

Article

Lorentz-Violating Gravity Models and the Linearized Limit

Michael Seifert

Department of Physics, Astronomy, and Geophysics, Connecticut College, New London, CT 06320, USA; mseifer1@conncoll.edu; Tel.: +1-860-439-5138

Received: 7 September 2018; Accepted: 10 October 2018; Published: 12 October 2018



Abstract: Many models in which Lorentz symmetry is spontaneously broken in a curved spacetime do so via a “Lorentz-violating” (LV) vector or tensor field, which dynamically takes on a vacuum expectation value and provides additional local geometric structure beyond the metric. The kinetic terms of such a field will not necessarily be decoupled from the kinetic terms of the metric, and will generically lead to a set of coupled equations for the perturbations of the metric and the LV field. In some models, however, the imposition of certain additional conditions can decouple these equations, yielding an “effective equation” for the metric perturbations alone. The resulting effective equation may depend on the metric in a gauge-invariant way, or it may be gauge-dependent. The only two known models yielding gauge-invariant effective equations involve differential forms; I show in this work that the obvious generalizations of these models do not yield gauge-invariant effective equations. Meanwhile, I show that a gauge-dependent effective equation may be obtained from any “tensor Klein–Gordon” model under similar assumptions. Finally, I discuss the implications of this work in the search for Lorentz-violating gravitational effects.

Keywords: Lorentz symmetry; linearized gravity; spontaneous symmetry breaking

1. Introduction

The prospect of Lorentz symmetry violation has received a fair amount of attention in recent years [1]. In the realm of particle physics on flat spacetime, the Standard Model Extension (SME) [2] has been developed as a descriptive framework for violations of Lorentz symmetry. The original Standard Model Lagrangian can be thought of as including all possible gauge-invariant and Lorentz-invariant combinations of local field operators up to some power-counting renormalizability cutoff, each with its own coefficient; it is then a matter of determining these coefficients via experiments. In general, a particular experiment will not be sensitive to a single coefficient in the Standard Model Lagrangian, but will instead place bounds on one or more combinations of these coefficients, with different experiments being sensitive to different combinations.

The SME “extends” the Standard Model by allowing the combinations of the field operators appearing in the Lagrange density to be Lorentz tensors instead of Lorentz scalars. The “minimal” SME contains all operators which are power-counting renormalizable; the “non-minimal” sector of the SME includes operators of higher powers as well. Since the overall Lagrange density must still be a scalar, this means that the “coefficients” of these new tensor operators must be tensors as well, contracted with the operators to form a Lorentz scalar. It then becomes an experimental question to measure the components of these coefficient tensors in some fiducial reference frame (usually taken to be the sun-centered reference frame [3].) As with the original Standard Model, a given experiment will be sensitive to some particular combinations of the components of the coefficient tensors. In the same sense that the original Standard Model action contains “all possible low-energy physics” that is consistent with the underlying gauge groups, locality, and Lorentz symmetry, the SME action contains

“all possible low-energy physics” that is consistent with the underlying gauge groups and locality, but including frame-dependent effects. The general philosophy of the particle physics SME has been to remain agnostic concerning the underlying details of the “new physics” that might give rise to Lorentz-violating effects and simply focus on the phenomenological consequences that can arise from these violations; it is designed to be a framework rather than a model.

Given the utility of this framework in the arena of particle physics, one might wonder whether a similar framework could be devised for gravitational physics as well. After all, both particle physics and general relativity take Lorentz symmetry as one of their fundamental axioms, and a violation of Lorentz symmetry could show up in either realm. However, the path from the action to experimental signatures is significantly less straightforward in the gravitational sector than in the particle physics sector. As a result, it is not as clear that the existing framework for the “gravity sector of the SME” actually captures all possible signatures for Lorentz violation in the same way that the “particle physics sector of the SME” captures all possible signatures for Lorentz violation in particle physics.

The purpose of the present work is to argue that the range of models that can be analysed within the “minimal gravity sector of the SME”, as it exists, is limited. In particular, there are only two known models in this category, and their obvious generalizations do not fall into the same category. In contrast, a broad class of models containing a Lorentz-violating field can be cast into a more general form which lies outside the minimal gravity sector of the SME. While it remains an open (and ill-defined) question whether these latter models are viable, these results suggest that a broader framework for Lorentz violation in the gravitational regime may be necessary.

2. Linearized Equations for Lorentz-Violating Gravity

A framework for the study of Lorentz symmetry violation in the context of gravity was developed by Kostelecký & Bailey [4], and much experimental work in the years since has searched for the effects described within this framework (see [5] for a review.) This framework attempts to follow the method taken by the particle physics sector of the SME by obtaining a Lorentz-violating equation of motion for the metric perturbations of a flat background, and then examining the phenomenology of the modified metrics. One begins by assuming a gravitational action of the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (R + uR + s^{ab} R_{ab}^T + t^{abcd} C_{abcd}) + \mathcal{L}_{\text{mat}} \right]. \quad (1)$$

Here, the first term is the standard Einstein–Hilbert action (with $\Lambda = 0$); the second through fourth terms are “small” couplings of tensors u , s^{ab} , and t^{abcd} to the Ricci scalar, trace-free Ricci tensor, and Weyl tensor, respectively; and the final term encodes the dynamics of the fields other than the metric (we use reduced Planck units, i.e., $c = 8\pi G = \hbar = 1$). The tensors u , s^{ab} , and t^{abcd} are “coupled” to the spacetime curvature in this action, much as the particle physics SME coefficients are coupled to the fermion and boson fields. One would therefore expect that an action of this form could give rise to Lorentz-violating effects in gravitational physics. (As in [4], we restrict attention here to a Riemann spacetime with vanishing torsion, rather than the more general case of Riemann–Cartan spacetime. We also restrict attention to the “minimal” sector of the gravitational SME, in which higher derivatives of the Riemann tensor do not appear in the Lagrangian).

However, some important differences between this action and the particle physics SME action immediately arise. In particular, it is difficult to write down an action that includes a fixed background tensor (“prior geometry”) in the context of a model with a dynamical metric. The equations of motion resulting from this prior geometry can easily violate the Bianchi identities, leading to a mathematically inconsistent model [6] (however, see [7] for a recent discussion of how these difficulties might be evaded).

For this reason, it has generally been assumed that the objects u , s^{ab} , and t^{abcd} in (1) are not fixed background tensors, but are instead constructed from some *dynamical* tensor which we denote as Ψ^{\dots} (the dots here denote a generic index structure). As this field is intended to dynamically provide us

with the extra geometric background structure required for Lorentz symmetry violation, we refer to Ψ^{\dots} as the *Lorentz-violating field* (or *LV field*). Similar approaches have been proposed for the spontaneous breaking of another spacetime symmetry, namely scale-invariance [8,9]. However, as the fields used there were scalars, the issues outlined in this work do not arise.

In this picture, diffeomorphism symmetry is broken spontaneously: while the underlying action is assumed to be diffeomorphism-invariant, a certain class of solutions to the equations of motion have some or all of this symmetry broken. Since the SME focuses on local field operators, it is natural to assume that the underlying theory arises from a locally constructed Lagrangian as well. Finally, I assume that the full equations of motion resulting from this action are second-order. The condition that the action be locally constructed and diffeomorphism-invariant implies that the Lagrangian must be locally constructed from the metric, the Riemann tensor (and its derivatives), and the symmetrized derivatives of the other fields [10]; the condition that the equations of motion be second-order then prohibits any derivatives of the Riemann tensor or more than two derivatives of the other fields from appearing in any term in the Lagrangian. The full action is thus of the following form:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \underbrace{\frac{\xi}{2} f(\Psi^{\dots}) R^{\dots}}_{\text{coupling}} + \underbrace{\frac{1}{2} (\nabla\Psi)(\nabla\Psi)}_{\text{kinetic}} + \underbrace{V(\Psi^{\dots})}_{\text{potential}} \right]. \quad (2)$$

In (2), we divided the terms that could appear in a general action into three broad categories; the action (2) could, in principle, include multiple expressions of each type. The “kinetic” terms determine the dynamics of Ψ^{\dots} ; by choosing different combinations of cross-contractions and traces of the tensor $\nabla\Psi$, we can obtain different dynamics for the LV field. The “coupling” terms allow for a direct coupling between Ψ^{\dots} and the curvature, with each term multiplied by a coupling constant ξ_i . By adjusting the size of ξ , we can hope to “tune” the Lorentz-violating effects. Finally, the “potential” term consists of a potential energy $V(\Psi^{\dots})$ for the field Ψ^{\dots} . Each of these terms must be a spacetime scalar, with all of the indices in each term fully contracted.

To investigate the linearized limit of an action of the form (2), we must first vary the action with respect to the metric g_{ab} and the LV field Ψ^{\dots} to obtain the equations of motion. An important question arises: which of the three types of term (kinetic, coupling, or potential) contribute when we vary the action with respect to g_{ab} ? It turns out that all three types of terms will vary when we vary the metric:

- The kinetic terms can vary in two ways when we change the metric. The more obvious way is that the full contraction of the tensor $\nabla\Psi$ with itself may require some raising and lowering of indices, which requires an implicit use of the metric or the inverse metric; these terms will change when the metric is varied. A more subtle point (but in some ways a more important one) is that the covariant derivative operator ∇_a also varies when the metric is varied. One could also consider Palatini-type theories, where the connection is viewed as a variable independent from the metric. As the correspondence between the “metric” and “Palatini” versions of a generalized gravity theory is not straightforward [11], I focus on the “metric” versions here for simplicity.
- The coupling terms explicitly depend on the Riemann tensor, which varies with the metric. As with the kinetic terms, they can also contain implicit factors of the metric or inverse metric that arise from index raising and lowering.
- The potential term must be a spacetime scalar constructed out of Ψ^{\dots} ; this will again usually require the raising and lowering of Ψ 's indices, and so, implicitly depends on the metric.

The variation of the kinetic terms and coupling terms is particularly problematic. For the kinetic terms, the variation of the fields will yield something of the form (with indices temporarily suppressed)

$$\begin{aligned} \delta \left(\frac{1}{2} (\nabla \Psi)(\nabla \Psi) \right) &\sim (\nabla \Psi) [\nabla(\delta \Psi) + \Psi(\nabla \delta g)] + (\nabla \Psi)(\nabla \Psi)(\delta g) \\ &\sim -(\nabla \nabla \Psi) \delta \Psi + [-\Psi(\nabla \nabla \Psi) + (\nabla \Psi)(\nabla \Psi)] \delta g \end{aligned} \quad (3)$$

which is integrated by parts in the second step. Note, in particular, that the coefficient of δg generically contains second derivatives of Ψ^{\dots} ; thus, second derivatives of Ψ^{\dots} will appear in the Einstein equation. Similarly, variation of the coupling terms will yield terms of the form

$$(\Psi R) \sim R(\delta \Psi) + \Psi(\delta R) \sim R(\delta \Psi) + \Psi(\nabla \nabla \delta g) \sim R(\delta \Psi) + (\nabla \nabla \Psi) \delta g. \quad (4)$$

All told, then, a generic action of the form (2) gives rise to equations of motion that are of schematically of the form

$$G + \zeta F_1(\Psi)R + \Psi(\nabla \nabla \Psi)' + (\nabla \Psi)(\nabla \Psi) + F_2(\Psi) = 0, \quad (5a)$$

$$(\nabla \nabla \Psi) + \zeta F_3(\Psi)R + F_4(\Psi) = 0, \quad (5b)$$

where G stands for the Einstein tensor, R stands for the Riemann tensor, and the F_i functions are some combinations (not necessarily invariants) of the LV field Ψ . The combinations of second derivatives that appear in the modified Einstein Equation (5a) will generically not be the same as those that appear in the equation of motion (5b) for Ψ itself; this is indicated by the use of the prime on the second derivatives of Ψ in the Einstein equation.

In the particle physics sector of the SME, the Lorentz-violating tensor coefficients are assumed to be constant throughout all of the flat spacetime, and to arise from the vacuum expectation value of the LV tensor field. We therefore require that there exists a solution to the equations of motion (5) with a flat metric ($g_{ab} = \eta_{ab}$) and a constant LV field ($\nabla \Psi = 0$). It is not too hard to see that if the metric is flat and the tensor field is constant, the equations of motion from (2) will reduce to the form

$$-\frac{1}{2}V(\Psi)\eta^{ab} + \left. \frac{\delta V}{\delta g_{ab}} \right|_{g \rightarrow \eta} = 0 \quad (6)$$

and

$$\left. \frac{\delta V}{\delta \Psi} \right|_{g \rightarrow \eta} = 0. \quad (7)$$

As a particularly simple example, suppose V is a single-argument function that depends only on a Lorentz invariant $X(\Psi)$ constructed from Ψ^{\dots} and the metric g_{ab} . In such an case, these equations become

$$-\frac{1}{2}V(X)\eta^{ab} + V'(X) \left. \frac{\delta X}{\delta g_{ab}} \right|_{g \rightarrow \eta} = 0 \quad (8)$$

and

$$V'(X) \left. \frac{\delta X}{\delta \Psi^{\dots}} \right|_{g \rightarrow \eta} = 0, \quad (9)$$

and are satisfied if $V(X) = 0$ and $V'(X) = 0$. By constructing the potential appropriately, we can find models where there exist backgrounds consisting of a flat metric and a constant but non-zero LV field. We denote the background value of Ψ^{\dots} as $\bar{\Psi}^{\dots}$.

The linearized limit of our model is found by looking at linearized perturbations about this background:

$$g_{ab} = \eta_{ab} + \epsilon h_{ab}, \quad \Psi^{\dots} = \bar{\Psi}^{\dots} + \epsilon \tilde{\psi}^{\dots}. \quad (10)$$

This results in a set of linearized equations of the form

$$(\delta G) + \zeta \bar{\Psi}(\delta R) + \bar{\Psi} \delta(\nabla \nabla \Psi)' + \dots = 0, \quad (11a)$$

$$\delta(\nabla \nabla \Psi) + \zeta \delta R + \dots = 0, \quad (11b)$$

where δG and δR stand for the linearized Einstein and Riemann tensors, respectively, and the ellipses in these equations contain terms that do not depend on the derivatives of h or $\tilde{\psi}$. Throughout this paper, we use δ to denote the linear-order perturbation of a quantity of the flat background.

Since our full Lagrangian (2) is diffeomorphism invariant, the linearized equations are invariant under the infinitesimal diffeomorphisms $h_{ab} \rightarrow h_{ab} + \mathcal{L}_v \eta_{ab}$ and $\tilde{\psi}^{\dots} \rightarrow \tilde{\psi}^{\dots} + \mathcal{L}_v \tilde{\Psi}^{\dots}$, where v_a is the vector field generating the diffeomorphism and \mathcal{L}_v is the Lie derivative along this vector field. In terms of familiar index notation, these transformations are

$$h_{ab} \rightarrow h_{ab} + \partial_a v_b + \partial_b v_a \quad (12)$$

$$\tilde{\psi}^{a_1 \dots a_k}_{b_1 \dots b_l} \rightarrow \tilde{\psi}^{a_1 \dots a_k}_{b_1 \dots b_l} - \sum_{i=1}^k \tilde{\Psi}^{a_1 \dots c \dots a_k}_{b_1 \dots b_l} \partial_c v^{a_i} + \sum_{i=1}^l \tilde{\Psi}^{a_1 \dots a_k}_{b_1 \dots c \dots b_l} \partial_{b_i} v^c. \quad (13)$$

These can be thought of as gauge transformations for the linearized fields; perturbations of this form do not contain any physical information.

The ultimate goal in this process is to obtain a second-order partial differential equation for h alone, without reference to the LV fluctuations $\tilde{\psi}$. To do this, we must be able to eliminate the terms that are dependent on $\tilde{\psi}$ and its derivatives from (11a). We have certain tools at our disposal for this task. We may impose gauge conditions on h_{ab} , rewrite antisymmetrized derivatives of Ψ in terms of the Riemann tensor, or apply the linearized tensor equation (11b). We may also choose to restrict our solution space by imposing boundary conditions on various combinations of fields. Depending on the exact forms of the equations (11), we can envision three possibilities:

1. We can obtain a linearized second-order equation for h_{ab} that is manifestly gauge-invariant. Since the linearized Riemann tensor contains all of the “gauge-invariant information” encoded in the linearized metric, this means that the resulting equation is simply of the form

$$\delta G_{ab} + \zeta \tilde{\Psi}^{\dots} \delta R \dots = 0. \quad (14)$$

2. The process for obtaining this equation may (and, in the examples below, will) require boundary conditions to be set, but should not rely on any particular choice of gauge. Equations of this type are sometimes taken as the starting point for the study of Lorentz violation in gravitational physics [12,13], while remaining agnostic about the underlying model of Lorentz violation. However, it has also been pointed out that not all LV gravity models lead to such an equation [14].
3. We cannot obtain a linearized second-order equation for h_{ab} alone that is gauge-invariant, but we can find some particular gauge that allows us to eliminate $\tilde{\psi}$ from the Equation (11b). Again, the imposition of some appropriate set of boundary conditions may be necessary.
3. The Equations (11a) and (11b) are inextricably coupled. This means that we cannot write down a second-order equation for the metric alone without knowing the behavior of the LV field as well. In such a case, it could, in principle, still be possible to solve Equation (11b) for $\tilde{\psi}$ in terms of h and whatever other fields are present by Green’s functions; one could then insert this solution for $\tilde{\psi}$ into (11a). However, the resulting equation would include some kind of integral over spacetime and would not result in a local second-order partial differential equation for h_{ab} .

As a shorthand, we can call these types of equations “Type I”, “Type II”, and “Type III”, respectively. In general, it is not immediately clear from simple examination of the action (2) whether a particular model will yield equations of a particular type; one must generally work through the above process to

find out. If we can obtain a Type I or Type II equation, we can, in principle, solve for the metric without having detailed knowledge of the LV field dynamics. In a model for which only Type III equations can be found, on the other hand, one must have knowledge of the LV field dynamics in order to figure out the weak-field limit; it is difficult to “remain agnostic” about the dynamics of the new fields (as one could with a Type I equation) and still make meaningful predictions.

3. *n*-Form Fields

As examples of models in which Type I equations can be obtained, I first consider the class of models where the LV field is an *n*-form field $\Psi_{a_1 \dots a_n}$: a completely antisymmetric tensor of rank $(n, 0)$. For what follows, it is useful to denote a sequence of indices with braces, for example, we denote

$$\Psi^{a_1 \dots a_n} \Psi_{a_1 \dots a_n} \equiv \Psi^{\{a\}} \Psi_{\{a\}}. \tag{15}$$

The number of indices in a pair of braces should usually be evident from the context.

A particular feature of *n*-forms is that it is possible to define a natural notion of differentiation (the exterior derivative) that is independent of the metric. If $\Psi_{\{a\}}$ is an *n*-form, then $(d\Psi)_{\{a\}}$ is an $(n + 1)$ -form defined in abstract-index notation as

$$(d\Psi)_{a\{b\}} = (n + 1) \nabla_{[a} \Psi_{\{b\}}]. \tag{16}$$

It is not hard to show that this quantity is independent of the derivative operator ∇_a used. One might imagine that this will make one’s life easier; since the derivative is independent of the metric, the terms involving the second derivatives of the LV field in (5a)—and therefore in (11a)—will not arise. However, while this property is necessary for obtaining a decoupled equation for the metric, it is not sufficient.

We can use the exterior derivative to construct a kinetic term for a Lorentz-violating *n*-form field Ψ :

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2(n + 1)} (d\Psi)_{\{a\}} (d\Psi)^{\{a\}} - V(\Psi^2) + \mathcal{L}_{\text{coupling}} \right], \tag{17}$$

where $\Psi^2 \equiv \Psi_{\{a\}} \Psi^{\{a\}}$ and

$$\mathcal{L}_{\text{coupling}} = \zeta_1 \Psi^{ab}{}_{\{e\}} \Psi^{cd\{e\}} R_{abcd} + \zeta_2 \Psi^a{}_{\{c\}} \Psi^{b\{c\}} R_{ab} + \zeta_3 \Psi^2 R. \tag{18}$$

In what follows, we also find it necessary to require that

$$4\zeta_1 + (n - 1)\zeta_2 = 0. \tag{19}$$

Both the bumblebee model described in [6] and the tensor models described in [15] fall into this category, using a one-form or a two-form (respectively) as the LV field. (The models in [15] can also include a potential that depends on the invariant $\Psi_{ab} \Psi_{cd} \epsilon^{abcd}$ as well as the tensor norm $\Psi_{ab} \Psi^{ab}$. In general, such an invariant only exists if $n = d/2$, where d is the dimension of the spacetime. The arguments which follow can be generalized to such cases straightforwardly). In the former work, the ζ_1 term does not exist and ζ_2 is unconstrained; in the latter work, the condition (19) (with $n = 2$) is found to be necessary to obtain a “decoupled” effective equation. The generalized bumblebee models described in [14] are also of this general form, with the modification that the indices in the kinetic term for the one-form Ψ_a are contracted using an “effective metric” constructed out of the spacetime metric g_{ab} and Ψ_a itself.

To understand how the full decoupling of the equations can be accomplished in this case, we must first look at the linearized equation of motion for the LV field. This is

$$\partial_d (d\tilde{\Psi})^{dab\{c\}} - \lambda \tilde{\Psi}^{ab\{c\}} \delta(\Psi^2) + 2\zeta_1 \tilde{\Psi}^{de\{c\}} \delta R^{ab}{}_{de} + 2\zeta_2 \tilde{\Psi}^{d[b\{c\}} \delta R^a]{}_d + 2\zeta_3 \tilde{\Psi}^{ab\{c\}} \delta R = 0, \tag{20}$$

where $\lambda \equiv V''(\Psi^2)$, $\delta(\Psi^2)$ is the linear variation in Ψ^2 , and the tensor $(d\psi)_{\{a\}} \equiv \delta(d\Psi)_{\{a\}}$ is defined similarly to (16). (Note that for any tensor $t_{\{b\}}$ which vanishes in the background, $\nabla_a t_{\{b\}} = \partial_a t_{\{b\}} + \mathcal{O}(\epsilon^2)$).

In general, the derivatives present in the linearized LV equation of motion (11b) (Equation (20) in the present case) are the only tools we have to eliminate the derivatives of $\tilde{\psi}$ in the linearized Einstein Equation (11a). However, the structure of the kinetic term for an n -form field leads to two auxiliary conditions which turn out to be crucial for the decoupling. First, if we take the divergence of (20), the kinetic term identically vanishes and we obtain

$$-\lambda \Psi^{ab\{c\}} \partial_a \delta(\Psi^2) + 2\zeta_1 \Psi^{de\{c\}} \partial_a \delta R^{ab}{}_{de} + 2\zeta_2 \Psi^{d[b\{c\}} \partial_a \delta R^a{}_d + 2\zeta_3 \Psi^{ab\{c\}} \partial_a \delta R = 0. \tag{21}$$

However, the Riemann tensor satisfies the Bianchi identity $\nabla_{[a} R_{bc]de} = 0$. At linear order, this becomes $\partial_{[a} \delta R_{bc]de} = 0$, and taking the traces of this equation implies that $2\partial_{[a} \delta R_{b]c} = \partial^d \delta R_{dcab}$ and $2\partial^a \delta R_{ab} = \partial_b \delta R$. Splitting up the antisymmetrizers in (21) and applying the Bianchi identities lets us rewrite the second and third terms as

$$\begin{aligned} & 2\zeta_1 \Psi^{de\{c\}} \partial_a \delta R^{ab}{}_{de} + 2\zeta_2 \Psi^{d[b\{c\}} \partial_a \delta R^a{}_d \\ &= \frac{\zeta_2}{n} \Psi^{dbc_1 \dots c_{n-2}} \partial_a \delta R + \frac{2[4\zeta_1 + (n-1)\zeta_2]}{n(n-1)} \left[\Psi^{adc_1 \dots c_{n-2}} \partial_{[a} \delta R_{d]}{}^b - \sum_{i=1}^{n-2} \Psi^{adc_1 \dots c_{n-2}} \partial_a \delta R_{d}{}^{c_i} \right]. \end{aligned} \tag{22}$$

Since we assume the condition (19) applies to the coupling constants, this means that (21) simplifies greatly:

$$\Psi^{ab\{c\}} \partial_a \left[-\lambda \delta(\Psi^2) + \left(\frac{\zeta_2}{n} + 2\zeta_3 \right) \delta R \right] = 0. \tag{23}$$

A possible solution to this equation (though not the only one) is that the quantity in square brackets vanishes:

$$\lambda \delta(\Psi^2) = \left(\frac{\zeta_2}{n} + 2\zeta_3 \right) \delta R. \tag{24}$$

I call this the “massive mode condition”, since it effectively requires that $\delta(\Psi^2)$ only deviates by a small amount (i.e., $\mathcal{O}(\zeta_i)$) from zero. Note, however, that this condition can only be applied if (19) holds; for a general set of coupling constants, imposing such a condition would be more poorly motivated.

The second condition can be found by taking the “curl” of (20). Since we now know that we can choose a solution such that $\lambda \delta(\Psi^2)$ is $\mathcal{O}(\zeta_i)$, the linearized Equation (20) reduces to

$$\partial^b (d\tilde{\psi})_{b\{a\}} = \mathcal{O}(\zeta_i). \tag{25}$$

If we take the antisymmetrized derivative of the left-hand side, we obtain

$$\partial_{[c} \partial^b (d\tilde{\psi})_{|b|a_1 \dots a_n]} = \partial_{[c} \partial^b \partial_{|b|} \tilde{\psi}_{a_1 \dots a_n]} - n \partial_{[c} \partial^b \partial_{a_1} \tilde{\psi}_{|b|a_2 \dots a_n]} = \square \partial_{[c} \tilde{\psi}_{a_1 \dots a_n]}, \tag{26}$$

where the second term in the middle step vanishes due to the antisymmetrization over the partial derivatives. Thus, we have

$$\square (d\tilde{\psi})_{\{a\}} = \mathcal{O}(\zeta_i). \tag{27}$$

As an aside, the identity (26) can also be derived elegantly using the language of differential forms. Define \underline{d} and $\underline{\delta}$ as the exterior derivative and codifferential on the manifold (note that this $\underline{\delta}$ is an operator that takes n -forms to $(n-1)$ -forms, and is distinct from the usage of δ to denote linearized perturbations in this work.) The kinetic term in (20) can be written as $\underline{\delta} \underline{d}\psi$. We then have $\underline{d}\underline{\delta}(\underline{d}\psi) = (\underline{d}\underline{\delta} + \underline{\delta}\underline{d})(\underline{d}\psi)$, since $\underline{d}^2 = 0$. This operator acting on $(\underline{d}\psi)$ is the so-called *Laplace–de Rham*

operator, which differs from the “conventional” tensor Laplacian $\square = \nabla^a \nabla_a$ by (at most) various contractions of the curvature with the n -form.

Equation (27) is a hyperbolic differential equation on our manifold, and via an appropriate choice of boundary conditions, we can require that the linearized field strength $(d\tilde{\psi})_{\{a\}}$ remains $\mathcal{O}(\xi_i)$ throughout our spacetime:

$$d\tilde{\psi}_{\{a\}} = \mathcal{O}(\xi_i). \tag{28}$$

I call this condition the “curl condition”. As with the massive mode condition (24), it must be noted that this condition is not true for the most general solution of the equation (27) from which it stems.

With these in mind, we can now attempt to decouple the n -form fluctuations from the metric fluctuations. The linearized Einstein equations in this case are of the form

$$\delta G_{ab} - n\lambda\delta(\Psi^2)\tilde{\Psi}_a^{\{c\}}\tilde{\Psi}_{b\{c\}} + \sum_i \xi_i \left[(\mathcal{A}_{(i)})_{ab}{}^{cdef} \delta R_{cdef} + (\mathcal{B}_{(i)})_{ab}{}^{cdef\{g\}} \delta(\nabla_{(c}\nabla_{d)}\Psi_{ef\{g\}}) \right] = 0, \tag{29}$$

where the $(\mathcal{A}_{(i)})_{ab}{}^{cdef}$ and $(\mathcal{B}_{(i)})_{ab}{}^{cdef\{g\}}$ tensors are constructed out of the background fields $\tilde{\Psi}_{\{a\}}$ and η_{ab} . (Recall from above that the antisymmetrized second derivative of $\Psi_{\{a\}}$ can be written in terms of contractions of the Riemann tensor with $\Psi_{\{a\}}$.) It is important here to note that there are no terms that arise involving the second derivatives of $\Psi_{\{a\}}$ at $\mathcal{O}(\xi^0)$. For a generic model, such terms would arise from the variation in the kinetic term for the LV field, since the covariant derivative operator is dependent on the metric. However, the field strength (16) is independent of the metric, and so it evades this obstacle.

Assuming the massive mode condition (24) holds, we can replace the second term in (29) with curvature terms multiplied by coefficients of $\mathcal{O}(\xi_i)$, and the third term is already of the desired form. It is the second derivatives of $\Psi_{\{a\}}$ that arise in (29) (the terms contracted with $(\mathcal{B}_{(i)})_{ab}{}^{cdef\{g\}}$) that we need to eliminate. Explicitly, these tensors are

$$(\mathcal{B}_{(1)})_{ab}{}^{cdef\{g\}} = 4\tilde{\Psi}_{(a}{}^{(c\{g\}}\eta_{b)}{}^{e\eta^{|d|}f} \tag{30}$$

$$(\mathcal{B}_{(2)})_{ab}{}^{cdef\{g\}} = \eta_{ab}\tilde{\Psi}^{(c|f\{g\}}\eta^{|d|e} - \tilde{\Psi}^{cf\{g\}}\eta^d{}_{(a}\eta^e{}_{b)} + \tilde{\Psi}_{(a}{}^{f\{g\}}\eta^{cd}\eta_{b)}{}^e - \tilde{\Psi}_{(a}{}^{f\{g\}}\eta^{ce}\eta_{b)}{}^d, \tag{31}$$

$$(\mathcal{B}_{(3)})_{ab}{}^{cdef\{g\}} = 2\tilde{\Psi}^{ef\{g\}}(\eta_{ab}\eta^{cd} - \eta_{(a}{}^c\eta_{b)}{}^d). \tag{32}$$

If we can show that these tensors contracted with $\delta(\nabla_{(c}\nabla_{d)}\Psi_{ef\{g\}})$ are automatically $\mathcal{O}(\xi_i)$, then we will be able to write down an approximate equation, correct to $\mathcal{O}(\xi_i)$, that only contains various contractions of the linearized Riemann tensor with the background values of $\tilde{\Psi}_{\{a\}}$. This is the desired form of the equations of motion underlying the gravitational sector of the SME.

Enforcing the massive mode condition (24), under which $\delta\Psi^2$ is $\mathcal{O}(\xi_i)$, implies that

$$\frac{1}{2}\delta(\nabla_a\Psi^2) = \delta(\Psi^{\{b\}}\nabla_a\Psi_{\{b\}}) = \tilde{\Psi}^{\{b\}}\delta(\nabla_a\Psi_{\{b\}}) \tag{33}$$

is $\mathcal{O}(\xi_i)$ as well. This allows us to eliminate any terms where we have contracted $\tilde{\Psi}^{ef\{g\}}$ with $\delta(\nabla_{(c}\nabla_{d)}\Psi_{ef\{g\}})$ in the linearized equations of motion. In particular, this guarantees that both terms arising from (32) do not contribute to the equations of motion at $\mathcal{O}(\xi_i)$.

Enforcing the curl condition (28) allows us to eliminate more of the above derivatives. We have

$$\tilde{\Psi}^{b_1\dots b_n}(d\tilde{\psi})_{ab_1\dots b_n} = \frac{1}{n+1}\tilde{\Psi}^{b_1\dots b_n} \left[\delta(\nabla_a\Psi_{b_1\dots b_n}) - n\delta(\nabla_{[b_1}\Psi_{|a|b_2\dots b_n]}) \right]. \tag{34}$$

Since we are assuming that $(d\tilde{\psi})_{\{a\}}$ is $\mathcal{O}(\xi_i)$ throughout the spacetime, and since the first term on the right-hand side is also $\mathcal{O}(\xi_i)$ via the massive mode condition, we conclude that

$$\tilde{\Psi}^{b_1\dots b_n}\delta(\nabla_{[b_1}\Psi_{|a|b_2\dots b_n]}) = \mathcal{O}(\xi_i) \tag{35}$$

as well. We conclude that any contraction of the form $\Psi^{cf\{g\}}$ or $\Psi^{df\{g\}}$ (and by symmetry, $\Psi^{ce\{g\}}$ or $\Psi^{de\{g\}}$) with $\delta(\nabla_c \nabla_d \Psi_{ef\{g\}})$ will also be of the form $\mathcal{O}(\xi_i)$; this eliminates the first two terms of (31).

What remains are (30) and the last two terms of (31), and the only tool we have not yet used is the equation of motion itself. The kinetic term in (20) can be rewritten as

$$\partial^p (d\tilde{\Psi})_{pqr\{s\}} = (n + 1)\delta(\nabla_c \nabla_d \Psi_{ef\{g\}})\eta^{cp}\eta_{[p}^d \eta_q^e \eta_r^f \eta_{s_1 \dots s_{n-2}}^{g_1 \dots g_{n-2}}], \tag{36}$$

where $\eta_{s_1 \dots s_n}^{g_1 \dots g_n} \equiv \eta_{s_1}^{g_1} \dots \eta_{s_n}^{g_n}$. If we contract (36) with the tensor $\eta_{(a}^q \tilde{\Psi}_{b)}^{r\{s\}}$, then we obtain the tensor $\delta(\nabla_c \nabla_d \Psi_{ef\{g\}})$ contracted with the tensor

$$(n + 1) \left[\eta^{cp} \eta_{(a}^q \tilde{\Psi}_{b)}^{r\{s\}} \right] \eta_{[p}^d \eta_q^e \eta_r^f \eta_{\{s\}}^{\{g\}} \} = \frac{1}{n!} \left(2\eta^{cd} \eta_{(a}^{[e} \tilde{\Psi}_{b)}^{f]\{g\}} - 2\eta^{c[e} \eta_{(a}^{|d|} \tilde{\Psi}_{b)}^{f]\{g\}} + 2\eta^{c[e} \eta_{(a}^f] \tilde{\Psi}_{b)}^{d\{g\}} + \dots \right), \tag{37}$$

where the ellipses stand for terms involving permutations where two or more of the indices $d, e,$ and f are attached to the background LV field $\tilde{\Psi}$, along with one of the free indices a or b . Such terms only exist if $n > 2$. These latter terms do not arise in linearized Einstein Equations (29). Further, they cannot be eliminated via the auxiliary conditions (33) or (35), as these conditions require $\tilde{\Psi}$ to be fully contracted with the perturbation $\delta(\nabla_c \nabla_d \Psi_{ef\{g\}})$. Thus, a Lorentz-violating model including an n -form LV field with $n > 2$ cannot be analysed in the SME gravitational framework.

The exceptions are the lowest-rank cases of $n = 1$ and $n = 2$. For $n = 1$, the ξ_1 term (30) does not exist, and the kinetic term in the LV equation of motion contracted with $\eta_{(a}^c \tilde{\Psi}_{b)}$ is simply

$$(\eta^{cd} \eta_{(a}^e \tilde{\Psi}_{b)} - \eta^{ce} \eta_{(a}^d \tilde{\Psi}_{b)})\delta(\nabla_c \nabla_d \Psi_e). \tag{38}$$

These terms are exactly the remaining terms in (31) in this case, and so all non-curvature terms in (29) are $\mathcal{O}(\xi^2)$ or higher. For $n = 2$, the remaining terms in (30) and (31) can be combined to equal the terms explicitly written out in (37) (recalling from (19) that $4\xi_1 = \xi_2$ in this case.) The remaining terms in (20) are all of the form $\mathcal{O}(\xi_i)$, either explicitly or via the massive mode condition, and thus, all non-curvature terms in (29) are $\mathcal{O}(\xi_i^2)$ or higher.

It is worth taking stock of all of the assumptions that have gone into obtaining these effective equations. Only in two cases ($n = 1$, and $n = 2$ in the case $4\xi_1 = \xi_2$) can this procedure be completed. In both cases, auxiliary conditions concerning the solutions—namely, (24) and (28)—are required to obtain a linearized metric equation that fully eliminates the LV field perturbations from the linearized Einstein equation. A general linearized solution will not satisfy these conditions, and will therefore not obey the effective equation derived by this procedure. Moreover, while the arguments leading to these auxiliary conditions can be generalized to higher-rank n -forms, the conditions are insufficient to eliminate the LV field perturbations from the equations of motion. It seems unlikely that a general action for a LV n -form field (17) can be reduced to an effective equation of the form (14).

4. Tensor Klein–Gordon Fields

It may occur that we cannot write an analog of the linearized Einstein equation solely in terms of the linearized Riemann tensor in a model containing an LV field. However, we can still hope to find an equation of motion for the linearized metric that, while not invariant under the “gauge transformations” $h_{ab} \rightarrow h_{ab} + \partial_{(a} \xi_{b)}$, is at least decoupled from any explicit dependence on the fluctuations of the LV field. Such an equation is what I called a “Type II” equation above.

An example of a class of models that can yield such equations are the “tensor Klein–Gordon” actions, of the form

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \nabla_a \Psi_{\{b\}} \nabla^a \Psi^{\{b\}} - V(\Psi^2) \right], \tag{39}$$

where $\Psi_{\{b\}}$ is now an arbitrary rank- $(n, 0)$ tensor field without any particular symmetry structure. Note the absence of any particular explicit coupling between the LV field $\Psi_{\{b\}}$ and the Riemann tensor in the Lagrangian. The linearized equation of motion for the LV field in such a model is

$$\eta^{cd} \delta(\nabla_c \nabla_d \Psi_{\{a\}}) - 2\lambda \bar{\Psi}_{\{a\}} \delta(\Psi^2) = 0, \quad (40)$$

with $\lambda = V''(\bar{\Psi}^2)$, as before. In terms of the perturbations $\tilde{\psi}_{\{a\}}$ and h_{ab} , the first term above becomes

$$\begin{aligned} \eta^{cd} \delta(\nabla_c \nabla_d \Psi_{\{a\}}) &= \eta^{cd} \left(\partial_c \partial_d \tilde{\psi}_{\{a\}} - \sum_i \partial_c \Gamma^f_{da_i} \bar{\Psi}_{a_1 \dots b \dots a_n} \right) \\ &= \square \left(\tilde{\psi}_{\{a\}} - \frac{1}{2} \sum_i \bar{\Psi}_{a_1 \dots b \dots a_n} h_{a_i b} \right) - \sum_i \partial^c \partial_{[a_i} h_{b]c} \bar{\Psi}_{a_1 \dots b \dots a_n}. \end{aligned} \quad (41)$$

However, in the standard harmonic gauge $\partial^b h_{ab} = \frac{1}{2} \partial_a h$ (where $h \equiv h^a_a$), we have

$$\partial^c \partial_{[a_i} h_{b]c} = \partial_{[a_i} \partial^c h_{b]c} = \frac{1}{2} \partial_{[a_i} \partial_{b]} h = 0. \quad (42)$$

Moreover, in terms of the perturbations, we have

$$\delta(\Psi^2) = 2\bar{\Psi}^{\{a\}} \tilde{\psi}_{\{a\}} - \sum_i \bar{\Psi}^{a_1 \dots a_i \dots a_n} \bar{\Psi}_{a_1 \dots b \dots a_n} h_{a_i b} = 2\bar{\Psi}^{\{a\}} \left(\tilde{\psi}_{\{a\}} - \frac{1}{2} \sum_i \bar{\Psi}_{a_1 \dots b \dots a_n} h_{a_i b} \right). \quad (43)$$

(The negative sign arises from $\delta(g^{ab}) = -\eta^{ac} \eta^{bd} h_{cd}$.) Putting this all together, the linearized equation of motion for the LV field is (in the harmonic gauge)

$$\square \left(\tilde{\psi}_{\{a\}} - \frac{1}{2} \sum_i \bar{\Psi}_{a_1 \dots c \dots a_n} h_{a_i c} \right) - 4\lambda \bar{\Psi}_{\{a\}} \bar{\Psi}^{\{b\}} \left(\tilde{\psi}_{\{b\}} - \frac{1}{2} \sum_i \bar{\Psi}_{b_1 \dots c \dots b_n} h_{b_i c} \right) = 0. \quad (44)$$

This is a hyperbolic equation for the quantity in parentheses above, which means that if we choose the appropriate boundary conditions, we can make the said quantity as small as we want:

$$\tilde{\psi}_{\{a\}} - \frac{1}{2} \sum_i \bar{\Psi}_{a_1 \dots c \dots a_n} h_{a_i c} \approx 0. \quad (45)$$

In other words, via a choice of boundary condition, the LV field perturbations $\tilde{\psi}_{\{a\}}$ can be written in terms of the background tensor values $\bar{\Psi}_{\{a\}}$ and the metric perturbations h_{ab} . The LV field perturbations can therefore be completely eliminated from the linearized Einstein Equation (11a) in favor of the metric perturbations. The result will be an equation which holds only in harmonic gauge, but which does not contain any explicit dependence on the LV field fluctuations. While such an equation will not fit into the gravitational framework of the SME, one could still analyze the resulting equation in terms of its predicted post-Newtonian effects, its predictions for gravitational wave propagation, and so forth. The linearized metric found in Section IV.B of [4], for example, is the linearized metric for a model containing a Lorentz-violating “vector Klein–Gordon” field, and in obtaining that metric, the same assumption (45) concerning the perturbations of the LV field is made.

This result for tensor Klein–Gordon models could be generalized to other gauges; for example, the result (42) would still hold in which $\partial^b \bar{h}_{ab}$ (where $\bar{h}_{ab} = h_{ab} - \frac{1}{2} \eta_{ab} h$) is equal to the gradient of some scalar function f , rather than vanishing as assumed in the harmonic gauge. Imposing a different gauge choice would yield an effective equation of motion for h_{ab} that appears superficially different from the equation obtained in the original gauge. However, the solutions to this new equation should simply be the solutions of the original equation modified by a gauge transformation; the choice of gauge would not cause a change in the physically measurable aspects of the metric.

Finally, we note that as with the differential forms in Section 3, the most general solutions to the linearized equations of motion will not obey (45), and will therefore not necessarily obey the effective equation derived from it.

5. Discussion

In general, the Euler–Lagrange equations for a model containing both a tensor field and a dynamical metric contain coupled kinetic terms, and if the tensor field takes on a vacuum expectation value in flat spacetime, this coupling will persist in the linearized equations about this background. One can try to evade this by using an n -form field for which the “natural” kinetic term for the field (involving the tensor norm of the field strength) does not couple directly to the metric perturbations. With such an equation of motion for the linearized metric in hand, one could proceed to follow the analysis of [4], looking for the observational signatures of Lorentz violation of the types derived there. However, this assumption is not sufficient to fully “decouple” the equations; one must also restrict attention to the solutions satisfying (24) and (35), and even in such instances, the decoupling only occurs when $n = 1$ or $n = 2$.

Even granting these assumptions, it appears that the great majority of models one can write down in which a vector or tensor field takes on a vacuum expectation value do not yield Type I equations. To date, only the two above-mentioned models have been found to yield equations of Type I, and their obvious generalizations to higher-rank differential forms fail to yield a decoupled, gauge-invariant equation for the linearized metric. (One could generalize the two-form fields in [15] in the same way that the “generalized bumblebee models” of [14] generalized the original bumblebee model [6]; see the discussion following (18). It seems likely that such a model could yield Type I equations as well, though this remains to be shown rigorously.) If the linearized equations are Type II or III, the form of the linearized metric will almost certainly differ from the predicted linearized metric in the SME, and thus, the experimental signatures of this model will not match those of the SME. Such models would have to be analyzed on a case-by-case basis from first principles. Indeed, in [4], the second derivatives of the fields that cannot be eliminated are implicitly placed into a divergence-free tensor Σ_{ab} . It is conjectured in that work that models in which Σ_{ab} does not vanish “may even be generic”; the current work lends credence to this conjecture.

In the particle physics sector of the SME, the size of the coefficients of the Lorentz-violating terms in the action bears directly on the size of the effects that are physically observed. One would therefore naïvely expect that an action of the form (2) containing a “small” coupling term would lead to “small” violations of Lorentz symmetry. For the known models [6,14,15] that yield Type I equations, this is effectively the case—small values of the coefficients ξ_i lead to small contributions to the linearized equations of motion.

In contrast, the size of the Lorentz-violating effects in a model yielding a Type II or Type III equation may not be determined by the size of the “coupling” terms in the action. For example, the tensor Klein–Gordon models described in Section 4 have no coupling terms at all, but give rise to an effective equation with a kinetic term of the form

$$\square h_{ab} + \bar{\Psi} \cdots \bar{\Psi} \cdots \mathcal{D} \dots [h] + \dots = 0, \quad (46)$$

where $\mathcal{D} \dots [h]$ is some second-order differential operator acting on h_{ab} , and the indices of the two $\bar{\Psi}$ tensors are contracted with this operator in some way. The difference between this equation of motion and the linearized Einstein equation is therefore governed by coefficients of $\mathcal{O}(E^2)$, where E is the energy scale of the LV field’s vacuum expectation value in reduced Planck units. Since these deviations would increase with the magnitude of the background field, one could, in principle, completely exclude such models via both precision gravitational experiments (thereby excluding sufficiently large energy scales) and accelerator experiments (excluding sufficiently small energy scales).

Of course, the gap between “sufficiently large” and “sufficiently small” may be quite large indeed. As a very rough estimate, one might assume that a linearized metric solving (46) would differ from the weak-field GR solution by deviations of $\mathcal{O}(E^2)$. Given that spacetime-anisotropic deviations from the weak-field GR solution are currently bounded to about one part in 10^{11} [1], this implies that the vacuum expectation value for Ψ could be no larger than about $10^{-5}m_p \approx 10^{12}$ GeV, nowhere near the current bounds from accelerator experiments.

It is also possible that a model that yields Type II or Type III equations will be phenomenologically non-viable for reasons other than the form of its linearized metric. The tensor Klein–Gordon fields in Section 4, for example, include a kinetic term that is of indeterminate sign, and do not contain any gauge degrees of freedom. This is likely to lead to a Hamiltonian that is unbounded below, running a strong risk of runaway solutions. Thus, while it may be possible to find a linearized metric in a Type II model (as was done for the vector Klein–Gordon model in [4]), such a solution would need to be checked for stability. Similarly, while a Type II equation can be reduced to a single equation involving the metric perturbation h_{ab} , the number of physical degrees of freedom of the metric may be different from those of a metric obeying a Type I equation; the lack of gauge symmetry in a Type II equation means that degrees of freedom that correspond to gauge transformations in a Type I equation may now become physically meaningful. The extra propagating modes arising in such a model could be used to constrain it or rule it out.

One might conjecture that all viable Lorentz-violating models of gravity yield equations of Type I. Unless we simply *define* a “viable model” to be one that can yield linearized equations of Type I—which is somewhat tautological, and requires clear motivation—we must have a clear idea of what we mean by “viable”. The criteria for a model to be viable on purely theoretical grounds are something of a judgement call. Most physicists would probably agree that a model with a Hamiltonian that is unbounded below is probably not a good one, but whether or not one should be interested in a model with extra metric degrees of freedom is more open to debate. Many other possible criteria for a “viable” model could be proposed, depending on the context one is interested in. Moreover, such a conjecture elides the fact that a given model may have some linearized solutions that obey such an equation and some that do not; in the known examples [6] and [15], extra conditions on the linearized solutions are needed to obtain a “Type I” equation. Without a more precise and well-motivated statement of the properties of a viable model, and justifications for any necessary auxiliary conditions on the linearized solutions, this “conjecture” is really more of a hope.

The SME framework has been incredibly useful for discussing the signatures of Lorentz violation in the realm of particle physics. Any self-consistent model of Lorentz violation in particle physics can be mapped onto one or more coefficients in the SME action (either in the minimal or the non-minimal versions of the framework), and there is a well-developed machinery that maps these coefficients into experimental results. In the realm of particle physics, therefore, the SME can legitimately claim to be a mature framework for the study of violations of Lorentz symmetry.

In contrast, the gravitational sector of the SME does not yet have this same status. Under certain assumptions, certain n -form models mentioned above do fit within the SME gravity framework developed in [4]. However, if one starts with a particular action involving a dynamical metric and a tensor field with a vacuum expectation value, one cannot easily discern which of the above three types it falls into. There is no way to determine this other than to go through the long process of deriving the perturbational equations of motion (11) and trying to eliminate the LV tensor fluctuations from the linearized Einstein equation. Even then, the process is not guaranteed to be straightforward. It may rely on subtle auxiliary conditions (such as the massive mode condition (24) or the curl condition (28)) that are not immediately evident from the linearized equations of motion and that a general linearized solution of the equations of motion may not satisfy. Finally, there is no guarantee that any particular model will result in a linearized equation that yields the same linearized gravity phenomenology as a Type I equation; the experimental signatures of Type II and Type III models can, in principle, be totally different from those of the well-studied Type I models. It is therefore unclear that the gravitational

sector of the SME has the same level of phenomenological universality that the particle physics sector of the SME enjoys.

Funding: This research received no external funding.

Acknowledgments: The author would like to acknowledge the helpful and thorough comments of one of the anonymous reviewers.

References

1. Kostelecký, V.A.; Russell, N. Data tables for Lorentz and CPT violation. *Rev. Mod. Phys.* **2011**, *83*, 11–31. [[CrossRef](#)]
2. Colladay, D.; Kostelecký, V.A. Lorentz-violating extension of the standard model. *Phys. Rev. D* **1998**, *58*, 116002. [[CrossRef](#)]
3. Bluhm, R.; Kostelecký, V.A.; Lane, C.D.; Russell, N. Probing Lorentz and CPT violation with space-based experiments. *Phys. Rev. D* **2003**, *68*, 125008. [[CrossRef](#)]
4. Bailey, Q.; Kostelecký, V. Signals for Lorentz violation in post-Newtonian gravity. *Phys. Rev. D* **2006**, *74*, 045001. [[CrossRef](#)]
5. Tasson, J.D. The standard-model extension and gravitational tests. *Symmetry* **2016**, *8*, 111. [[CrossRef](#)]
6. Kostelecký, V.A. Gravity, Lorentz violation, and the standard model. *Phys. Rev. D* **2004**, *69*, 105009. [[CrossRef](#)]
7. Bluhm, R. Explicit versus spontaneous diffeomorphism breaking in gravity. *Phys. Rev. D* **2015**, *91*, 065034. [[CrossRef](#)]
8. Guendelman, E.I. Scale invariance, new Inflation and decaying Λ -terms. *Mod. Phys. Lett. A* **1999**, *14*, 1043–1052. [[CrossRef](#)]
9. Guendelman, E.I.; Nishino, H.; Rajpoot, S. Two-measure approach to breaking scale-invariance in a standard-model extension. *Phys. Lett. Sect. B Nuclear Elem. Part. High-Energy Phys.* **2017**, *765*, 251–255. [[CrossRef](#)]
10. Iyer, V.; Wald, R.M. Some properties of the Noether charge and a proposal for dynamical black hole entropy. *Phys. Rev. D* **1994**, *50*, 846–864. [[CrossRef](#)]
11. Iglesias, A.; Kaloper, N.; Padilla, A.; Park, M. How (not) to use the Palatini formulation of scalar-tensor gravity. *Phys. Rev. D Part. Fields Gravit. Cosmol.* **2007**, *76*, 1–11. [[CrossRef](#)]
12. Kostelecký, V.A.; Tasson, J.D. Constraints on Lorentz violation from gravitational Čerenkov radiation. *Phys. Lett. Sect. B Nucl. Elem. Part. High-Energy Phys.* **2015**, *749*, 551–559. [[CrossRef](#)]
13. Kostelecký, V.A.; Mewes, M. Testing local Lorentz invariance with gravitational waves. *Phys. Lett. Sect. B Nuclear Elem. Part. High-Energy Phys.* **2016**, *757*, 510–514. [[CrossRef](#)]
14. Seifert, M.D. Vector models of gravitational Lorentz symmetry breaking. *Phys. Rev. D* **2009**, *79*, 124012. [[CrossRef](#)]
15. Altschul, B.; Bailey, Q.G.; Kostelecký, V.A. Lorentz violation with an antisymmetric tensor. *Phys. Rev. D* **2010**, *81*, 065028. [[CrossRef](#)]



© 2018 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).