Article

Application of the Fuzzy CODAS Method Based on Fuzzy Envelopes for Hesitant Fuzzy Linguistic Term Sets: A Case Study on a Personnel Selection Problem

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Abstract: Fuzzy multi-criteria decision-making (MCDM) methods are useful and reliable for multi-criteria selection problems under uncertain and imprecise situations. In these methods, if decision-makers hesitate among several linguistic terms, hesitant fuzzy linguistic term sets (HFLTSs), represented by a set of successive linguistic terms instead of single linguistic terms, may be more appropriate to make evaluations. The notion of a fuzzy envelope for the HFLTSs is a beneficial tool that can be directly applied to fuzzy MCDM methods to elicit comparative linguistic expressions (CLEs). The aim of this study is to present a methodology that combines the fuzzy CODAS (COMBIBATIVE Distance-based Assessment) method with the fuzzy envelope of HFLTs based on CLEs to solve a personnel selection problem. In order to examine the feasibility of the presented methodology, a case study on blue-collar personnel selection in a manufacturing firm is conducted. A sensitivity analysis is performed to demonstrate the stability and validity of the ranking results. Furthermore, the ranking results of the presented methodology are compared with various fuzzy MCDM methods, including fuzzy EDAS, fuzzy TOPSIS, fuzzy WASPAS, fuzzy ARAS, and fuzzy COPRAS. The results show that the presented methodology is efficient and stable for solving personnel selection problems in a hesitant fuzzy environment.

Keywords: hesitant fuzzy linguistic term set; fuzzy envelope; comparative linguistic expression; fuzzy MCDM; fuzzy CODAS; personnel selection

1. Introduction

In a competitive business environment, firms need to constantly change and develop in order to survive in the long term. This need leads firms to reconsider all their resources and harmonize their structures with dynamic processes. The concept of human resource management (HRM) should be considered more carefully because of the rapidly increasing number of competitive enterprises that exist under today’s competitive conditions. When HRM is considered as a chain, the most important link of this chain is completing the process of personnel selection. Personnel selection is the process of selecting the best candidate with the qualifications required to perform a particular job [1]. If firms carry out personnel selection effectively, it can be expected that the standards of other HRM functions, such as training, performance appraisal, career planning, and wage management, could increase. In short, the success of the personnel selection process is most obviously derived from the performance evaluation outputs. Personnel selection is a complex problem, affected by a wide range of factors,
including significant changes in society, businesses, jobs, and rules [2]. Therefore, it is very important to select the right personnel for the right job.

The personnel selection problem can be handled by considering three main objectives: (i) defining the selection criteria and their relative importance, (ii) ensuring an appropriate numerical scale for evaluating alternative personnel with respect to the criteria, and (iii) obtaining a comparative ranking using a dependable method [3]. In light of these explanations, personnel selection can be seen as a kind of decision-making problem with multiple and often contradictory criteria due to its complex and multi-dimensional structure. Thus, multi-criteria decision-making (MCDM) methods are one of the most suitable approaches to solve such problems. So far, many MCDM methods have been introduced by several authors, such as Simple Additive Weighting (SAW) [4], Analytical Hierarchy Process (AHP) [5], Analytical Network Process (ANP) [6], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [7], Elimination Et Choix Traduisant laRéalité or Elimination and Choice Translating Reality (ELECTRE) [8], Preference Ranking Organization Methods for Enrichment Evaluations (PROMETHEE) [9], Complex Proportional Assessment (COPRAS) [10], Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [11], Multi-Objective Analysis by Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) [12], Weighted Aggregated Sum Product Assessment (WASPAS) [13], the Additive Ratio Assessment (ARAS) method [14], TOMada de Decisao Interativa e Multicrit’evio (TODIM) [15], Evaluation based on distance from average solution (EDAS) [16], and COmbinative Distance-based Assessment (CODAS) [17].

An analysis of the literature shows that various MCDM methods, such as AHP [18], SAW [19], ANP [20], TOPSIS [21], and PROMETHEE [22], have been applied to personnel selection problems. However, decision problems involving individual judgments and subjectivity present serious risks to the provision and testing of a resolution. This leads decision-makers to systematic solutions in the course of problem-solving. The employers shape personnel selection criteria in line with their own needs, in terms of their own policies, for personnel selection situations in which they are to choose a new position or to move their existing personnel to a higher position. These assessments are used to provide solutions under the conditions of uncertain and incomplete information that we often encounter in daily life.

The fuzzy set theory proposed by Zadeh [23] appears to be an essential tool to provide a decision framework that incorporates the imprecise judgments that are inherent in the personnel selection process [1]. Based on fuzzy set theory, Bellman and Zadeh [24] introduced the fuzzy MCDM methodology, which has been subsequently widely accepted and used to solve many decision-making problems. In many studies, personnel selection has been considered as an MCDM problem in a fuzzy environment owing to its complexity and importance for business success. Some researchers have used fuzzy MCDM methods in personnel selection problems as shown in Table 1. It can be observed that the fuzzy TOPSIS method has been more popular among the fuzzy MCDM methods due to its simplicity, ease of use, and flexibility [25].
Table 1. Some of the literature on fuzzy multi-criteria decision-making (MCDM) methods for personnel selection.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Applied Method(s)</th>
</tr>
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<tbody>
<tr>
<td>Liang and Wang [26]</td>
<td>A two-stage fuzzy MCDM</td>
</tr>
<tr>
<td>Yaakob and Kawata [27]</td>
<td>A fuzzy linguistic evaluation</td>
</tr>
<tr>
<td>Chen [28]</td>
<td>Fuzzy TOPSIS</td>
</tr>
<tr>
<td>Tsao and Chue [29]</td>
<td>Improved fuzzy MCDM algorithm</td>
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<tr>
<td>Capaldo and Zollo [31]</td>
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<td>Huang et al. [32]</td>
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<tr>
<td>Saghaﬁan and Hejazi [33]</td>
<td>A modiﬁed Fuzzy TOPSIS</td>
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<tr>
<td>Golec and Kahya [34]</td>
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</tr>
<tr>
<td>Mahdavi et al. [35]</td>
<td>Fuzzy TOPSIS</td>
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<td>Gungör et al. [36]</td>
<td>Fuzzy AHP</td>
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<td>Ayub et al. [37]</td>
<td>Fuzzy ANP</td>
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<tr>
<td>Polychroniou and Giannikos [38]</td>
<td>Fuzzy TOPSIS</td>
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<td>Kelemenis and Askounis [39]</td>
<td>Fuzzy TOPSIS</td>
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<td>Kelemenis et al. [40]</td>
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<td>Baležentis et al. [41]</td>
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<td>Kabak et al. [42]</td>
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<td>Sang et al. [46]</td>
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<td>Chang [47]</td>
<td>Fuzzy Delphi, ANP, and TOPSIS</td>
</tr>
</tbody>
</table>

Since the introduction of fuzzy sets by Zadeh [23], some extensions have been developed, such as type-2 fuzzy sets [48,49] and interval type-2 fuzzy sets [50], type-n fuzzy sets [51], intuitionistic fuzzy sets [52,53] and interval-valued intuitionistic fuzzy sets [54], vague sets or intuitionistic fuzzy sets [55,56], fuzzy multisets [57,58], nonstationary fuzzy sets [59], neutrosophic fuzzy sets [60], and hesitant fuzzy sets [61,62]. Because the judgments and preferences of decision-makers are affected by uncertainty, the use of definite and precise numbers in linguistic judgments is unreasonable [63,64]. Therefore, various fuzzy MCDM methods based on the aforementioned extensions of the fuzzy sets have been proposed by several authors for personnel selection problems. For example, Dursun and Karsak [1] proposed the fuzzy TOPSIS with 2-tuples and applied it to a numerical example reported in Liang and Wang [26]. Zhang and Liu [65] introduced an intuitionistic fuzzy MCDM method based on a gray relational analysis for choosing the best personnel. Boran et al. [66] presented an intuitionistic fuzzy-number-based TOPSIS method for selecting a sales manager in a manufacturing company. Wan et al. [67] introduced the VIKOR method, in which the score of alternatives is stated by triangular intuitionistic fuzzy numbers for selecting a department manager in an investment company. Yu et al. [68] studied a hesitant fuzzy group decision-making method using some aggregation operators and applied it to personnel evaluation. Liu et al. [69] proposed an extended VIKOR method using interval 2-tuple linguistic variables and implemented it in personnel selection. Ji et al. [70] presented a projection-based TODIM method using multi-valued neutrosophic numbers for application to personnel selection problems. Qin et al. [71] introduced some hesitant fuzzy aggregation operators based on Frank triangular norms for implementation in human resource selection. Heidary Dahooie et al. [2] presented a competency framework to select the best information technology expert by integrating two MCDM methods, SWARA and grey ARAS. Efe and Kurt [72] presented a possibility-degree-based TOPSIS method using interval type-2 trapezoidal fuzzy numbers for the personnel selection problem in an assembly line of a textile firm. Yeni and Özçelik [73] presented the interval-valued Atanassov intuitionistic fuzzy CODAS (IVAIF-CODAS) method and applied it to a personnel selection problem.
As seen from the literature review, personnel selection problems have been handled in a fuzzy environment. In many of these problems, evaluations of the criteria and alternatives are carried out by using single linguistic expressions. However, since the complexity of such problems is quite high due to the lack of knowledge, the evaluations based on single linguistic expressions will not be meaningful. Therefore, decision-makers will prefer to use comparative linguistic expressions (CLEs), which are close to human beings’ cognitive model, instead of single linguistic expressions for reflecting their preferences in decision-making problems in a more flexible and richer way [74,75]. The concept of a hesitant fuzzy linguistic term set (HFLTS) is introduced to build complex linguistic expressions for modeling the decision-makers’ hesitations. These complex linguistic expressions provide flexibility to decision-makers in order to explain their preferences via context-free grammars that generate CLEs in decision-making problems. The linguistic decision-making solving scheme consists of four steps: (i) selecting the linguistic term set and the semantics, (ii) selecting the aggregation operator for linguistic information, (iii) aggregation, and (iv) exploitation [75].

In this study, we aimed to select the best personnel among six blue-collar personnel working in a manufacturing firm in Turkey with respect to conflicting multiple criteria evaluated by decision-makers (experts). In the current research, HFLTs based on CLEs are used to evaluate criteria and alternatives considering expert opinions. In order to rank the alternatives, a fuzzy extension of the CODAS method, one of the newest MCDM methods, developed by Keshavarz Ghorabaee et al. [76], is utilized. A sensitivity analysis is implemented in order to show the stability of the ranking results obtained from the fuzzy CODAS method. For validating the presented method, the results of the fuzzy CODAS method using HFLTs based on CLEs are compared with the results of several fuzzy MCDM methods, including fuzzy EDAS, fuzzy TOPSIS, fuzzy WASPAS, fuzzy ARAS, and fuzzy COPRAS.

The rest of this study is organized as follows. In Section 2, definitions for hesitant fuzzy sets (HFSs), HFLTs, the fuzzy envelope for the HFLTs, and trapezoidal fuzzy numbers are given. In Section 3, the Fuzzy CODAS method and the presented methodology are explained in detail. Section 4 includes a real-life application to personnel selection using the presented methodology and comparative results from the other considered fuzzy MCDM methods. A discussion and the conclusions of the study are given in the last section.

2. Preliminaries

In this section, HFSs, HFLTs, the fuzzy envelope for the HFLTs, and trapezoidal fuzzy numbers are explained.

2.1. Hesitant Fuzzy Set (HFS)

The theory of hesitant fuzzy sets (HFSs) was introduced by Torra [62]. In the study, basic operations and properties of HFSs were given. It was shown that the envelope of an HFS is an intuitionistic fuzzy set and operations of HFSs are consistent with intuitionistic fuzzy sets. HFSs are quite helpful in cases where decision-makers hesitate to ensure their preferences in a decision-making process. Hesitant fuzzy sets and intuitionistic fuzzy sets have been applied in various fields (e.g., [77–84]).

Given a reference set \( X \), an HFS on \( X \) is a function \( h \) that returns a subset of values in \([0, 1]\). Let \( M = \{ \mu_1, \mu_2, ..., \mu_n \} \) be a set of membership functions of \( n \) fuzzy sets. A HFS \( h_M \) associated with \( M \) is defined as the union of membership functions of a set of fuzzy sets as follows:

\[
h_M(x) : = \bigcup_{\mu \in M} \{ \mu(x) \}.
\]

The basic concepts and operations of HFSs are given as follows:
(i) The lower bound \( h^- (x) \) and the upper bound \( h^+ (x) \) of a HFS \( h \) are defined by:

\[
h^-(x) = \min h(x)
\]
where:

\[ h^+(x) = \max h(x) \tag{3} \]

(ii) The complement of an HFS \( h \) is defined as:

\[ h^c(x) = \bigcup_{\gamma \in h(x)} (1 - \gamma). \tag{4} \]

(iii) The union of two HFSs \( h_1 \) and \( h_2 \) is considered as:

\[ h_1 \cup h_2(x) = \{ h(x) \in (h_1(x) \cup h_2(x)) | h(x) \geq \max(h_1^-(x), h_2^-(x)) \}. \tag{5} \]

(iv) The intersection between two HFSs \( h_1 \) and \( h_2 \) is given by:

\[ h_1 \cap h_2(x) = \{ h(x) \in (h_1(x) \cap h_2(x)) | h(x) \leq \min(h_1^+(x), h_2^+(x)) \}. \tag{6} \]

(v) The envelope of an HFS \( h \) is defined as:

\[ A_{\text{env}(h)} = \{ x, \mu_A(x), \nu_A(x) \} \tag{7} \]

where: \( A_{\text{env}(h)} \) is the intuitionistic fuzzy set of \( h \) with \( \mu_A(x) = h^-(x) \) and \( \nu_A(x) = 1 - h^+(x) \).

2.2. Hesitant Fuzzy Linguistic Term Sets (HFLTSs)

Hesitant fuzzy linguistic term sets (HFLTSs), introduced by Rodriguez et al. [74], are based on the fuzzy linguistic approach and HFSs. HFLTSs aim to reveal linguistic information when experts hesitate between various linguistic terms to explain their evaluations. In the study of Rodriguez et al. [74], the definition of linguistic expressions that are more similar to human beings’ expressions was semantically represented by HFLTSs and generated by a context-free grammar.

An HFLTS \( H_S \) is an ordered finite subset of the consecutive linguistic terms of a linguistic term set \( S = \{s_0, s_1, ..., s_N\} \). Given a linguistic term set, \( S = \{s_0 : \text{nothing}, s_1 : \text{very bad}, s_2 : \text{bad}, s_3 : \text{medium}, s_4 : \text{good}, s_5 : \text{very good}, s_6 : \text{perfect}\} \) and a linguistic variable \( \vartheta \), an example of an HFLTS can be considered as \( H_S(\vartheta) = \{\text{medium}, \text{good}, \text{very good}\} \).

The basic concepts and operations of HFLTSs are given as follows [62,74,77,85]:

(i) An empty HFLTS and a full HFLTS for a linguistic variable \( \vartheta \) are given in Equations (8) and (9) respectively:

\[ H_S(\vartheta) = \{\} \tag{8} \]

\[ H_S(\vartheta) = S. \tag{9} \]

(ii) The lower bound \( H_{S^}\) and the upper bound \( H_{S^} \) of an HFLTS \( H_S \) are defined by:

\[ H_{S^\} = \min(s_i) = s_j, s_i \in H_S \text{ and } s_i \geq s_j, \forall i \tag{10} \]

\[ H_{S^} = \max(s_i) = s_j, s_i \in H_S \text{ and } s_i \leq s_j, \forall i. \tag{11} \]

(iii) The complement of an HFLTS \( H_S \) is given as:

\[ H_S^c = S - H_S = \{s_i | s_i \in S \text{ and } s_i \notin H_S\}. \tag{12} \]

(iv) The union of two HFLTSs \( H_S^1 \) and \( H_S^2 \) is as follows:

\[ H_S^1 \cup H_S^2 = \{s_i | s_i \in H_S^1 \text{ or } s_i \in H_S^2\}. \tag{13} \]
(v) The intersection between two HFLTSs $H^1_S$ and $H^2_S$ is as follows:

$$H^1_S \cap H^2_S = \{ s_i | s_i \in H^1_S \text{ and } s_i \in H^2_S \}.$$  

(14)

(vi) The envelope of an HFLTS $H_S$ is a linguistic interval defined by:

$$env(H_S) = [H^-_S, H^+_S], \quad H^-_S \leq H^+_S.$$  

(15)

For example, envelope of the HFLTS $H_S(\theta) = \{\text{medium, good, very good}\}$ is obtained as $env(H_S(\theta)) = [\text{medium, very good}]$.

(vii) To compare the two HFLTSs $H^1_S$ and $H^2_S$, the concept of the envelope of the HFLTS is used as follows:

$$H^1_S(\theta) > H^2_S(\theta) \text{ iff } env(H^1_S(\theta)) > env(H^2_S(\theta))$$  

(16)

$$H^1_S(\theta) = H^2_S(\theta) \text{ iff } env(H^1_S(\theta)) = env(H^2_S(\theta)).$$  

(17)

2.3. Fuzzy Envelope for HFLTSs

A context-free grammar $G_H$ is a way of describing linguistic terms and CLEs by the HFLTSs with ease. So, the grammar $G_H$ was defined for generating the CLEs in cases where the experts hesitate between several linguistic terms to explain their evaluations in decision-making problems. The concept of envelope for an HFLTS is introduced in order to more easily compare the linguistic terms in the HFLTS, which is a linguistic term subset [74]. Recently, Liu et.al [86] have proposed the type-2 fuzzy envelope of HFLTSs for the representation of CLEs.


The general process of the fuzzy envelope for CLEs and HFLTSs is explained as follows [87]:

(i) Given an HFLTS $H_S = \{s_i, s_{i+1}, ..., s_j\}$, all linguistic terms $s_k \in S$, $k = i, ..., j$ can be defined by trapezoidal fuzzy numbers as $A^k = T[a^k_L, a^k_M, a^k_R, a^k_z], k = 0, 1, ..., g$, because it is sufficient to seize the uncertainty of the CLEs. Therefore, the set of all linguistic terms in the HFLTS is aggregated as

$$T = \{a^1_L, a^1_M, a^{i+1}_L, a^i_M, a^{i+1}_R, a^{i+2}_L, a^{i+2}_M, a^{i+2}_R, ..., a^g_L, a^g_M, a^g_R\}.$$  

(18)

For simplification, a special case $a^{k-1}_R = a^k_M = a^{k+1}_L, k = 1, 2, ..., g - 1$ was considered in Liu and Rodriguez [87] and $T = \{a^i_L, a^i_M, a^{i+1}_M, a^i_R\}$ was obtained.

(ii) To represent CLEs based on an HFLTS $H_S$, the trapezoidal membership function $A = T(a, b, c, d)$ was used. Because $s_i = \min H_S$ and $s_j = \max H_S$, the left ($a$) and the right ($b$) limits of $A$ from the left limit of $s_i$ and the right limit of $s_j$ were obtained by

$$a = \min\{a^i_L, a^i_M, a^{i+1}_M, a^i_R\} = a^i_L$$  

(19)

$$d = \max\{a^i_L, a^i_M, a^{i+1}_M, a^i_R\} = a^i_R.$$  

(20)

Using the ordered weighted operator (OWA) aggregation, $b$ and $c$ were obtained as

$$b = OWA^W\{a^i_M, a^{i+1}_M, a^i_R\}$$  

(21)

$$c = OWA^W\{a^i_M, a^{i+1}_M, a^i_R\}.$$  

(22)

(iii) Linguistic terms may have a different importance reflected by OWA weights, $W^s, W^t$, due to hesitation between the linguistic terms associated with an HFLTS.
(iv) The fuzzy envelope for an HFLTS $H_S$ by using the fuzzy trapezoidal membership function $T(a, b, c, d)$ is obtained as follows:

$$\text{env}(H_S) = T(a, b, c, d).$$  \hspace{1cm} (23)

The fuzzy envelope $\text{env}(H_S)$ of an HFLTS represents the CLEs by means of the fuzzy membership functions aggregating the linguistic terms $[75,87]$. 

The fuzzy envelope for CLEs: Let $G_H$ be a context-free grammar to generate linguistic expressions and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. Each element of $G_H = \{V_N, V_T, I, P\}$ is explained as

$$V_N = \{\langle \text{primary term} \rangle, \langle \text{composite term} \rangle, \langle \text{unary relation} \rangle, \langle \text{conjunction} \rangle\} \hspace{1cm} (24)$$

$$V_T = \{\langle \text{lower than} \rangle, \langle \text{greater than} \rangle, \langle \text{between} \rangle, \langle \text{and} \rangle, s_0, s_1, \ldots, s_g\} \hspace{1cm} (25)$$

$$I \in V_N \hspace{1cm} (26)$$

$$P = \begin{cases} 
I ::= \langle \text{primary term} \rangle | \langle \text{composite term} \rangle,
\langle \text{composite term} \rangle ::= \langle \text{unary relation} \rangle \langle \text{primary term} \rangle | \langle \text{binary relation} \rangle \langle \text{primary term} \rangle 
\langle \text{primary term} \rangle \langle \text{conjunction} \rangle \langle \text{primary term} \rangle 
\langle \text{unary relation} \rangle ::= \langle \text{lower than} \rangle | \langle \text{greater than} \rangle,
\langle \text{binary relation} \rangle ::= \langle \text{between} \rangle,
\langle \text{conjunction} \rangle ::= \langle \text{and} \rangle 
\end{cases} \hspace{1cm} (27)$$

where $V_N$ is the set of non-terminal symbols; $V_T$ is the set of terminals’ symbols; $I$ is the starting symbol; $P$ is the set of production rules defined in an extended Backus–Naur Form; and the symbol “|” denotes alternative elements.

The CLEs “$ll$” generated by the context-free grammar $G_H$ are transformed into the HFLTS according to their meanings via the transformation function $E_{G_H} : ll \rightarrow H_S$, as follows $[74,75]$:

$$E_{G_H}(s_i) = \{s_i | s_i \in S\} \hspace{1cm} (28)$$

$$E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\} \hspace{1cm} (29)$$

$$E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\} \hspace{1cm} (30)$$

$$E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\} \hspace{1cm} (31)$$

$$E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\} \hspace{1cm} (32)$$

$$E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}. \hspace{1cm} (33)$$

These linguistic expressions are illustrated in Figure 1 $[74,85]$. 
Figure 1. The hesitant fuzzy linguistic term set (HFLTS) with the linguistic expressions.

For example, considering the linguistic term set \( S \),

\[
S = \{ s_0 : \text{nothing}(N), s_1 : \text{very bad}(VB), s_2 : \text{bad}(B), s_3 : \text{medium}(M), s_4 : \text{good}(G), s_5 : \text{very good}(VG), s_6 : \text{perfect}(P) \}
\]

the fuzzy envelope for the HFLTS \( H_{s_1} = \{ s_3, s_4, s_5, s_6 \} \) based on \( l_{1,1} = \text{at least} \ s_3 \) is obtained by the following:

The elements of the \( H_{s_1} \) are considered as \( T = \{ a^3_M, a^3_M, a^3_M, a^6_H \} \). The fuzzy envelope of \( H_{s_1} \) is denoted as \( \text{env}(H_{s_1}) = T(a_1, b_1, c_1, d_1) \), which is a trapezoidal fuzzy number. The parameters of the fuzzy envelope of \( H_{s_1} \) are obtained as:

\[
a_1 = \min \{ a^3_L, a^3_M, a^3_M, a^6_R \} = a^3_L = 0.33
\]

\[
d_1 = \max \{ a^3_L, a^3_M, a^3_M, a^6_R \} = a^6_R = 1
\]

\[
b_1 = \text{OWA}_{W^2}(a^3_M, a^3_M, a^3_M, a^6_H)
\]

\[
= \left( \frac{1}{4} \right)^3 a^3_M + \left( 1 - \frac{1}{4} \right) \left( \frac{3}{4} \right)^2 a^3_M + \left( 1 - \frac{3}{4} \right) \left( \frac{1}{4} \right)^2 a^3_M + \left( \frac{3}{4} \right)^3 a^3_M
\]

\[
= \left( \frac{3}{4} \right)^3 + (1 - \frac{3}{4}) (\frac{3}{4})^2 0.83 + (1 - \frac{1}{4}) (\frac{1}{4})^2 0.67 + (1 - \frac{1}{4}) 0.50 = 0.64625 \approx 0.65
\]

by using the associated OWA weighting vector, which is computed as follows:

\[
W^2 = (a^{g-i}, (1-a)a^{g-i-1}, (1-a)a^{g-i-2}, (1-a))^T \quad \text{for} \ i = 3, \ g = 6, \ a = \frac{g-i}{6}
\]

and

\[
c_1 = \text{OWA}_{W^2}(a^3_M, a^3_M, a^3_M, a^6_M)
\]

\[
= a^6_M = 1
\]

Thus, the fuzzy envelope for \( H_{s_1} \) is obtained as \( \text{env}(H_{s_1}) = T(0.33, 0.65, 1, 1) \), which is a trapezoidal fuzzy number.

An example of an HFLTS (in red color) and its fuzzy envelope (in blue color) is shown in Figure 2 [88].
2.4. Trapezoidal Fuzzy Numbers

The fuzzy set theory proposed by Zadeh [23] provides a mathematical tool to address uncertainties about human cognitive processes, such as thinking and reasoning [76]. A fuzzy set \( \tilde{A} \) in the universe of discourse \( X = \{x_1, x_2, ..., x_n\} \) is represented by its membership function \( \mu_{\tilde{A}}(x) \) as follows:

\[
\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) | x \in X \} \quad (34)
\]

where \( x \in X \) indicates the elements belonging to \( X \), and \( \mu_{\tilde{A}}(x) : X \rightarrow [0, 1] \).

**Definition 1.** A fuzzy number is a special case of a convex and normalized fuzzy subset of the real line \( \mathbb{R} \) that ensures \( \sup \mu_{\tilde{A}}(x) = 1 \) and \( \mu_{\tilde{A}}(x) : \mathbb{R} \rightarrow [0, 1] \).

**Definition 2.** A fuzzy number \( \tilde{A} \) is defined as a trapezoidal fuzzy number if it has a membership function as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \\
\frac{(a_4 - x)}{(a_4 - a_3)}, & a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases} \quad (35)
\]

**Definition 3.** Arithmetic operations on positive trapezoidal fuzzy numbers \( \tilde{A} = (a_1, a_2, a_3, a_4) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4) \) and crisp number \( k \), are given as follows [76,89]:

**Addition:**

\[
\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \quad (36)
\]

\[
\tilde{A} + k = (a_1 + k, a_2 + k, a_3 + k, a_4 + k) \quad (37)
\]

**Subtraction:**

\[
\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \quad (38)
\]

\[
\tilde{A} - k = (a_1 - k, a_2 - k, a_3 - k, a_4 - k) \quad (39)
\]

**Multiplication:**

\[
\tilde{A} \otimes \tilde{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4) \quad (40)
\]

\[
\tilde{A} \times k = \begin{cases} 
(a_1 \times k, a_2 \times k, a_3 \times k, a_4 \times k), & \text{if } k \geq 0 \\
(a_4 \times k, a_3 \times k, a_2 \times k, a_1 \times k), & \text{if } k < 0
\end{cases} \quad (41)
\]

**Division:**

\[
\tilde{A}_1 \circ \tilde{A}_2 = (a_1/a_4, a_2/a_3, a_3/b_2, a_4/b_1) \quad (42)
\]

\[
\tilde{A}/k = \begin{cases} 
(a_1/k, a_2/k, a_3/k, a_4/k), & \text{if } k > 0 \\
(a_4/k, a_3/k, a_2/k, a_1/k), & \text{if } k < 0
\end{cases} \quad (43)
\]
Definition 4. The defuzzified (crisp) value of a trapezoidal fuzzy number $\tilde{A}$ is defined as follows [76,89]:

$$\kappa(\tilde{A}) = \frac{1}{3}(a_1 + a_2 + a_3 + a_4 - \frac{a_3a_4 - a_1a_2}{(a_3 + a_4) - (a_1 + a_2)}).$$ (44)

Definition 5. The Euclidean distance ($d_E$) and Hamming ($d_H$) distance between two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ are defined as follows, respectively [76,89]:

$$d_E(\tilde{A}, \tilde{B}) = \left( \frac{(a_1 - b_1)^2 + 2(a_2 - b_2)^2 + 2(a_3 - b_3)^2 + (a_4 - b_4)^2}{6} \right)^{\frac{1}{2}}. (45)$$

$$d_H(\tilde{A}, \tilde{B}) = \frac{|a_1 - b_1| + 2|a_2 - b_2| + 2|a_3 - b_3| + |a_4 - b_4|}{6}. (46)$$

3. Proposed Methodology

In this section, the proposed methodology based on the fuzzy CODAS method using fuzzy envelopes for HFLTSs is introduced. The proposed methodology, which has four phases, is clearly shown, followed by the fuzzy CODAS method.

3.1. Representation of the Proposed Methodology

The proposed methodology is based on the fuzzy CODAS method using fuzzy envelopes for HFLTSs that consider CLEs. The representation of the proposed methodology is given in four phases as depicted in Figure 3.

![Figure 3. The procedure of the proposed methodology.](image-url)
Each of the phases can be explained as follows: In the first phase, the selection criteria and alternatives are defined by decision-makers. The evaluations for the criteria and the alternatives with respect to the criteria using CLEs are carried out in Phase 2. In Phase 3, the CLEs are transformed into the HFLTSs and their fuzzy envelopes for both the criteria and alternatives. The fuzzy CODAS method, including nine steps, is applied in Phase 4.

3.2. The Fuzzy CODAS Method

The combinative distance-based assessment (CODAS) method was introduced by Keshavarz Ghorabaee et al. [17] as an efficient method to solve MCDM problems. The CODAS method basically utilizes a combinative form of the Euclidean distance and Taxicab distance for calculation of the evaluation ratings of alternatives. In order to detect the desirability of an alternative, the Euclidean distance as the primary and the Taxicab distance as the secondary measure are computed according to the negative-ideal point. The alternative with a greater distance is a more preferred alternative [90].

A fuzzy extension of the CODAS method was recently developed by Keshavarz Ghorabaee et al. [76] to deal with uncertainty in decision-making problems and was applied to a market segment selection problem. Unlike the CODAS method, fuzzy-based Euclidean and fuzzy-based Hamming distances are utilized in the fuzzy CODAS method for fuzzy problems. Linguistic variables and trapezoidal fuzzy numbers have been used in the fuzzy CODAS method.

The crisp or fuzzy version of the CODAS method has been used by just a few authors. Panchal et al. [91] implemented the CODAS method in the urea fertilizer industry. Bolturk [92] used the CODAS method that is based on pythagorean fuzzy sets for supplier selection in manufacturing. Mathew and Sahu [93] employed the CODAS method in material handling equipment selection. Peng and Garg [94] used the CODAS method that is based on interval-valued fuzzy soft sets in mine emergency decision-making. Bolturk and Kahraman [90] applied the interval-valued intuitionistic fuzzy CODAS method to wave energy facility location selection. Yeni and Özçelik [73] presented an interval-valued Atanassov intuitionistic fuzzy CODAS method for group decision-making processes and applied it to a personnel selection problem.

The steps of the fuzzy CODAS method for MCDM problems are given as follows [76]:

Step 1. Construct the fuzzy decision matrix as \( \tilde{X} = \left[ \tilde{x}_{ij} \right]_{m \times n} \), \( i = 1, 2, ..., m; j = 1, 2, ..., n \), where each \( \tilde{x}_{ij} \) indicates the fuzzy performance score for the alternative \( A_j \) with respect to the criterion \( C_i \) by decision-makers. The \( \tilde{x}_{ij} \) is obtained from the fuzzy envelope of the HFLTS for CLEs and stated as \( \tilde{x}_{ij} = \text{env}(H_{S_{ij}}) = T(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}) \). \( T \) denotes the trapezoidal fuzzy number.

Step 2. Obtain the fuzzy weight vector of the criteria as \( \tilde{W} = \left[ \tilde{w}_j \right]_{1 \times n} \), \( j = 1, 2, ..., n \), where each \( \tilde{w}_j \) identifies the fuzzy criterion’s importance by decision-makers. The \( \tilde{w}_j \) is handled by using the fuzzy envelope of the HFLTS for CLEs and denoted as \( \tilde{w}_j = \text{env}(H_{U_{ij}}) = T(w_{j1}, w_{j2}, w_{j3}, w_{j4}) \). \( T \) denotes the trapezoidal fuzzy number.

Step 3. Determine the fuzzy normalized decision matrix, \( \tilde{N} = \left[ \tilde{n}_{ij} \right]_{m \times n} \), \( i = 1, 2, ..., m; j = 1, 2, ..., n \), by using

\[
\tilde{n}_{ij} = \begin{cases} 
\tilde{x}_{ij} / \max_i(\tilde{x}_{ij}), & j \in B \\
1 - \tilde{x}_{ij} / \max_i(\tilde{x}_{ij}), & j \in C 
\end{cases} 
\]  

(47)

In Equation (47), \( B \) and \( C \) denote the benefit criteria and cost criteria, respectively.

Step 4. Construct the fuzzy weighted normalized decision matrix values of \( \tilde{R} = \left[ \tilde{r}_{ij} \right]_{m \times n} \), \( i = 1, 2, ..., m; j = 1, 2, ..., n \) as follows:

\[
\tilde{r}_{ij} = w_j \otimes \tilde{n}_{ij}, \quad 0 < \mathcal{G}(\tilde{w}_j) < 1. 
\]  

(48)
Step 5. Determine the fuzzy negative ideal solution $\tilde{NS} = \left[ \tilde{n}_{ij} \right]_{1 \times m}$ as follows:

$$\tilde{n}_{ij} = \min_i \tilde{r}_{ij} \quad (49)$$

where $\min_i \tilde{r}_{ij} = \left\{ \tilde{r}_{ij} \mid \kappa(\tilde{r}_{ij}) = \min_i \kappa(\tilde{r}_{ij}), k = 1, 2, ..., n \right\}$.

Step 6. Compute the Euclidean distance ($ED_i$) and Hamming distance ($HD_i$) of each alternative from the fuzzy negative ideal solution by using

$$ED_i = \sum_{j=1}^{m} d_E(\tilde{r}_{ij}, \tilde{n}_{ij}) \quad (50)$$

$$HD_i = \sum_{j=1}^{m} d_H(\tilde{r}_{ij}, \tilde{n}_{ij}). \quad (51)$$

Step 7. Define the relative assessment matrix $RA = [p_{ik}]_{m \times n}$ as follows:

$$p_{ik} = (ED_i - ED_k) + (t(ED_i - ED_k) \times (HD_i - HD_k)), \quad k = 1, 2, ..., n \quad (52)$$

where $t$ indicates a threshold function according to the threshold parameter $\theta$ set by the decision-maker, defined as:

$$t(x) = \begin{cases} 1, & \text{if } |x| \geq \theta \\ 0, & \text{if } |x| < \theta \end{cases}. \quad (53)$$

Step 8. Calculate the assessment score of each alternative as follows:

$$AS_i = \sum_{k=1}^{m} p_{ik}. \quad (54)$$

Step 9. Rank the alternatives according to the decreasing values of $AS_i$, $i = 1, ..., m$. The alternative with the highest assessment score is the most desirable alternative.

4. A Case Study of a Blue-Collar Personnel Selection Problem

In this section, the proposed methodology is applied to a blue-collar personnel selection problem for a manufacturing firm in Turkey. Six blue-collar personnel working in the same position at the firm are handled as the alternatives. The evaluation criteria affecting the blue-collar personnel selection of the firm are determined by the five key decision-makers (experts) who are competent to make decisions for the firm. The roles of the five experts are the board chairman, the human resources and public relations manager, the production manager, the purchasing and logistics supervisor, and the accounting and finance manager.

4.1. Application of the Proposed Methodology

The proposed methodology, including the fuzzy CODAS method, which uses the fuzzy envelopes of the HFLTSs based on CLEs, is applied to select the best blue-collar personnel among the alternatives and is explained in four phases as follows.

Phase 1. Define the criteria and alternatives for the personnel selection problem:

The aim of the personnel selection problem is to find the best personnel for the firm. During the selection and evaluation process, the curricula vitae (CVs) of the six blue-collar personnel were examined and more than one interview was conducted with each candidate. As a result of interviews with all blue-collar personnel working in the firm, 11 criteria affecting the selection process were determined by the five decision-makers and are shown in Figure 4. All the criteria, except for the salary expectation (CS), are defined as benefit criteria. The salary expectation is evaluated as the cost
criterion since the salary requested by a candidate is lower than that requested by the other candidates, which will contribute to the preference of that candidate.

Figure 4. The framework for the selection of blue-collar personnel.

Phase 2. Provide evaluations of the criteria and the alternatives using CLEs:
From the interview results with the blue-collar personnel, the evaluations of the criteria were made by using linguistic expressions and their corresponding triangular fuzzy numbers, which are given in Table 2 and Figure 5.

Table 2. The linguistic expressions and corresponding triangular fuzzy numbers used for evaluations.

<table>
<thead>
<tr>
<th>Linguistic Expressions for the Criteria</th>
<th>Linguistic Expressions for the Alternatives</th>
<th>Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definitely low (DL)</td>
<td>Nothing (N)</td>
<td>(0, 0, 0.17)</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>Very bad (VB)</td>
<td>(0, 0.17, 0.33)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>Bad (B)</td>
<td>(0.17, 0.33, 0.5)</td>
</tr>
<tr>
<td>Middle (M)</td>
<td>Medium (M)</td>
<td>(0.33, 0.5, 0.67)</td>
</tr>
<tr>
<td>High (H)</td>
<td>Good (G)</td>
<td>(0.5, 0.67, 0.83)</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>Very Good (VG)</td>
<td>(0.67, 0.83, 1)</td>
</tr>
<tr>
<td>Definitely high (DH)</td>
<td>Perfect (P)</td>
<td>(0.83, 1, 1)</td>
</tr>
</tbody>
</table>

Linguistic expressions are given in a linguistic term set (U) as follows:

\[
U = \left\{ u_0 = \text{definitely low(DL)}, u_1 = \text{very low(VL)}, u_2 = \text{low(L)}, u_3 = \text{medium(M)}, \right. \\
.\left. u_4 = \text{high(H)}, u_5 = \text{very high(VH)}, u_6 = \text{definitely high(DH)} \right\}
\]

Based on the CLEs made by the five experts, aggregated evaluations of the criteria were performed and are shown in Table 3.
The linguistic term set,

\[ S = \{ s_0 : \text{nothing}(N), s_1 : \text{very bad}(VB), s_2 : \text{bad}(B), s_3 : \text{medium}(M), s_4 : \text{good}(G), s_5 : \text{very good}(VG), s_6 : \text{perfect}(P) \} \]

shown in Table 2 and Figure 5, is also used to evaluate the alternatives with respect to the criteria provided from the five experts. The aggregated evaluations of the alternatives with respect to the criteria are shown in Table 5. The fuzzy envelopes of the HFLTSs were obtained by using the general computational process introduced by Liu and Rodriguez [87] and explained in Section 2.3. As shown in Table 5, the fuzzy decision matrix was obtained from the fuzzy envelopes of HFLTSs.
Table 3. The aggregated evaluations of both the criteria and the alternatives using comparative linguistic expressions (CLEs) from the experts.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEs</td>
<td>between L and H</td>
<td>DH</td>
<td>at least L</td>
<td>between M and VH</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and M</td>
<td>between H and VH</td>
<td>at least H</td>
<td>between L and VH</td>
<td>at least VH</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>M</td>
<td>at least VG</td>
<td>at least M</td>
<td>at least G</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between VH and M</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>M</td>
<td>at least G</td>
<td>between M and VG</td>
<td>M</td>
<td>between B and M</td>
<td>between M and G</td>
<td>between G and VH</td>
<td>between G and VH</td>
<td>at least G</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>M</td>
<td>at least G</td>
<td>between M and VG</td>
<td>M</td>
<td>between B and M</td>
<td>between M and G</td>
<td>between G and VH</td>
<td>between G and VH</td>
<td>at least G</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>G</td>
<td>at least G</td>
<td>between M and VG</td>
<td>G</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between G and VH</td>
<td>between G and VH</td>
<td>at least G</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>M</td>
<td>at least VG</td>
<td>between M and VG</td>
<td>M</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between G and VH</td>
<td>between G and VH</td>
<td>at least G</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>M</td>
<td>between M and VG</td>
<td>between M and VG</td>
<td>M</td>
<td>between M and G</td>
<td>between M and G</td>
<td>between G and VH</td>
<td>between G and VH</td>
<td>at least G</td>
<td>between M and G</td>
<td>at least G</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. The HFLTSs and their fuzzy envelopes, defuzzied weights, normalized weights, and rank of the criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>HFLTS</th>
<th>Fuzzy Envelopes</th>
<th>The Defuzzied Weight</th>
<th>The Normalized Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>[L, M, H]</td>
<td>(0.17, 0.47, 0.53, 0.83)</td>
<td>0.500</td>
<td>0.068</td>
<td>9</td>
</tr>
<tr>
<td>C2</td>
<td>[DH]</td>
<td>(0.83, 1.00, 1.00, 1.00)</td>
<td>0.943</td>
<td>0.128</td>
<td>1</td>
</tr>
<tr>
<td>C3</td>
<td>[L, M, H, VH, DH]</td>
<td>(0.17, 0.18, 1.00, 1.00)</td>
<td>0.587</td>
<td>0.080</td>
<td>7</td>
</tr>
<tr>
<td>C4</td>
<td>[M, H, VH]</td>
<td>(0.33, 0.64, 0.70, 1.00)</td>
<td>0.667</td>
<td>0.091</td>
<td>6</td>
</tr>
<tr>
<td>C5</td>
<td>[DL, VL, L, M, H, VH]</td>
<td>(0.00, 0.00, 0.80, 1.00)</td>
<td>0.452</td>
<td>0.061</td>
<td>10</td>
</tr>
<tr>
<td>C6</td>
<td>[M, H, VH, DH]</td>
<td>(0.33, 0.65, 1.00, 1.00)</td>
<td>0.737</td>
<td>0.100</td>
<td>5</td>
</tr>
<tr>
<td>C7</td>
<td>[L, M]</td>
<td>(0.17, 0.33, 0.50, 0.67)</td>
<td>0.418</td>
<td>0.057</td>
<td>11</td>
</tr>
<tr>
<td>C8</td>
<td>[H, VH]</td>
<td>(0.50, 0.67, 0.83, 1.00)</td>
<td>0.750</td>
<td>0.102</td>
<td>4</td>
</tr>
<tr>
<td>C9</td>
<td>[H, VH, DH]</td>
<td>(0.50, 0.85, 1.00, 1.00)</td>
<td>0.822</td>
<td>0.112</td>
<td>3</td>
</tr>
<tr>
<td>C10</td>
<td>[B, VH]</td>
<td>(0.17, 0.43, 0.73, 1.00)</td>
<td>0.583</td>
<td>0.079</td>
<td>8</td>
</tr>
<tr>
<td>C11</td>
<td>[VH, DH]</td>
<td>(0.67, 0.97, 1.00, 1.00)</td>
<td>0.889</td>
<td>0.121</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 5. The HFLTs for the alternatives and the fuzzy decision matrix obtained from the fuzzy envelopes of the HFLTs.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria</th>
<th>The HFLTs Generated from the CLEs for the Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>C1</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C5</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C6</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C7</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C8</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C9</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C10</td>
<td>(M, G, VG, P)</td>
</tr>
<tr>
<td></td>
<td>C11</td>
<td>(M, G, VG, P)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria</th>
<th>The Fuzzy Decision Matrix Formed by the Fuzzy Envelopes of the HFLTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>C1</td>
<td>(0.33, 0.50, 0.67, 0.97)</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>(0.67, 0.97, 0.00, 0.68)</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>(0.33, 0.65, 0.50, 0.85)</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>(0.33, 0.50, 0.67, 0.83)</td>
</tr>
<tr>
<td></td>
<td>C5</td>
<td>(0.50, 0.85, 0.00, 0.68)</td>
</tr>
<tr>
<td></td>
<td>C6</td>
<td>(0.33, 0.50, 0.67, 0.83)</td>
</tr>
<tr>
<td></td>
<td>C7</td>
<td>(0.33, 0.64, 0.50, 0.67)</td>
</tr>
<tr>
<td></td>
<td>C8</td>
<td>(0.33, 0.50, 0.50, 0.67)</td>
</tr>
<tr>
<td></td>
<td>C9</td>
<td>(0.33, 0.50, 0.67, 0.83)</td>
</tr>
<tr>
<td></td>
<td>C10</td>
<td>(0.33, 0.50, 0.67, 0.83)</td>
</tr>
<tr>
<td></td>
<td>C11</td>
<td>(0.33, 0.50, 0.67, 0.83)</td>
</tr>
</tbody>
</table>
Phase 4. Apply the fuzzy CODAS method to aggregate the evaluations represented by fuzzy envelopes:

The fuzzy CODAS method was used to carry out the determination of the ranking results of the alternatives. According to Equation (47), the fuzzy normalized decision matrix was constituted. Then, the fuzzy weighted normalized decision matrix was obtained by using Equation (48) considering the fuzzy weights given in Table 4.

The fuzzy negative-ideal solution was computed by Equation (49). The Euclidean distance and Hamming distance of each alternative were computed by Equations (50) and (51), respectively. These calculations are given in Table 5.

The relative assessment matrix (RA) was calculated by Equation (52). In the analysis, the threshold parameter was taken as $\theta = 0.02$. The assessment score of each alternative ($AS_i$) was computed by using Equation (54), and the elements of the relative assessment matrix are given in Table 6. The ranking of the alternatives was made based on decreasing values of the assessment scores shown in Table 7. The ranking results showed that Alternative 4 is the best among the alternatives.
Table 6. The fuzzy weighted normalized matrix, negative-ideal solution, and distances.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>ED (_i)</th>
<th>HD (_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(0.08, 0.35, 0.40, 0.83)</td>
<td>(0.10, 1.10)</td>
<td>(1.36, 1.36)</td>
<td>(0.85, 1.22)</td>
<td>(−0.12, −0.43)</td>
<td>(1.22, 1.22)</td>
<td>(0.35, 0.90)</td>
<td>(0.93, 1.12)</td>
<td>(0.93, 1.33)</td>
<td>(0.55, 1.00)</td>
<td>(0.08, 0.32)</td>
<td>(0.41, 0.79)</td>
<td>2.643</td>
</tr>
<tr>
<td>A2</td>
<td>(0.08, 0.35, 0.40, 0.83)</td>
<td>(1.10, 1.10)</td>
<td>(0.95, 1.36)</td>
<td>(0.43, 0.82)</td>
<td>(0.11, −0.15)</td>
<td>(1.22, 1.22)</td>
<td>(0.36, 0.90)</td>
<td>(0.93, 1.12)</td>
<td>(0.89, 1.11)</td>
<td>(0.73, 1.24)</td>
<td>(0.10, 1.22)</td>
<td>2.331</td>
<td>2.062</td>
</tr>
<tr>
<td>A3</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.46, 0.93)</td>
<td>(0.13, 0.39)</td>
<td>(0.00, 0.00)</td>
<td>(0.13, 0.51)</td>
<td>(0.28, 0.50)</td>
<td>(0.22, 0.57)</td>
<td>(0.08, 0.32)</td>
<td>(0.41, 0.79)</td>
<td>(0.10, 1.22)</td>
<td>2.149</td>
<td>1.976</td>
</tr>
<tr>
<td>A4</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.13, 0.39)</td>
<td>(0.00, 0.00)</td>
<td>(0.13, 0.51)</td>
<td>(0.28, 0.50)</td>
<td>(0.22, 0.57)</td>
<td>(0.08, 0.41)</td>
<td>(0.41, 1.00)</td>
<td>3.006</td>
<td>2.769</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.13, 0.39)</td>
<td>(0.00, 0.00)</td>
<td>(0.13, 0.51)</td>
<td>(0.28, 0.50)</td>
<td>(0.22, 0.57)</td>
<td>(0.08, 0.32)</td>
<td>(0.41, 0.79)</td>
<td>(0.10, 1.22)</td>
<td>2.249</td>
<td>2.066</td>
</tr>
<tr>
<td>A6</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.08, 0.45, 0.56, 1.24)</td>
<td>(0.13, 0.39)</td>
<td>(0.00, 0.00)</td>
<td>(0.13, 0.51)</td>
<td>(0.28, 0.50)</td>
<td>(0.22, 0.57)</td>
<td>(0.08, 0.32)</td>
<td>(0.41, 0.79)</td>
<td>(0.10, 1.22)</td>
<td>0.651</td>
<td>0.512</td>
</tr>
</tbody>
</table>

Table 7. The relative assessment matrix, appraisal scores, and ranking of the alternatives.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>RA</th>
<th>AS (_i)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>0.640</td>
<td>3.870</td>
</tr>
<tr>
<td>A2</td>
<td>−0.640</td>
<td>0.268</td>
<td>1.383</td>
</tr>
<tr>
<td>A3</td>
<td>−0.908</td>
<td>−0.268</td>
<td>0</td>
</tr>
<tr>
<td>A4</td>
<td>0.743</td>
<td>1.383</td>
<td>1.650</td>
</tr>
<tr>
<td>A5</td>
<td>−0.717</td>
<td>−0.077</td>
<td>0.191</td>
</tr>
<tr>
<td>A6</td>
<td>−3.870</td>
<td>−3.230</td>
<td>−2.962</td>
</tr>
</tbody>
</table>
4.2. Sensitivity Analysis

A sensitivity analysis was carried out to investigate the validation of the proposed methodology with respect to the 11 cases shown in Table 8. In order to constitute different cases for the criteria, the CLE of a given criterion was changed (shown in grey background color in Table 8) while the CLEs of the other criteria remained fixed. For instance, in Case 1, the CLE ‘between $L$ and $H$’ of criterion C1 was replaced with ‘$M$’, while the other 10 criteria remained same.

According to the sensitivity analysis of different cases for the fuzzy CODAS method, the ranking results given in Table 8 show that A4 is the best alternative and A6 is the worst alternative in the blue-collar personnel selection problem. Despite the small changes in ranking results, the ranking of the alternatives has good stability against different cases. While the rankings of the alternatives A4, A1, and A6 remain unchanged for all the cases, the rankings of the alternatives A2, A3, and A5 are similar in many cases. Thus, it can be said that the results of the proposed methodology are stable and the initial ranking result is reliable, since the best alternative (A4) and the worst alternative (A6) are relatively insensitive to the changes in criteria weights.
Table 8. The sensitivity analysis for different cases of criteria weights for the proposed methodology.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>Ranking of the Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original CLEs</td>
<td>between L and H</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A2 &gt; A3 &gt; A5 &gt; A6</td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>M</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A2 &gt; A3 &gt; A5 &gt; A6</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>between L and H</td>
<td>between L and M</td>
<td>at least L</td>
<td>between M and VH</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A2 &gt; A3 &gt; A5 &gt; A6</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>between L and H</td>
<td>between L and M</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A2 &gt; A3 &gt; A5 &gt; A6</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>between L and H</td>
<td>between L and M</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
</tr>
<tr>
<td>Case 6</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 7</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 8</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 9</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 10</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A5 &gt; A2 &gt; A3 &gt; A6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 11</td>
<td>DH</td>
<td>at least L</td>
<td>between L and M</td>
<td>at most VH</td>
<td>at least M</td>
<td>between L and VH</td>
<td>at least H</td>
<td>between VH and L</td>
<td>at least VH</td>
<td>A4 &gt; A1 &gt; A2 &gt; A3 &gt; A5 &gt; A6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3. Comparative Analysis

In order to demonstrate the validity of the ranking results of the fuzzy CODAS method, the results were compared with several other fuzzy MCDM methods (fuzzy EDAS [89], fuzzy TOPSIS [20], fuzzy WASPAS [95], fuzzy ARAS [96], and fuzzy COPRAS [97,98]). In addition, the ranking results of the considered fuzzy MCDM methods were also compared with the ranking results of the performance scores determined by the five experts in each interview as shown in Table 9.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Fuzzy CODAS</th>
<th>Fuzzy EDAS</th>
<th>Fuzzy TOPSIS</th>
<th>Fuzzy WASPAS</th>
<th>Fuzzy ARAS</th>
<th>Fuzzy COPRAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>A3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>A4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>A6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

As seen from Table 9, A4 is the best alternative and A6 is the worst alternative according to the considered fuzzy MCDM methods. The ranking results of the fuzzy CODAS method are completely consistent with those of the fuzzy TOPSIS method. The literature review showed that the classical or fuzzy version of the TOPSIS method is the most commonly used MCDM method in personnel selection problems. With respect to the ranking results, it can be said that, like the fuzzy TOPSIS method, the fuzzy CODAS method is an efficient method for multi-criteria personnel selection problems.

The Spearman’s correlation coefficients were calculated for evaluating the correlations between the ranking results of the considered fuzzy MCDM methods and the performance scores shown in Table 10.

<table>
<thead>
<tr>
<th>Fuzzy MCDM Methods</th>
<th>Performance Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy CODAS</td>
<td>0.714</td>
</tr>
<tr>
<td>Fuzzy EDAS</td>
<td>0.714</td>
</tr>
<tr>
<td>Fuzzy TOPSIS</td>
<td>0.829</td>
</tr>
<tr>
<td>Fuzzy WASPAS</td>
<td>0.943</td>
</tr>
<tr>
<td>Fuzzy ARAS</td>
<td>0.714</td>
</tr>
<tr>
<td>Fuzzy COPRAS</td>
<td>0.943</td>
</tr>
</tbody>
</table>

Table 10. The Spearman’s correlation coefficients between the fuzzy MCDM methods and the performance scores.

It can be concluded from Table 10 that the proposed method is consistent with the other methods because all the correlation coefficients are greater than 0.70.

5. Discussion and Conclusions

In today’s rapidly changing manufacturing environment, competition conditions have created the need to examine the concept of HRM. When HRM is considered as a chain, the most important link of this chain is to complete the process of personnel selection. Therefore, qualified personnel selection has been extremely important for the organizational success of firms. In some real-life situations, such as personnel selection problems, decision-makers generally prefer linguistic evaluations rather than exact numerical assessments due to uncertainty and vagueness in decision-making problems. The fuzzy set theory presents an ideal tool to capture the vagueness and uncertainty that exists in MCDM problems. Various extensions of fuzzy sets have been proposed for MCDM problems in different fields. Hesitant fuzzy sets (HFSs) are one of the most popular extensions of fuzzy sets and are
frequently used in MCDM methods. Hesitant fuzzy linguistic term sets (HFLTSs) based on the fuzzy linguistic approach and HFSs are useful for explaining experts’ evaluations.

In the current study, a methodology based on the fuzzy CODAS method using fuzzy envelopes for HFLTSs with respect to CLEs is presented in order to select the best blue-collar personnel working in a manufacturing firm in Turkey. CODAS is one of the recently developed distance-based MCDM methods, such as EDAS, ARAS, WASPAS, and COPRAS. In the method, a combinative form of Euclidean and Taxicab distances for alternatives with respect to the negative-ideal point is utilized for ranking the alternatives. The literature review shows that the CODAS method has great potential for various extensions of fuzzy sets, such as type-2, hesitant, and intuitionistic sets.

The fuzzy extension of the CODAS method combines the two kinds of distance, i.e., the Euclidean distance and the Hamming distance, and utilizes only the negative-ideal solution in the evaluation process. Thus, the fuzzy CODAS method can be differentiated from the other fuzzy MCDM methods in these two respects. The advantage of the fuzzy CODAS method is to provide more accurate ranking results using a combination of the Euclidean distance and the Hamming distance rather than only one type of distance.

The contribution of this study is to take into consideration fuzzy envelopes of HFLTSs for CLEs in the fuzzy CODAS method. In the solution process of the study, experts have evaluated both the criteria and the alternatives with respect to the criteria by using CLEs. The CLEs are transformed into HFLTSS, and then fuzzy envelopes for the HFLTSs are obtained by using trapezoidal fuzzy numbers. Afterwards, the fuzzy CODAS method is applied to rank the blue-collar personnel to select the best one. According to the ranking results, it is determined that Alternative 4 is the best and Alternative 6 is the worst amongst the alternatives. In order to verify the proposed methodology, a sensitivity analysis was performed by taking into account different cases based on criteria weights using CLEs. From the sensitivity analysis, it is shown that the ranking results of the fuzzy CODAS method are stable with small changes in many cases. The rankings of the alternatives A4, A1, and A6 are the same for all the cases, while the others are similar in many cases. It can be concluded that the results of the proposed methodology are steady almost all cases.

The validity of the ranking results obtained from the fuzzy CODAS method was also shown by means of several other fuzzy MCDM methods (fuzzy EDAS, fuzzy TOPSIS, fuzzy WASPAS, fuzzy ARAS, and fuzzy COPRAS). The ranking results of the fuzzy MCDM methods were compared with each other and also performance scores. The performance scores were determined by the experts as a result of interviews with the blue-collar personnel. The comparative analyses indicate that A4 is the best alternative and A6 is the worst alternative with respect to the considered fuzzy MCDM methods. In order to compare the ranking results of the fuzzy MCDM methods, the Spearman’s correlation coefficients were calculated. Since the obtained ranking results of the fuzzy CODAS method are almost consistent with the results of the other fuzzy MCDM methods used for comparison, the presented methodology can be used as an alternative MCDM method under fuzziness conditions where the information is vague, incomplete, or uncertain.

The practicability of CLEs based on context-free grammars and HFLTSs was shown for a multi-criteria personnel selection problem by allowing experts’ opinions. Also, the usefulness of fuzzy envelopes for HFLTSs was demonstrated by adopting the fuzzy CODAS method and the other considered fuzzy MCDM methods. In summary, the proposed methodology aims to provide a realistic way to address the personnel selection problems in the real world and guide researchers working on this subject. For further studies, the proposed methodology of this study could be applied in other fields; e.g., supplier selection, facility location selection, and renewable energy selection. In addition, other extensions of fuzzy sets, such as pythagorean fuzzy sets, neutrosophic fuzzy sets, and interval-valued intuitionistic fuzzy sets, could be adopted in different MCDM methods for solving personnel selection problems. The presented methodology could also be extended for type-2 fuzzy envelopes of HFLTSs that take into account CLEs for solving different MCDM problems.
Author Contributions: The contribution of all the authors to the creation of this article (including development of the concept, application and analysis of the experimental results, and writing of the article) is equal.

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