( P \) be the set of SR passengers who request one-off transportation requests. To formulate the time consistent schedule routing problem, let \( M \) denote a set of days of the planning horizon. A directed graph \( G = (N, A) \) with node set \( N = \{0, 1, \ldots, 2n + 1\} \) and arc set \( A \), is considered. Node 0 represents the origin depot and node \( 2n+1 \) represents the destination depot. Subsets \( N_s \subset N \) and \( N_f \subset N \) are, respectively, the pickup and drop-off nodes for all passenger. For each SR passenger \( p \), \( s(p) \) is the set of pickup nodes of passenger index \( p \) and \( f(p) \) is the set of drop-off nodes: for each SDAR passenger \( q \), \( s(q) \) is the set of pickup node of passenger index \( q \) and \( f(q) \) is the set of the drop-off node with \( N_s = s(p) \cup s(q) \) and \( N_f = f(p) \cup f(q) \). Each passenger order is denoted by \( i \in N_s \cup N_f \). These notations are used to represent transportation orders and the corresponding passenger types.

Let \( b^m_o \) be a variable indicating the order arrival time of a passenger, denoted by \( o \), and let \( b_i^m \) be the planned arrival time for passengers on day \( m \). Let \( C \) be the maximum number of time-classes allowed for every customer and \( c_{s(p)}^m \) and \( c_{f(p)}^m \) be the pickup node time class and the drop-off node time class for the SR passenger \( p \) on day \( m \). When the difference between the passenger’s order arrival time in day \( m \) and the order arrival time in another day \( m' \) is greater or equal to \( L \), i.e., \( (|b_i^m - b_i^{m'}| \geq L) \), whereas \( L \) is a time parameter to determine a time class, then an additional class would be needed. The objective of the stage-one decision model is to identify the minimal number of time class by determining \( b_o^m \) and \( b_i^{m'} \).

\[
TW(C) = \min C \quad (1)
\]
\[
|b_o^m - b_i^{m'}| \geq L \Rightarrow C_m \neq C_{m'}, m, m' \in M \quad (2)
\]
\[
C_m \leq C \quad m \in M \quad (3)
\]
\[
C_m \in \{1, \ldots, |M|\} \quad m \in M \quad (4)
\]
\[
C \in \{1, \ldots, |M|\} \quad (5)
\]

This decision model is almost the same with the time consistency defined by Feillet et al. [20] and Tellez et al. [21]. We adopt this decision model for handling the time consistency requirement of SR service orders, and these requirements will become part of the constraints in the vehicle routing decisions of integrating them with SDAR service orders in stage two.

### 4.3. Formulating the Stage-Two Decision Model: Vehicle Routing

In stage two, the decision model is aimed to identify an optimal route plan for SR and SDAR service orders using a set of vehicles denoted by \( V \). To identify the optimal route plan, we need to define the capacity of each vehicle, the load and the service time of a service order, the traveling time of pickup and drop-off locations related to early pickup and late drop-off time tolerances as well as their
relationship with the time classes. The capacity of each vehicle, \( v \in V \), is denoted by \( Z \). Let \( K_i^{m} \) be the load of vehicle \( v \) upon leaving node \( i \) on day \( m \). It should be noted that all vehicles are not assumed to be active on each day. Let \( R_i^{m} \) = 1 if the SR passenger \( p \) requests service at day \( m \), \( R_i^{m} \) = 0 otherwise. Each node \( i \in N \) is associated with a space \( k_i^{m} \) needed with a service time \( T_i \), where \( k_0^{m} = k_2^{m} = 0, k_{s(u)}^{m} = -k_{f(u)}^{m}, \forall u \in P \cup Q \), and \( T_0 = T_{2n+1} = 0 \). The service time is the time required to load the passenger in a vehicle, and it is assumed as a constant time for each passenger. Further, it is assumed that vehicles are not allowed to wait for passengers in the decision model. Instead, it is assumed that each passenger has a tolerance, which is the difference between the order time and actual arrival time for each customer order.

The objective is to minimise the total travelling time during the planning horizon with time-consistent constraints for both SR and SDAR service orders. The decision variable is denoted by \( x_{i,j,p} \), which is the binary variable equal to one if arc \((i, j)\) is traveled by vehicle \( v \) on day \( m \), equal to zero otherwise. Let \( b_i^{m} \) be the time at which node \( i \), \( (i \in N) \) is reached on day \( m \) and let \( t_{ij}^{m} \) be the travelling time from node \( i \) to node \( j \) on day \( m \). The binary variables \( y_{m,m'}^{f(p)} \) are set to 1 if the pickup time for customer \( p \) on days \( m \) and \( m' \) are in the increasing order and they are in the same time-class \((|b_i^{m} - b_i^{m'}| \leq L)\), and 0 otherwise, whereas \( L \) is a time parameter for determining a time class. Each SDAR passenger has a departure time window \([c_i^{m}(q) - \tau_i^{m}(q), r_i^{m}(q)]\) and an arrival time window \([m_i^{m}(f(q)) + f_i^{m}(q), f_i^{m}(q)]\), where \( \tau_i^{m}(q) \) and \( f_i^{m}(q) \) are the tolerance of early pickup and late drop-off for passenger \( q \). A large number, denoted by \( H \), is used in the time-related constraints in the model.

The objective function that minimises the time cost in the planning horizon is given by

\[
\text{Min } z = \sum_{m \in M} \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} c_{i,j,v}^{m} x_{i,j,v}^{m} \tag{6}
\]

subject to

\[
\text{s.t. } \sum_{k \in K} \sum_{p \in P} x_{s(p),i,k}^{m} = R_i^{m} \quad p \in P, m \in M \tag{7}
\]

\[
\sum_{m \in M} \sum_{v \in V} \sum_{p \in P} x_{s(p),i,v}^{m} = 1 \quad q \in Q \tag{8}
\]

\[
\sum_{j \in N} x_{0,i,v}^{m} \leq 1 \quad v \in V, m \in M \tag{9}
\]

\[
\sum_{i \in N} x_{1,i,v}^{m} \leq 1 \quad v \in V, m \in M \tag{10}
\]

\[
\sum_{j \in N} x_{i,j,v}^{m} - \sum_{j \in N} x_{i,j,v}^{m} = 0 \quad i \in N \cup N_f, v \in V, m \in M \tag{11}
\]

\[
\sum_{j \in N} x_{u(i),j,v}^{m} - \sum_{j \in N} x_{u(i),j,v}^{m} = 0 \quad u \in P \cup Q, v \in V, m \in M \tag{12}
\]

\[
|b_{s(p)}^{m} - b_{s(p)}^{m'}| \leq L + H y_{s(p)}^{m,m'} \quad p \in P, m, m' \in M \tag{13}
\]

\[
|b_{f(p)}^{m} - b_{f(p)}^{m'}| \leq L + H y_{f(p)}^{m,m'} \quad p \in P, m, m' \in M \tag{14}
\]

\[
c_{s(p)}^{m} + 1 \leq y_{s(p)}^{m,m'} + H(1 - y_{s(p)}^{m,m'}) \quad p \in P, m, m' \in M \tag{15}
\]

\[
c_{f(p)}^{m} + 1 \leq y_{f(p)}^{m,m'} + H(1 - y_{f(p)}^{m,m'}) \quad p \in P, m, m' \in M \tag{16}
\]

\[
y_{s(p)}^{m,m'} \in [0,1] \quad p \in P, m, m' \in M
\]
\[ y_{f(p)}^{m,m'} \in \{0, 1\} \quad p \in P, \quad m, m' \in M \] (18)
\[ c_{f(p)}^m \leq C_p \quad p \in P, \quad m \in M \] (19)
\[ c_{f(p)}^m \leq C_p \quad p \in P, \quad m \in M \] (20)
\[ b_{s(q)}^m \geq \tau_{s(q)}^m - \tau_q \quad q \in Q, \quad m \in M \] (21)
\[ b_{f(q)}^m \leq b_{s(q)}^m + \tau_q \quad q \in Q, \quad m \in M \] (22)
\[ b_j^m \geq b_i^m + T_i + t_{i,j}^m - H(1 - x_{i,j,v}^m) \quad i, j \in N, \quad m \in M, \quad v \in V \] (23)
\[ k_{i,v}^m \geq (k_{i,v}^m + k_j^m) x_{i,j,v}^m \quad i, j \in N, \quad m \in M, \quad v \in V \] (24)
\[ \max\{0, k_i^m\} \leq k_{i,v}^m \leq \min\{Z, Z + k_i^m\} \quad i \in N, \quad m \in M, \quad v \in V \] (25)
\[ c_{f(p)}^m + \tau_{s(p)}^m \in \{1, \ldots, C_p\} \quad p \in P, \quad m \in M \] (26)
\[ x_{i,j,v}^m \in \{0, 1\} \quad i, j \in N, \quad m \in M, \quad v \in V \] (27)
\[ R_p^m \in \{0, 1\} \quad p \in P, \quad m \in M \] (28)
\[ b_i^m, k_i^m \geq 0 \quad i \in N, \quad m \in M \] (29)

Constraint (7) is that SR passengers will be served if, and only if, he or she requires SR service at node \(i\) on day \(m\). Constraint (8) is that SDAR passengers must be served once. Constraint (9), (10) guarantee that vehicle \(v\) leaves the depot and return to the sink node once, allowing that some vehicles do not operate in some days. Constraint (11) is the flow conservation. Constraint (12) guarantee each O-D pair is visited by the same vehicle. Constraints (13) and (14) are the pickup time requirement and the arrival time requirement. Constrains (13)–(20) are the time class constraints of the SR passengers; \(C_p\) is the maximum number of time class for SR passenger \(p\), which is obtained from Equation (1) in stage one. Constraints (19) and (20) limit the maximum number of time class for both the pickup time and drop-off time allowed for each SR passenger \(p\). Constraints (21) and (22) are the time window constraint for earlier pickup and later drop-off for the SDAR passenger, respectively. Constraints (23) and (24) guarantee the consistency of the time and prevent subtour in the network. Constraint (25) ensures the load of service order not exceeding the vehicle capacity. The capacity at node \(i\) at day \(m\) should be less than the minimum amount between the capacity \(Z\) and \(Z + k_i^m\) and greater than the maximum amount between 0 and \(k_i^m\). Constraints (26)–(29) define the range of variables.

5. Numerical Illustration and Case Study

For the illustration and evaluation of the proposed model that integrates the SR and SDAR service orders, we developed an experiment with a decision support system of the vehicle routing problem using real operational data. The identification of the time-class consistency as the stage one decision is solved by a program created using Python, and the outputs of the stage one decision are used for the stage two decision model involving vehicle routing for the integrated service orders. In the case study, we collaborated with an accessible transportation organisation, which provides different paratransit services including SR and DAR.

5.1. Numerical Illustration

To illustrate the proposed idea, two cases are considered: (i) a route plan for three SR service orders and (ii) a route plan for three SR service orders and two SDAR service orders. The two examples are summarised in Tables 1 and 2, respectively; the tables also include information such as passenger no., route sequence no. of route plan by pickup point (PP) and drop point (DP), pickup time (PT), drop-off time (DT), and value of consistent time-class for PT and DT.
Table 1. Original route plan contains schedule route (SR) passengers only.

<table>
<thead>
<tr>
<th>Passenger</th>
<th>PP</th>
<th>DP</th>
<th>Mon. PT</th>
<th>Mon. DT</th>
<th>Tue. PT</th>
<th>Tue. DT</th>
<th>Wed. PT</th>
<th>Wed. DT</th>
<th>Thur. PT</th>
<th>Thur. DT</th>
<th>Fri. PT</th>
<th>Fri. DT</th>
<th>No. of PT</th>
<th>No. of DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR 1</td>
<td>1</td>
<td>6</td>
<td>9:00</td>
<td>10:10</td>
<td>9:00</td>
<td>10:10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9:00</td>
<td>10:10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SR 2</td>
<td>2</td>
<td>7</td>
<td>9:30</td>
<td>10:30</td>
<td>9:30</td>
<td>10:30</td>
<td>9:30</td>
<td>10:30</td>
<td>10:10</td>
<td>11:00</td>
<td>9:30</td>
<td>10:30</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2. Integrating SR and SDAR service orders (maximum time classes = 2).

<table>
<thead>
<tr>
<th>Passenger</th>
<th>PP</th>
<th>DP</th>
<th>Mon. PT</th>
<th>Mon. DT</th>
<th>Tue. PT</th>
<th>Tue. DT</th>
<th>Wed. PT</th>
<th>Wed. DT</th>
<th>Thur. PT</th>
<th>Thur. DT</th>
<th>Fri. PT</th>
<th>Fri. DT</th>
<th>No. of PT</th>
<th>No. of DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR 1</td>
<td>1</td>
<td>6</td>
<td>9:00</td>
<td>10:10</td>
<td>9:00</td>
<td>10:25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9:00</td>
<td>10:10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SR 2</td>
<td>2</td>
<td>7</td>
<td>9:30</td>
<td>10:30</td>
<td>9:30</td>
<td>10:00</td>
<td>9:30</td>
<td>10:50</td>
<td>11:00</td>
<td>10:30</td>
<td>9:30</td>
<td>10:30</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>SDAR 1</td>
<td>3</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>9:50</td>
<td>11:15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SDAR 2</td>
<td>4</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10:00</td>
<td>11:35</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3a and Table 1 show the original one-week route plan and the timesheet for the three SR passengers (SR 1, SR 2, and SR 3). As summarised in Table 2, there are two potential SDAR passengers, SDAR 1 and SDAR 2, who place an order to be picked up at 9:50 a.m. on Tuesday and at 10:00 a.m. on Wednesday, respectively. The integration rule acts as follows.

1. On Tuesday, to integrate SDAR 1 (Figure 3b), the pickup time of the order of SR 3 is adjusted to 10:15 a.m., and the drop-off time is adjusted to 11:50 a.m. This adjustment does not violate the time class constrains because the renewed pickup time class for SR 3 is {9:50, 9:50, 9:50 a.m.} and {10:15 and 10:30 a.m.};

2. On Wednesday, the order of SDAR 2 (Figure 3c) is placed into the system. If SDAR 2 is integrated into this route plan, the pickup time of SDAR 2 will be at 10:25 a.m., and the time class of each SR passengers remains 2.

This example shows the possibility of integrating an SDAR service order into a route of SR passengers for the sake of serving an increased number of passengers while maintaining the time consistency of the SR passengers.
5.2. Case Study and Experiment Results

Our case study collaborated with an accessible transportation organisation that faces the challenges of the increasing demand of passengers due to ageing population in Hong Kong. The territory with 2755 km$^2$ area consists of Hong Kong Island, the Kowloon Peninsula, the New Territories, Lantau Island, and over 200 other islands. Of the total area, 1073 km$^2$ (414 sq mi) is land. From 1988 to 2018, the size and share of the elderly (aged 65 and over) population rose continuously from 0.46 million and 8.2% to 1.27 million and 17.9%, respectively [43]. It is expected that the elderly population will almost double from 1.27 million and 17.9% to 2.44 million between 2018 and 2038. The collaborated organisation provides the accessible transportation services of SR, DAR, feeder, and pooled DAR to satisfy the increasing demand of people with travel difficulty because of aging population. The users of the DAR or pooled DAR services need to make their requests at least a day prior to the service date, while users of the SR service need to make their request at least a month prior to the service date. Service availability depends on the availability of the vehicle, as these four services share the same pool of vehicles. Most vehicles are used for SR service during the weekdays and more vehicles are available for DAR service during the weekends. In other words, people requesting DAR service during the weekdays may not be served because most vehicles are occupied to provide SR service. This causes a high rejection rate of service requests for DAR service during the weekdays, due to the limited number of vehicles as well as the inflexible vehicle scheduling approach. We evaluated how many vehicles could be released to increase the opportunity to serve more people, as well as the effects on the travelling time of DAR passengers in the integrated model using a flexible vehicle scheduling system.

Further analysis was conducted with the collaborated organisation’s operation data to evaluate the benefit of a flexible vehicle routing system that can support the integrated SDAR with SR services. In a random sample of seven days of SR services, the average number of passenger trips per day was 1276, which was handled by an average number of 90 vehicles per day. Based on this selected period, all SDAR service orders were retrieved, and these orders accounted for 189 passenger trips per day. Those SDAR service orders were handled by an average number of 49 vehicles. On average, each vehicle is used to serve 14 passenger trips for the SR service, while only 3.86 passenger trips are served for the SDAR service. In terms of traveling time, most trips are less than one and half hours. The objective of the analysis is to evaluate the number of vehicles that can be saved by using the integrated SDAR with the SR.

Figure 4 shows the number of vehicles used in the case of integrating SDAR with SR for comparison. When compared with the original operation, the number of vehicles used for the integrated service ranges between 94–108 per day, and the average number of vehicles is 100.7. This implies that an operational efficiency improvement of 27.4%—as measured by the number of vehicles—can be achieved. Apart from the benefit of integrated service, we also evaluate whether this increased operational efficiency would result in a much longer travelling time for an SDAR service order.
These experimental results indicate the benefit of the integrated service. Accordingly, the change in the route is mild because there is only one additional pickup of the SDAR passenger. However, it is crucial to review the route plan using a digital map.

Figure 4. Number of vehicles used for the comparison of integrated service.

Figure 5 shows the overall travelling time of the integrated SDAR service orders. When compared with the original SDAR service order, the overall travelling time of the SDAR service order in the integrated mode increased by 2 min on average. In fact, the travelling time of the SDAR service order did not always increase in the integrated mode, as some of the SDAR service orders had a better fit with the SR passenger than that in the original route plan with the other SDAR passenger. However, it is crucial to review the route plan using a digital map.

Figure 5. Traveling time of original and new SDAR order.

Figure 6 shows the digital map for a planning manager to preview the change in route in the integrated service. Accordingly, the change in the route is mild because there is only one additional pickup of the SDAR passenger. These experimental results indicate the benefit of the integrated service using a flexible vehicle routing system, which not only provides a flexible option of integrating different paratransit services, but also enables a planning manager to visualise the proposed route in a digital map.
6. Discussion

The case study and experimental results show the practical contributions of our study, however there are certain limitations as well. First, the results are based on the assumption of a high user acceptance rate of the new service option. However, some passengers would prefer individual convenience to the benefit of all people in the entire society. For such passengers, the paratransit service would be offered at a market price without any subsidy from the government. Second, the current system design focuses on the planning phase of scheduling service orders, but it would be further enhanced to consider the possibility of serving ad-hoc order requests in real time as well. More information communication technologies would be adopted in the design of our proposed system. Finally, the system performance is currently measured by the number and utilisation of vehicles, and it does not directly indicate the improvement in carbon emission. It is advised to include the measurement system of carbon emission in vehicle to reflect the macro view of sustainability.

Our study not only helps the practitioners, but also contributes to the literature. When compared with other related studies, most of the previous studies focussed on the macro-view of social sustainability in terms of economic, environmental, and social effects via survey and traffic model. In contrast, our study focuses on the design of a transportation system which aims to sustain the community transportation organisation under the pressure of increasing demand by using fewer resources. This addresses the research gap in achieving sustainability at both social and the organisation levels. In addition, our study proposes an integrated decision model which connects individual services...
in a unified system. This integrated decision model can also help other logistics and transportation organisations to integrate several other services for sustainable operation.

7. Conclusions

The increasing demand for paratransit services will drive the development of a flexible vehicle scheduling system for accessible transportation organization, so that more people can be served with fewer vehicles. We investigated the system design with three module layers and customised the process for multiple types of paratransit services and the decision model of integrating schedule route service with a DAR service. Based on the experimental analysis, we found that 27.4% of vehicles could be released to serve more people with travel disability. This improved operational efficiency does not compensate for the longer travelling time of SDAR passengers because of the optimisation techniques applied in the flexible vehicle scheduling system. The flexible system is designed to support the planning manager visualising any change in the route plan in a digital map for better decision and communication. Further, it is believed that these findings will facilitate sustainable and flexible operations for logistics and transportation organisations.

Future research could be conducted to deal with price-responsive demand with uncertainties. This study currently focuses on the order planning processes which have a set of orders in the database for vehicle scheduling, but it does not investigate the re-scheduling processes for new orders and change requests occur in real time. The scope of system design and decision complexity will be broader and higher, as it requires a communication mechanism between a planning manager and drivers in the network. In terms of the vehicle scheduling decision model, this problem is regarded as a dynamic and stochastic vehicle routing problem and the challenge will be how to satisfy the time consistency of SR passengers and ad-hoc order requests of SDAR passengers at the same time.


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