Article

Stability Analysis of Tunnel Face Reinforced with Longitudinal Fiberglass Dowels Together with Steel Pipe Umbrella

Kaihang Han 1,2,*, Xuetao Wang 1,2, Xuetao Wang 1,2,*, Beibei Hou 3, Cheng-yong Cao 1,2 and Xing-Tao Lin 1,2

1 College of Civil and Transportation Engineering, Shenzhen University, Shenzhen 518060, China; hankaihang@szu.edu.cn (K.H.); cy-cao@szu.edu.cn (C.-y.C.); xtlin@szu.edu.cn (X.-T.L.)
2 Underground Polis Academy, Shenzhen University, Shenzhen 518060, China
3 China Jingye Engineering Company Limited, Beijing 100088, China; houbeibei0311@hotmail.com
* Correspondence: xuetao.wang@szu.edu.cn

Received: 23 November 2020; Accepted: 11 December 2020; Published: 13 December 2020

Abstract: When tunnels are constructed under difficult geotechnical conditions in urban areas, tunnel face stability is one of the main issues to be addressed. To ensure tunnel face stability and reduce the impact of tunneling on adjacent structures, a few alternative procedures of ground reinforcement should be adopted, which includes reinforcing the soil ahead of the face using longitudinal fiberglass dowels alone or together with a steel pipe umbrella. It is of great academic value and engineering signification to reasonably determine the limit reinforcement density of these ground reinforcements. In this paper, an analytical prediction model is proposed by using the limit analysis method to analyze the tunnel face stability, and the favorable effects of longitudinal fiberglass dowels and steel pipe umbrella on tunnel face stability are investigated quantitatively. The analytical prediction model consists of a wedge ahead of the tunnel face, distributed force acting on the wedge exerted by overlying ground, and the support forces stem from the longitudinal fiberglass dowels. Moreover, sensitivity analysis is conducted to study the effect of the depth of cover, the tunnel shape, the reinforcement installation interval and the reduction factor on the required limit reinforcement density.

Keywords: face stability; shallow tunneling method; limit analysis; cohesive–frictional soils; reduction effect; reinforcement density

1. Introduction

The stability of the tunnel face is one of the main problems to be solved when the tunnel passes through complicated geological conditions [1,2]. The shallow tunneling method is mainly applied to urban subways, municipal underground pipe networks and other shallow-buried underground structures. This method is mostly used in quaternary soft strata, and the excavation methods include the positive step method, single side wall method, middle wall method (also known as CD method and CRD method), double side wall method (eyeglasses method), etc. The shallow tunneling method has the advantages of flexibility; little influence on the ground building, road and underground pipe network; less land for demolition; no disturbance to nearby communities; no pollution to the urban environment, and so on. In order to ensure the face stability of the tunnels constructed using the shallow tunneling method, some flexible measures should be taken to improve the stability of the tunnel face under poor ground conditions. The commonly used pre-reinforcement techniques for tunnel construction are shown in Table 1.
Table 1. Classification of pre-reinforcement techniques for tunnel construction.

<table>
<thead>
<tr>
<th>Pre-Reinforcement Technology</th>
<th>Construction Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stable Arch</td>
</tr>
<tr>
<td>Advance bolt</td>
<td>✓</td>
</tr>
<tr>
<td>Pipe shed</td>
<td>✓</td>
</tr>
<tr>
<td>Horizontal rotary jet pile</td>
<td>✓</td>
</tr>
<tr>
<td>Core soil reserved for annular excavation</td>
<td>✓</td>
</tr>
<tr>
<td>Shotcrete on excavated surface</td>
<td>✓</td>
</tr>
<tr>
<td>Anchor of excavated surface</td>
<td>✓</td>
</tr>
<tr>
<td>Grouting of excavated surfaces</td>
<td>✓</td>
</tr>
<tr>
<td>Anchor bolt reinforcement</td>
<td>✓</td>
</tr>
<tr>
<td>Reinforcement of locked pile</td>
<td>✓</td>
</tr>
<tr>
<td>Grouting reinforcement of arch foot</td>
<td>✓</td>
</tr>
<tr>
<td>Temporary inverted arch</td>
<td>✓</td>
</tr>
<tr>
<td>Drainage measures</td>
<td>✓</td>
</tr>
<tr>
<td>Surface drainage</td>
<td>✓</td>
</tr>
<tr>
<td>Drainage</td>
<td>✓</td>
</tr>
<tr>
<td>Waterproofing measures</td>
<td>✓</td>
</tr>
<tr>
<td>Grouting</td>
<td>✓</td>
</tr>
<tr>
<td>Freeze</td>
<td>✓</td>
</tr>
<tr>
<td>Contact grouting</td>
<td>✓</td>
</tr>
<tr>
<td>Full section grouting</td>
<td>✓</td>
</tr>
<tr>
<td>Joint grouting</td>
<td>✓</td>
</tr>
<tr>
<td>Surface pre-grouting</td>
<td>✓</td>
</tr>
</tbody>
</table>

Numerical, experimental and theoretical analytical methods have been developed to study the stability of tunnel faces reinforced with auxiliary methods. Based on the centrifugal test, it was found that the extrusion displacement of soil on the excavation surface decreases when the bolt is arranged in the outer area of the excavation surface, and the optimal bolt arrangement length is approximately twice the length from the excavation surface to the fracture surface [3]. A series of scale model tests and three-dimensional finite element simulations were carried out to study the constraints of reinforcement parameters such as density, length and stiffness on excavation face deformation. The results indicated that when the reinforcement element is installed, the reinforcement effect of the excavation face is obvious, and there is a critical value to exert the maximum effect of the reinforcement [4,5]. Centrifugal test equipment and discrete element simulation were used to investigate the influence of typical auxiliary bolt (front bolt, vertical pre-reinforcement bolt and advance support bolt) on the stability of the excavation face. The results showed that when the length exceeds 0.5 $D$, the front bolt will play a beneficial role, and the front bolt placed on the upper section of the tunnel is more effective than the lower section [6]. The effectiveness of the pre-reinforcement of the excavation face of deep buried tunnel was studied by numerical simulation methods, and the results were compared with the measured data [7]. The stability of the tunnel excavation face under the pipe umbrella was studied, and the support pressure needed to ensure the stability of the tunnel excavation face was deduced by upper limit theory and limit equilibrium theory. The results showed that under the condition of no water, the effect of multi-stage grouting on the supporting force needed to ensure the stability of the excavation face is not obvious. When seepage is considered, the steel pipe umbrella has a relatively strong influence on underwater tunnels. [8]. A series of centrifugal tests were carried out on the
excavation robot to study the effect of tube shed support and excavation method (full section excavation or ring excavation with reserved core soil) on the displacement above the excavation face of the tunnel. The model test showed that the maximum settlement of the full section of the excavated strata under the action of the pipe shed is one quarter of that without the action of the pipe umbrella [9]. A series of large-scale model tests are carried out to study the reinforcement mechanism of the tunnel roof tube shed [10]. The supporting system structure of the pipe shed is analyzed, and the bending moment and shear force of the pipe shed are analyzed using finite element software. Moreover, they compared the results with general commercial software to verify the reliability of the proposed finite element calculation [11]. An analytical study on the stability of the tunnel excavation face strengthened by the auxiliary method includes an equivalent homogenized material concept [12–14] and the support forces exerted by the individual bolts [15–19].

In urban subway tunnel excavation, retaining core soil is widely used to partially realize the retaining wall effect, which can control the stability of the tunnel face under the general stratum conditions. However, in special stratum conditions, such as large water inflow and sand layers with relatively small water barrier thickness, the core soil of the tunnel face is often difficult to retain, and soil collapse occurs frequently. Subsequent measures to deal with the collapse, such as grouting small pipes, have become the conventional means to maintain normal excavation of the working face of the tunnel under such stratum conditions. This cannot be said to be a passive face of soil pre-reinforcement. A large number of engineering practices have also shown that the use of longitudinal fiberglass dowels in the tunnel face is an active and effective method to pre-reinforce the front soil (c.f., Figure 1). It is of great academic value and engineering significance to reasonably determine the limit reinforcement density of these ground reinforcements. In this paper, based on the limit analysis method, an analytical prediction model is proposed to analyze the favorable effects of longitudinal fiberglass dowels and steel pipe umbrella on face stability. Moreover, sensitivity analysis is carried out to study the effect of the depth of cover, the tunnel shape, the reinforcement installation interval and the reduction factor on the required reinforcement density.

![Figure 1](image_url). Face failure reinforced with longitudinal fiberglass dowels together with steel pipe umbrella.

2. Stability Analysis of the Tunnel Face Reinforced with Longitudinal Fiberglass Dowels Together with Steel Pipe Umbrella

2.1. The New Analytical Prediction Model

In this paper, a new analytical prediction model is proposed to investigate the stability of a tunnel face reinforced with auxiliary methods, as shown in Figure 2a. The tunnel is a rigid cylinder with a diameter of \( D \), an underground cover depth of \( C \) and a surcharge applied on the ground surface of \( \sigma_s \). The analytical prediction model consists of a wedge ahead of the tunnel face, distributed force acting on the wedge exerted by overlying ground, and the support forces stem from the longitudinal fiberglass dowels. On the one hand, the advanced pre-reinforcement structure of the steel pipe umbrella is deemed the beam on the Winkler elastic foundation, which shows that the existence of the steel pipe umbrella effectively reduces the vertical pressure exerted by overlying ground. On the other
hand, the strengthening effect of the longitudinal fiberglass dowels depends primarily on the tensile bearing capacity of the bolt or the bond strength of the ground–bolt interface. The ground extrusion in front of the tunnel face is effectively reduced for the installation of longitudinal fiberglass dowels. The mechanical force analysis of the new analytical prediction model is shown in Figure 2b.

Figure 2. The new failure mechanism. (a) Concept map; (b) Mechanical force analysis.

2.2. Power of the External Loads $P_e$

2.2.1. Power of the Soil Unit Weight

The power of self-weight of the wedge body is as follows:

$$P_w = G[V \cos(\varphi + \beta)] = \left(\frac{1}{2}H^2 \tan \beta \gamma \right)[V \cos(\varphi + \beta)] = \frac{1}{2}H^2 \tan \beta \cos(\varphi + \beta)V$$  \hspace{1cm} (1)

where $G$ is the weight of the wedge, $V$ is the velocity of the wedge, $B$ is the width of the wedge, $H$ is the height of the wedge, $\varphi$ is the angle of internal friction of the ground, and $\beta$ is the inclination of the slope and the horizontal plane, as rendered in Figure 2b.

2.2.2. Power Induced by the Friction on Both Sides

The power of the friction on both sides is as follows:

$$P_{T_s} = 2 \int_0^H \left[c + \left((H - z)\gamma + \tilde{\sigma}_s \tan \varphi \right) z \tan \beta \right] V \cos \varphi \, dz = -\left(c + K \tan \varphi \frac{2z_0 + \gamma H}{3}\right) H^2 \tan \beta \cos \varphi V$$  \hspace{1cm} (2)
where \( c \) is the cohesion of the ground, \( z \) is the vertical coordinate, and \( K \) is the lateral pressure coefficient.

2.2.3. Power Induced by the Vertical Stress on the Sliding Wedge Body

Basic ideas and assumptions of the mechanical model for advanced small pipes are as follows:

1) Both the beam theory and the theory of beam on Winkler elastic foundation are adopted to investigate the mechanical behavior and characteristics of advanced small pipes in tunnel construction. Beam theory is used to analyze the advanced small pipes that are embedded in soil in the front of the tunnel face, whereas the theory of beam on Winkler elastic foundation is used to analyze the advanced small pipes behind the tunnel face, as rendered in Figure 3.

2) It is assumed that the fixed end \( A \) has certain vertical displacement \( y_0 \), which is a known value and is considered as the measured vault subsidence value.

3) The length of the advanced small pipes consists of two parts, which includes unsupported span (length in \( 1.5a \), includes excavation footage \( 1.0a \) and the length \( 0.5a \) due to support delay effect) and the length of the wedge (length in \( l \), as depicted in Figure 3). The symbol \( a \) denotes the excavation footage—that is, the length of the tunnel for each excavation.

4) In order to simplify the analysis, the horizontal projection length of advanced small pipes is considered.

The load \( q \) (uniform distribution) above the tunnel face will entirely act on the collapse slope surface of the tunnel face within length \( l \). According to the analysis above, the stability analysis models of the upper-bound solution of the tunnel face with the following conditions are established, as follows:

1) The remaining length \( l_e \) of the pipe in soil is longer than the length \( l \) of the wedge (Type I). The stability analysis model I is shown in Figure 3a. The subgrade reaction force that acts on the wedge is \( p \) (triangular distribution).

2) The remaining length \( l_e \) of the pipe in soil is shorter than the length \( l \) of the wedge (Type II). The stability analysis model II is shown in Figure 3b. The load that acts on the wedge can be divided into two parts, one is the subgrade reaction force \( p \) (trapezium distribution) along the pipe and the other is the uniform load \( q \) that acts on the wedge.

\[ y_0 = \begin{cases} 1 & \text{if } A \text{ is Type I}, \\ 0 & \text{if } A \text{ is Type II}. \end{cases} \]

Figure 3. Mechanical model of the steel pipe umbrella. (a) Type I; (b) Type II.

1) For Type I, the control differential equations of the reinforced foundation beam for different segments are obtained as follows:

\[
\begin{align*}
AO: & \quad \frac{d^2 y}{dx^2} = \frac{h_b(x)}{EI}, \\
OB: & \quad \frac{d^4 y}{dx^4} + 4\lambda^4 y = \frac{h_b(x)}{EI}, \\
BC: & \quad \frac{d^4 y}{dx^4} + 4\lambda^4 y = 0 
\end{align*}
\]

where \( y \) is the deflection of the beam, \( \lambda \) is the characteristic coefficient of the beam, \( E \) is the elasticity modulus of the beam material, and \( I \) is the moment of inertia of the beam section.
The deflection equations are obtained as follows:

\[
\begin{aligned}
    AO : y_1 &= qbx^4 / (24EI) + C_1 x^3 + C_2 x^2 + C_3 x + C_4 \\
    OB : y_2 &= y_1 + y_3 \\
    BC : y_3 &= e^{\lambda x} (C_5 \cos \lambda x + C_6 \sin \lambda x) + e^{-\lambda x} (C_7 \cos \lambda x + C_8 \sin \lambda x) + q/K
\end{aligned}
\]

where \(y_1, y_2\) and \(y_3\) denote the deflections of the beams \(AO, OB\) and \(BC\), respectively; \(C_1, C_2, C_3, C_4, C_5, C_6, C_7\) and \(C_8\) present the undetermined coefficients of differential equations; \(y_1\) is a particular solution to a differential equation for beams \(OB\).

Boundary conditions are as follows:

\[
\begin{aligned}
    y_3 |_{x=0} &= y_0, \quad \theta_1 |_{x=0} = \theta_2 |_{x=0} = 0 \\
    y_1 |_{x=0} &= y_0, \quad \theta_1 |_{x=0} = \theta_2 |_{x=0} = 0
\end{aligned}
\]

where \(\theta, M\) and \(Q\) denote the angle of rotation, bending moment and shear force of the beams, respectively.

According to the above boundary conditions, the following equations are obtained to calculate the unknown parameters in Equation (4):

\[
\begin{bmatrix}
    A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\
    A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\
    A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\
    A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\
    A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\
    A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66}
\end{bmatrix}
\begin{bmatrix}
    C_1 \\
    C_2 \\
    C_3 \\
    C_4 \\
    C_5 \\
    C_6
\end{bmatrix}
= \begin{bmatrix}
    B_1 \\
    B_2 \\
    B_3 \\
    B_4 \\
    B_5 \\
    B_6
\end{bmatrix}
\]

where

\[
\begin{aligned}
    A_{11} &= 27a^3, A_{12} = -18a^2, A_{13} = 12a, A_{14} = -8A_{15} = 0, A_{16} = 0, B_1 = \frac{27a^4}{24EI} - 8y_0, \\
    A_{21} &= 27a^2, A_{22} = -12a, A_{23} = 4A_{24} = 0, A_{25} = 0, A_{26} = 0, B_2 = \frac{27a^3}{24EI}, \\
    A_{31} &= 0, A_{32} = 0, A_{33} = A_{34} = 1, A_{35} = -1, A_{36} = 0, B_3 = \frac{1}{2}[\cosh(\lambda l) \cos(\lambda l)], \\
    A_{41} &= 0, A_{42} = 0, A_{43} = 1, A_{44} = 0, A_{45} = -1, A_{46} = \frac{1}{2}[\sinh(\lambda l) \cos(\lambda l) - \cosh(\lambda l) \sin(\lambda l)], \\
    A_{51} &= 0, A_{52} = 1, A_{53} = 0, A_{54} = 0, A_{55} = 0, A_{56} = \lambda l, B_5 = \frac{1}{2}[\sinh(\lambda l) \sin(\lambda l)], \\
    A_{61} &= 3, A_{62} = 0, A_{63} = 0, A_{64} = 0, A_{65} = \lambda l, A_{66} = \lambda l^2, B_6 = \frac{1}{2}[\sinh(\lambda l) \cos(\lambda l) + \cosh(\lambda l) \sin(\lambda l)].
\end{aligned}
\]

\[ (2) \] For Type II, the control differential equations of the reinforced foundation beam for different segments are obtained as follows:

\[
\begin{aligned}
    AO : \frac{d^4 y}{dx^4} &= \frac{b q(x)}{EI} \\
    OB : \frac{d^4 y}{dx^4} + 4\lambda^4 y &= \frac{b q(x)}{EI}
\end{aligned}
\]

The deflection equations are obtained as follows:

\[
\begin{aligned}
    AO : y_1 &= qbx^4 / (24EI) + C_1 x^3 + C_2 x^2 + C_3 x + C_4 \\
    OB : y_2 &= e^{\lambda x} (C_5 \cos \lambda x + C_6 \sin \lambda x) + e^{-\lambda x} (C_7 \cos \lambda x + C_8 \sin \lambda x) + q/K
\end{aligned}
\]
Boundary conditions are as follows:

\[
\begin{align*}
    y_1\big|_{x=-\frac{h_0}{2}} &= y_0, \quad \theta_1\big|_{x=-\frac{h_0}{2}} = 0 \\
    y_1\big|_{x=0} &= y_2, \quad \theta_1\big|_{x=0} = \theta_2 \big|_{x=0} \\
    M_1\big|_{x=l_o} &= M_2 \big|_{x=l_o}, \quad Q_1\big|_{x=l_o} = Q_2 \big|_{x=l_o} \\
    M_2\big|_{x=l_o} &= y_0, \quad Q_2\big|_{x=l_o} = 0
\end{align*}
\]  

(10)

According to the above boundary conditions, the following equations are obtained to calculate the unknown parameters in Equation (9):

\[
\begin{bmatrix}
    A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\
    A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\
    A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\
    A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\
    A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\
    A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} \\
    A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} \\
    A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88}
\end{bmatrix}
\begin{bmatrix}
    C_1 \\
    C_2 \\
    C_3 \\
    C_4 \\
    C_5 \\
    C_6 \\
    C_7 \\
    C_8
\end{bmatrix}
= \begin{bmatrix}
    B_1 \\
    B_2 \\
    B_3 \\
    B_4 \\
    B_5 \\
    B_6 \\
    B_7 \\
    B_8
\end{bmatrix},
\]  

(11)

where

\[
\begin{align*}
    A_{11} &= 27\alpha_0^2, \quad A_{12} = -18\alpha_0^2, \quad A_{13} = 12\alpha_0^2, \quad A_{14} = -8\alpha_0^2, \quad A_{15} = 4\alpha_0^2, \quad A_{16} = 2\alpha_0^2, \quad A_{17} = 0, \quad A_{18} = 0, \quad B_1 = \frac{27\alpha_0^4}{164} - 8\xi_0, \\
    A_{21} &= 27\alpha_0^2, \quad A_{22} = -12\alpha_0^2, \quad A_{23} = 6\alpha_0^2, \quad A_{24} = 2\alpha_0^2, \quad A_{25} = 0, \quad A_{26} = 0, \quad A_{27} = 0, \quad A_{28} = 0, \quad B_2 = \frac{27\alpha_0^4}{164}, \\
    A_{31} &= 0, \quad A_{32} = 0, \quad A_{33} = 0, \quad A_{34} = 0, \quad A_{35} = -1, \quad A_{36} = 1, t_A = 0, \quad A_{37} = 1, \quad A_{38} = -1, \quad B_3 = \frac{\xi}{12}, \\
    A_{41} &= 0, \quad A_{42} = 0, \quad A_{43} = 0, \quad A_{44} = 0, \quad A_{45} = -1, \quad A_{46} = 1, \quad A_{47} = 1, \quad A_{48} = -1, \quad B_4 = 0, \\
    A_{51} &= 0, \quad A_{52} = 0, \quad A_{53} = 0, \quad A_{54} = 0, \quad A_{55} = -1, \quad A_{56} = -1, \quad A_{57} = 0, \quad A_{58} = 0, \quad B_5 = 0, \\
    A_{61} &= 0, \quad A_{62} = 0, \quad A_{63} = 0, \quad A_{64} = -1, \quad A_{65} = -1, \quad A_{66} = -1, \quad A_{67} = -1, \quad A_{68} = 0, \\
    A_{71} &= 0, \quad A_{72} = 0, \quad A_{73} = 0, \quad A_{74} = 0, \quad A_{75} = -\xi t_2, \quad A_{76} = \xi t_2 \sin(\lambda l_e), \quad A_{77} = -\xi t_2 \cos(\lambda l_e), \quad A_{78} = -\xi t_2 \sin(\lambda l_e), \\
    A_{81} &= -\xi t_2 \cos(\lambda l_e), \quad A_{82} = 0, \quad A_{83} = 0, \quad A_{84} = 0, \quad A_{85} = 0, \quad A_{86} = 0, \quad A_{87} = 0, \quad A_{88} = 0.
\end{align*}
\]  

(12)

The average ground reaction forces are calculated as follows:

\[
\sigma_0 = \frac{\int_{0}^{l_1} f_0 \min[l, l_1] p(x) dx}{\min[l, l_1]} = \frac{\int_{l_2}^{l_1} y_0 2 \min[l, l_1] dx}{\min[l, l_1]} \leq \sigma_{\theta},
\]  

(13)

Power of the vertical stress on the sliding wedge body is calculated as follows:

\[
P_{\theta} = (\sigma_{\theta} H B \tan \beta) |\cos(\varphi + \beta)| V,
\]  

(14)

2.2.4. Power Induced by the Longitudinal Fiberglass Dowels

According to the work by Anagnostou and Perazzelli [17], the reinforcement effect of longitudinal fiberglass dowels depends on the tensile axial force, and there are three distributions of support pressure induced by the longitudinal fiberglass dowels for different wedge angles $\beta$, as shown in Figure 4.
Based on these assumptions, the tensile axial force $s$ induced by the longitudinal fiberglass dowels is as follows:

$$
\begin{align*}
\beta & \leq \beta_1, \\
\beta_1 & \leq \beta \leq \beta_2, \quad s(z) = \begin{cases} 
    s_2(z) = n\tau_m z \tan \beta, & (z_1 \leq z \leq H) \\
    s_3(z) = 0, & (z_2 \leq z \leq H) \\
    s_1(z), & (0 \leq z \leq z_1) \\
    s_2(z), & (z_1 \leq z \leq z_2) \\
    s_1(z), & (0 \leq z \leq z_1)
\end{cases},
\end{align*}
$$

(15)

where $n$ is the reinforcement density of the longitudinal fiberglass dowels, $d$ is the diameter of the longitudinal fiberglass dowels, and $\tau_m$ is the bond strength of the soil–grout interface of the longitudinal fiberglass dowels.

\[
\begin{align*}
\beta_1 &= \arctan\left(\frac{L'}{2H}\right), \\
\beta_2 &= \arctan\left(\frac{L'}{H}\right), \\
z_1 &= \frac{0.5L'}{\tan \beta}, \\
z_2 &= \frac{L'}{\tan \beta} = 2z_1,
\end{align*}
\]

(16)

Power of the longitudinal fiberglass dowels reads for different conditions as follows:

\[
\begin{align*}
P_s &= -s_1(H)H/2BV = -n\tau_m \tan \beta \frac{H^2}{2}BV, \quad [\beta \leq \beta_1], \\
\end{align*}
\]

(17)

\[
\begin{align*}
P_s &= -s_2(z) \frac{z^2 + 3z_1 - H(H-z_1)}{2}BV, \quad [\beta_1 \leq \beta \leq \beta_2] \\
P_s &= -s_3(z) \frac{z^2}{2}BV, \quad [\beta \geq \beta_2]
\end{align*}
\]

(18)

2.3. Dissipation Power on Discontinuity Surface $P_v$

Dissipation power on the discontinuity surface is obtained as follows:

\[
P_v = c \frac{H}{\cos \beta} \cos \phi BV,
\]

(20)
2.4. Critical Reinforcement Density of Longitudinal Fiberglass Dowels

Based on the upper-bound limit theory, the power of external force is equal to the dissipation of internal force in the analytical prediction model, as follows:

$$P_{\omega} + P_{f_v} + P_{\sigma_v} + P_s = P_{\omega}, \quad (21)$$

Since there are three distributions of support pressure induced by the longitudinal fiberglass dowels for different wedge angles $\beta$ (c.f., Figure 4), the critical reinforcement density ($n_1$, $n_2$ and $n_3$ corresponding to three distributions of support pressure) of longitudinal fiberglass dowels is calculated:

$$n_{cr} = \max(n_1, n_2, n_3), \quad (22)$$

where

$$n_1 = \left\{ \begin{array}{ll}
\beta \leq \beta_1; \\
\frac{1}{1/2} H^2 B \tan \beta \cos (\varphi + \beta) - (c + K \tan \frac{2\beta + \varphi}{3}) H^2 \tan \beta \cos (\varphi + \beta) + \frac{H}{\cos \varphi \cos \beta} \right. \\
\left. \frac{\sin \alpha_1 \tan \beta}{H \cos \gamma} \right\},
$$

$$n_2 = \left\{ \begin{array}{ll}
\beta_1 \leq \beta \leq \beta_2; \\
\frac{1}{1/2} H^2 B \tan \beta \cos (\varphi + \beta) - (c + K \tan \frac{2\beta + \varphi}{3}) H^2 \tan \beta \cos (\varphi + \beta) + \frac{H}{\cos \varphi \cos \beta} \right. \\
\left. \frac{\sin \alpha_1 \tan \beta}{H \cos \gamma} \right\},
$$

$$n_3 = \left\{ \begin{array}{ll}
\beta \geq \beta_2; \\
\frac{1}{1/2} H^2 B \tan \beta \cos (\varphi + \beta) - (c + K \tan \frac{2\beta + \varphi}{3}) H^2 \tan \beta \cos (\varphi + \beta) + \frac{H}{\cos \varphi \cos \beta} \right. \\
\left. \frac{\sin \alpha_1 \tan \beta}{H \cos \gamma} \right\}.
$$

3. Sensitivity Analysis

3.1. Longitudinal Fiberglass Dowels in the Excavation Face Alone

3.1.1. The Influence Rules of the Cover Depth on Limit Reinforcement Density

Figure 5 renders the influence rules of the cover depth $h$ on limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$. The results indicate that when the cover depth is greater than the width of the tunnel face, the limit reinforcement density does not increase significantly.

![Figure 5](image-url)
3.1.2. The Influence Rules of the Tunnel Shape on Limit Reinforcement Density

Figure 6 shows the influence rules of the different tunnel shapes on the limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$. The results are markedly different between shapes A, B and C. Specifically, shape C needs a lower reinforcement density than shape A, though they have the same height, which agrees with common sense that the C-D tunneling method is more stable than the full-face tunneling method. Moreover, a comparison between the reinforcement required in the cases of tunnel shapes B and C indicates that the C-D tunneling method is more stable than the benching tunneling method.

![Figure 6](image)

**Figure 6.** The influence rules of the different tunnel shapes on limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$.

3.1.3. The Influence Rules of the Reinforcement Installation Interval on Limit Reinforcement Density

Figure 7 depicts the influence rules of the two installation intervals $l$ on the limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$. The results show that large installation intervals of bolts require greater reinforcement density $n_{cr}$.

![Figure 7](image)

**Figure 7.** The influence rules of the two installation intervals $l$ on limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$. 
3.2. Longitudinal Fiberglass Dowels in the Excavation Face Together with Pre-Supports

The grouting pipe roof bears the vertical stress component induced by overlying soil pressure and seepage force that act on the tunnel crown, which leads to a decrease in the limit support pressure of the tunnel face. Figure 8 indicates the effect of the reduction factor (RF) on the required reinforcement density $n_{cr}$.

![Figure 8](image_url)

**Figure 8.** The influence rules of the reduction factor (RF) on limit reinforcement density $n_{cr}$ with the variation of the cohesion $c$.

4. Conclusions

This paper proposed an analytical prediction model by using the limit analysis method to analyze the tunnel face stability. Moreover, the sensitivity analysis is conducted to investigate the effect of the depth of cover, the tunnel shape, the reinforcement installation interval and the reduction factor on the limit reinforcement density. The main conclusions are as follows:

1. The advanced pre-reinforcement structure of the steel pipe umbrella is considered as the beam on the Winkler elastic foundation, which shows that the existence of the steel pipe umbrella effectively reduces the vertical pressure exerted by overlying ground. Under general conditions, with the increase in the length of the pre-reinforcement, its promoting effect on tunnel face stability is obvious. However, when the surplus length of the pre-reinforcement structure reaches the critical fracture length, the length of the pre-reinforcement structure on the stability of the tunnel face is no longer a key factor.

2. The strengthening effect of the longitudinal fiberglass dowels depends primarily on the tensile bearing capacity of the bolt or the bond strength of the ground–bolt interface. The ground extrusion in the front of the tunnel face is effectively reduced for the installation of longitudinal fiberglass dowels. Moreover, the limit reinforcement density of longitudinal fiberglass dowels is assessed under specific lengths with or without the consideration of the steel pipe umbrella.

3. The results indicate that the required reinforcement density does not increase significantly when the cover depth is greater than the width of the face. The C-D tunneling method is more stable than the full-face tunneling method and benching tunneling method. Moreover, the results show that large installation intervals of bolts require greater reinforcement density.

**Author Contributions:** Conceptualization, K.H. and X.W.; Data curation, B.H., C.-y.C. and X.-T.L.; Funding acquisition, K.H.; Investigation, K.H.; Methodology, K.H. and X.W.; Supervision, K.H.; Validation, X.W.; Writing—original draft, K.H., X.W., C.-y.C. and X.-T.L.; Writing—review and editing, K.H., X.W., B.H., C.-y.C. and X.-T.L. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Natural Science Foundation of China. (No. 51908371).
Acknowledgments: The authors are deeply thankful to the reviewers and editor for their valuable suggestions to improve the quality of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References


Publisher’s Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).