A New Model for Determining Weight Coefficients of Criteria in MCDM Models: Full Consistency Method (FUCOM)

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Abstract: In this paper, a new multi-criteria problem solving method—the Full Consistency Method (FUCOM)—is proposed. The model implies the definition of two groups of constraints that need to satisfy the optimal values of weight coefficients. The first group of constraints is the condition that the relations of the weight coefficients of criteria should be equal to the comparative priorities of the criteria. The second group of constraints is defined on the basis of the conditions of mathematical transitivity. After defining the constraints and solving the model, in addition to optimal weight values, a deviation from full consistency (DFC) is obtained. The degree of DFC is the deviation value of the obtained weight coefficients from the estimated comparative priorities of the criteria. In addition, DFC is also the reliability confirmation of the obtained weights of criteria. In order to illustrate the proposed model and evaluate its performance, FUCOM was tested on several numerical examples from the literature. The model validation was performed by comparing it with the other subjective models (the Best Worst Method (BWM) and Analytic Hierarchy Process (AHP)), based on the pairwise comparisons of the criteria and the validation of the results by using DFC. The results show that FUCOM provides better results than the BWM and AHP methods, when the relation between consistency and the required number of the comparisons of the criteria are taken into consideration. The main advantages of FUCOM in relation to the existing multi-criteria decision-making (MCDM) methods are as follows: (1) a significantly smaller number of pairwise comparisons (only \( n - 1 \)), (2) a consistent pairwise comparison of criteria, and (3) the calculation of the reliable values of criteria weight coefficients, which contribute to rational judgment.

Keywords: multi-criteria decision-making; criteria weights; FUCOM; AHP; BWM

1. Introduction

Determining the weights of criteria is one of the key problems that arise in multi-criteria analysis models. The problem of choosing an appropriate method of determining criteria weights in problems of multi-criteria decision-making (MCDM) is a very important stage, which complicates the decision-making process. Taking into account the fact that the weights of criteria can significantly influence the outcome of the decision-making process, it is clear that particular attention must be paid to the objectivity factors of criteria weights.

Roberts and Goodwin [1] provide an overview of the studies in which the advantages and disadvantages of the individual methods of determining the weights of criteria are considered.
The authors whose literature was consulted [1–4] agree that the values of criteria weights are significantly conditioned by the methods of their determination. Moreover, there is no agreement upon the best method of determining criteria weights, nor is there agreement upon the method of the direct determination of the “real” set of weights. In the literature, however, there is agreement that the weights calculated by applying certain methods are more accurate than the weights obtained by the methods of a direct weight assignment based on the expert’s understanding of the significance of criteria.

By studying the available literature, it can be noticed that there is no unique classification of the methods of determining criteria weights and that it is carried out on several grounds, in accordance with the author’s perceptions and needs for solving a specific practical problem. Thus, the literature by [5] provides the classification of the methods of determining criteria weights, which is objective and subjective in relation to whether weights are calculated indirectly, on the basis of the outcomes (consequences), or they are obtained directly from the decision-maker. The classification of the methods of determining criteria weights, which is more extensive to a slightly greater extent, can be found in the literature by [4], where the methods of determining criteria weights are divided into: statistical and algebraic, holistic and decomposed, direct and indirect, and compensatory and non-compensatory. In algebraic methods, the $n$ weight is calculated based on an $n - 1$ set of judgments (conclusions) by using a simple system of equations. Statistical methods include a regression analysis that can also provide guidance on the choice of weights. Decomposed procedures are based on the comparison of the one-to-one pair of criteria at a time, whereas in holistic methods, the decision-maker considers both the criteria and the alternatives when expressing his/her preferences and makes the overall assessment of the alternatives. In direct methods, the decision-maker compares two criteria by using a ratio scale, whereas in indirect methods, criteria weights are calculated based on the decision-maker’s preferences.

Based on the concept of compensation and trade-offs among criteria, methods can be classified as compensatory and non-compensatory [6]. Compensatory methods are used for the aggregation of partial values in the methods of multi-attribute utility theory, whereas non-compensatory methods are used to aggregate partial values in outranking methods. The compensatory methods most commonly used are: (1) the trade-off method [7], which discovers the decision-maker’s dilemmas through a pairwise examination of criteria; (2) the swing method [8], which implies the construction of the two extreme hypothetical scenarios W and B, where the first (W) presents the worst values of all criteria, and the second scenario (B) corresponds to the best values; (3) the SMART method (the Simple Multi-Attribute Rating Technique) [9], which implies the procedure for determining criteria weights by comparing criteria with the best, and the worst, criterion from a defined set of criteria; (4) the SMARTER (SMART Exploiting Ranks) method, whose authors Edwards and Barron [9], presented a new version of the SMART method that uses the centroid method to determine criteria weights.

Contrary to compensatory methods, non-compensatory methods mainly reflect the global values of the relative importance of criteria. Non-compensatory methods do not pay particular attention to the impact of the range of a specific decision-making context, which is important in constructing partial relations of preferences. The most commonly used non-compensatory methods are the following ones: (1) the point allocation method [10], which is one of the simplest methods used to determine criteria weights, where, according to the priority of criteria, a decision-maker distributes a certain number of points to each criterion; (2) the direct rating method [11], in which the decision-maker first ranks all the criteria according to their significance, and on the basis of the criterion rank, the decision-maker assigns a weight to each criterion; (3) the methods of pairwise comparisons, where the decision-maker compares each criterion with others and determines the level of preferences for each pair of such criteria. The ordinal scale helps to determine the preference value of one criterion against the other. One of the most commonly applied methods based on pairwise comparisons is the Analytic Hierarchy Process (AHP) method [12]. The methods of pairwise comparisons, in addition to the AHP method, also include the DEMATEL method [13] and the Best Worst Method (BWM) [14]; (4) the resistance to change method [1], which has the elements of the swing method and the pairwise comparison
The resistance to change method starts from the assumption that each criterion has two opposite sides (poles) of performance (the desirable and the undesirable) and the priorities of criteria are defined on the basis of these settings.

In addition to this classification, the majority of authors suggest the classification of the models of determining the weights of criteria into subjective and objective models [15,16]. Subjective approaches reflect the decision-maker’s subjective opinion and intuition. By such an approach, the decision-maker directly influences the outcome of the decision-making process, since the weights of criteria are determined based on the information obtained from the decision-maker or from the experts involved in the decision-making process. Objective approaches are based on determining the weights of criteria on the basis of the information contained in a decision-making matrix applying certain mathematical models. Objective approaches neglect the decision-maker’s opinion.

In a subjective approach, the decision-maker or experts give their opinion on the significance of criteria for a certain decision-making process in accordance with their system of preferences. There are many ways to obtain the weights of criteria by applying a subjective approach, which can vary in the number of the participants in the weighting process, the applied methods and the forming of the final weights of criteria. Subjective approaches are mainly based on the pairwise comparisons of criteria or the ranking of criteria. The most well-known objective methods are the following: the entropy method [17], the CRITIC method (Criteria Importance Through Intercriteria Correlation) [18] and the FANMA method, named after its authors [19].

This paper presents a new subjective model for determining the weights of criteria: the Full Consistency Method (FUCOM). The FUCOM algorithm is based on the pairwise comparisons of criteria, where only the \( n - 1 \) comparison in the model is necessary. The model implies the implementation of a simple algorithm with the ability to validate the model by determining the deviation from full consistency (DFC) of the comparison. The consistency of the model is defined on the basis of the satisfaction of mathematical transitivity conditions. One of the characteristics of the developed new method is the lowering of decision-maker’s subjectivity, which leads to consistency or symmetry in the weight values of the criteria. Since FUCOM belongs to the group of subjective models, the most well-known subjective models are presented in the following section. Through the overview of the literature, the advantages and disadvantages of the existing models are highlighted and the gap filled by the new model in the literature is emphasized.

Taking into account all of the foregoing, there is a need for a method that requires (1) a small number of the pairwise comparisons of criteria, (2) a possibility of defining DFC of the comparison, and (3) the appreciation of transitivity in the pairwise comparison of criteria. Accordingly, FUCOM has been developed and the following goals of this paper have been set. The first goal of the paper is to present a new model for determining the weights of criteria that requires only the \( n - 1 \) pairwise comparison of criteria by applying any scale (either integer or decimal). The second goal is to define a model that allows the calculation of the comparison consistency degree and the validation of the results, by fully respecting the conditions of mathematical transitivity. The third goal of the paper is to define a model that enables the calculation of the reliable values of the weight coefficients of criteria that contribute to a rational judgment. The fourth contribution of the paper is the comparison of the FUCOM method with other common subjective methods such as the BWM model and the AHP model. The advantages of FUCOM in relation to the existing subjective models in the literature are hereinafter explained in detail.

The rest of the paper is organized in the following manner: in the paper’s second chapter a review of applications of subjective MCDM methods are given, while in the paper’s third chapter, an algorithm and the application of the FUCOM example are presented. In the fourth chapter of the paper, a comparison of the FUCOM results with the results obtained by means of the application of the AHP model and the BWM is performed, and a discussion about the FUCOM results is given. The fifth chapter provides the concluding observations and the directions for future research.
2. Review of Applications of Subjective MCDM Methods in Different Studies

The best-known methods from the group of the subjective methods of determining the weighting values of criteria are the following: the AHP method [12], the DEMATEL (Decision-making Trial and Evaluation Laboratory) method [13], the SWARA (Step-Wise Weight Assessment Ratio Analysis) method [20], and the BWM [14]. Each of these methods has a wide application in the various areas of science and technology, as well as in solving real-life problems. The AHP method was used in [21] to make a strategic decision in a transport system; i.e., for the purpose of the reconfiguration of the railway infrastructure in the port of Trieste. In [22], this method was used to determine the significance of the criteria in evaluating different transitivity alternatives in transport in Catania. In [23], the AHP method was used to identify and evaluate defects in the passenger transport system, whereas in [24], it was used to select an alternative to the electronic payment system. Stević et al. [25] carried out a site selection of a logistics center in Bosnia and Herzegovina by applying the AHP method. In [26], the DEMATEL method analyzed the risk in mutual relations in logistics outsourcing, whereas a combination of the AHP and the DEMATEL methods in [27] was also used in the field of risk; i.e., the integration of logistical information. The integration of the DEMATEL method is not rare, so in [28], together with the ANP and the DEA, a decision was made on the choice of the 3PL logistics provider. In terms of applying the SWARA method for the purpose of determining the weighting values of criteria in the field of transport and logistics for the observed period since 2015, it has not been noticed. Only its fuzzy form [29] was used to select the 3PL in the sustainable network of reverse logistics and a rough form [30] for the purpose of determining the significance of criteria to the procurement of railroad wagons. When the application of the BWM is concerned, the situation is similar—in its rough form, and in combination with the Rough SAW method, it was applied in [31] in a logistics company.

It is impossible not to notice that, despite the fact that it is a relatively old method, the AHP method is still used in a large number of publications in its crisp form [32–60]. This confirms the conclusions by Zavadskas et al. [16] that, in the literature, the AHP method is the method most commonly used to determine the weights of criteria and/or rank alternatives. The validation of the results in the AHP model are based on the degree of consistency, whose value is limited to max 0.10. Since it is necessary to respect mathematical transitivity in the pairwise comparisons of criteria, the deviation from transitivity results in an increase in inconsistency. In the AHP method, it is required to make the $n(n - 1)/2$ pairwise comparisons of criteria [45]. A large number of comparisons complicate the application of the model, especially in the cases of a larger number of criteria. According to some authors [15], it is almost impossible to perform completely consistent pairwise comparisons in the AHP method if there are more than nine criteria. This problem is usually overcome by dividing criteria into subcriteria, which further complicates the model.

The DEMATEL method was also used in a large number of studies [61–92], but its main disadvantage is a lack of consistency measure; i.e., the inability to validate the results obtained. Therefore, the DEMATEL method is mainly used to determine the interaction among criteria and the diagram of relations [93,94]. The DEMATEL method is often used in order to determine the weights of criteria in combination with the ANP (Analytic Network Process) method [95]. This partly eliminates a disadvantage. The AHP and ANP methods have many aggregation procedures to obtain a preference vector from pairwise comparisons matrix. The SWARA method is applied because of its simplicity and a small number of steps. However, the SWARA method, like DEMATEL, does not have the ability to determine the consistency degree of the comparisons obtained. For this reason, the SWARA method is much less used in the literature than the two previously mentioned methods (the AHP and DEMATEL), which is evident from [96–113]. The BWM is the method that has increasingly been applied over a short time [114–137]. Some authors [122–124,128,134,138–140] see this method as an adequate substitute for the AHP. Its major advantage is a smaller number of pairwise comparisons $(2n - 3)$ compared to the AHP. However, the degree of consistency in this method ranges to one, which in certain cases reflects a high degree of subjectivity. Razei [14] proposes the determining of the interval values of weight coefficients as a solution to this problem and he suggests the determining of the mean values...
of intervals and taking that value as the final value of the weight of criteria. Nevertheless, due to the inconsistency of the results, there is no guarantee that the optimal values of weight coefficients will be within the defined intervals.

3. Full Consistency Method (FUCOM)

The problems of multi-criteria decision-making are characterized by the choice of the most acceptable alternative out of a set of the alternatives presented on the basis of the defined criteria. A model of multi-criteria decision-making can be presented by a mathematical equation
\[
\max\{f_1(x), f_2(x), ..., f_n(x)\}, \ n \geq 2, \text{ with the condition that } x \in A = \{a_1, a_2, ..., a_m\}; \text{ where } n \text{ represents the number of the criteria, } m \text{ is the number of the alternatives, } f_j \text{ represents the criteria (} j = 1, 2, ..., n) \text{ and } A \text{ represents the set of the alternatives } a_i (i = 1, 2, ..., m). \text{ The values } f_{ij} \text{ of each considered criterion } f_j \text{ for each considered alternative } a_i \text{ are known, namely } f_{ij} = f_j(a_i), \ \forall (i, j); \ i = 1, 2, ..., m; \ j = 1, 2, ..., n. \text{ The relation shows that each value of the attribute depends on the } j\text{th criterion and the } i\text{th alternative.}

Real problems do not usually have the criteria of the same degree of significance. It is therefore necessary that the significance factors of particular criteria should be defined by using appropriate weight coefficients for the criteria, so that their sum is one. Determining the relative weights of criteria in multi-criteria decision-making models is always a specific problem inevitably accompanied by subjectivity. This process is very important and has a significant impact on the final decision-making result, since weight coefficients in some methods crucially influence the solution. Therefore, particular attention in this paper is paid to the problem of determining the weights of criteria, and the new FUCOM model for determining the weight coefficients of criteria is proposed. This method enables the precise determination of the values of the weight coefficients of all of the elements mutually compared at a certain level of the hierarchy, simultaneously satisfying the conditions of the comparison consistency, too.

In real life, pairwise comparison values \(a_{ij} = w_i / w_j\) (where \(a_{ij}\) shows the relative preference of criterion \(i\) to criterion \(j\)) are not based on accurate measurements, but rather on subjective estimates. There is also a deviation of the values \(a_{ij}\) from the ideal ratios \(w_i / w_j\) (where \(w_i\) and \(w_j\) represents criterion weights of criterion \(i\) and criterion \(j\)). If, for example, it is determined that \(A\) is of much greater significance than \(B\), \(B\) of greater importance than \(C\), and \(C\) of greater importance than \(A\), there is inconsistency in problem solving and the reliability of the results decreases. This is especially true when there are a large number of the pairwise comparisons of criteria. FUCOM reduces the possibility of errors in a comparison to the least possible extent due to: (1) a small number of comparisons \((n - 1)\) and (2) the constraints defined when calculating the optimal values of criteria. FUCOM provides the ability to validate the model by calculating the error value for the obtained weight vectors by determining DFC. On the other hand, in the other models for determining the weights of criteria (the BWM, the AHP models), the redundancy of the pairwise comparison appears, which makes them less vulnerable to errors in judgment, while the FUCOM methodological procedure eliminates this problem.

In the following section, the procedure for obtaining the weight coefficients of criteria by using FUCOM is presented.

**Step 1.** In the first step, the criteria from the predefined set of the evaluation criteria \(C = \{C_1, C_2, ..., C_n\}\) are ranked. The ranking is performed according to the significance of the criteria; i.e., starting from the criterion that is expected to have the highest weight coefficient to the criterion of the least significance. Thus, the criteria ranked according to the expected values of the weight coefficients are obtained:

\[
C_{j(1)} > C_{j(2)} > \cdots > C_{j(k)}
\]  

(1)

where \(k\) represents the rank of the observed criterion. If there is a judgment of the existence of two or more criteria with the same significance, the sign of equality is placed instead of “>” between these criteria in the expression (1).
Step 2. In the second step, a comparison of the ranked criteria is carried out and the comparative priority \( \phi_{k/(k+1)} \), \( k = 1, 2, ..., n \), where \( k \) represents the rank of the criteria) of the evaluation criteria is determined. The comparative priority of the evaluation criteria \( \phi_{k/(k+1)} \) is an advantage of the criterion of the \( C_j(k) \) rank compared to the criterion of the \( C_j(k+1) \) rank. Thus, the vectors of the comparative priorities of the evaluation criteria are obtained, as in the expression (2):

\[
\Phi = \left( \phi_{1/2}, \phi_{2/3}, ..., \phi_{k/(k+1)} \right)
\]

where \( \phi_{k/(k+1)} \) represents the significance (priority) that the criterion of the \( C_j(k) \) rank has compared to the criterion of the \( C_j(k+1) \) rank.

The comparative priority of the criteria is defined in one of the two ways defined in the following parts:

(a) Pursuant to their preferences, decision-makers define the comparative priority \( \phi_{k/(k+1)} \) among the observed criteria. Thus, for example, if two stones \( A \) and \( B \), which, respectively, have the weights of \( w_A = 300 \) grams and \( w_B = 255 \) grams are observed, the comparative priority \( \phi_{A/B} \) of Stone \( A \) in relation to Stone \( B \) is \( \phi_{A/B} = 300/255 = 1.18 \). Also, if the weights \( A \) and \( B \) cannot be determined precisely, but a predefined scale is used (e.g., from 1 to 9), then it can be said that stones \( A \) and \( B \) have weights \( w_A = 8 \) and \( w_B = 7 \), respectively. Then the comparative priority \( \phi_{A/B} \) of Stone \( A \) in relation to Stone \( B \) can be determined as \( \phi_{A/B} = 8/7 = 1.14 \). This means that stone \( A \) in relation to stone \( B \) has a greater priority (weight) by 1.18 (in the case of precise measurements); i.e., by 1.14 (in the case of application of measuring scale). In the same manner, decision-makers define the comparative priority among the observed criteria \( \phi_{k/(k+1)} \). When solving real problems, decision-makers compare the ranked criteria based on internal knowledge, so they determine the comparative priority \( \phi_{k/(k+1)} \) based on subjective preferences. If the decision-maker thinks that the criterion of the \( C_j(k) \) rank has the same significance as the criterion of the \( C_j(k+1) \) rank, then the comparative priority is \( \phi_{k/(k+1)} = 1 \).

(b) Based on a predefined scale for the comparison of criteria, decision-makers compare the criteria and thus determine the significance of each individual criterion in the expression (1). The comparison is made with respect to the first-ranked (the most significant) criterion. Thus, the significance of the criteria \( \alpha_{C_i(j)} \) for all of the criteria ranked in Step 1 is obtained. Since the first-ranked criterion is compared with itself (its significance is \( \alpha_{C_i(1)} = 1 \)), a conclusion can be drawn that the \( n-1 \) comparison of the criteria should be performed.

For example: a problem with three criteria ranked as \( C_2 > C_1 > C_3 \) is being subjected to consideration. Suppose that the scale \( \alpha_{C_i(j)} \in [1, 9] \) is used to determine the priorities of the criteria and that, based on the decision-maker’s preferences, the following priorities of the criteria \( \alpha_{C_2} = 1 \), \( \alpha_{C_1} = 3.5 \) and \( \alpha_{C_3} = 6 \) are obtained. On the basis of the obtained priorities of the criteria and condition \( \frac{w_i}{w_{k+1}} = \phi_{k/(k+1)} \) we obtain following calculations \( \frac{w_2}{w_1} = \frac{3.5}{1} \) i.e., \( w_2 = 3.5 \cdot w_1 \), \( \frac{w_3}{w_2} = \frac{6}{3.5} \) i.e., \( w_3 = 1.714 \cdot w_2 \). In that way, the following comparative priorities are calculated: \( \phi_{C_2/C_1} = 3.5/1 = 3.5 \) and \( \phi_{C_1/C_3} = 6/3.5 = 1.714 \) (expression (2)).

As we can see from the example shown in Step 2b, the FUCOM model allows the pairwise comparison of the criteria by means of using integer, decimal values or the values from the predefined scale for the pairwise comparison of the criteria.

Step 3. In the third step, the final values of the weight coefficients of the evaluation criteria \( (w_1, w_2, ..., w_n)^T \) are calculated. The final values of the weight coefficients should satisfy the two conditions:

(1) that the ratio of the weight coefficients is equal to the comparative priority among the observed criteria \( \phi_{k/(k+1)} \) defined in Step 2; i.e., that the following condition is met:

\[
\frac{w_k}{w_{k+1}} = \phi_{k/(k+1)}
\]
(2) In addition to the condition (3), the final values of the weight coefficients should satisfy the condition of mathematical transitivity; i.e., that $\psi_{k/(k+1)} \otimes \psi_{(k+1)/(k+2)} = \psi_{k/(k+2)}$. Since $\psi_{k/(k+1)} = \frac{w_k}{w_{k+1}}$ and $\psi_{(k+1)/(k+2)} = \frac{w_{k+1}}{w_{k+2}}$, that $\frac{w_k}{w_{k+1}} \otimes \frac{w_{k+1}}{w_{k+2}} = \frac{w_k}{w_{k+2}}$ is obtained. Thus, yet another condition that the final values of the weight coefficients of the evaluation criteria need to meet is obtained, namely:

$$\frac{w_k}{w_{k+2}} = \psi_{k/(k+1)} \otimes \psi_{(k+1)/(k+2)}$$

(4)

Full consistency i.e., minimum DFC ($\chi$) is satisfied only if transitivity is fully respected; i.e., when the conditions of $\frac{w_k}{w_{k+1}} = \psi_{k/(k+1)}$ and $\frac{w_k}{w_{k+2}} = \psi_{k/(k+1)} \otimes \psi_{(k+1)/(k+2)}$ are met. In that way, the requirement for maximum consistency is fulfilled; i.e., DFC is $\chi = 0$ for the obtained values of the weight coefficients. In order for the conditions to be met, it is necessary that the values of the weight coefficients $(w_1, w_2, ..., w_n)^T$ meet the condition of $|\frac{w_k}{w_{k+1}} - \psi_{k/(k+1)}| \leq \chi$ and $|\frac{w_k}{w_{k+2}} - \psi_{k/(k+1)} \otimes \psi_{(k+1)/(k+2)}| \leq \chi$, with the minimization of the value $\chi$. In that manner the requirement for maximum consistency is satisfied.

Based on the defined settings, the final model for determining the final values of the weight coefficients of the evaluation criteria can be defined.

$$\min \chi$$

s.t.

$$|\frac{w_j}{w_{j+1}} - \psi_{j/(j+1)}| \leq \chi, \forall j$$

$$|\frac{w_j}{w_{j+2}} - \psi_{j/(j+1)} \otimes \psi_{(j+1)/(j+2)}| \leq \chi, \forall j$$

$$\sum_{j=1}^n w_j = 1, \forall j$$

$$w_j \geq 0, \forall j$$

(5)

By solving the model (5), the final values of the evaluation criteria $(w_1, w_2, ..., w_n)^T$ and the degree of DFC ($\chi$) are generated. In order to achieve a better understanding of the presented model, two simple examples will demonstrate the process of determining weight coefficients by applying FUCOM. In the first example, the procedure for determining the comparative priority ($q_{k/(k+1)}$) is shown by applying Step 2a, whereas in the second example, $q_{k/(k+1)}$ is determined by applying Step 2b.

**Example 1.** The determination of the criteria weight coefficients will be presented through the example of the evaluation of transport-manipulative means in the logistics centers. There are four criteria identified for the evaluation of forklifts: the purchase price ($C_1$), the manufacturer’s warranty ($C_2$), the service network ($C_3$), and the maximum load capacity ($C_4$). As previously described, FUCOM was implemented through the following steps:

**Step 1.** In the first step, the decision-makers performed the ranking of the criteria: $C_1 > C_2 > C_3 > C_4$.

**Step 2.** In the second step (Step 2a), based on the decision-maker’s preferences, the comparative priorities of the ranked criteria were determined and the vector of the comparative priorities of the evaluation criteria was obtained (Table 1).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{k/(k+1)}$</td>
<td>1.00</td>
<td>1.08</td>
<td>1.25</td>
<td>1.45</td>
</tr>
</tbody>
</table>

**Step 3.** The final values of the weight coefficients should meet the following two conditions:
The comparison was based on the scale coefficients can be defined as: 

$$\phi = \begin{cases} \frac{w_1}{w_2} = 1.08, \frac{w_2}{w_3} = 1.25 \text{ and } \frac{w_3}{w_4} = 1.45 \end{cases}$$

Thus, that \( \frac{w_1}{w_2} = 1.08, \frac{w_2}{w_3} = 1.25 \) and \( \frac{w_3}{w_4} = 1.45 \) is obtained.

### Table 2. Priorities of criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C2</th>
<th>C1</th>
<th>C4</th>
<th>C3</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{C(i)} )</td>
<td>1</td>
<td>2.1</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

(2) In addition to the condition (3), the final values of the weight coefficients should meet the condition of mathematical transitivity; i.e., the condition (4). Thus, that \( \frac{w_1}{w_2} = 1.08 \cdot 1.25 = 1.35 \) and \( \frac{w_2}{w_4} = 1.25 \cdot 1.45 = 1.81 \) is obtained.

Regarding the defined limitations, on the basis of the expression (5), a finite model for determining the weight coefficients meeting the condition of maximum consistency can be defined.

$$\min \chi \quad s.t. \begin{cases} \left| \frac{w_1}{w_2} - 1.08 \right| \leq \chi, \left| \frac{w_2}{w_3} - 1.25 \right| \leq \chi, \left| \frac{w_3}{w_4} - 1.45 \right| \leq \chi, \\
\frac{w_1}{w_4} - 1.35 \leq \chi, \left| \frac{w_2}{w_4} - 1.81 \right| \leq \chi, \\
\sum_{j=1}^{4} w_j = 1, \ w_j \geq 0, \ \forall j \end{cases}$$

By solving this model, the final values of the weight coefficients \((0.315, 0.291, 0.233, 0.161)^T\) and DFC of the results \(\chi = 0.00\) are obtained. If the obtained values of the weight coefficients are compared by applying the expression (3), the values of the vector \(\Phi\) given in Table 1 are obtained.

**Example 2.** An example in which a car buyer evaluated the considered alternatives by using the following five criteria: Quality \((C_1)\), Price \((C_2)\), Comfort \((C_3)\), Safety Level \((C_4)\), and Interior \((C_5)\) were considered.

**Step 1.** In the first step, the decision-makers performed the ranking of the criteria: \(C_2 > C_1 > C_4 > C_3 > C_5\).

**Step 2.** In the second step (Step 2b), the decision-maker performed the pairwise comparison of the ranked criteria from Step 1. The comparison was made with respect to the first-ranked \(C_2\) criterion. The comparison was based on the scale \([1,9]\). Thus, the priorities of the criteria \(\omega_{C(i)}\) for all of the criteria ranked in Step 1 were obtained (Table 2).

Based on the obtained priorities of the criteria, the comparative priorities of the criteria are calculated: \(\varphi_{C_2/C_1} = 2.1/1 = 2.1, \varphi_{C_3/C_4} = 3/2.1 = 1.43, \varphi_{C_4/C_3} = 3/3 = 1 \) and \(\varphi_{C_5/C_5} = 7/3 = 2.33\).

**Step 3.** The final values of weight coefficients should meet the following two conditions:

1. The final values of the weight coefficients should meet the condition (3); i.e., that \( \frac{w_1}{w_2} = 2.1, \frac{w_2}{w_3} = 1.43, \frac{w_3}{w_4} = 1 \) and \( \frac{w_4}{w_5} = 2.33\).

2. In addition to the condition (3), the final values of the weight coefficients should meet the condition of mathematical transitivity; i.e., that \( \frac{w_1}{w_2} = 2.1 \cdot 1.43 = 3.00, \frac{w_2}{w_4} = 1.43 \cdot 1 = 1.43 \) and \( \frac{w_4}{w_5} = 1 \cdot 2.33 = 2.33\). By applying the expression (5), the final model for determining the weight coefficients can be defined as:

$$\min \chi \quad s.t. \begin{cases} \left| \frac{w_1}{w_2} - 2.1 \right| \leq \chi, \left| \frac{w_2}{w_3} - 1.43 \right| \leq \chi, \left| \frac{w_3}{w_4} - 1 \right| \leq \chi, \left| \frac{w_4}{w_5} - 2.33 \right| \leq \chi, \\
\left| \frac{w_1}{w_4} - 3.00 \right| \leq \chi, \left| \frac{w_2}{w_4} - 1.43 \right| \leq \chi, \left| \frac{w_3}{w_5} - 2.33 \right| \leq \chi, \\
\sum_{j=1}^{5} w_j = 1, \ w_j \geq 0, \ \forall j \end{cases}$$
By solving this model, the final values of the weight coefficients \((0.437, 0.208, 0.146, 0.146, 0.063)^T\) and DFC of the results \(\chi = 0.00\) are obtained.

The aim of applying all the multi-criteria models is to select the best (most desirable, most significant) alternative, in other words alternative with the best final value of the criterion function. The total value of the criterion function \(f_i (i = 1, 2, \ldots, m)\) of the alternative \(i\), can be obtained using different methods. However, FUCOM can be successfully transformed into a classic multi-criteria model by adding the expression (6) that is presented in the next section. The values of the weight coefficients of the criteria obtained by FUCOM and which meet the condition that \(w_j \geq 0\) and \(\sum_{j=1}^{n} w_j = 1\) can also be used to determine the finite values of the criterion functions applying the expression (6)

\[
f_i = \sum_{j=1}^{n} w_j x_{ij}
\]

where \(w_j\) represents optimal values of weight coefficients obtained using FUCOM, while \(x_{ij}\) represents the values of alternatives according to optimization criteria in the initial decision matrix \(X = [x_{ij}]_{m \times n}\).

By the application of a simple additive weighted value function (6), which is the basic model for most MCDM methods, the FUCOM algorithm is transformed into a classical multi-criteria model that can be used to evaluate \(m\) alternative solutions by \(n\) optimization criteria.

With multi-criteria decision making, the authors recommend the complete FUCOM method to be applied to each decision-maker in particular. After obtaining the weight values of the criteria for all decision makers it is necessary to perform their simplification by applying some aggregators.

### 4. Discussion and Comparisons

In this chapter, based on the presented methodology, the advantages of FUCOM that make it a reliable and interesting MCDM model are distinguished. The benefits of FUCOM are shown by comparing it with the well-known methodologies for determining the weight coefficients of criteria. For the purpose of the comparison, the BWM and the AHP methods were put in focus, since the validity of both methodologies is based on meeting the conditions of mathematical transitivity and the pairwise comparison of the criteria. Depending on the satisfaction degree of transitivity requirements in the BWM and the AHP models, the consistency of the obtained results; i.e., the optimality of the solution is estimated. Taking into account the fact that FUCOM is methodologically based on assessing the comparative priority of criteria and on meeting the conditions of transitivity, the comparison by applying the BWM and the AHP models represents a logical step for comparing results and validating the model. Each of the selected advantages was analyzed through the examples discussed in the literature in which the BWM and the AHP approaches were applied.

(1) In comparison with similar subjective models (the AHP and the BWM methods) for determining the weight the coefficients of criteria, FUCOM only requires the \(n - 1\) pairwise comparison of criteria. When the application of the AHP method is concerned, it is necessary to perform the \(n(n - 1)/2\) pairwise comparison of criteria, whereas the application of the BWM method requires \(2n - 3\) comparisons. An increase in the number of criteria in the BWM and the AHP models produces a significant increase in the number of pairwise comparisons (Table 3), which greatly complicates the mathematical formulation of the models mentioned.

<table>
<thead>
<tr>
<th>MCDM Method</th>
<th>The Number of Criteria ((n)) and the Required Number of Pairwise Comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n = 2)</td>
</tr>
<tr>
<td>AHP ((n(n - 1)/2))</td>
<td>1</td>
</tr>
<tr>
<td>BWM ((2n - 3))</td>
<td>1</td>
</tr>
<tr>
<td>FUCOM ((n - 1))</td>
<td>1</td>
</tr>
</tbody>
</table>
The research study [19] has shown that decision-makers can successfully perform pairwise comparisons up to a maximum of seven criteria, and exceptionally up to nine criteria. However, an increase in the number of criteria affects an increase in the required number of pairwise comparisons, which can significantly affect the consistency of the results obtained. Based on the relations shown in Table 3, it is noted that FUCOM has a significant advantage in relation to the BWM and the AHP methods, whereas simultaneously it provides results identical to those obtained by the application of the considered methodologies.

Example 3. In this example, the problem of a task-oriented resource allocation in cloud computing was considered, inclusive of the application of the AHP method [141]. The comparison was made in pairs of eight criteria, with a total of 28 pairwise comparisons performed (Table 4).

Table 4. The pairwise comparison of eight criteria in the AHP model [141].

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>wj</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>2</td>
<td>1/2</td>
<td>2</td>
<td>1/2</td>
<td>2</td>
<td>0.1111</td>
</tr>
<tr>
<td>C2</td>
<td>1/2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>0.0556</td>
</tr>
<tr>
<td>C3</td>
<td>2</td>
<td>1/4</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0.2222</td>
</tr>
<tr>
<td>C4</td>
<td>1/2</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>0.0556</td>
</tr>
<tr>
<td>C5</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0.2222</td>
</tr>
<tr>
<td>C6</td>
<td>1/2</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>0.0556</td>
</tr>
<tr>
<td>C7</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>0.2222</td>
</tr>
<tr>
<td>C8</td>
<td>1/2</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>0.0556</td>
</tr>
</tbody>
</table>

CR = 0.000

The model shown in Table 5 can be successfully solved applying BWM. Based on the data in Table 5 (the fifth or seventh row in the table), the Best-to-Others (BO) vector $A_B = (2, 4, 1, 4, 1, 4, 1, 4)^T$ is formed. Also, based on the data in Table 5 (fourth, sixth or eighth columns), the Other-to-Worst (OW) vector $A_W = (2, 1, 4, 1, 4, 1, 4, 1)^T$ is formed. By introducing the BO and OW vectors in the BWM model, the identical values of the weight coefficients of the criteria are obtained, as well as in the AHP method (Table 4) with the degree of consistency CR = 0.

Table 5. The priorities of the criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C3</th>
<th>C5</th>
<th>C7</th>
<th>C1</th>
<th>C2</th>
<th>C4</th>
<th>C6</th>
<th>C8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{C_{(i)}}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

By applying FUCOM, the same values of the weight coefficients with only seven pairwise comparisons of the criteria were obtained, whereas the AHP method for this example requires 28 pairwise comparisons. In order for the FUCOM mathematical model to form, the data displayed in Table 4 (i.e., the comparisons made for the criterion bearing the highest weight coefficient (Criteria $C_3$, $C_5$ and $C_7$)) were used. In this example, there are three most influential criteria, so we choose the comparisons made for one of the three criteria. Based on the comparisons for the $C_7$ criterion in Table 4 (the data from the seventh row of Table 4), the criteria are possible to rank in the following manner: $C_3 = C_5 = C_7 > C_1 > C_2 = C_4 = C_6 = C_8$ and the priorities of the criteria can be determined (Table 5).

Based on the obtained priorities of the criteria, the comparative priorities of the criteria are calculated: $\varphi_{T_5/T_3} = \varphi_{T_5/T_1} = 1/1 = 1$, $\varphi_{T_7/T_1} = 2/1 = 2$, $\varphi_{T_7/T_2} = 4/2 = 2$ and $\varphi_{T_7/T_4} = \varphi_{T_7/T_6} = \varphi_{T_7/T_8} = 4/4 = 1$, and, the model for determining the optimal values of the weight coefficients:
\[
\min \chi \quad \begin{cases}
\frac{w_3}{w_5} - 1 \leq \chi, & \frac{w_5}{w_7} - 1 \leq \chi, & \frac{w_7}{w_1} - 2 \leq \chi, & \frac{w_1}{w_2} - 2 \leq \chi, & \frac{w_2}{w_4} - 1 \leq \chi,
\frac{w_4}{w_6} - 1 \leq \chi, & \frac{w_6}{w_8} - 1 \leq \chi, & \frac{w_8}{w_3} - 1 \leq \chi, & \frac{w_3}{w_7} - 1 \leq \chi, & \frac{w_7}{w_2} - 4 \leq \chi,
\sum_{j=1}^{8} w_j = 1, & w_j \geq 0, \forall j
\end{cases}
\]

By solving the model, the identical values of the weight coefficients as those obtained in the AHP model are obtained, with DFC \( \chi = 0.00 \) and with only seven pairwise comparisons of criteria.

Table 6 shows the values of weight coefficients of the criteria obtained by AHP, BWM, and FUCOM models for Example 3.

Table 6. Results of AHP, BWM, and FUCOM models implementation (Example 3).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>AHP ((w_j))</th>
<th>BWM ((w_j))</th>
<th>FUCOM ((w_j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.1111</td>
<td>0.1111</td>
<td>0.1111</td>
</tr>
<tr>
<td>C_2</td>
<td>0.0556</td>
<td>0.0556</td>
<td>0.0556</td>
</tr>
<tr>
<td>C_3</td>
<td>0.2222</td>
<td>0.2222</td>
<td>0.2222</td>
</tr>
<tr>
<td>C_4</td>
<td>0.0556</td>
<td>0.0556</td>
<td>0.0556</td>
</tr>
<tr>
<td>C_5</td>
<td>0.2222</td>
<td>0.2222</td>
<td>0.2222</td>
</tr>
<tr>
<td>C_6</td>
<td>0.0556</td>
<td>0.0556</td>
<td>0.0556</td>
</tr>
<tr>
<td>C_7</td>
<td>0.2222</td>
<td>0.2222</td>
<td>0.2222</td>
</tr>
<tr>
<td>C_8</td>
<td>0.0556</td>
<td>0.0556</td>
<td>0.0556</td>
</tr>
<tr>
<td>CR</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

By using all three models (AHP, BWM, FUCOM) on the same example from the literature, it has been shown that FUCOM gives the simplest solution with only seven comparisons, followed by BWM with thirteen comparisons and eventually AHP with 23 comparisons.

Example 4. When buying a car, the buyer considers five criteria: the quality \((C_1)\), the price \((C_2)\), comfort \((C_3)\), safety \((C_4)\) and the style \((C_5)\). By using the BWM method, the Best–to-Others (BO) and the Others–to-Worst (OW) vectors are obtained [118], \(A_B = (2, 1, 4, 2, 8)^T\) and \(A_W = (4, 8, 2, 4, 1)^T\). By solving the BWM, the optimal values of the weight coefficients are obtained:

\[w_1 = 0.2105, \ w_2 = 0.4211, \ w_3 = 0.1053, \ w_4 = 0.2105, \ w_5 = 0.0526,\]

and the degree of consistency \(CR = 0.00\).

Based on the References data [118], a pairwise comparison matrix of the AHP model (Table 7) was formed and the values of weight coefficients of the criteria were obtained with the degree of consistency \(CR = 0.029\).

Table 7. Pairwise comparisons of criteria - AHP method.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(C_4)</th>
<th>(C_5)</th>
<th>(w_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1)</td>
<td>1.000</td>
<td>0.333</td>
<td>3.000</td>
<td>1.000</td>
<td>5.000</td>
<td>0.2017</td>
</tr>
<tr>
<td>(C_2)</td>
<td>3.000</td>
<td>1.000</td>
<td>5.000</td>
<td>3.000</td>
<td>7.000</td>
<td>0.4641</td>
</tr>
<tr>
<td>(C_3)</td>
<td>0.333</td>
<td>0.200</td>
<td>1.000</td>
<td>0.333</td>
<td>3.000</td>
<td>0.0888</td>
</tr>
<tr>
<td>(C_4)</td>
<td>1.000</td>
<td>0.333</td>
<td>3.000</td>
<td>1.000</td>
<td>5.000</td>
<td>0.2017</td>
</tr>
<tr>
<td>(C_5)</td>
<td>0.200</td>
<td>0.143</td>
<td>0.333</td>
<td>0.200</td>
<td>1.000</td>
<td>0.0436</td>
</tr>
</tbody>
</table>
Using the AHP method, similar values of the weight coefficients of the criteria are obtained, as with BWM, but with a significantly higher number of pairwise comparisons. Differences in the values of weight coefficients between AHP and BWM are due to the incomplete consistency of results in the AHP model (CR\textsubscript{AHP} = 0.029, CR\textsubscript{BWM} = 0.000).

To determine the weight coefficients by using FUCOM, only the comparisons obtained in the BO vector of the BWM are used. Based on the BO vectors, the criteria are possible to rank as follows: \( C_2 > C_1 = C_4 > C_3 > C_5 \), with only \( n-1 \) comparison, and the priorities of the criteria can be determined (Table 8).

**Table 8. The priorities of the criteria.**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( \omega_{C_{(i)}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_2 )</td>
<td>1</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>2</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>2</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>4</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>8</td>
</tr>
</tbody>
</table>

Thus, the comparative priorities of the criteria are obtained, namely \( \varphi_{C_2/C_1} = 2/1 = 2 \), \( \varphi_{C_1/C_4} = 2/2 = 1 \), \( \varphi_{C_4/C_3} = 4/2 = 2 \), and the model is formed.

\[
\begin{align*}
\min \chi \\
\text{s.t.} \begin{cases}
\frac{w_2}{w_1} - 2 \leq \chi, \\
\frac{w_2}{w_4} - 2 \leq \chi, \\
\frac{w_2}{w_4} - 2 \leq \chi, \\
\frac{w_5}{w_3} - 2 \leq \chi, \\
\frac{w_5}{w_3} - 4 \leq \chi, \\
\sum_{j=1}^{5} w_j = 1, \\
w_j \geq 0, \quad \forall j
\end{cases}
\end{align*}
\]

By solving the model, the identical values of the weight coefficients and \( \chi = 0.00 \), as in the BWM, are obtained, with only four pairwise comparisons of the criteria. FUCOM was also tested on other examples from the literature in which the BWM and the AHP models were used [118,142], and the optimal values of the weight coefficients of the criteria were obtained with the \( n-1 \) number of the pairwise comparisons.

Table 9 shows the values of weight coefficients of the criteria obtained by AHP, BWM, and FUCOM models for Example 4.

**Table 9. Results of the AHP, BWM, and FUCOM models application (Example 4).**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>AHP ((w_j))</th>
<th>BWM ((w_j))</th>
<th>FUCOM ((w_j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>0.2017</td>
<td>0.2105</td>
<td>0.2105</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.4641</td>
<td>0.4211</td>
<td>0.4211</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>0.0888</td>
<td>0.1053</td>
<td>0.1053</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>0.2017</td>
<td>0.2105</td>
<td>0.2105</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>0.0436</td>
<td>0.0526</td>
<td>0.0526</td>
</tr>
</tbody>
</table>

CR 0.029 0.000 0.000

From Table 9 it is noted that the optimal values of the weight coefficients of the criteria are given by BWM and FUCOM, while the AHP model has smaller deviations from the optimal values. The solution obtained by the AHP model is also acceptable, since the values of the degree of consistency are within the allowed limits, or CR \( \leq 0.1 \) [12]. The simplest solution with only four comparisons is given by FUCOM, followed by the BWM with seven comparisons and eventually AHP with 23 comparisons.

(2) FUCOM allows satisfying the complete consistency of the model, by respecting the conditions of transitivity. The BWM and the AHP models are based on adherence to mathematical transitivity; i.e., on meeting the conditions that \( a_{ij} \otimes a_{jk} = a_{ik} \) (where \( a_{ij} \) shows the relative preference of criterion \( i \) to criterion \( j \) and \( a_{jk} \) shows the relative preference of criterion \( j \) to criterion \( k \)). If the pairs are compared, the obtained relation reads \( a_{13} = 3 \) and \( a_{34} = 6 \); then, in order to meet the condition of transitivity,
$a_{14}$ should have the value $a_{14} = 18$. However, since the scale $a_{ij} \in [1, 9]$ is applied in both models, in the largest number of the cases of the pairwise comparison, $a_{14}$ obtains the maximum value from the scale; i.e., $a_{14} = 9$. From the above example, it is noted that BWM and AHP models in the case of pairwise comparison deliberately allow certain deviations and ignore total transitivity. However, the deviation from transitivity results in a decrease in the consistency of the model, which further affects the reliability of the results. On the other hand, FUCOM always strives for the maximum consistency of results, which is one of the key conditions in a rational judgment. Meeting the conditions of consistency affects the reliability of results; i.e., the optimality of weight coefficients.

For example, in a research study conducted by Sener et al. [143], the GIS-AHP model for the selection of a solid waste disposal site is applied. In the research, ten criteria were used, and when the weight coefficients were being determined by using the AHP model, the consistency $CR = 0.03908$ was obtained. The obtained values of the weight coefficients are close to the optimal values, but they are not optimal since the consistency is $CR \neq 0$. If FUCOM is applied for the same problem, the optimal values of the weight coefficients are obtained, with DFC equal to zero. A similar situation is found in the other examples [14,118,130,144–149] in which the BWM and the AHP models are used, where the weights of the criteria are obtained with the degree of consistency $CR \neq 0$. By applying FUCOM to the same problems, the optimal values of the weight coefficients with the full consistency of the results (DFC = 0) are obtained. Through the examples from the References [14,118,130,144–149] tested so far, FUCOM has shown a high consistency of the results, since the DFC values have been approximately equal to zero (DFC $\approx 0$). The model for determining the weight coefficients of the criteria requires the reliable reproduction of expert opinions, without introducing any additional imprecision and deviations. Therefore, the authors suggest that the values of DFCs for which the weight coefficients are acceptable, or close to optimal values, should be in the interval $DFC \in [0, 0.025]$.

(3) As already noted above, the BWM use the scale $[1, 9]$ for the pairwise comparison in most cases. Since BWM only use the integer values from the interval $[1, 9]$, the smallest possible relation between the best and the next value in the ranking is two. This means that, in the case of full consistency (CR = 0), the best criterion in relation to the next one will have an advantage minimally twice as great. Since the minimum value is $a_{ij} = 2$, it means that the ratio $\frac{w_i}{w_j} = 2$. This ratio in the BWM is violated only when the degree of consistency is different from zero; i.e., when the condition of the optimality of weight coefficients is violated. In FUCOM, as shown in Example 1, this ratio solely depends on the decision-maker’s preference since the preference is not defined on the basis of a predefined scale, but on the basis of the subjective assessment instead.

(4) The weight coefficients obtained by applying FUCOM are more reliable and contribute to the rational judgment. Compared to the BWM, FUCOM provides the more reliable values of weight coefficients since comparisons are made with a higher degree of consistency. In the BWM, in the case of insufficient consistency ($\xi^* \geq 0.145$), multi-optimality occurs [118]. In the case of multi-optimality in the BWM, it is recommended that the interval values of weight coefficients should be determined [118]. After forming the interval values of weight coefficients, the mean value of the interval is determined, which is further taken as the optimal value of weight coefficients. However, it does not guarantee that the central part of the interval represents the optimal values of such weight coefficients. The optimal value may be closer to the left or the right limit of the interval. In the cases of a greater inconsistency of the results, it may happen that the optimal values of the weight coefficients of the criteria are not included in the defined interval values. This occurs when the left and the right interval limits are defined for the relations significantly deviating from transitivity conditions; i.e., in the cases of a greater model inconsistency. Since in FUCOM the values of the weight coefficients of criteria are obtained on the basis of a small number of pairwise comparisons $(n - 1)$, the values of weight coefficients are obtained with the maximum satisfaction of model consistency and with a high degree of optimality. The above-mentioned problem will be accounted for in the following example.
Example 5. The same problem [118] as in Example 4 is considered, with the BO and the OW values of the vector: $A_B = (2, 1, 4, 3, 8)^T$ and $A_W = (4, 8, 2, 3, 1)^T$. By applying the BWM, the values of the weight coefficients are obtained, $w_1 = 0.2285$, $w_2 = 0.4489$, $w_3 = 0.1102$, $w_4 = 0.1573$, $w_5 = 0.0551$ and $\xi^* = 0.145$. Since $\xi^* = 0.145$, the values obtained are not completely consistent, which results in the occurrence of the multi-optimality of the solution. It is therefore necessary that the optimal interval values of the weight coefficient should be determined [118]. By solving the BWM, the following interval values of the weight coefficients are obtained:

- $w_1 = [0.2145, 0.2289]$, $w_1^{(center)} = 0.2217$;
- $w_2 = [0.4461, 0.4571]$, $w_2^{(center)} = 0.4516$;
- $w_3 = [0.1085, 0.1176]$, $w_3^{(center)} = 0.1131$;
- $w_4 = [0.1563, 0.1602]$, $w_4^{(center)} = 0.1582$;
- $w_5 = [0.0548, 0.0561]$, $w_5^{(center)} = 0.0554$.

By applying FUCOM in the given example, the optimal values of the weight coefficients with a high consistency of the model are obtained as follows: $w_1 = 0.2262$, $w_2 = 0.4527$, $w_3 = 0.1134$, $w_4 = 0.1509$, $w_5 = 0.0556$ and $\chi = 0.001$. The consistency of the results obtained by the application of the FUCOM model is much higher than that of the results obtained by applying the BWM. We note that the values of the FUCOM weight coefficients are within the defined intervals obtained by using the BWM, except for the criterion $C_4$. For the criterion $C_4$, the optimal value of the weight coefficient $w_4 = 0.1509$ (obtained by FUCOM) is not covered by the interval of the criterion $C_4$ ($w_4 = [0.1563, 0.1602]$, $w_4^{(center)} = 0.1582$) defined by the BWM. For the remaining criteria, the optimum values of the weight coefficients are within the defined intervals, but they deviate from the central parts of the interval, which are recommended in the BWM as the optimal values of weight coefficients.

5. Conclusions

The results of the research study are clearly indicative of the justification for the development of a new credible model for determining the weight coefficients of criteria. On the one hand, FUCOM is based on a partly simple mathematical apparatus, and as such is expected to be affirmatively promoted by other authors. On the other hand, the model allows obtaining the credible and reliable weight coefficients that contribute to the rational judgment and obtaining credible results when making decisions. Therefore, the application of this model is significant.

Firstly, FUCOM is a tool that helps executives to deal with their own subjectivity in prioritizing criteria through a simple algorithm and by applying an acceptable scale. Secondly, FUCOM allows obtaining optimal weight coefficients with a possibility of validating them by the consistency of the results. Thirdly, the application of the FUCOM model allows us to obtain the optimum values of weight coefficients by using a simple mathematical apparatus that allows our favoring certain criteria in evaluating phenomena in accordance with the decision-maker’s current requirements and minimizing the risk of decision-making. In addition, FUCOM provides us with the optimal values of weight coefficients and reduces the subjective impact and inconsistency of experts’ preferences to the final values of criterion weights. By comparing FUCOM with other models in the third section of the paper, the robustness and objectivity of the model are demonstrated. The enviable stability of the obtained results is presented. One of the most important advantages is obtaining the same results as in the BWM and the AHP models by only performing the $n - 1$ comparison of criteria. Moreover, it has been shown that the model is flexible and suitable for application to various measuring scales for the purpose of representing experts’ preferences.

In the paper is shown the comparison of the newly developed FUCOM method with respect to the subjective methods like BWM and AHP. Of all the examples in which the comparison was performed, it can be concluded that the FUCOM method gives better results than those mentioned above, in particular in terms of consistency. However, it is necessary to take into account the fact that there is
a large difference in the number of criteria comparisons, especially when it comes to a relationship with the AHP method. Therefore, it can be expected that in certain cases there will be different results of the same problem that is solved by different methods. In the examples and comparisons with the AHP method presented in this paper, there was no such case, but such a possibility should not be excluded from consideration. As with other subjective models for determining the weight of the criteria (AHP, BWM, SWARA, DEMATEL etc.) and with the FUCOM model there is a subjective influence of the decision maker on the final values of the weight of the criteria. This particularly refers to the first and second steps of FUCOM in which decision-makers rank the criteria according to their personal preferences and make pairwise comparisons of ranked criteria. However, unlike analyzed subjective models, FUCOM has showed significantly less variations in the obtained values of the weight coefficients of the criterion than the optimum values. In a large number of tests the obtained values of the weight coefficients of the criteria were equal to the optimal values; i.e., the deviation was \( DFC \approx 0 \). From this it can be concluded that the FUCOM algorithm leads to negligible additional deviations, resulting in more reliable results that are in many cases equal to optimal values.

Taking into account the advantages of FUCOM, the need for software development and implementation for real-world applications is imposed. This will make the model significantly closer to users and will enable the exploitation of all of the benefits stated in the paper. We also suggest that the proposed model should be used in some other real-world applications and comparison of the results obtained by applying other MCDM methods. One of the directions for future research should be towards improving the validation of the model results. Finally, we propose that this model should be expanded through the application of different uncertainty theories, such as neutrosophic and fuzzy sets, rough numbers, grey theory, etc. The extension of FUCOM by using the theories of uncertainty will enable the processing of experts’ preferences even when comparisons are made on the basis of the data that are partially or even very little-known. This would enable an easier expression of the decision-maker’s preferences, simultaneously respecting the subjectivity and lack of information on particular phenomena.

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