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Class of Analytic Functions Defined by q-Integral Operator in a Symmetric Region

Lei Shi [1], Mohsan Raza [2,*], Kashif Javed [2], Saqib Hussain [3] and Muhammad Arif [4]

1 School of Mathematics and Statistics, Anyang Normal University, Anyang 455002, China
2 Department of Mathematics, Government College University, Faisalabad 38000, Pakistan
3 Department of Mathematics, COMSATS University Islamabad, Abbottabad Campus 22010, Pakistan
4 Department of Mathematics, Abdul Wali Khan University Mardan, 23200 Mardan, Pakistan
* Correspondence: mohsan976@yahoo.com

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Abstract: The aim of the present paper is to introduce a new class of analytic functions by using a q-integral operator in the conic region. It is worth mentioning that these regions are symmetric along the real axis. We find the coefficient estimates, the Fekete–Szegö inequality, the sufficiency criteria, the distortion result, and the Hankel determinant problem for functions in this class. Furthermore, we study the inverse coefficient estimates for functions in this class.

Keywords: analytic functions; q-integral operator; conic region

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1. Introduction

Let $\mathcal{A}$ denote the class of functions f of the form:

$$f(z) = z + \sum_{m=2}^{\infty} a_m z^m, \quad z \in \mathbb{D}. \tag{1}$$

which are analytic in $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{S}$ denotes a subclass of $\mathcal{A}$, which contains univalent functions in $\mathbb{D}$. Let f be a univalent function in $\mathbb{D}$. Then, its inverse function f^{-1} exists in some disc $|w| < r \leq 1/4$, of the form:

$$f^{-1}(w) = w + B_2 w^2 + B_3 w^3 + \cdots . \tag{2}$$

For any analytic functions f of the form (1) and g of the form:

$$g(z) = z + \sum_{m=2}^{\infty} b_m z^m, \qquad z \in \mathbb{D}, \tag{3}$$

the convolution (Hadamard product) is given as:

$$(f * g)(z) = z + \sum_{m=2}^{\infty} a_m b_m z^m, \ (z \in \mathbb{D}).$$

Let f and g be analytic functions in $\mathbb{D}$. Then, f is said to be subordinate to g, written as $f(z) \prec g(z)$, if there exists a function w analytic in $\mathbb{D}$ with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$. Moreover, if g is univalent in $\mathbb{D}$, then the following equivalent relation holds:

$$f(z) \prec g(z) \iff f(0) = g(0) \quad \text{and} \quad f(\mathbb{D}) \subset g(\mathbb{D}).$$

The classes of k-uniformly starlike and k-uniformly convex functions were introduced by Kanas and Wiśniowska [1,2]. A function $f \in \mathcal{S}$ is in $k - \mathcal{ST}$, if and only if:

$$\Re \frac{zf'(z)}{f(z)} > k \left| \frac{zf'(z)}{f(z)} - 1 \right|,$$

where $k \in [0, \infty)$ and $z \in \mathbb{D}$. Similarly, for $k \in [0, \infty)$, a function $f \in \mathcal{S}$ is in $k - \mathcal{UCV}$, if and only if:

$$\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > k \left| \frac{zf''(z)}{f'(z)} \right|.$$

In particular, the classes $0 - \mathcal{ST} = \mathcal{ST}$ and $0 - \mathcal{UCV} = \mathcal{UCV}$ are the familiar classes of uniformly-starlike and uniformly-convex functions, respectively. These classes have been studied extensively. For some details, see [1–5].

Recently, a vivid interest has been shown by many researchers in quantum calculus due to its wide-spread applications in many branches of sciences especially in mathematics and physics. Among the contributors to the study, Jackson was the first to provide the basic notions and established results for the theory of q-calculus [6,7]. The idea of the q-derivative was first time used by Ismail et al. [8], and they introduced the q-extension of the class of starlike functions. A remarkable usage of the q-calculus in the context of geometric function theory was basically furnished, and the basic (or q-) hypergeometric functions were first used in geometric function theory in a book chapter by Srivastava (see, for details, p. 347 of [9]). The idea of q-starlikeness was further extended to certain subclasses of q-starlike functions. Recently, the q-analogue of the Ruscheweyh operator was introduced in [4], and it was studied in [10]. Many researchers contributed to the development of the theory by introducing certain classes with the help of q-calculus. For some details about these contributions, see [11–25]. We contribute to the subject by studying the q-integral operator in the conic region.

Now, we write some notions and basic concepts of q-calculus, which will be useful in our discussions. Throughout our discussion, we suppose that $q \in (0,1)$, $\mathbb{N} = \{1, 2, 3, \cdots\}$, and $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$, unless otherwise mentioned.

Definition 1. *Let $q \in (0,1)$. Then, the q-number $[t]_q$ is defined as:*

$$[t]_q = \begin{cases} \frac{1-q^t}{1-q}, & t \in \mathbb{C}, \\ \sum\limits_{j=0}^{m-1} q^j = 1 + q + q^2 + \cdots + q^{m-1}, & t = m \in \mathbb{N}. \end{cases}$$

Definition 2. *Let $q \in (0,1)$. Then, the q-factorial $[m]_q!$ is defined as:*

$$[m]_q! = \begin{cases} 1, & m = 0, \\ \prod\limits_{j=1}^{m} [j]_q, & m \in \mathbb{N}. \end{cases}$$

Definition 3. *Let $q \in (0,1)$. Then, the q-Pochhammer symbol $[t]_{m,q}$, $(z \in \mathbb{C},\ m \in \mathbb{N}_0)$ is defined as:*

$$[t]_{m,q} = \frac{(q^t; q)_m}{(1-q)^m} = \begin{cases} 1, & m = 0, \\ [t]_q [t+1]_q [t+2]_q \cdots [t+m-1]_q & m \in \mathbb{N}. \end{cases}$$

Furthermore, the gamma function in the q-analogue is defined by the following relation:

$$\Gamma_q(1) = 1 \text{ and } \Gamma_q(t+1) = [t]_q \Gamma_q(t).$$

Definition 4. *Let $q \in (0,1)$. Then, the q-derivative D_q of a function f is defined as:*

$$D_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{z(1-q)}, & z \neq 0, \\ f'(0) & z = 0 \end{cases} \tag{4}$$

provided that $f'(0)$ exists.

We observe that:

$$\lim_{q\to 1^-} D_q f(z) = \lim_{q\to 1^-} \frac{f(z) - f(qz)}{z(1-q)} = f'(z).$$

From Definition 4 and (1), it is clear that:

$$D_q f(z) = 1 + \sum_{m=2}^{\infty} [m]_q \, a_m z^{m-1}.$$

Now, take the function:

$$F_{q,\mu+1}(z) = z + \sum_{m=2}^{\infty} \Lambda_m z^m, \tag{5}$$

where $\mu > -1$, $\Lambda_m = \frac{[\mu+1]_{m-1,q}}{[m-1]_q!}$ and $z \in \mathbb{D}$. Now, consider a function $F_{q,\mu+1}^{(-1)}$ by:

$$F_{q,\mu+1}^{(-1)}(z) * F_{q,\mu+1}(z) = zD_q f(z),$$

then the q-Noor integral operator is define by:

$$\mathrm{I}_q^{\mu} f(z) = F_{q,\mu+1}^{(-1)}(z) * f(z) = z + \sum_{m=2}^{\infty} \Phi_{m-1} a_m z^m, \ (\mu > -1, \, z \in \mathbb{D}), \tag{6}$$

where:

$$\Phi_{m-1} = \frac{[m]_q!}{[\mu+1]_{m-1,q}}. \tag{7}$$

It is clear that $\mathrm{I}_q^0 f(z) = zD_q f(z)$ and $\mathrm{I}_q^1 f(z) = f(z)$. From (6), we obtain:

$$[\mu+1, q]\mathrm{I}_q^{\mu} f(z) = [\mu, q]\mathrm{I}_q^{\mu+1} f(z) + q^{\mu} zD_q \left(\mathrm{I}_q^{\mu+1} f(z)\right). \tag{8}$$

The q-Noor integral operator was recently defined by Arif et al. [26]. By taking $q \to 1^-$, the operator defined in (6) coincides with the Noor integral operator defined in [27,28]. For some details about the q-analogues of various differential operators, see [29–33]. The main aim of the current paper is to study the q-Noor integral operator by defining a class of analytic functions. Now, we introduce it as follows:

Definition 5. *A function f belongs to the class $\mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$, $\gamma \in \mathbb{C} - \{0\}$, if:*

$$\Re\left\{\frac{1}{\gamma}\left(\frac{zD_q \mathrm{I}_q^{\mu} f(z)}{\mathrm{I}_q^{\mu} f(z)} - 1\right) + 1\right\} > k\left|\frac{1}{\gamma}\left(\frac{zD_q \mathrm{I}_q^{\mu} f(z)}{\mathrm{I}_q^{\mu} f(z)} - 1\right)\right|, \ \mu > -1, \, k \in [0, \infty), \, z \in \mathbb{D}. \tag{9}$$

Geometric Interpretation

Let $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$. Then, $\frac{zD_q \mathrm{I}_q^{\mu} f(z)}{\mathrm{I}_q^{\mu} f(z)}$ assumes all the values in the domain $\Delta_{k,\gamma} = h_{k,r}(\mathbb{D})$ such that:

$$\Delta_{k,\gamma} = \gamma \Delta_k + (1 - \gamma),$$

where:

$$\Delta_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\},$$

or equivalently,

$$\frac{zD_q \mathrm{I}_q^{\mu} f(z)}{I_q^{\mu} f(z)} \prec h_{k,\gamma}(z). \tag{10}$$

The boundary $\partial\Delta_{k,\gamma}$ of the above region is the imaginary axis when $k = 0$. It is a hyperbola in the case of $k \in (0,1)$. When $k \in [0,1)$, we have:

$$h_{k,\gamma}(z) = 1 + \frac{2\gamma}{1-k^2} \left\{ \left(\frac{2}{\pi} arccosk \right) arctanh\sqrt{z} \right\}, \quad z \in \mathbb{D}.$$

In the case of $k = 1$, $\partial\Delta_{k,\gamma}$ is a parabola, and in this case:

$$h_{1,\gamma}(z) = 1 + \frac{2\gamma}{\pi} \left(log\frac{1+\sqrt{z}}{1-\sqrt{z}} \right)^2, \quad z \in \mathbb{D}.$$

When $k > 1$, $\partial\Delta_{k,\gamma}$ is an ellipse and:

$$h_{k,\gamma}(z) = 1 + \frac{2\gamma}{k^2-1} sin \left(\frac{\pi}{2\mathcal{F}(s)} \int_0^{v(z)/\sqrt{s}} \left(1-y^2\right)^{-1/2} \left(1-(sy)^2\right)^{-1/2} dy \right) + \frac{2\gamma}{1-k^2},$$

where $v(z) = \frac{z-\sqrt{s}}{1-\sqrt{sz}}$, $0 < s < 1$, $z \in \mathbb{D}$, and z is selected so that $k = \cosh\left(\frac{\pi\mathcal{F}'(s)}{4\mathcal{F}(s)}\right)$, where $\mathcal{F}$ is the first kind of Legendre's complete elliptic integral and $\mathcal{F}'$ is the complementary integral of $\mathcal{F}$; see [1,2]. Kanas and Wiśniowska [1,2] showed that the function $h_{k,\gamma}(\mathbb{D})$ is convex and univalent. All the curves discussed above have a vertex at $(k+\gamma)/(k+1)$. Now, it is clear that the domain $\Delta_{k,\gamma}$ is the right half plane for $k = 0$, hyperbolic for $k \in (0,1)$, parabolic when $k = 1$, and elliptic when $k > 1$. It is worth mentioning that the domain $\Delta_{k,\gamma}$ is symmetric with respect to the real axis. The function $h_{k,\gamma}(\mathbb{D}) = \Delta_{k,\gamma}$ is the extremal function in many problems for the classes of uniformly-starlike and uniformly-convex functions. For more about the conic domain; see [3,34].

Let $\mathcal{P}$ denote the class of functions h of the form:

$$h(z) = 1 + \sum_{m=1}^{\infty} c_m z^m, \quad z \in \mathbb{D}, \tag{11}$$

which are analytic with a positive real part in $\mathbb{D}$. If $k \in [0,\infty)$, $\gamma \in \mathbb{C} - \{0\}$, then the class $\mathcal{P}\left(h_{k,\gamma}\right)$ can be defined as:

$$\mathcal{P}(h_{k,\gamma}) = \left\{ h \in \mathcal{P} : h(\mathbb{D}) \subset \Delta_{k,\gamma} \right\}.$$

Lemma 1 ([35])**.** *Let $k \in [0,\infty)$ and $h_{k,\gamma}$ be introduced above. If:*

$$h_{k,\gamma}(z) = 1 + \sum_{m=1}^{\infty} Q_m z^m, \tag{12}$$

then:

$$Q_1 = \begin{cases} \frac{2\gamma A^2}{1-k^2}, & 0 \le k < 1, \\ \frac{8\gamma}{\pi^2}, & k = 1, \\ \frac{\pi^2\gamma}{4\sqrt{s}(k^2-1)R^2(s)(1+s)}, & k > 1, \end{cases} \tag{13}$$

and:

$$Q_2 = \begin{cases} \frac{A^2+2}{3} Q_1 & 0 \leq k < 1, \\ \frac{2}{3} Q_1 & k = 1, \\ \frac{4R^2(s)\left(s^2+6s+1\right)-\pi^2}{24\sqrt{s}R^2(s)(1+s)} Q_1 & k > 1, \end{cases} \tag{14}$$

where:

$$A = \frac{2\cos^{-1} k}{\pi},$$

and $0 < s < 1$, *which is selected so that* $k = cosh\left(\frac{\pi \mathcal{F}'(s)}{\mathcal{F}(s)}\right)$.

Let:

$$f_{k,\gamma}(z) = z + \sum_{m=2}^{\infty} A_m z^m$$

be the extremal function in class $\mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$ and $h_{k,\gamma}$ be of the form (12). Then, these functions can be related by the relation:

$$\frac{zD_q \mathrm{I}_q^{\mu} f_{k,\gamma}(z)}{\mathrm{I}_q^{\mu} f_{k,\gamma}(z)} = h_{k,\gamma}(z). \tag{15}$$

From (15) , we have:

$$zD_q \mathrm{I}_q^{\mu} f_{k,\gamma}(z) = p_{k,\gamma}(z) \mathrm{I}_q^{\mu} f_{k,\gamma}(z).$$

Furthermore:

$$z + \sum_{m=2}^{\infty} [m]_q \Phi_{m-1} A_m z^m = \left(\sum_{m=0}^{\infty} Q_m z^m\right)\left(z + \sum_{m=2}^{\infty} \Phi_{m-1} A_m z^m\right).$$

Equating the coefficients of z^m in the above relation, we obtain:

$$[m]_q \Phi_{m-1} A_m = \Phi_{m-1} A_m + \sum_{j=1}^{m-1} \Phi_{j-1} A_j Q_{m-j}$$

and:

$$A_m = \frac{1}{q[m-1]_q \Phi_{m-1}} \sum_{j=1}^{m-1} \Phi_{j-1} A_j Q_{m-j}. \tag{16}$$

This implies that:

$$A_2 = \frac{Q_1}{q\Phi_1}, \tag{17}$$

$$A_3 = \frac{Q_1^2 + qQ_2}{q^2(1+q)\Phi_2}, \tag{18}$$

$$A_4 = \frac{1}{(1+q+q^2)q\Phi_3}\left\{Q_3 + \frac{Q_1 Q_2}{q} + \frac{Q_1^3 + qQ_1Q_2}{q^2(1+q)}\right\}. \tag{19}$$

Lemma 2 ([36]). *If* $h \in \mathcal{P}$ *satisfies* (11) *, then:*

$$|c_2 - vc_1^2| \leq 2 \max\{1; |2v-1|\} \quad (v \in \mathbb{C}).$$

Lemma 3 ([37]). *If* $h \in \mathcal{P}$ *satisfies* (11) *, then:*

$$|c_n - c_{n-m}c_m| < 2, n > m, \ n = 1, 2, 3, \cdots.$$

Lemma 4 ([38]). *If $h \in \mathcal{P}$ satisfies (11), then:*

$$|c_3 - 2c_1c_2 + c_1^3| \leq 2.$$

2. Main Results

Theorem 1. *If $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$, then:*

$$|a_2| \leq A_2, \quad |a_3| \leq A_3, \tag{20}$$

and:

$$|a_4| \leq \frac{Q_1}{4q\,[3]_q\,\Phi_3}\{|F| + |(E - 2F)| + |(F - E + 4)|\}, \tag{21}$$

where:

$$E = 4 - \frac{4Q_2}{Q_1} - \frac{2Q_1}{q} - \frac{2Q_1}{q[2]_q}, \tag{22}$$

with:

$$F = 1 + \frac{Q_3}{Q_1} - \frac{2Q_2}{Q_1} + \frac{1 + [2]_q}{q[2]_q}(Q_2 - Q_1) + \frac{Q_1^2}{q[2]_q}. \tag{23}$$

Proof. Suppose that:

$$\frac{zD_q\mathrm{I}_q^{\mu} f(z)}{\mathrm{I}_q^{\mu} f(z)} = p(z), \tag{24}$$

where p is analytic in $\mathbb{D}$. Then, from (24), we have:

$$zD_q\mathrm{I}_q^{\mu} f(z) = p(z)\mathrm{I}_q^{\mu} f(z).$$

Consider:

$$p(z) = 1 + \sum_{m=1}^{\infty} p_m z^m \tag{25}$$

and $\mathrm{I}_q^{\mu} f(z)$ is given in the relation (6). Then:

$$z + \sum_{m=2}^{\infty} [m]_q \Phi_{m-1} a_m z^m = \left(\sum_{m=0}^{\infty} p_m z^m\right)\left(z + \sum_{m=2}^{\infty} \Phi_{m-1} a_m z^m\right).$$

It follows from the above relation that:

$$[m]_q \Phi_{m-1} a_m = \Phi_{m-1} a_m + \sum_{j=1}^{m-1} \Phi_{j-1} a_j p_{m-j}$$

and:

$$a_m = \frac{1}{q[m-1]_q \Phi_{m-1}} \sum_{j=1}^{m-1} \Phi_{j-1} a_j p_{m-j}. \tag{26}$$

Furthermore, consider the function:

$$h(z) = (1 + w(z))(1 - w(z))^{-1} = 1 + c_1 z + c_2 z^2 + \cdots. \tag{27}$$

Then, h is analytic in $\mathbb{D}$ with $Re(h(z)) > 0$. By using (12) and (27) , we have:

$$\begin{aligned} p(z) &= p_{k,\gamma}\left(\frac{-1+h(z)}{1+h(z)}\right) = 1+\frac{1}{2}c_1Q_1z+\left(\frac{1}{2}c_2Q_1+\frac{1}{4}c_1^2(Q_2-Q_1)\right)z^2 \\ &+\left\{\frac{1}{8}(Q_1-2Q_2+Q_3)c_1^3+\frac{1}{2}(Q_2-Q_1)c_2c_1+\frac{1}{2}Q_1c_3\right\}z^3+\cdots. \end{aligned} \tag{28}$$

Now, from (26) and (28) , we obtain:

$$a_2 = \frac{p_1}{q\Phi_1} = \frac{c_1Q_1}{2q\Phi_1}. \tag{29}$$

Now, using the fact that $|c_m| \leq 2$, we get:

$$|a_2| = \left|\frac{p_1}{q\Phi_1}\right| = \left|\frac{c_1Q_1}{2q\Phi_1}\right| \leq \frac{|Q_1|}{q\Phi_1} = \frac{Q_1}{q\Phi_1} = A_2.$$

Similarly:

$$a_3 = \frac{1}{q[2]_q\Phi_2}\{p_2+p_1a_2\Phi_1\} = \frac{qp_2+p_1^2}{(1+q)\,q^2\Phi_2}. \tag{30}$$

In view of the relation $|p_1|^2+|p_2| \leq Q_1^2+Q_2$ (see [5]) and (17), we obtain:

$$\begin{aligned} |a_3| = \frac{|qp_2+p_1^2|}{(1+q)\,q^2\Phi_2} &\leq \frac{q\left(|p_2|+|p_1^2|\right)+(1-q)\left|p_1^2\right|}{(1+q)\,q^2\Phi_2} \\ &\leq \frac{q\left(|Q_2|+|Q_1^2|\right)+(1-q)\left|Q_1^2\right|}{(1+q)\,q^2\Phi_2} \\ &\leq \frac{q\,|Q_2|+\left|Q_1^2\right|}{(1+q)\,q^2\Phi_2} = A_3, \end{aligned}$$

which implies the required result. Now, equating the coefficients of z^3, we have:

$$a_4 = \frac{Q_1}{8\,[3]_q\,q\Phi_3}(4c_3-Ec_1c_2+Fc_1^3), \tag{31}$$

where E and F are given by (22) and (23), respectively. This implies that:

$$\begin{aligned} |a_4| &= \frac{Q_1}{8q\,[3]_q\,\Phi_3}\left|F(c_3-2c_1c_2+c_1^3)+(E-2F)(c_3-c_1c_2)+(F-E+4)c_3\right| \\ &\leq \frac{Q_1}{8q\,[3]_q\,\Phi_3}\left|F(c_3-2c_1c_2+c_1^3)\right|+\left|(E-2F)(c_3-c_1c_2)\right|+\left|(F-E+4)c_3\right| \\ &\leq \frac{Q_1}{2q\,[3]_q\,\Phi_3}|F|+|(E-2F)|+|(F-E+4)|, \end{aligned}$$

where we have used Lemmas 3 and 4. □

Theorem 2. *Let $0 \leq k < \infty$, $q \in (0,1)$, and $\gamma \in \mathbb{C}-\{0\}$. If $f \in \mathcal{K}-\mathcal{UST}_q^{\mu}(\gamma)$ of the form (1), then:*

$$|a_m| \leq \frac{Q_1\,(Q_1+q)\left(Q_1+q\,[2]_q\right)\ldots\left(Q_1+q\,[m-2]_q\right)}{q^{m-1}\Phi_{m-1}\prod\left(1+q+\ldots+q^{k-1}\right)}, \quad m \geq 2.$$

Proof. The result is clearly true for $m = 2$. That is:

$$|a_2| \leq \frac{Q_1}{q} = A_2.$$

Let $m \geq 2$, and suppose that the relation is true for $j \leq m-1$, then we obtain:

$$\begin{aligned}
|a_m| &= \frac{1}{q\,[m-1]_q\,\Phi_{m-1}}\left|p_{m-1} + \sum_{j=2}^{m-1}\Phi_{j-1}a_j p_{m-j}\right| \\
&\leq \frac{1}{q\,[m-1]_q\,\Phi_{m-1}}\left\{Q_1 + \sum_{j=2}^{m-1}\Phi_{j-1}|a_j|Q_1\right\} \\
&\leq \frac{1}{q\,[m-1]_q\,\Phi_{m-1}}Q_1\left\{1 + \sum_{j=2}^{m-1}\Phi_{j-1}|a_j|\right\} \\
&\leq \frac{1}{q\,[m-1]_q\,\Phi_{m-1}}Q_1\left\{1 + \sum_{j=2}^{m-1}\Phi_{j-1}\frac{Q_1\,(Q_1+q)\left(Q_1+q\,[2]_q\right)\ldots\left(Q_1+q\,[j-2]_q\right)}{q^{j-1}\Phi_{j-1}\prod\left(1+q+\ldots+q^{k-1}\right)}\right\},
\end{aligned}$$

where we applied the induction hypothesis to $|a_j|$ and the Rogosinski result $|p_m| \leq Q_1$ (see [39]). This implies that:

$$|a_m| \leq \frac{1}{q\,[m-1]_q\,\Phi_{m-1}}Q_1\left\{1 + \sum_{j=2}^{m-1}\frac{Q_1\,(Q_1+q)\left(Q_1+q\,[2]_q\right)\ldots\left(Q_1+q\,[j-2]_q\right)}{q^{j-1}\prod\left(1+q+\ldots+q^{k-1}\right)}\right\}.$$

Applying the principal of mathematical induction, we find:

$$\begin{aligned}
&1 + \sum_{j=2}^{m-1}\frac{Q_1\,(Q_1+q)\left(Q_1+q\,[2]_q\right)\ldots\left(Q_1+q\,[j-2]_q\right)}{q^{j-1}\prod\left(1+q+\ldots+q^{k-1}\right)} \\
= &\quad \frac{Q_1\,(Q_1+q)\left(Q_1+q\,[2]_q\right)\ldots\left(Q_1+q\,[m-2]_q\right)}{q^{m-2}\prod\left(1+q+\ldots+q^{k-2}\right)}.
\end{aligned}$$

Hence, the desired result. □

Theorem 3. *If $f \in \mathcal{A}$ is given in (1) and the inequality:*

$$\sum_{m=2}^{\infty}\left\{q[m-1]_q(k+1) + |\gamma|\right\}\Phi_{m-1}|a_m| \leq |\gamma| \tag{32}$$

holds true for some $0 \leq k < \infty$, $q \in (0,1)$ and $\gamma \in \mathbb{C} - \{0\}$, then $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$.

Proof. Using (9), we have:

$$k\left|\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)} - 1\right)\right| - \Re\left\{\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)} - 1\right)\right\} < 1.$$

This implies that:

$$\begin{aligned}
&k\left|\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right)\right|-\Re\left\{\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right)\right\}\\
&\leq\frac{k}{|\gamma|}\left|\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right|+\frac{1}{|\gamma|}\left|\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right|\\
&\leq\frac{(k+1)}{|\gamma|}\left|\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right|.
\end{aligned}$$

We see that:

$$\begin{aligned}
\left|\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right| &= \left|\frac{z+\sum_{m=2}^{\infty}[m]_q\,\Phi_{m-1}a_mz^m-z-\sum_{m=2}^{\infty}\Phi_{m-1}a_mz^m}{z+\sum_{m=2}^{\infty}\Phi_{m-1}a_mz^m}\right|\\
&=\left|\frac{\sum_{m=2}^{\infty}q[m-1]_q\Phi_{m-1}a_mz^m}{z+\sum_{m=2}^{\infty}\Phi_{m-1}a_mz^m}\right|\\
&\leq\frac{\sum_{m=2}^{\infty}q[m-1]_q\Phi_{m-1}|a_m|}{1-\sum_{m=2}^{\infty}\Phi_{m-1}|a_m|}.
\end{aligned}$$

From the above, we have:

$$\begin{aligned}
&k\left|\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right)\right|-\Re\left\{\frac{1}{\gamma}\left(\frac{zD_q\mathrm{I}_q^{\mu}f(z)}{\mathrm{I}_q^{\mu}f(z)}-1\right)\right\}\\
&\leq\frac{(k+1)}{|\gamma|}\frac{\sum_{m=2}^{\infty}q[m-1,q]\Phi_{m-1}|a_m|}{1-\sum_{m=2}^{\infty}\Phi_{m-1}|a_m|}\\
&\leq 1.
\end{aligned}$$

This completes the proof. □

Theorem 4. *If $f\in\mathcal{K}-\mathcal{UST}_q^{\mu}(\gamma)$, then $f(\mathbb{D})$ contains an open disk of radius:*

$$\frac{q\,(1+q)}{q\,|Q_1|\,[\mu+1]_q+2q\,(1+q)},$$

where Q_1 is defined by (11).

Proof. Let $w_0\in\mathbb{C}$ and $w_0\neq 0$ with $f(z)\neq w_0$ in $\mathbb{D}$. Then:

$$f_1(z)=w_0f(z)\,(w_0-f(z))^{-1}=z+\left(\frac{1}{w_0}+a_2\right)z^2+....$$

Since $f_1\in\mathcal{S}$,

$$\left|\frac{1}{w_0}+a_2\right|\leq 2.$$

Now, by applying Theorem 1, we obtain:

$$|\frac{1}{w_0}|\leq 2+\frac{2\,|Q_1|\,[\mu+1]_q}{q\,(1+q)}.$$

Hence:

$$|w_0|\geq\frac{q\,(1+q)}{|Q_1|\,q[\mu+1]_q+2q\,(1+q)}.$$

□

Theorem 5. *If* $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$*, then:*

$$\mathrm{I}_q^{\mu} f(z) \prec z exp \int_0^z \frac{h_{k,\gamma}(w(\xi)) - 1}{\xi} d\xi, \tag{33}$$

where w *is analytic in* $\mathbb{D}$ *with* $w(0) = 0$ *and* $|w(z)| < 1$*. Moreover, for* $|z| = \rho$*, we have:*

$$\left(exp \int_0^1 \frac{h_{k,\gamma}(-\rho) - 1}{\rho} d\rho \right) \leq \left| \frac{\mathrm{I}_q^{\mu} f(z)}{z} \right| \leq \left(exp \int_0^1 \frac{h_{k,\gamma}(\rho) - 1}{\rho} d\rho \right),$$

where $h_{k,\gamma}$ *is given in* (10)*.*

Proof. From (10), we obtain:

$$\frac{D_q \mathrm{I}_q^{\mu} f(z)}{\mathrm{I}_q^{\mu} f(z)} = \frac{h_{k,\gamma}(w(z)) - 1}{z} + \frac{1}{z},$$

for a function w, which is analytic in $\mathbb{D}$ with $w(0) = 0$ and $|w(z)| < 1$. Integrating the above relation with respect to z, we have:

$$\mathrm{I}_q^{\mu} f(z) \prec z exp \int_0^z \frac{h_{k,\gamma}(w(\xi)) - 1}{\xi} d\xi. \tag{34}$$

Since the function $h_{k,\gamma}$ is univalent and maps the disk $|z| < \rho (0 < \rho \leq 1)$ onto a convex and symmetric region with respect to the real axis,

$$\frac{k + \gamma}{\gamma + 1} < h_{k,\gamma}(-\rho|z|) \leq \Re\{h_{k,\gamma}(w(\rho z))\} \leq h_{k,\gamma}(\rho|z|). \tag{35}$$

Using the above inequality, we have:

$$\int_o^1 \frac{h_{k,\gamma}(-\rho|z|) - 1}{\rho} d\rho \leq \Re \int_o^1 \frac{h_{k,\gamma} w(\rho z) - 1}{\rho} d\rho \leq \int_o^1 \frac{h_{k,\gamma}(\rho|z|) - 1}{\rho} d\rho, \quad z \in \mathbb{D}.$$

Consequently, the subordination (24) implies that:

$$\int_0^1 \frac{h_{k,\gamma}(-\rho|z|) - 1}{\rho} d\rho \leq log \left| \frac{\mathrm{I}_q^{\mu} f(z)}{z} \right| \leq \int_0^1 \frac{h_{k,\gamma}(\rho|z|) - 1}{\rho} d\rho.$$

Furthermore, the relations $h_{k,\gamma}(-\rho) \leq h_{k,\gamma}(-\rho|z|)$, $h_{k,\gamma}(\rho|z| \leq h_{k,\gamma}(\rho)$ leads to:

$$\left(exp \int_0^1 \frac{h_{k,\gamma}(-\rho|z|) - 1}{\rho} d\rho \right) \leq \left| \frac{\mathrm{I}_q^{\mu} f(z)}{z} \right| \leq \left(exp \int_0^1 \frac{h_{k,\gamma}(\rho|z|) - 1}{\rho} d\rho \right).$$

This completes the proof. □

Theorem 6. *Let* $k \in [0, \infty)$ *and* $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$ *of the form* (1)*. Then:*

$$|a_3 - \sigma a_2^2| \leq \frac{|Q_1|}{2q\,[2]_q \Phi_2} max\{1; |2v - 1|\}, \quad \sigma \in \mathbb{C},$$

where:

$$v = \frac{1}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q} + \frac{\sigma Q_1 \Phi_2 (1+q)}{q\Phi_1^2}\right). \tag{36}$$

The values of Q_1 and Q_2 are given by (13) and (14), respectively, and that of Φ_2 is given in (7).

Proof. If $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$, then using (29) and (30), we have:

$$a_2 = \frac{Q_1 c_1}{2q\Phi_1},$$

$$a_3 = \frac{1}{4q[2]_q\Phi_2}\left\{2c_2Q_1 + c_1^2(Q_2 - Q_1) + \frac{Q_1^2c_1^2}{q}\right\},$$

which together imply that:

$$\begin{aligned}\left|a_3 - \sigma a_2^2\right| &= \frac{1}{4q[2]_q\Phi_2}\left|\left\{\left(2c_2Q_1 + c_1^2(Q_2 - Q_1)\right) + \frac{Q_1^2c_1^2}{q}\right\} - \frac{\sigma Q_1^2c_1^2}{4q^2\Phi_1^2}\right| \\ &= \frac{Q_1}{4q[2]_q\Phi_2}\left|c_2 - vc_1^2\right|,\end{aligned}$$

where v is defined by (36). Applying Lemma 2, we have the desired result. □

Theorem 7. *If $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$ is given in (1), then:*

$$|a_2a_3 - a_4| \leq \frac{|Q_1|}{4q\,[3]_q\,\Phi_3}\left\{|A| + |(B-2A)| + |A - B + 4|\right\},$$

where:

$$B = E + \frac{2Q_1\Phi_3\,[3]_q}{q\,[2]_q\,\Phi_1\Phi_2}, \quad A = F + \frac{Q_1\Phi_3\,[3]_q}{q\,[2]_q\,\Phi_1\Phi_2}\left(Q_2 - Q_1 + \frac{Q_1^2}{q}\right),$$

with E and F given in (22) and (23), respectively.

Proof. By using (29)–(31), it is easy to see that:

$$\begin{aligned}|a_2a_3 - a_4| &= \frac{|-Q_1|}{8q\,[3]_q\,\Phi_3}\left|4c_3 - Bc_1c_2 + Ac_1^3\right| \\ &= \frac{|Q_1|}{8q\,[3]_q\,\Phi_3}\left|(A - B + 4)c_3 + (B - 2A)(c_3 - c_1c_2) + A(c_3 - 2c_1c_2 + c_1^3)\right| \\ &\leq \frac{|Q_1|}{4q\,[3]_q\,\Phi_3}\left\{|A| + |(B-2A)| + |A - B + 4|\right\},\end{aligned}$$

where we used Lemmas 3 and 4. This completes the proof. □

Theorem 8. *If $k \in [0, \infty)$ and letting $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$ and having the inverse coefficients of the form (2), then the following results hold:*

$$|B_2| \leq \frac{|Q_1|}{q\Phi_1},$$

$$|B_3| \leq \frac{|Q_1|}{q\,[2]_q\,\Phi_2} max\left\{1; \left|\frac{Q_1H}{q} + \frac{Q_2}{Q_1}\right|\right\},$$

and:

$$H = \frac{2[2]_q \Phi_2}{\Phi_1^2} - 1. \tag{37}$$

Proof. Since $f(f^{-1}(\omega)) = \omega$; therefore, using (2) , we have:

$$B_2 = -a_2, \quad B_3 = 2a_2^2 - a_3.$$

Putting the value of a_2 and a_3 in the above relation, it follows easily that:

$$B_2 = -a_2 = -\frac{c_1 Q_1}{2q\Phi_1}. \tag{38}$$

Using the coefficient bound $|c_1| \leq 2$, we can write:

$$|B_2| = \left| \frac{-c_1 Q_1}{2q\Phi_1} \right| \leq \frac{|Q_1|}{q\Phi_1}. \tag{39}$$

Now with the help of Lemma 2, we obtain:

$$\begin{aligned} B_3 &= 2a_2^2 - a_3 \\ &= -\frac{Q_1}{2q\,[2]_q\,\Phi_2} \left\{ c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\right) - \frac{c_1^2 Q_1}{q\Phi_1^2}[2]_q \Phi_2 \right\} \\ &= -\frac{Q_1}{2q\,[2]_q\,\Phi_2} \left\{ c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\left(\frac{2[2]_q\Phi_2}{\Phi_1^2} - 1\right)\right) \right\} \\ &= -\frac{Q_1}{2q\,[2]_q\,\Phi_2} \left\{ c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1 H}{q}\right) \right\}. \end{aligned} \tag{40}$$

Taking the absolute value of the above relation, we have:

$$\begin{aligned} |B_3| &\leq \frac{|Q_1|}{q\,[2]_q\,\Phi_2} \left| c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1 H}{q}\right) \right| \\ &\leq \frac{|Q_1|}{q\,[2]_q\,\Phi_2} max \left\{ 1; \left| \frac{Q_1 H}{q} + \frac{Q_2}{Q_1} \right| \right\}. \end{aligned}$$

□

Theorem 9. *If $f \in \mathcal{K} - \mathcal{UST}_q^{\mu}(\gamma)$ with inverse coefficients given by (2), then for a complex number λ, we have:*

$$|B_3 - \lambda B_2^2| \leq \frac{|Q_1|}{q\,[2]_q\,\Phi_2} max \left\{ 1; \left| \left(\frac{(2-\lambda)\,[2]_q\,\Phi_2 Q_1}{q\Phi_1^2} - 1 \right) \frac{Q_1}{q} + \frac{Q_2}{Q_1} \right| \right\}.$$

Proof. From (38) and (40), we have:

$$\begin{aligned} B_3 - \lambda B_2^2 &= \frac{c_1^2 Q_1^2}{2q^2\Phi_1^2} - \frac{Q_1}{2q\,[2]_q\,\Phi_2}\left(c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\right)\right) - \frac{\lambda c_1^2 Q_1^2}{4q^2\Phi_1^2} \\ &= \frac{c_1^2 Q_1^2}{4q^2\Phi_1^2}(2-\lambda) - \frac{Q_1}{2q\,[2]_q\,\Phi_2}\left(c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\right)\right) \\ &= -\frac{Q_1}{2q\,[2]_q\,\Phi_2}\left\{ c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\left(\frac{(2-\lambda)\,[2]_q\,\Phi_2 Q_1}{q\Phi_1^2} - 1\right)\right)\right\}. \end{aligned}$$

Now, by applying Lemma 2, the absolute value of the above equation becomes:

$$
\begin{aligned}
|B_3 - \lambda B_2^2| &\leq \frac{|Q_1|}{2q\,[2]_q\,\Phi_2}\left|c_2 - \frac{c_1^2}{2}\left(1 - \frac{Q_2}{Q_1} - \frac{Q_1}{q}\left(\frac{(2-\lambda)\,[2]_q\,\Phi_2 Q_1}{q\Phi_1^2} - 1\right)\right)\right| \\
&\leq \frac{|Q_1|}{q\,[2]_q\,\Phi_2} max\left\{1; \left|\left(\frac{(2-\lambda)\,[2]_q\,\Phi_2 Q_1}{q\Phi_1^2} - 1\right)\frac{Q_1}{q} + \frac{Q_2}{Q_1}\right|\right\}.
\end{aligned}
$$

This completes the proof. □

3. Future Work

The idea presented in this paper can easily be implemented to introduce some more subfamilies of analytic and univalent functions connected with different image domains.

4. Conclusions

In this article, we defined a new class of analytic functions by using the q-Noor integral operator. We investigated some interesting properties, which are useful to study the geometry of the image domain. We found the coefficient estimates, the Fekete–Szegö inequality, the sufficiency criteria, the distortion result, and the Hankel determinant problem for this class.

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