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Energy-Momentum Relocalization, Surface Terms, and Massless Poles in Axial Current Matrix Elements

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Abstract: The energy-momentum relocalization in classical and quantum theory is addressed with specific impact on non-perturbative QCD and hadronic structure. The relocalization is manifested in the existence of canonical and symmetric (Belinfante and Hilbert) energy momentum tensors (EMT). The latter describes the interactions of hadrons with classical gravity and inertia. Canonical EMT, in turn, is naturally emerging due to the translation invariance symmetry and appears when spin structure of hadrons is considered. Its relation to symmetric Hilbert and Belinfante EMTs requires the possibility to neglect the contribution of boundary terms for the classical fields. For the case of quantum fields this property corresponds to the absence of zero-momentum poles of matrix element of the axial current dual to the spin density. This property is satisfied for quarks manifesting the symmetry counterpart of $U_A(1)$ problem and may be violated for gluons due to QCD ghost pole.

Keywords: gravity; relocalization; topology; boundary; poles

1. Introduction

The space-time symmetry related to the energy-momentum and angular momentum conservation is manifested in field theory as the appearance of energy-momentum and spin currents (see [1] and Ref. therein). Their definition is not unique. The structure of Lagrangian immediately defines, after the application of the Noether theorem, the canonical densities. Passing to the quantum operators and their matrix elements one may analyse how the fundamental fields are manifested in the spin structure of elementary particles.

From the other side, the interaction of particles with gravity involves the symmetruc Hilbert tensor resulting from the variation with respect to metric and the symmetry property naturally emerges after the absorption of spin density to the orbital one using the Belinfante procedure [1].

The interplay of both forms of Energy-Momentum Tensor (EMT) is especially important for hadrons, as in the absence of mathematically rigorous confinement theory their spin structure is an important problem of non-perturbative Quantum Chromodynamics (QCD). The same non-perturbative effects are responsible for the most of the visible mass of the Universe, and, therefore, for its gravitational interaction.

These interactions of hadrons with gravity (and inertia, due to equivalence principle) are encoded in their gravitational formfactors [2–6] (see also [7] and Ref. therein). They define the macrospic properties of all objects, and, as it appeared more recently, the responce of hadrons to fastest ever rotation and acceleration emerging in heavy-ion collisions (see [8] and Ref. therein). Indeed, the angular velocity of quark-gluon matter in the non-central heavy-ion collisions corresponds to the change of the velocity of the order of speed of light c at the distance of order of Compton wavelength $l_C, \omega \sim c/l_C$, which is some 25 orders of magnitude larger than angular velocity of Earth rotation. By coincidence, the acceleration a of this matter which is of the order of $a \sim c^2/l_C$ is larger than Earth's gravity g by almost the same factor.

One may wonder, why the highly non-inertial frame formed by quark-gluon matter in heavy-ion collisions can have any impact on the observables measured by the detector located in the laboratory frame. The non-inertial matter will play a role if its interaction with hadrons may be considered as a quantum measurement which is certainly true if particle spin (essentially quantum object!) or Hawking–Unruh radiation [8] are considered. Let us also note here that the main outcome of equivalence principle (EP) for spin motion in the gravitational field, the equality of classical and quantum rotators (orbital and spin angular momenta) precession frequencies (see [7] and Ref. therein), becomes trivial for the rotating frame (like Earth) if spin is considered just as some vector remaining constant [9] in the inertial frame and rotating in the frame of the Earth like Foucault pendulum. The non-trivial meaning would emerge if the quantum measurement of the spin in the rotating frame is considered and its similarity to pendulum and orbital angular momentum (AM) is the manifestation of EP.

The interaction with gravity, as it was already mentioned, is described by the symmetric Hilbert EMT representing (like Belinfante and any symmetric EMT) the angular momentum (AM) as the orbital one. This form of AM allows one to derive the EP as low-energy theorem (see [7] and Ref. therein) by making use of momentum and angular momentum conservation. Like in QED, the global symmetry puts the restrictions for the interaction (defined by local one) for small momenta. In distinction from QED, where only terms of zero order in momenta are fixed, while the linear ones (momenta) are dynamical, in gravity, because of more complicated gauge group, the momenta are also fixed and anomalous gravitomagnetic moment is absent, which is another formulation [2] of EP.

At the same time, in the analysis of hadronic spin structure the canonical expressions naturally appear. The spin of fundamental fields plays the special role in the physical interpretation of QCD hadronic structure. The interplay of various forms of EMT and AM are discussed in detail in the problems of hadronic structure [10] and heavy-ion collisions [11].

The procedure of relocalization [1] changing the local quantities but preserving the conserved (angular) momenta, requires the possibility to discard the surface terms, which is usually assumed. At the same time, their consideration when generalized to the case of quantum operators was earlier found [12–14] to lead to some non-trivial constraints for matrix elements of axial currents. Here we develop these ideas and put them into modern context. As a result, we find the constraints for the zero-mass poles in matrix elements of singlet axial current (dual to quark spin desnity) leading to an amazing interplay between general symmetry properties of relocalization and very specific topologocal QCD dynamics.

2. Boundary Terms in Coordinate and Momentum Space

Let us start with the following expression for the quark–gluon angular momentum density

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{5,\sigma} + x^{\nu}T^{\mu\rho} - x^{\rho}T^{\mu\nu}. \quad (1)$$

The first term in the right hand side (r.h.s.) is just the canonical quark spin tensor dual to singlet axial current. Note that the energy–momentum tensor here accumulates also the quark orbital momentum as well as the total gluon angular momentum. We may proceed further along this way and express the quark spin in the orbital form with the simultaneous change of the energy–momentum tensor to the one suggested by Belinfante long ago

$$M_B^{\mu,\nu\rho} = x^{\nu}T_B^{\mu\rho} - x^{\rho}T_B^{\mu\nu}. \quad (2)$$

As the conservation of the angular momentum

$$\partial_\mu M_B^{\mu,\nu\rho} = 0 \quad (3)$$

immediately leads to the symmetry of $T^{\mu\rho}$ (so that symmetric Hilbert tensor may also be considered in such a role), the latter implies that

$$\epsilon_{\mu\nu\rho\alpha} M_B^{\mu,\nu\rho} = 0. \tag{4}$$

One might conclude that the totally antisymmetric quark spin tensor is somehow cancelled and does not contribute to the total angular momentum [15]. This is also the manifestation of the general belief that axial current and angular momentum represent the different aspects of spin structure. Still, it appears possible to extract some quantitative information about their interplay.

The Belinfante (and Hilbert) EMT lead to the same AM as canonical one so that:

$$\int d^3x M_B^{0,\nu\rho} = \int d^3x M^{0,\nu\rho}. \tag{5}$$

We assume (which was done only tacitly in [12]) that also the stronger condition is valid

$$\int d^3x M_B^{\mu,\nu\rho} = \int d^3x M^{\mu,\nu\rho}, \tag{6}$$

so that

$$\epsilon_{\mu\nu\rho\alpha} \int d^3x M^{\mu,\nu\rho} = 0. \tag{7}$$

Substituting here the definition (1) one get

$$\int d^3x (3 J_5^\alpha(x) + 2\epsilon^{\mu\nu\rho\alpha} x_\nu T_{A.\mu\alpha}(x)) = 0, \tag{8}$$

where $T_A^{\mu\alpha} = (T^{\mu\alpha} - T^{\alpha\mu})/2$ is the antisymmetric part of energy momentum tensor responsible for the separate non-conservation of orbital and spin AM. The conservation of total AM results in the relation

$$\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\partial_\mu J_\sigma^5 = T_A^{\rho\nu}. \tag{9}$$

Note that we consider the weak gravitational fields and the derivatives, as well as quantum states in what follows, correspond to flat space. The consideration of strong fields may be achieved by applying the Dirac equation in curved space (see [7] and Ref. therein).

Set of Equations (8) and (9) allows one to exclude either spin (J_5) or orbital (T_A) AM. The latter is easier, as one can use the local Equation (9). Furthermore, axial current operator is related to many observables.

Excluding EMT antisymmetric part by making use of the conservation of AM density (9), one get in the case of classical fields:

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \int d^3x \partial^\rho (J_5^\alpha x^\nu) = 0. \tag{10}$$

This is in fact the way to represent the surface terms whose neglect is necessary to apply the Belinfante procedure.

Passing to the most interesting case of quantum operators one should switch the Equations (8) and (after incorporating the AM conservation at operator level) (10) between particle (nucleon) states with the momenta P and $P+q$.

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \int d^3x \langle P|\partial^\rho (J_5^\alpha x^\nu)|P+q\rangle = 0. \tag{11}$$

Expressing the local operator $J_5^\alpha(x) = exp(i\hat{P}x) J_5^\alpha(0) exp(-i\hat{P}x)$ by action of shift operator $exp(i\hat{P}x)$ allows one to perform the integration resulting in appearance of $\delta(\vec{q})$. Now $\frac{\partial}{\partial x^\mu}$ is substituted by $-iq_\mu$ and x^μ by $i\frac{\partial}{q_\mu}$ acting on that $\delta^3(\vec{q})$. The latter is, by definition, equal, up to a sign, to the derivative acting on the matrix element. As a result, one obtains the following constraint:

$$q^\mu \frac{\partial}{\partial q^\alpha}|_0\langle P|J_5^\alpha|P+q\rangle = q^\alpha \frac{\partial}{\partial q^\alpha}|_0\langle P|J_5^\mu|P+q\rangle. \quad (12)$$

It is a quantum counterpart of (10) and it is natural that surface terms in coordinate space correspond to zero momenta. To make it more clear, let us multiply both sides by q_μ:

$$q^2 \frac{\partial}{\partial q^\alpha}\langle P|J_5^\alpha|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1)q_\gamma\langle P|J_5^\gamma|P+q\rangle. \quad (13)$$

This equality is obviously valid up to the second and higher powers of q. Note that the differential operator in the r.h.s. subtracts the terms linear in q from the divergence matrix element proportional to sq for the pure kinematical reasons.

What can be dangerous is the pole for $q^2 \to 0$ which naturally appears for anomalous axial current already in perturbation theory for massless fermions [16]. For massless quarks, due to t'Hooft consistency principle, in the case of non-singlet currents these poles correspond to the exchange of the massless Goldstone mesons.

The exception is provided by singlet channel where η' remains massive manifesting the famous $U_A(1)$ problem and the correspondent pole is absent (see, e.g., [17] and Ref. therein):

$$\langle P,S|J_{5,\mu}(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu(Sq)G_2, \quad (14)$$

$$q^2 G_2|_0 = 0. \quad (15)$$

The G_2 pole term, if present, would provide a contribution linear in q to the l.h.s.

Therefore, solution of $U_A(1)$ problem provides simulatenously the necessary dynamical mechanism for relocalization of massless quarks spin. This, in turn, leads to relation of conservation laws and canonical EMT with Belinfante and Hilbert EMT, supporting the emergence of equivalence principle as low-energy theorem [7].

3. Problems with Relocalization for Gluons

The situation is changed in the case of gluons. The relevant matrix element of topological current

$$\langle P,S|K_\mu^5(0)|P+q,S\rangle = 2MS_\mu\tilde{G}_1(q^2) + q_\mu(Sq)\tilde{G}_2(q^2), \quad (16)$$

$$q^2\tilde{G}_2(q^2)|_0 \neq 0, \quad (17)$$

contains the contribution $\tilde{G}_2$ of the relevant Kogut–Susskind ghost (or instanton [18]) pole [19] which is fully responsible [20] for the value of the forward matrix element of anomaly-free quark gluon current $J_5^\mu - K^\mu$.

The consideration of topological current as dual to spin is naturally supported by the studies of bosonic anomalies in gravitational field [21] which may be relevant also for consideration of rotating quark-gluon matter in heavy-ion collisions [22]. Therefore one has a contradiction between kinematics of Relocalization Invariance (RI) requiring the absence of surface terms (corresponding to zero-mass poles of matrix elements) and instanton-type dynamics requiring their presence. The possible outcomes are the following

(i) If RI is indeed violated the coupling of nucleons to gravity (described by the formfactors of Belinfante EMT [13,23]) may be unconstrained by the form of conservation laws in terms of canonical EMT. In extreme case, assuming that just the canocnical form is related to translational invariance, this might result in the violation of Equivalence Principle for nucleons at several percent level which may be tested experimentally and is probably already excluded by the data.

(ii) One may assume "Hadronic censorship" leading to the absence of the ghost pole: in this case the matrrix element

$$< P,S|J_5^\mu - K^\mu|P,S >= 0$$

in the chiral limit. Bearing in mind the smallness of gluon spin one should mostly attribute the quark spin to the (predominantly strange) quark mass. This may explain the relative smallness of quark spin ("Spin Crisis") and may be checked, say, by lattice calculations of pseudoscalar quark densities.

(iii) The simplest solution would be the impossibility to separate spin and orbital momenta of gluons in the meaningful way.

4. Discussion

We found the relation between general space-time symmetry responsible for interactions of hadrons with gravity and the specific QCD dynamics. As a result, the quark spin relocalization is supported by solution of $U_A(1)$ problem and the same non-perturbative dynamics may spoil the extraction of totally antisymmetric gluon spin density.

The future studies, besides the exploration of mentioned in the previous section alternatives may include following developments:

(i) investigation of boundary terms in hydrodynamic approximation;

(ii) exploration of the role of boundary terms (spoiling the transition of spin to orbital AM) for twisted states, which might be obtained also at high energies (see [24] and Ref. therein) and provide the complementary description of Transverse Momentum Dependent parton correlators.

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