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The Relation between Fundamental Constants and Particle Physics Parameters

Rodger I. Thompson

Steward Observatory and Department of Astronomy, University of Arizona, Tucson, AZ 85721, USA; rit@email.arizona.edu

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Abstract: The observed constraints on the variability of the proton to electron mass ratio μ and the fine structure constant α are used to establish constraints on the variability of the Quantum Chromodynamic Scale and a combination of the Higgs Vacuum Expectation Value and the Yukawa couplings. Further model dependent assumptions provide constraints on the Higgs VEV and the Yukawa couplings separately. A primary conclusion is that limits on the variability of dimensionless fundamental constants such as μ and α provide important constraints on the parameter space of new physics and cosmologies.

Keywords: fundamental constants; particle physics; cosmology

1. Introduction

Over the past two decades, there has been a renewed interest in measuring fundamental dimensionless constants such as the proton to electron mass ratio μ and the fine structure constant α in the early universe. Modulo some possibilities discussed at this conference, impressive constraints on the variation of μ and α have been established over time periods that span a significant fraction of the age of the universe. Here, the implications of those constraints are examined in terms of the stability of three basic physics parameters: the Quantum Chromodynamic Scale Λ_{QCD}, the Higgs Vacuum Expectation Value ν and the Yukawa Couplings h. Previous reports of a possible variation of α [1] spurred significant efforts to account for the variation in terms of varying Λ_{QCD}, ν and h [2–10]. These efforts form the basis of the present work except that the process is reversed in that constraints on the variability of the physics parameters is established in terms of the limits on the variability of μ and α. Most of the earlier efforts limited the variability to only one of the physics parameters, usually Λ_{QCD}. In contrast, the work of [5] considered the possibility of all three parameters varying. As such, it is the primary reference in this work.

2. Observational Constraints

There are relatively few constraints on the variation of μ since molecular spectra provide the primary limits on the variability of μ [11] and only a few molecular spectra at high redshift exist. On the other hand, radio observations of molecular spectra at moderate redshifts provide a significantly tighter constraint on μ than exists for α. The current limits on both μ and α are given below.

2.1. μ Constraints

The constraints on $\frac{\Delta\mu}{\mu}$ come from optical observations of redshifted electronic transitions of molecular hydrogen and from radio observations of methanol and ammonia molecules. The majority of the H_2 observations are at redshifts greater than two where the ultraviolet rest frame transitions enter the optical bands and the current radio observations are all at redshifts less than one. The radio observations, however, are significantly more accurate than the optical H_2 observations. The radio observations of methanol in PKS1830-211 [12] ($\frac{\Delta\mu}{\mu} = (-2.9 \pm 10) \times 10^{-8}$) at a redshift of 0.88582 is currently the tightest constraint on a variation of μ. In spite of the relatively low redshift, the look back time is greater than half the age of the universe. Figure 1 and Table 1 show all of the constraints on the variation of μ with 1σ error bars. The radio observations are shown separately in Figure 2 since they are barely visible in Figure 1.

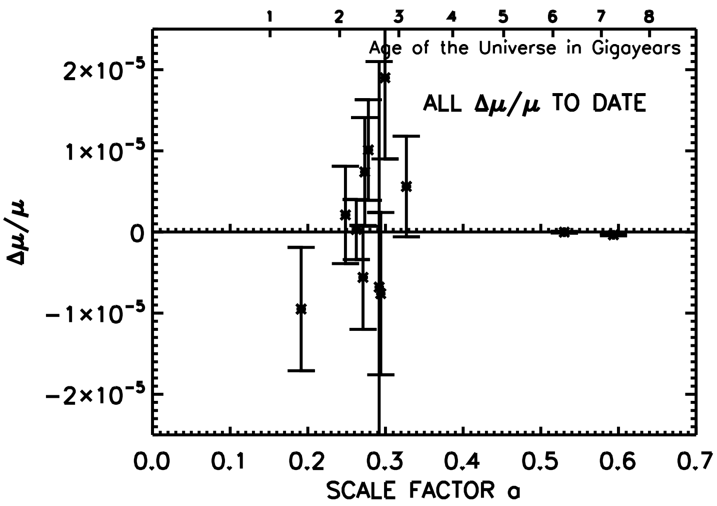

Figure 1. All of the observational constraints on $\Delta\mu/\mu$ from radio ($z < 1$) and optical ($z > 1$) observations plotted versus the scale factor $a = 1/(1+z)$. All constraints are at the 1σ level. The low redshift radio constraints are difficult to see at the scale of this plot. The age of the universe in gigayears is plotted on the top axis and in Figure 2.

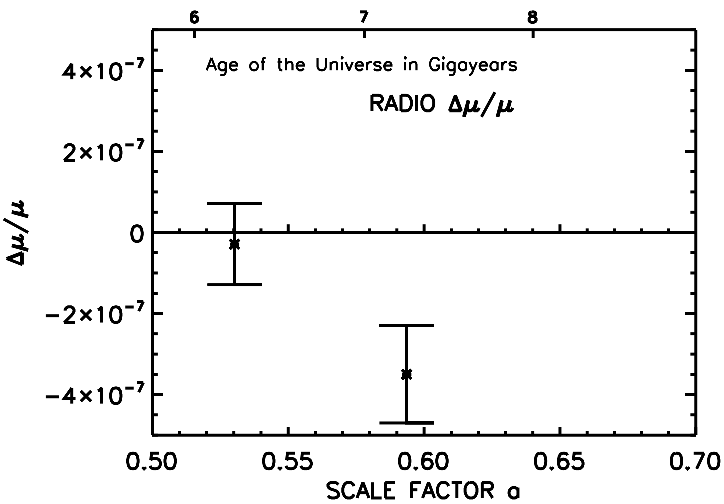

Figure 2. The low redshift radio $\Delta\mu/\mu$ constraints at $z = 0.6874$ and $z = 0.88582$ plotted versus the scale factor $a = 1/(1+z)$. The error bar at $z = 0.6874$ is 1σ, however, the error bar at $z = 0.88582$ ($a = 0.53$) includes systematic effects that increase the error to $\pm 10^{-7}$. That is the primary constraint utilized in this work.

Table 1. Current best observational constraints on $\frac{\Delta\mu}{\mu}$.

Object	Redshift	$\Delta\mu/\mu$	1σ error	Ref.
J1443+2724	4.224	-9.5×10^{-6}	$\pm 7.6 \times 10^{-6}$	[13]
Q0347-383	3.0249	2.1×10^{-6}	$\pm 6. \times 10^{-6}$	[14]
Q0528-250	2.811	3.0×10^{-7}	$\pm 3.7 \times 10^{-6}$	[15]
Q J0643-5041	2.659	7.4×10^{-6}	$\pm 6.7 \times 10^{-6}$	[16]
Q0405-443	2.5974	10.1×10^{-6}	$\pm 6.2 \times 10^{-6}$	[17]
Q2348-011	2.426	-6.8×10^{-6}	$\pm 27.8 \times 10^{-6}$	[18]
He0027-1836	2.402	-7.6×10^{-6}	$\pm 1.0 \times 10^{-5}$	[19]
Q01232+082	2.34	1.9×10^{-5}	$\pm 1.0 \times 10^{-5}$	[20]
J2123-005	2.059	5.6×10^{-6}	$\pm 6.2 \times 10^{-6}$	[21]
PKS1830-211	0.88582	-2.9×10^{-8}	$\pm 5.7 \times 10^{-8}$	[12]
B0218+357	0.6847	-3.5×10^{-7}	$\pm 1.2 \times 10^{-7}$	[22]

2.2. α Constraints

Contrary to the case with μ, there are several thousand measurement of the fine structure splitting at many redshifts of which several hundred are appropriate for testing for a variation of α. In two cases [1,23], temporal changes in α are reported with [23] reporting both spatial and temporal changes at the 10^{-5} level. (See the contributions by Webb et al. in these proceedings.) Subsequent work by [24], however, sees no variation at the $\frac{\Delta\alpha}{\alpha} = (0.4 \pm 1.7) \times 10^{-6}$ at the 1σ level where the error is the rms of the statistical and systematic errors. This is a significantly lower constraint than the reported variation by [23] of $\approx(-6.4 \pm 1.2) \times 10^{-6}$ and consistent with no change in α. Reference [24] attributes the difference to known wavelength calibration errors in the previous analysis in [1,23]. For the purposes of this work, the limits on a variation of α discussed in [24] are taken as the primary limit.

3. The Dependence of Fundamental Constants on the Physics Parameters

The numerical values of both α and μ depend on the values of the physics parameters Λ_{QCD}, ν and h. The following follows the discussion of [5] done for a different purpose but relevant to the current analysis. The connection between the fundamental constants and the physics parameters is probably most obvious for the proton to electron mass ratio μ where the physics parameters set the mass of the proton and electron.

3.1. The Proton to Electron Mass Ratio

The fractional change of μ, $\frac{d\mu}{\mu}$ by simple mathematics is

$$\frac{d\mu}{\mu} = \frac{dm_p}{m_p} - \frac{dm_e}{m_e}. \tag{1}$$

The fractional change of the electron mass is easy since it is a fundamental particle whose mass is set by the Higgs VEV and the electron Yukawa coupling such that $m_e = h_e \nu$, therefore

$$\frac{dm_e}{m_e} = \frac{dh_e}{h_e} + \frac{d\nu}{\nu}. \tag{2}$$

The mass of the proton, however, is much more complicated since it is a composite particle but the fractional change is easier since the ratio eliminates some of the terms. In [5], the fractional change of the proton mass is given by

$$\frac{dm_p}{m_p} = a\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} + b(\frac{dh}{h} + \frac{d\nu}{\nu}). \tag{3}$$

Both a and b are scalars of order unity whose sum by dimensional requirements should equal one to ensure that the proton has units of mass. Combining (2) and (3) gives

$$\frac{d\mu}{\mu} = a\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} + (b-1)(\frac{dh}{h} + \frac{d\nu}{\nu}). \tag{4}$$

Here, the common assumption that although the Yukawa couplings have different values their fractional changes $\frac{dh}{h}$ should be the same is employed. Next, using $(a+b) = 1$ b is eliminated and the fractional change of μ is

$$\frac{d\mu}{\mu} = a[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - (\frac{dh}{h} + \frac{d\nu}{\nu})]. \tag{5}$$

3.2. The Fine Structure Constant

Without additional information, (5) only constrains the combination of Λ_{QCD}, ν, and h rather than any of them individually. The observational constraints on $\frac{d\alpha}{\alpha}$ can provide some of that information since it constrains a different combination of the three parameters. From [5] $\frac{d\alpha}{\alpha}$ depends on the three parameters as

$$\frac{d\alpha}{\alpha} = R^{-1}[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - \frac{2}{9}(\frac{dh}{h} + \frac{d\nu}{\nu})]. \tag{6}$$

A new model dependent parameter R is introduced that can range between unity and in excess of 100. In [5], R is taken as 36 based on unification arguments. For the purposes of this work, that value is accepted partially since it is in the midrange of currently acceptable values. Both (5) and (6) effectively depend on two variables, $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$ and $(\frac{dh}{h} + \frac{d\nu}{\nu})$ which provides a mechanism for solving for $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$ as a function of the constraints on the variance of μ and α and the model dependent parameters.

The Physics of R

Various authors use different models and assumptions to set the value of R. An example is [7] who assumes that the variation in α is produced by temporal changes of the GUT unification scale M_U which also makes the physics parameters time variable. In this model R is given by

$$R = \frac{2\pi}{9\alpha}\frac{\Delta b_3}{\frac{5}{3}\Delta b_1 + \Delta b_2} \tag{7}$$

where the b_i are the beta function coefficients that scale $Q = \beta_i M < M_U$. At the unification scale M_U all of the beta functions are unified to b_U. Δb_i is defined as $\Delta b_i \equiv b_U - b_i$. The different gauge couplings $\alpha_i(Q)$ $(i = 1,2,3)$ are then given by

$$(\alpha_i(Q))^{-1} = (\alpha_U(M_U))^{-1} - \frac{b_i}{2\pi}ln(\frac{Q}{M_U}) \tag{8}$$

The GUT scale M_U is allowed to change but $\alpha_U(M_{Pl})$ and M_{Pl} are held constant where M_{Pl} is the Planck mass. At the unification scale, R is given by

$$R = \frac{2\pi}{9\alpha}\frac{b_U + 3}{\frac{8}{3}b_U - 12} \tag{9}$$

As b_U becomes either positively or negatively large, the value of R approaches 36, the value used in [5].

4. Observational Constraints on $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$

The combination of constraints on the fractional variation of two fundamental constants μ and α provides the opportunity to put constraints on the fractional variation of Λ_{QCD}. Eliminating $(\frac{dh}{h} + \frac{dv}{v})$ from (5) and (6) yields

$$\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} = \frac{d\alpha}{\alpha}\frac{(b-1)R}{[(b-1)-\frac{2}{9}a]} + \frac{d\mu}{\mu}\frac{2}{9[(b-1)-\frac{2}{9}a]} \tag{10}$$

where both factors a and b from (3) have been retained. In [5] $a = 0.76$ and $b = 0.24$. Again invoking $(a+b) = 1$ (10) simplifies to

$$\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} = \frac{9R}{7}\frac{d\alpha}{\alpha} - \frac{2}{7a}\frac{d\mu}{\mu}. \tag{11}$$

Eliminating $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$ provides a constraint on $(\frac{dh}{h} + \frac{dv}{v})$ of

$$(\frac{dh}{h} + \frac{dv}{v}) = (\frac{9}{7})[R\frac{d\alpha}{\alpha} - \frac{1}{a}\frac{d\mu}{\mu}]. \tag{12}$$

Note that the leading terms on the right hand side of (11) and (12) are identical.

Individual Constraints on $\frac{\Delta v}{v}$ and $\frac{\Delta h}{h}$

At the expense of additional model dependence, it is possible to individually constrain $\frac{dv}{v}$ and $\frac{dh}{h}$. In the standard model, there is an established relationship between the fractional variation of the Higgs VEV v and the fractional variation of the Yukawa couplings h given by [5].

$$\frac{dv}{v} = S\frac{dh}{h}. \tag{13}$$

The value of S is model dependent so the constraints on $\frac{dv}{v}$ and $\frac{dh}{h}$ are doubly model dependent. These constraints come from using (13) in (12). The $\pm$ in the $\frac{\Delta\alpha}{\alpha}$ and $\frac{\Delta\mu}{\mu}$ terms in Equations (14) and (15) simply reflect that limits on $\frac{d\alpha}{\alpha}$ and $\frac{d\mu}{\mu}$ are plus or minus the quoted error. As stated in Section 5, the terms are combined as a root mean square when evaluated.

$$\frac{\Delta v}{v} = \frac{9}{7}\frac{S}{(1+S)}[(R\frac{\pm\Delta\alpha}{\alpha}) - \frac{1}{a}(\frac{\pm\Delta\mu}{\mu})]. \tag{14}$$

Similarly the limit on $\Delta h/h$ is

$$\frac{\Delta h}{h} = \frac{9}{7}\frac{1}{(1+S)}[(R\frac{\pm\Delta\alpha}{\alpha}) - \frac{1}{a}(\frac{\pm\Delta\mu}{\mu})]. \tag{15}$$

5. Constraints from a Given Model

Since the discussion has followed the work of [5], the model used in that reference is used as an example for producing constraints on the physics parameters. It is also a constraint on the particular model as well. The values for the model dependent parameters in [5] are $a = 0.76$, $b = 0.24$ (which add to one), $R = 36$ and $S = 160$. Note that with this parameter set and the observational constraints, the common leading term, the α term, in (11) and (12) dominate the constraints on the parameters. The constraints are

$$\Delta\Lambda_{QCD}/\Lambda_{QCD} \leq \pm 7.9 \times 10^{-5} \tag{16}$$

$$\Delta v/v \leq \pm 7.9 \times 10^{-5} \tag{17}$$

$$\Delta h/h \leq \pm 4.9 \times 10^{-7}. \tag{18}$$

The look back time for the constraints is the average look back time of the α observations at a redshift of 1.54 equal to 9.4 gigayears or roughly 70% of the age of the universe.

The observational constraints are evaluated via rms summing of the 1σ α and μ terms for each parameter. This assumes that the 1σ errors are centered on zero rather than the "measured" values of the observations. It was noted earlier that the R model parameter has a range of model values. Figure 3 shows the variation of the limit on $\Delta\Lambda_{QCD}/\Lambda_{QCD}$ as a function of R.

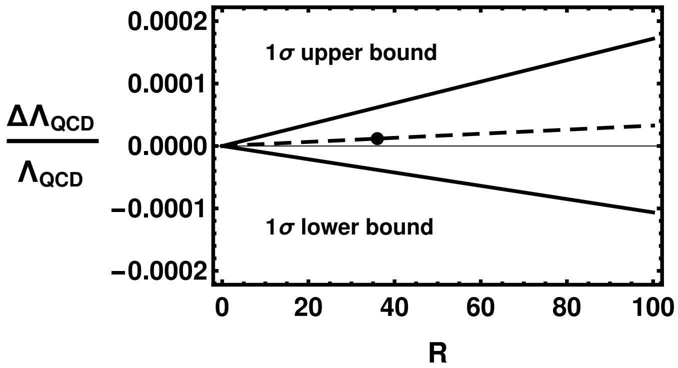

Figure 3. The figure indicates the 1σ variation of the limit on $d\Lambda_{QCD}/\Lambda_{QCD}$ as a function of the model parameter R. The dashed line indicates the limit on $d\Lambda_{QCD}/\Lambda_{QCD}$ if the measured values of $d\alpha/\alpha$ and $d\mu/\mu$ are used rather than the limits. The dot is at $R = 36$ which is the example value. Note that although it is not apparent at the scale of the figure the limit on $d\Lambda_{QCD}/\Lambda_{QCD}$ at $R = 0$ is not zero but rather the small $\frac{2}{7a}\frac{d\mu}{\mu}$ term in (11) that does not depend on R.

6. A Model Dependent Limit on $\frac{\Delta\alpha}{\alpha}$

The observational constraints on the physics parameters are dominated by the α term both because of its less stringent observational limit and by the larger coefficient in the model of [5]. That model, however, predicts a relationship between a variation of α and a variation of μ given by

$$\frac{d\alpha}{\alpha} = \frac{1}{R}\frac{d\mu}{\mu}. \tag{19}$$

The model therefore predicts that the fractional variation of α should be smaller than the variation of μ by a factor of 1/36. It is of some interest to see how the constraints on the physics parameters change if this model dependent limit of $\frac{\Delta\alpha}{\alpha} \leq \pm 2.8 \times 10^{-9}$ on the fractional change of α is imposed. The new constraint on $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$ is

$$\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}} \leq [\pm(2.8 \times 10^{-9})\frac{9R}{7} \pm 10^{-7}\frac{2}{7a}] \leq \pm 1.7 \times 10^{-7}. \tag{20}$$

Similar replacements in (14) and (15) yield constraints on the time variation of ν and h of

$$\frac{\Delta\nu}{\nu} \leq (\frac{9}{7})(\frac{S}{S+1})[\pm R(2.8 \times 10^{-9}) \pm \frac{10^{-7}}{a}] \leq \pm 2.1 \times 10^{-7} \tag{21}$$

$$\frac{\Delta h}{h} \leq (\frac{9}{7})(\frac{1}{S+1})[\pm R(2.8 \times 10^{-9}) \pm \frac{10^{-7}}{a}] \leq \pm 1.3 \times 10^{-9}. \tag{22}$$

The net result of the model dependent limit on $\frac{d\alpha}{\alpha}$ is a very stringent set of limits on the fractional change of the parameters at the look back time of the μ constraints which is greater than half the age of the universe. This constraint severely limits the parameter space of theories that predict significant fractional changes in Λ_{QCD}, ν or h.

7. Conclusions

It is difficult to test for time variability of the primary particle physics parameters such as the Quantum Chromodynamic Scale, the Higgs Vacuum Expectation Value and the Yukawa couplings. It is, however, relatively easy to use spectra of objects in the early universe to test for time variability of dimensionless fundamental constants whose numerical values depend on the physics parameters. Individually, the constants only constrain a combination of the parameters but combining the observational constraints on the variability of two or more constants provides model dependent constraints on the fractional variability of the QCD scale and a combination of the fractional variability of the Higgs VEV and the Yukawa couplings. Introduction of an additional model dependent parameter sets limits on the fractional variability of the Higgs VEV and the Yukawa couplings separately.

The constraints on the fractional time variability of the physics parameters limits the parameter space of new physics theories that require a time variability of any of the three basic physics parameters. It is recommended that observed constraints on the variability of dimensionless fundamental constants become another important tool in evaluating the validity of non-standard physics and cosmology theories.

Conflicts of Interest: The author declares no conflict of interest.

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