symmetry

MDPI

Article

Unsettling Physics in the Quantum-Corrected Schwarzschild Black Hole

Valerio Faraoni * and Andrea Giusti

Department of Physics & Astronomy, Bishop's University, 2600 College Street, Sherbrooke, QC J1M 1Z7, Canada; agiusti@ubishops.ca

* Correspondence: vfaraoni@ubishops.ca; Tel.: +1-819-822-9600; Fax: +1-819-822-9611

Received: 25 June 2020; Accepted: 17 July 2020; Published: 1 August 2020

check for updates

Abstract: We study a quantum-corrected Schwarzschild black hole proposed recently in Loop Quantum Gravity. Prompted by the fact that corrections to the innermost stable circular orbit of Schwarzschild diverge, we investigate time-like and null radial geodesics. Massive particles moving radially outwards are confined, while photons make it to infinity with infinite redshift. This unsettling physics, which deviates radically from both Schwarzschild (near the horizon) and Minkowski (at infinity) is due to repulsion by the negative quantum energy density that makes the quasilocal mass vanish as one approaches spatial infinity.

Keywords: quantum-corrected black hole; Loop Quantum Gravity; pseudo-Newtonian potential

1. Introduction

Einstein's theory of gravity is plagued by singularities, which occur generically if the matter fields satisfy reasonable energy conditions [1]. These singularities are not necessarily avoided even if quantum matter evades these energy conditions. However, it is believed that quantizing the geometry will cure the singularity problem. Loop Quantum Gravity and Loop Quantum Cosmology propose possible solutions in which quantum geometry avoids the classical singularities by making the curvature invariants finite and effectively discretizing spacetime. Referring specifically to black hole singularities, the search for quantum-corrected black holes that are singularity-free has a long history, beginning with phenomenological attempts that were not solidly rooted in established theories, and later developing with full quantum gravity constructs. We refer the reader to the relevant literature, which is too long to summarize here. In broad strokes, one approach has been to quantize matter in curved space with the hope that this will be sufficient to avoid singularities, through the violation of the classical energy conditions. However, whether this approach can be successful is now doubtful. Even if the energy conditions cherished by classical relativity are violated, there is no guarantee that singularities will be avoided.

The Loop Quantum Gravity approach is different, in that it attempts to quantize the geometry itself, and this more radical approach can, and should (at least in principle) be connected to the macroscopic world. For black holes, this statement means to connect to the physics in regions outside and around the horizons, as well as to spatial infinity far away from the black hole horizon. In Loop Quantum Gravity, the spacetime geometry is fundamentally discrete, which is expressed by the fact that there is an area operator $\hat{\Delta}$ with a minimum eigenvalue that is strictly positive. All considerations about quantum black holes revolve around this basic fact and incorporate it somehow in the effective description of black holes. This discreteness of the geometry and the spectrum of the area operator must, therefore, leave some traces also in the macroscopic description of a black hole at scales much larger than the Planck scale. Indeed, this is the case. Naturally, quantum-correcting the geometry of general relativity in the interior region (i.e., inside the black hole horizon) leads to changes at the horizon and outside of it, when the interior geometry is matched to the exterior one (see, e.g., [2–9]). An unwanted effect is that, in some proposals, quantum-correcting the interior to cure the singularity may lead to large quantum effects in low-curvature regions [10–15]. More in general, it is interesting to understand quantum corrections in the exterior regions which are, in principle, accessible to observers.

A quantum-corrected black hole which deviates from the Schwarzschild solution of general relativity was proposed recently in Refs. [16–18], with the explicit purpose of providing a singularity-free black hole which is also free of problems induced by large quantum corrections in the low-curvature regions. In this solution, which corrects the prototypical black hole, i.e., the Schwarzschild geometry, loop corrections to the Schwarzschild geometry are quantified by a small dimensionless parameter ϵ, which is mass-dependent. The explicit mass dependence is given by [18]

$$\epsilon = \sqrt{1+\gamma^2\delta_b^2} - 1\,, \tag{1}$$

where $\gamma \simeq 0.2375$ is the Barbero–Immirzi parameter (see [16–18]),

$$\delta_b = \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\,\gamma^2 m} \right)^{1/3}, \tag{2}$$

Δ is the minimum positive eigenvalue of the area operator, given in terms of the Planck mass l_{Pl} by $\Delta \simeq 5.17\, l_{Pl}^2$, and m is the black hole mass. Hence, when the macroscopic black hole properties are studied and one can take the limit $\delta_b \to 0$, it is

$$\epsilon \simeq \left(\frac{\gamma^2\Delta}{16\pi} \right)^{1/3} \frac{1}{m^{2/3}}\,. \tag{3}$$

In the case of a solar mass black hole, this parameter is estimated to have values of order $\epsilon \sim 10^{-26}$.

To first order in the small parameter ϵ, the quantum-corrected black hole geometry of [16–18] is described by the static and spherically symmetric line element (We follow the notations of Refs. [1,19] and we use units in which Newton's constant G and the speed of light are unity (but we occasionally restore G).) (see Equations (4.8)–(4.10) of Ref. [18])

$$ds^2 = -\left(\frac{r}{r_S}\right)^{2\epsilon} \left[1 - \left(\frac{r_S}{r}\right)^{1+\epsilon}\right] dt^2 + \frac{dr^2}{1-\left(\frac{r_S}{r}\right)^{1+\epsilon}} + r^2 d\Omega_{(2)}^2\,, \tag{4}$$

where $d\Omega_{(2)}^2 = d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$ is the line element on the unit 2-sphere, $r_S = 2m$, and $m > 0$ is a mass parameter analogous to the Schwarzschild mass. The line element (4) reduces to the Schwarzschild one when $\epsilon \to 0$. This happens even for small black holes [18], but even more so when macroscopic black holes of, say, stellar mass are considered, due to the mass dependence $\epsilon \sim m^{-2/3}$.

Since r is the areal radius, the possible horizons are located by the roots of $g^{rr} = \nabla^c r \nabla_c r = 0$. There is a unique event horizon in the geometry (4) and it coincides with the Schwarzschild horizon at $r_S = 2m$. Contrary to Schwarzschild, there is an effective energy density outside the horizon, given by [18]

$$\rho = -\frac{\epsilon}{8\pi r^2}\left(\frac{r_S}{r}\right)^{1+\epsilon}, \tag{5}$$

which is negative and purely quantum-mechanical in origin.

In principle, correcting gravity due to geometry quantization or other reasons has implications for massive and massless particles and for fluids surrounding black holes and forming accretion disks around them. There is, therefore, much current interest in using observations of black holes to test deviations from general relativity or possibly detect scalar hair [20–31].

Since particle motion near black hole horizons is relativistic and, in general, complicated, astrophysicists have introduced pseudo-Newtonian potentials to simplify the problem but still provide an effective description (at least for certain purposes) of time-like geodesics. Naturally, particles orbiting black holes in circular orbits are of special interest, and pseudo-Newtonian potentials can provide significant simplifications when studying their motions [32–47]. Despite cheating many of the difficulties, pseudopotentials are still remarkably accurate in determining circular orbits. The phase space of massive test particles in the Schwarzschild geometry is very similar to that derived from the associated pseudo-potential [48]. Indeed, the pseudo-potential is precisely defined so that it preserves the equilibrium points of the relevant dynamical system [32,34]. The Paczynski–Wiita potential [32,34] was the first to be introduced in astrophysics and it locates exactly the innermost stable circular orbit (ISCO) and the marginally bound orbit of Schwarzschild, and it reproduces the Keplerian angular momentum $L(r)$. It is not as accurate in reproducing the Keplerian angular velocity and the radial epicyclic frequency, but it gives approximations that are nevertheless useful in some instances [32,34]. In the following section, we derive the pseudo-Newtonian potential associated with the quantum-corrected black hole (4) and we show that the quantum correction to the Schwarzschild ISCO diverges. This fact prompts us to investigate radial time-like and null geodesics, to find that massive particles are confined by the negative quantum energy density and photons making it to infinity are infinitely redshifted. In Section 3 we show that the Misner-Sharp-Hernandez/Hawking-Hayward quasilocal mass for this geometry vanishes as one approaches spatial infinity. We also report the Kodama quantities defined in spherical symmetry. Section 4 contains the conclusions.

2. Pseudo-Newtonian Potential for the Quantum-Corrected Black Hole

An analogue of the Paczynski–Wiita pseudo-Newtonian potential for the Schwarzschild black hole [32,34] can be introduced for any static and spherically symmetric black hole [49]. This pseudo-Newtonian potential is [49]

$$\Phi(r) = \frac{1}{2}\left(1 + \frac{1}{g_{00}}\right), \tag{6}$$

which in our case becomes

$$\Phi(r) = \frac{1}{2}\left[1 - \left(\frac{r_S}{r}\right)^{2\epsilon} \frac{1}{1 - \left(\frac{r_S}{r}\right)^{1+\epsilon}}\right], \tag{7}$$

that approaches $1/2$ as $r \to \infty$ instead of vanishing.

Using the expansion

$$a^{\alpha\epsilon+\beta} = a^{\beta}\left[1 + \alpha\epsilon \ln a + \mathcal{O}(\epsilon^2)\right], \tag{8}$$

with $a > 0$ and $\alpha, \beta \in \mathbb{R}$, one obtains, to first order,

$$\Phi(r) = -\frac{r_S}{2(r - r_S)}\left[1 + \epsilon \frac{r}{r_S}\frac{(2r - r_S)}{(r - r_S)} \ln\left(\frac{r_S}{r}\right) + \mathcal{O}(\epsilon^2)\right] \tag{9}$$

$$= -\frac{m}{r - 2m}\left[1 + \epsilon \frac{r}{m}\frac{(r - m)}{(r - 2m)} \ln\left(\frac{2m}{r}\right)\right] + \mathcal{O}(\epsilon^2). \tag{10}$$

As a check, one notes that for $\epsilon \to 0$ this pseudo-potential reduces to

$$\Phi_0(r) = -\frac{r_S}{2r(1 - r_S/r)} = -\frac{m}{r - 2m}, \tag{11}$$

which is the well-known Paczynski–Wiita pseudo-Newtonian potential for the Schwarzschild black hole [32,34]. The ϵ-expanded pseudo-potential (10) diverges as $r \to \infty$ instead of vanishing. Already at this stage one notices some problems with the asymptotics. The pseudo-Newtonian potential is defined also for non-asymptotically flat metrics (e.g., (anti-)de Sitter and Schwarzschild-(anti-)de Sitter), and the fact that the ϵ-expanded pseudo-potential diverges signals problems with asymptotic flatness, which are discussed below.

The radii of the circular orbits in the metric (4) are the roots of the equation [34,49]

$$\frac{d\Phi}{dr} = \frac{L^2}{r^3}, \tag{12}$$

where L is the angular momentum per unit mass of the particle on the circular orbit. This equation is what justifies the introduction of the (ϵ-expanded) pseudo-Newtonian potential in the first place [32,34]. Using

$$\frac{d\Phi}{dr} = \frac{r_S}{2\,(r-r_S)^2} + \frac{\epsilon}{2\,(r-r_S)^3}\left[(2r-r_S)\,(r-r_S) + (3r-r_S)\,r_S\,\ln\left(\frac{r_S}{r}\right)\right], \tag{13}$$

the equation locating the circular orbits of massive test particles becomes

$$\frac{r_S}{2\,(r-r_S)^2} + \frac{\epsilon}{2\,(r-r_S)^3}\left[(2r-r_S)\,(r-r_S) + (3r-r_S)\,r_S\,\ln\left(\frac{r_S}{r}\right)\right] = \frac{L^2}{r^3}. \tag{14}$$

To zero order, these circular orbits satisfy

$$\frac{r_S}{r_0} - \frac{2L^2}{r_0^2}\left(1-\frac{r_S}{r_0}\right)^2 = 0, \tag{15}$$

while the radius of a perturbed orbit is $r = r_0 + \delta r_0$ with $|\delta r_0/r_0| = \mathcal{O}(\epsilon)$. Given the smallness of the parameter ϵ quantifying the quantum gravity corrections to the Schwarzschild black hole, a linear expansion is an excellent approximation. By inserting the perturbed circular orbit radius $r_0 + \delta r_0$, expanding Equation (14) to first order, and taking advantage of the zero order Equation (15), one obtains

$$\frac{\delta r_0}{r_0} = \epsilon\, r_0^3\, \frac{(2r_0-r_S)\,(r_0-r_S) + (3r_0-r_S)\,r_S\,\ln(r_S/r_0)}{2\left[3L^2(r_S-r_0)^3 + r_S\,r_0^4\right]}. \tag{16}$$

Using again the zero order Equation (15) to substitute for L^2 yields

$$\frac{\delta r_0}{r_0} = \epsilon\, \frac{(2r_0-r_S)\,(r_0-r_S) + r_S\,(3r_0-r_S)\ln(r_S/r_0)}{r_S(3r_S-r_0)}; \tag{17}$$

the percent correction to the radii of the circular orbits is of first order in the parameter ϵ. The ISCO of the Schwarzschild black hole lies at $r_0 = 6m$, for which the correction diverges. This divergence shows that the ϵ-correction has unwanted large effects no matter how small the parameter ϵ, and is consistent with similar phenomenology found in Ref. [50] for a different quantum-corrected black hole. To gain more insight, let us consider radial time-like and null geoesics.

Begin with a time-like radial geodesic followed by a particle of mass m, 4-velocity u^c, and 4-momentum $p^c = mu^c$: since $t^a \equiv (\partial/\partial t)^a$ is a time-like Killing vector, the energy E of the particle is conserved along the geodesic, $g_{ab}p^a t^b = -E$, yielding

$$\frac{dt}{d\tau} = \frac{\bar{E}}{|g_{00}|}, \tag{18}$$

where $\bar{E} \equiv E/m$ is the particle energy per unit mass and τ is the proper time along the geodesic. Substituting into the normalization $u_c u^c = -1$ gives

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{r_S}{r}\right)^{2\epsilon}\left\{\bar{E}^2 - \left(\frac{r}{r_S}\right)^{2\epsilon}\left[1-\left(\frac{r_S}{r}\right)^{1+\epsilon}\right]\right\}. \tag{19}$$

When the particle is at large distances from the horizon as $r \to +\infty$, one obtains

$$\left(\frac{dr}{d\tau}\right)^2 \rightarrow -1\,, \tag{20}$$

which is absurd. The coordinate velocity of the particle is

$$v \equiv \frac{dr}{dt} = \frac{dr}{d\tau}\frac{d\tau}{dt} = \frac{dr}{d\tau}\frac{\bar{E}}{|g_{00}|}\,, \tag{21}$$

which yields

$$\begin{aligned} v^2 &\equiv \left(\frac{dr}{dt}\right)^2 = \left(\frac{r}{r_S}\right)^{2\epsilon}\left[1-\left(\frac{r_S}{r}\right)^{1+\epsilon}\right]^2\left\{1-\frac{1}{\bar{E}^2}\left(\frac{r}{r_S}\right)^{2\epsilon}\left[1-\left(\frac{r_S}{r}\right)^{1+\epsilon}\right]\right\} \\ &\approx -\frac{1}{\bar{E}^2}\left(\frac{r_S}{r}\right)^{4\epsilon}\,; \end{aligned} \tag{22}$$

also, the coordinate velocity becomes imaginary at sufficiently large radii, while it tends to zero as $r \rightarrow +\infty$. The physical meaning is that the particle cannot be located at $r = \infty$ or at large radii. This surprising fact can be explained as follows: although decaying slightly faster than $1/r^3$, the negative energy density (5) repels a massive particle located at finite r and prevents it from reaching infinity. To make an analogy, consider the (uncorrected) Schwarzschild metric with negative mass parameter: a massive test particle will be repelled from a finite radius and go to infinity, but in the present quantum-corrected geometry the test particle is instead located at a finite radius and is repelled by the effect of the negative quantum energy density far away. While the *local* effect of this negative energy density is negligible, the *accumulated* effect of all the negative mass from this finite radius to infinity "seen" by the particle repels if *from infinity* and keeps it confined.

It is now natural to ask what happens to an outgoing radial photon emitted at a finite radius. Let λ be an affine parameter along radial null geodesics. Then, energy conservation for these photons reads

$$\frac{dt}{d\lambda} = \frac{E}{|g_{00}|} = \left(\frac{r_S}{r}\right)^{2\epsilon}\frac{E}{1-(r_S/r)^{1+\epsilon}}\,, \tag{23}$$

and substitution into the normalization $u_c u^c = 0$ gives

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2\left(\frac{r_S}{r}\right)^{2\epsilon}\,. \tag{24}$$

As $r \rightarrow +\infty$, the photon slows down, $dr/d\lambda \rightarrow 0$ and gets "tired" (i.e., infinitely redshifted), $dt/d\lambda \rightarrow 0$. This behavior of test particles shows that true asymptotic flatness is not achieved and there are lingering effects of the quantum corrections in the geometry. Indeed, these effects become more important as one goes further away from the horizon and "sees" more negative mass coming from the density (5).

Another effect induced by the negative energy density is that it will be impossible to introduce Painlevé-Gullstrand coordinates [51,52] for the line element (4). In fact, these coordinates are associated with observers starting at spatial infinity with zero velocity [53] and they cannot be introduced in regions of negative quasilocal energy [54].

3. Kodama Vector, Misner-Sharp-Hernandez Mass, and Kodama Temperature

The quantum black hole metric (4) is already written in the Abreu-Nielsen-Visser gauge [55–58],

$$ds^2 = -e^{-2\Psi}\left(1-\frac{2M_{\rm MSH}}{r}\right)dt^2 + \frac{dr^2}{1-2M_{\rm MSH}/r} + r^2 d\Omega_{(2)}^2 \tag{25}$$

employing the areal radius as the radial coordinate. Here we have

$$\Psi = \epsilon \ln \left(\frac{r_S}{r} \right) ; \tag{26}$$

this redshift function diverges as $r \to +\infty$ instead of going to zero as in Minkowski space. M_{MSH} is the Misner-Sharp-Hernandez mass of general relativity [59,60], which formally is always defined in a spherically symmetric geometry by

$$1 - \frac{2GM_{\text{MSH}}}{R} = \nabla^c R \nabla_c R = g^{RR} , \tag{27}$$

where R is the areal radius (which is a scalar and is well defined in spherical symmetry by using the area A of the 2-spheres which are orbits of the rotational Killing vector field, $R = \sqrt{A/(4\pi)}$). (The last equality in (27) holds in a coordinate system where R is the radial coordinate, which is our case.) In spherical symmetry, the more general Hawking-Hayward quasilocal energy [61,62] reduces to the Misner-Sharp-Hernandez mass [63].

In our case, the Misner-Sharp-Hernandez mass contained in a sphere of radius r is

$$M_{\text{MSH}}(r) = \frac{r}{2}(1 - g^{rr}) = \frac{r}{2} \left(\frac{r_S}{r} \right)^{1+\epsilon} , \tag{28}$$

that, to the first order in ϵ, reduces to

$$\begin{aligned} M_{\text{MSH}}(r) &= \frac{r_S}{2} \left[1 + \epsilon \ln \left(\frac{r_S}{r} \right) \right] + \mathcal{O}(\epsilon^2) \\ &= m \left[1 + \epsilon \ln \left(\frac{2m}{r} \right) \right] + \mathcal{O}(\epsilon^2) , \end{aligned} \tag{29}$$

where m is the Misner-Sharp-Hernandez mass of the unperturbed general relativity (Schwarzschild) black hole, which coincides with the Schwarzschild mass. In regions near the horizon, the mass M_{MSH} of the quantum-corrected black hole is smaller than that of the original Schwarzschild black hole, which is attributed to the negative quantum energy density (5). Moreover, while the Misner-Sharp-Hernandez mass of Schwarzschild is the same (m) at any radius $r \geq r_S$, the quantum correction introduces a logarithmic dependence of M_{MSH} on the radius at finite values of $r \geq r_S$. Moreover, from Equation (28) one finds that $M_{\text{MSH}}(r) \to 0$ as $r \to +\infty$. This prevents massive particles from reaching infinity and even tires photons, which reach infinity with zero frequency.

The relation between pseudo-Newtonian potential and Misner-Sharp-Hernandez mass is of some interest. The inversion of Equation (28) gives

$$m = \frac{r}{2} \left(\frac{2M_{\text{MSH}}}{r} \right)^{\frac{1}{1+\epsilon}} \tag{30}$$

that, to first order in ϵ, yields

$$m = M_{\text{MSH}} \left[1 - \epsilon \ln \left(\frac{2M_{\text{MSH}}}{r} \right) \right] \tag{31}$$

which, substituted into the expression (10) of the (ϵ-expanded) pseudo-potential, yields

$$\Phi(r) = -\frac{M_{\text{MSH}}}{r - 2M_{\text{MSH}}} \left[1 + \epsilon \frac{r}{M_{\text{MSH}}} \ln \left(\frac{2M_{\text{MSH}}}{r} \right) \right] . \tag{32}$$

For the Schwarzschild black hole, instead, one has [49]

$$\Phi_{Schw}(r) = -\frac{M_{\text{MSH}}}{r - 2M_{\text{MSH}}} ; \tag{33}$$

the difference arises because, for the quantum-corrected black hole, $g_{00}\, g_{11} \neq -1$ (cf. Ref. [49]). Black holes that satisfy the condition $g_{00}\, g_{11} = -1$ (which has attracted some attention early on [64]), including the Schwarzschild geometry, have special geometric properties explored in [65]. The condition $g_{00}\, g_{11} = -1$ characterizes spacetimes in which the (double) projection of the Ricci tensor onto radial null vectors l^a vanishes, $R_{ab} l^a l^b = 0$ [65]. An equivalent characterization is that the restriction of the Ricci tensor to the (t, R) subspace is proportional to the restriction of the metric g_{ab} to this subspace [65]. A third characterization is that the areal radius is an affine parameter along radial null geodesics [65]. These characterizations are valid also in higher-dimensional spacetimes. Solutions of the Einstein equations that satisfy this condition include [65] vacuum, electrovacuum with both Maxwell and non-linear Born–Infeld electrodynamics, and a spherical global monopole called "string hedgehog" [66,67]. Quantum-correcting the Schwarzschild black hole spoils all these geometric properties. This is an indication that $g_{00}\, g_{11} = -1$ is a rather fragile property and will likely be spoiled by all methods to quantum-correct classical metrics that satisfy it (notably, Schwarzschild), even if they are static like the geometry (4) and have the same horizons of the uncorrected counterpart (which is not guaranteed in general since one can expect horizons to be dynamical or to fluctuate).

Let us consider now the Kodama quantities associated with spherical symmetry [68]. The Kodama vector, always defined in spherical symmetry, is given by

$$K^a = \mathrm{e}^{\Psi} \left(\frac{\partial}{\partial t} \right)^a \tag{34}$$

in the Abreu-Nielsen-Visser gauge [55–57], therefore its components are

$$K^\mu = \left(\left(\frac{r_S}{r} \right)^\epsilon, 0, 0, 0 \right) \tag{35}$$

and it is parallel, but not equal, to the time-like Killing vector. The Kodama four-current in this gauge is [55–57]

$$J^\mu = \frac{2\,\mathrm{e}^{\Psi}}{r^2} \left(-\frac{dM_{\mathrm{MSH}}}{dr}, \frac{dM_{\mathrm{MSH}}}{dt}, 0, 0 \right) = \left(\frac{\epsilon}{r^2} \left(\frac{r_S}{r} \right)^{2\epsilon+1}, 0, 0, 0 \right) \simeq \epsilon \left(\frac{r_S}{r^3}, 0, 0, 0 \right). \tag{36}$$

The Kodama temperature at the horizon $r = r_S = 2m$ is proportional to the Kodama surface gravity κ_{Kodama}:

$$\begin{aligned} T_{\mathrm{Kodama}} &= \left. \frac{\kappa_{\mathrm{Kodama}}}{2\pi} \right|_H = \frac{\mathrm{e}^{-\Psi(r_H)}}{2\pi} \left[\frac{1 - 2M'_{\mathrm{MSH}}(r_H)}{2 r_H} \right] \\ &= \frac{1}{2\pi} \left(\frac{1+\epsilon}{2\, r_S} \right) \\ &= \frac{1}{8\pi m} \left(1 + \epsilon \right) . \end{aligned} \tag{37}$$

Since the metric is static, the Kodama temperature coincides with the Killing temperature found in [18] using Euclidean methods associated with periodicity in Euclidean time.

4. Conclusions

We have examined the pseudo-Newtonian potential for the black hole of Refs. [16–18] derived in Loop Quantum Gravity. The simple relation (33) between pseudo-Newtonian potential and this quasilocal mass becomes complicated (albeit only to first order in the quantum corrections) and makes the pseudo-potential stabilize to $1/2$ as $r \to +\infty$. The deviation from the corresponding Schwarzschild ISCO is dramatic: to first order in ϵ, the percent correction to the radius of this orbit diverges, which signals severe deviations from Einstein's theory. Prompted by this fact, we have examined radial time-like and null geodesics, finding that massive test particles in radial motion starting from finite radii never make it to infinity, while outgoing photons arrive to infinity with infinite redshift. We trace these effects to the repulsion of the negative quantum energy density (5). Unlike in Schwarzschild spacetime, the Misner-Sharp-Hernandez/Hawking-Hayward quasilocal mass is position-dependent and vanishes as $r \to +\infty$.

Quantum-correcting the Schwarzschild black holes changes its asymptotics and makes it lose some of its peculiar features in the region outside the horizon. One should recover with an excellent approximation general relativity in the strong gravity region and Minkowskian physics as $r \to +\infty$, but this is not the case. In other words, even extremely small quantum corrections alter radically the physics of Minkowski space far away from the horizon. The problems discussed in previous literature with quantum-correcting black holes [10–13] seems to persist in this Loop Quantum Gravity black hole, even though it was supposed to be free from such problems.

Another problem, although not as important, consists of understanding why the physics deviates so strongly from the Schwarzschild physics near the horizon and from the Minkowski one at spatial infinity. The answer possibly lies in the fact that a proper limit of spacetimes as a parameter (in our case, ϵ) varies should not be based on coordinates, but should be done in an invariant way. Long ago, Geroch warned that the limit of the Schwarzschild geometry as the mass diverges is either the Minkowski space or a Kasner space [69]. A coordinate-independent approach based on the Cartan scalars has been developed in general relativity [70] and then applied to the limit of Brans–Dicke gravity when the Brans–Dicke parameter ω diverges [71]. This aspect will be explored in future work.

Author Contributions: Both authors (V.F., A.G.) contributed in equal measure to the design and implementation of this research. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported by the Natural Sciences and Engineering Research Council of Canada (Grant No. 2016-03803 to V.F.) and by Bishop's University. The work of A.G. has been carried out in the framework of the activities of the Italian National Group for Mathematical Physics (GNFM, INdAM).

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Wald, R.M. *General Relativity*; Chicago University Press: Chicago, IL, USA, 1984.
2. Hayward, S.A. Formation and evaporation of regular black holes. *Phys. Rev. Lett.* **2006**, *96*, 031103. [CrossRef] [PubMed]
3. Frolov, V.P. Notes on nonsingular models of black holes. *Phys. Rev. D* **2016**, *94*, 104056. [CrossRef]
4. De Lorenzo, T.; Pacilio, C.; Rovelli, C.; Speziale, S. On the Effective Metric of a Planck Star. *Gen. Relat. Gravit.* **2015**, *47*, 41. [CrossRef]
5. De Lorenzo, T.; Giusti, A.; Speziale, S. Non-singular rotating black hole with a time delay in the center. *Gen. Relat. Gravit.* **2016**, *48*, 31.
6. Giusti, A. On the corpuscular theory of gravity. *Int. J. Geom. Meth. Mod. Phys.* **2019**, 16, 1930001. [CrossRef]
7. Casadio, R.; Giugno, A.; Giusti, A.; Lenzi, M. Quantum corpuscular corrections to the Newtonian potential. *Phys. Rev. D* **2017**, *96*, 044010. [CrossRef]
8. Casadio, R.; Lenzi M., Micu, O. Bootstrapping Newtonian gravity. *Phys. Rev. D* **2018**, *98*, 104016. [CrossRef]
9. Casadio, R.; Lenzi, M.; Micu, O. Bootstrapped Newtonian stars and black holes. *Eur. Phys. J. C* **2019**, *79*, 894. [CrossRef]
10. Boehmer, C.G.; Vandersloot, K. Loop quantum dynamics of Schwarzschild interior. *Phys. Rev. D* **2007**, *76*, 1004030. [CrossRef]
11. Chiou, D.W. Phenomenological loop quantum geometry of the Schwarzschild black hole. *Phys. Rev. D* **2008**, *78*, 064040. [CrossRef]
12. Corichi, A.; Singh, P. Loop quantum dynamics of Schwarzschild interior revisited. *Class. Quantum Grav.* **2016**, *33*, 055006. [CrossRef]
13. Olmedo, J.; Saini, S.; Singh, P. From black holes to white holes: A quantum gravitational symmetric bounce. *Class. Quantum Grav.* **2017**, *34*, 225011. [CrossRef]

14. Bouhmadi-López, M.; Brahma, S.; Chen, C.-Y.; Chen, P.; Yeom, D.-H. Asymptotic non-flatness of an effective black hole model based on loop quantum gravity. *arXiv* **2020**, arXiv:1902.07874.
15. Khodadi, M.; Allahyari, A.; Vagnozzi, S.; Mota, D.F. Black holes with scalar hair in light of the Event Horizon Telescope. *arXiv* **2020**, arXiv:2005.05992.
16. Ashtekar, A.; Olmedo J.; Singh, P. Quantum Transfiguration of Kruskal Black Holes. *Phys. Rev. Lett.* **2018** *121*, 241301. [CrossRef]
17. Ashtekar, A.; Olmedo, J.; Singh, P. Quantum Extension of the Kruskal Space-time. *Phys. Rev. D* **2018**, *98*, 126003. [CrossRef]
18. Ashtekar, A.; Olmedo, J. Properties of a recent quantum extension of the Kruskal geometry. *arXiv* **2020**, arXiv:2005.02309.
19. Carroll, S.M. *Spacetime and Geometry: An Introduction to General Relativity*; Addison Wesley: San Francisco, CA, USA, 2004.
20. The Event Horizon Telescope Collaboration. First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. *Astrophys. J. Lett.* **2019**, *875*, L1. [CrossRef]
21. Psaltis, D.; Perrodin, D.; Dienes, K.R.; Mocioiu, I. Kerr Black Holes Are Not Unique to General Relativity. *Phys. Rev. Lett.* **2008**, *100*, 091101. *Erratum* **2008**, *100*, 119902. [CrossRef]
22. Jain, B.; Khoury, J. Cosmological tests of gravity. *Ann. Phys.* **2010**, *325*, 1479–1516. [CrossRef]
23. Clifton, T.; Ferreira, P.G.; Padilla, A.; Skordis, C. Modified gravity and cosmology *Phys. Rep.* **2012**, *513*, 1–189. [CrossRef]
24. Berti, E.; Cardoso, V.; Gualtieri, L.; Horbatsch, M.; Sperhake, U. Numerical simulations of single and binary black holes in scalar-tensor theories: Circumventing the no-hair theorem. *Phys. Rev. D* **2013**, *87*, 124020. [CrossRef]
25. Baker, T.; Psaltis, D.; Skordis, C. Linking tests of gravity on all scales: From the strong-field regime to cosmology. *Astrophys. J.* **2015**, *802*, 63. [CrossRef]
26. Berti, E.; Barausse, E.; Cardoso, V.; Gualtieri, L.; Pani, P.; Sperhake, U.; Stein, L.C.; Wex, N.; Yagi, K.; Baker, T.; et al. Testing general relativity with present and future astrophysical observations. *Class. Quantum Grav.* **2015**, *32*, 243001. [CrossRef]
27. Sotiriou, T.P.; Faraoni, V. Black Holes in Scalar-Tensor Gravity. *Phys. Rev. Lett.* **2010**, *108*, 081103. [CrossRef]
28. Horbatsch, M.W.; Burgess, C.P. Cosmic black-hole hair growth and quasar OJ287. *J. Cosmol. Astropart. Phys.* **2012**, *1205*, 010. [CrossRef]
29. Cardoso, V.; Carucci, I.P.; Pani, P.; Sotiriou, T.P. Black Holes with Surrounding Matter in Scalar-Tensor Theories. *Phys. Rev. Lett.* **2013**, *111*, 111101. [CrossRef]
30. Herdeiro, C.A.R.; Radu, E. Kerr Black Holes with Scalar Hair. *Phys. Rev. Lett.* **2014**, *112*, 221101. [CrossRef]
31. Sotiriou, T.P.; Zhou, S.-Y. Black Hole Hair in Generalized Scalar-Tensor Gravity. *Phys. Rev. Lett.* **2014**, *112*, 251102. [CrossRef]
32. Paczynski, B.; Wiita, P.J. Thick Accretion Disks and Supercritical Luminosities. *Astron. Astrophys.* **1980**, *88*, 23–31.
33. Kovár, J.; Stuchlík, Z.; Slaný, P. Pseudo-Newtonian gravitational potential of Schwarzschild black hole in the presence of quintessence. In *Proceedings of RAGtime 10–13, 15–17/20–22/15–17/14–16 September 2008/2009/2010/2011, Opava, Czech Republic*; Stuchlík, Z., Török, G., Pecháček, T., Eds.; Silesian University: Opava, Czech Republic, 2014; pp. 133–141.
34. Abramowicz, M.A. The Paczynski-Wiita potential: A step-by-step "derivation". *Astron. Astrophys.* **2009**, *500*, 213–214. [CrossRef]
35. Abramowicz, M.A. Calvani, M.; Nobili, L. Thick accretion disks with super-Eddington luminosities. *Astrophys. J.* **1980**. *242*, 772. [CrossRef]
36. Nowak, M.A.; Wagoner, R.V. Diskoseismology: Probing Accretion Disks. I. Trapped Adiabatic Oscillations. *Astrophys. J.* **1991**, *378*, 656–664. [CrossRef]
37. Artemova, I.V.; Bjornsson, G.; Novikov, I.D. Modified Newtonian Potentials for the Description of Relativistic Effects in Accretion Disks around Black Holes. *Astrophys. J.* **1996**, *461*, 565. [CrossRef]
38. Semerak, O.; Karas, V. Pseudo-Newtonian models of a rotating black hole field. *Astron. Astrophys.* **1999**, *343*, 325.
39. Kluzniak, W.; Lee, W.H. The swallowing of a quark star by a black hole. *Mon. Not. R. Astron. Soc.* **2002**, *335*, L29. [CrossRef]

40. Mukhopadhyay, B.; Misra, R. Pseudo-Newtonian Potentials to Describe the Temporal Effects on Relativistic Accretion Disks around Rotating Black Holes and Neutron Stars. *Astrophys. J.* **2003**, *582*, 347. [CrossRef]
41. Ivanov, R.I.; Prodanov, E.M. Pseudo-Newtonian potential for charged particle in Kerr–Newman geometry. *Phys. Lett. B* **2005**, *611*, 34–38. [CrossRef]
42. Chakrabarti, S.K.; Mondal, S. Studies of accretion flows around rotating black holes—I. Particle dynamics in a pseudo-Kerr potential. *Mon. Not. R. Astron. Soc.* **2006**, *369*, 976–984. [CrossRef]
43. Ghosh, S.; Mukhopadhyay, B. Generalized Pseudo-Newtonian Potential for Studying Accretion Disk Dynamics in Off-Equatorial Planes around Rotating Black Holes: Description of a Vector Potential. *Astrophys. J.* **2007**, *667*, 367. [CrossRef]
44. Wegg, C. Pseudo-Newtonian Potentials for Nearly Parabolic Orbits. *Astrophys. J.* **2012**, *749*, 183. [CrossRef]
45. Stuchlík, Z, Slaný, P.; Kovár, J. Pseudo-Newtonian and general relativistic barotropic tori in Schwarzschild-de Sitter spacetimes. *Class. Quantum Grav.* **2009**, *26*, 215013. [CrossRef]
46. Stuchlík, Z.; Kovár, J. Pseudo-Newtonian gravitational potential for Schwarzschild-de Sitter spacetimes. *Int. J. Mod. Phys. D* **2008**, *17*, 2089–2105. [CrossRef]
47. Tejeda, E.; Rosswog, S. Generalized Newtonian description of particle motion in spherically symmetric spacetimes. *arXiv* **2014**, arXiv:1402.1171.
48. Witzany, V.; Semerák, O.; Suková, P. Free motion around black holes with discs or rings: Between integrability and chaos—IV. *Mon. Not. R. Astron. Soc.* **2015**, *451*, 1770–1794. [CrossRef]
49. Faraoni, V.; Belknap-Keet, S.D.; Lapierre-Léonard, M. Paczynski-Wiita-like potential for any static spherical black hole in metric theories of gravity, *Phys. Rev. D* **2016**, *93*, 044042. [CrossRef]
50. Casadio, R.; Giusti, A.; Mentrelli, A. Orbits in a stochastic Schwarzschild geometry. *Phys. Rev. D* **2019**, *100*, 02403. [CrossRef]
51. Painlevé, P. La mécanique classique et la théorie de la relativité. *C. R. Acad. Sci. (Paris)* **1921**, *173*, 677–680.
52. Gullstrand, A. Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie. *Arkiv. Mat. Astron. Fys.* **1912**, *16*, 1.
53. Martel, K.; Poisson, E. Regular coordinate systems for Schwarzschild and other spherical spacetimes. *Am. J. Phys.* **2001**, *69*, 476–480. . [CrossRef]
54. Faraoni, V.; Vachon, G. When Painlevé-Gullstrand coordinates fail. *arXiv* **2020**, arXiv:2006.10827.
55. Nielsen, A.B.; Visser, M. Production and decay of evolving horizons. *Class. Quantum Grav.* **2006**, *23*, 4637–4658. [CrossRef]
56. Abreu, G.; Visser, M. Kodama time: Geometrically preferred foliations of spherically symmetric spacetimes. *Phys. Rev. D* **2010**, *82*, 044027. [CrossRef]
57. Faraoni, V. *Cosmological and Black Hole Apparent Horizons*; Lecture Notes in Physics Series vol. 907; Springer: New York, NY, USA, 2015.
58. Giusti, A.; Faraoni, V. Quasilocal mass in scalar-tensor gravity: Spherical symmetry. *arXiv* **2020**, arXiv:2005.04770.
59. Misner, C.W.; Sharp, D.H. Relativistic Equations for Adiabatic, Spherically Symmetric Gravitational Collapse. *Phys. Rev.* **1964**, *136*, B571. [CrossRef]
60. Hernandez, W.C.; Misner, C.W. Observer Time as a Coordinate in Relativistic Spherical Hydrodynamics. *Astrophys. J.* **1966**, *143*, 452. [CrossRef]
61. Hawking, S. Gravitational Radiation in an Expanding Universe. *J. Math. Phys.* **1968**, *9*, 598–604. [CrossRef]
62. Hayward, S.A. Quasilocal gravitational energy. *Phys. Rev. D* **1994**, *49*, 831–839. [CrossRef]
63. Hayward, S.A. Gravitational energy in spherical symmetry. *Phys. Rev. D* **1996**, *53*, 1938–1949. [CrossRef]
64. Bondi, H.; Kilmister, C.W. Note on Schiff's Paper on Relativity. *Am. J. Phys.* **1960**, *28*, 508. [CrossRef]
65. Jacobson, T. When is $g_{tt}g_{rr} = -1$?. *Class. Quantum Grav.* **2007**, *24*, 5717. [CrossRef]
66. Barriola, M.; Vilenkin, A. Gravitational Field of a Global Monopole. *Phys. Rev. Lett.* **1989**, *63*, 341–343. [CrossRef] [PubMed]
67. Guendelman, E.I.; Rabinowitz, A. Gravitational field of a hedgehog and the evolution of vacuum bubbles. *Phys. Rev. D* **1991**, *44*, 3152–3158. [CrossRef]
68. Kodama, H. Conserved Energy Flux for the Spherically Symmetric System and the Backreaction Problem in the Black Hole Evaporation. *Prog. Theor. Phys.* **1980**, *63*, 1217–1228. [CrossRef]
69. Geroch, R. Limits of spacetimes. *Commun. Math. Phys.* **1969**, *13*, 180–193. [CrossRef]

70. Paiva, F.M.; Reboucas, M.; MacCallum, M. On limits of spacetimes—A coordinate-free approach. *Class. Quantum Grav.* **1993**, *10*, 1165. [CrossRef]
71. Paiva, F.M.; Romero, C. The limits of Brans-Dicke spacetimes: A coordinate-free approach. *Gen. Relat. Gravit.* **1993**, *25*, 1305–1317. [CrossRef]