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Matter Accretion Versus Semiclassical Bounce in Schwarzschild Interior

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Abstract: We discuss the properties of the previously constructed model of a Schwarzschild black hole interior where the singularity is replaced by a regular bounce, ultimately leading to a white hole. We assume that the black hole is young enough so that the Hawking radiation may be neglected. The model is semiclassical in nature and uses as a source of gravity the effective stress-energy tensor (SET) corresponding to vacuum polarization of quantum fields, and the minimum spherical radius is a few orders of magnitude larger than the Planck length, so that the effects of quantum gravity should still be negligible. We estimate the other quantum contributions to the effective SET, caused by a nontrivial topology of spatial sections and particle production from vacuum due to a nonstationary gravitational field and show that these contributions are negligibly small as compared to the SET due to vacuum polarization. The same is shown for such classical phenomena as accretion of different kinds of matter to the black hole and its further motion to the would-be singularity. Thus, in a clear sense, our model of a semiclassical bounce instead of a Schwarzschild singularity is stable under both quantum and classical perturbations.

Keywords: general relativity; semiclassical gravity; quantum corrections; bounce solution; Schwarzschild black hole; particle creation

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1. Introduction

The existence of singularities in various solutions of general relativity (GR) as well as many alternative classical theories of gravity, describing black holes or the early Universe, is an undesirable but apparently inevitable feature. On the other hand, one can hardly believe that the curvature invariants or the densities and temperatures of matter that appear in such singularities can really reach infinite values. There is therefore a more or less common hope that a future theory of gravity valid at very large curvatures, high energies, small length and time scales will be free of singularities, and that such a theory should take into account quantum phenomena.

The existing numerous attempts to avoid singularities can be basically classified as follows (see also references therein for each item):

(a) In GR, "exotic" sources of gravity are invoked, violating the standard energy conditions, for example, phantom scalar fields; in classical extensions of GR, using quantities of geometric origin (torsion, nonmetricity, extra dimensions) whose effective stress-energy tensors (SETs) can have similar "exotic" properties [1–11]; it has also been argued that the effects of rotation in GR can also play the role of exotic matter (see [12–14]).

(b) In semiclassical gravity, where the geometry is treated classically and obeys the equations of GR or an alternative classical theory, using averages of quantum fields of matter as sources of gravity with possible "exotic" properties [15–19].

(c) Diverse models of quantum gravity are also often translated into the language of classical geometry and lead to nonsingular spacetimes describing both regular black hole interiors and early stages of the cosmological evolution [20–23].

One can notice that the singularity problems in black hole physics and Big Bang cosmology are quite similar. For example, the Schwarzschild singularity is located in a nonstationary "T-region", where the metric can be written as that of a homogeneous anisotropic cosmology, a special case of Kantowski–Sachs models. It is therefore natural that the same tools are used in attempts to attack these problems.

Classical nonsingular models in cosmology, black hole and wormhole physics are quite popular, but the "exotic" components that are necessarily present in those models require certain conjectures so far not confirmed by observations or experiments, and their consideration is often justified as a kind of phenomenological description of underlying quantum effects.

Many models of quantum gravity, in their representations in the language of classical geometry, lead to nonsingular cosmologies and black hole models, but most frequently such models reach the values of curvatures and densities close to the Planck scale. However, more surprising is the considerable diversity of their predictions, depending on various leading ideas employed in such models.

Thus, several scenarios in the framework of Loop Quantum Gravity (LQG) predict a bounce close to the Planck scale and a transition from a black hole to a white hole [21,22,24,25]. In particular, in [24,25], the authors considered quantum corrections to the Oppenheimer-Snyder collapse scenario. Much earlier, a similar scenario with singularity avoidance was considered by [26] on the basis of a quasiclassical approximation of the Wheeler-DeWitt equation.

Unlike that, application of the so-called polymerization concept to the interior of a Schwarzschild black hole [27,28], also removing the singularity, leads to a model with a single horizon and a Kantowski–Sachs cosmology with an asymptotically constant spherical radius at late times. (This geometry is partly similar to the classical black universes with a phantom scalar [8–10], but in the latter the late-time Kantowski–Sachs cosmology tends to de Sitter isotropic expansion.)

Some of the scenarios (see [29]) even lead to a quantum-corrected effective metric with an unconventional asymptotic behavior, although it is claimed that the quantum correction to the black hole temperature is quite negligible for sufficiently large black holes, and that the metric is asymptotically flat in a precise sense.

A consideration of homogeneous gravitational collapse of dust and radiation with LQG effects leads in [30] to the avoidance of both a final singularity and an event horizon, so that the outcome is a dense compact object instead of a black hole.

Let us also mention a study of black hole evaporation process by Ashtekar [31] using as guidelines: (i) LQG; (ii) simplified models with concrete results; and (iii) semiclassical effects. The author discussed various issues concerning the information loss problem and the final fate of evaporating black holes; one of his conclusions is that LQG effects do not appreciably change the semiclassical picture outside macroscopic black holes.

A comprehensive review of quantum gravity effects in gravitational collapse and black holes was provided by Malafarina [20] in 2017, and we here only mention a few results of interest and some papers that appeared later than this review. However, even this short list shows how diverse the results and conclusions can be depending on the particular approach. All that may be a manifestation of a so far uncertain status of quantum gravity.

Since matter can manifest its quantum properties at the atomic or macroscopic scales (as exemplified by lasers or the Casimir effect), one may hope that singularities in cosmology or black holes may be prevented at length scales much larger than the Planck one. This would look more attractive both from the observational viewpoint and also theoretically since the corresponding results, at least today, look more confident than those obtained with quantum gravity.

The black hole studies in the framework of semiclassical gravity ([17–19,32], and many others) mostly focus on the consequences of the Hawking black hole evaporation and the related information paradox. Their conclusions seem promising from the viewpoint of singularity avoidance. Thus, in [32], it is concluded that the black hole evaporation ultimately leads to emergence of an inner macroscopic region that hides the lost information and is separated from the external world. According to [19], the evaporation process even prevents the emergence of an event horizon. Thus, after formation of a large spherically symmetric black hole by gravitational collapse, the classical $r = 0$ singularity is replaced by an initially small regular core, whose radius grows with time due to increasing entanglement between Hawking radiation quanta outside and inside the black hole, and by the Page time (when half the black hole mass has evaporated), all quantum information stored in the interior is free to escape to the outer space.

However, there remains a question of what is happening inside a large black hole when it has just formed, and the evaporation process is too slow to immediately launch the above processes. Indeed, an approximate expression for the full evaporation time is $t_{\rm evap} \propto M^3$, where M is the initial black hole mass; it then follows that the Page time is $\frac{7}{8}t_{\rm evap}$, and if M is the solar mass, we have $t_{\rm evap} \approx 2.1 \times 10^{67}$ years. In other words, any astrophysical black hole (except for very light primordial ones) is at this initial stage of its evaporation. Moreover, under realistic conditions, its mass much faster grows due to accretion than decreases by evaporation.

In our study, we try to answer the following question: What is the internal geometry of such a large and "young" black hole if its Hawking evaporation can be neglected, but the impact of quantum fields that are present in a vacuum form is taken into account? In other words: If a body (a particle, a planet or a spacecraft) falls into such a black hole, what is the geometry it meets there?

More specifically, we are considering the neighborhood of a would-be Schwarzschild singularity ($r = 0$) in the framework of semiclassical gravity and explore a possible emergence of a bounce instead of the singularity. We can recall that in any spacetime region there always exist quantum oscillations of all physical fields. We do not assume any particular composition of these fields, considering only their vacuum polarization effects. In such a simplified statement of the problem, we showed [33] that there is a wide choice of the free parameters of the model that provide a possible implementation of such a scenario. The SET used to describe the vacuum polarization of quantum fields is taken in the form of of a linear combination of the tensors ${}^{(1)}H_\mu^\nu$ and ${}^{(2)}H_\mu^\nu$ obtained by variation of the curvature-quadratic invariants R^2 and $R_{\mu\nu}R^{\mu\nu}$ in the effective action in agreement with the renormalization methodology of quantum field theory in curved spacetimes [34,35]. In this scenario, in the internal Kantowski–Sachs metric, the spherical radius r evolves to a regular minimum instead of zero, while its longitudinal scale has a regular maximum instead of infinity. The free parameters of the model can be chosen so that the curvature scale does not reach the Planck scale but remains a few orders smaller (for example, on the GUT scale), sufficiently far from the necessity to include quantum gravity effects. The whole scenario is assumed to be time-symmetric with respect to the bouncing instant, therefore, as in many other papers, we are describing a smooth transition from black to white hole.

The nonlocal part of the effective SET of quantum fields in the Schwarzschild interior, depending on the whole history and mainly represented by particle production from vacuum, was estimated in [36], and it was shown that its contribution in the vicinity of a bounce is many orders of magnitude smaller than that of ${}^{(1)}H_\mu^\nu$ and ${}^{(2)}H_\mu^\nu$.

In the present paper, after a brief representation of the results of [33,36], we try to find out whether or not there are classical phenomena that could potentially destroy the bounce, namely, accretion of different kinds of matter which is always present near astrophysical black holes and whose density increases as it further moves inside the horizon towards the would-be singularity. It turns out that this accretion is also unable to affect the bounce due to its negligibly small contribution to the total SET.

The paper is structured as follows. Section 2 summarizes the problem statement and the assumptions made. In Section 3, we describe the bouncing solution to the field equations. In Section 4, we estimate the nonlocal contribution to the effective SET. Section 5 is devoted to calculations of the spherically symmetric accretion of the CMB radiation and massive matter to a Schwarzschild black hole. Section 6 is a brief discussion.

2. Field Equations and Assumptions

2.1. Near-Bounce Geometry

Considering a generic static, spherically symmetric black hole in its interior region (beyond the horizon), also called a T-region, we can write its metric in the general Kantowski–Sachs form

$$ds^2 = d\tau^2 - e^{2\gamma(\tau)}dx^2 - e^{2\beta(\tau)}d\Omega^2, \tag{1}$$

where τ is the natural time coordinate in the corresponding reference frame and x is a spatial coordinate that "inherits" the time coordinate of the static region after crossing the horizon; $d\Omega^2$ is, as usual, the metric on a unit sphere $\mathbb{S}^2$. It is a homogeneous anisotropic cosmological model with the topology $\mathbb{R} \times \mathbb{S}^2$ of its spatial sections.

Assuming that quantum effects can appreciably change the spacetime geometry only if the latter is very strongly curved, at smaller curvatures, even in a T-region ($r < 2m$), we can use with sufficient accuracy the Schwarzschild solution, which then takes the form

$$ds^2 = \left(\frac{2m}{T} - 1\right)^{-1} dT^2 - \left(\frac{2m}{T} - 1\right) dx^2 - T^2 d\Omega^2, \tag{2}$$

where $m = GM$, G being Newton's constant of gravity and M the black hole mass. We use the units $\hbar = c = 1$. Compared to the conventional expression, we have changed the notation, $r \to T$, to emphasize that in the T-region the coordinate r is temporal. Furthermore, at $T \ll 2m$, passing on to the Kantowski–Sachs cosmological time by putting $\sqrt{T/(2m)}dT = d\tau$, we obtain an asymptotic form of the metric in the notations of (1):

$$ds^2 = d\tau^2 - \left(\frac{4}{3}m\right)^{2/3} \tau^{-2/3} dx^2 - \left(\frac{9}{2}m\right)^{2/3} \tau^{4/3} d\Omega^2, \tag{3}$$

which is valid at $\tau/m \ll 1$. It is the Schwarzschild metric at approach to the singularity $\tau \to 0$, at which the scale along the x axis is infinitely stretched while the spheres $x = \text{const}$ are shrinking to zero.

In this study, our basic assumption will be that quantum field effects do not allow the space time to approach too close to the singularity $r \equiv e^{\beta} = 0$ (or $\tau = 0$ in (3)) but, instead, stop the contraction of r at $\tau = 0$ at some regular minimum value $r = r_0 > 0$, while the scale factor e^{γ} along the x axis simultaneously turns to a regular maximum. Then, at small τ, in agreement with (2) and (3), the metric takes the form

$$ds^2\Big|_{\text{bounce}} \simeq d\tau^2 - \frac{2m}{r_0}(1 - \bar{c}^2\tau^2)dx^2 - r_0^2(1 + \bar{b}^2\tau^2)d\Omega^2 \tag{4}$$

where $r_0, \bar{b}, \bar{c}$ are positive constants with appropriate dimensions. To obtain (4) directly from (2), we replace $T \ll 2m$ with r_0, then we accordingly replace $2m/T - 1 \approx 2m/T$ with $2m/r_0$ and transform the temporal part of the metric to $d\tau^2$. The terms $\propto \tau^2$ are added according to our minimum and maximum assumptions. Equivalently, the metric (4) can be obtained from (3) by putting the radius squared (the factor before $d\Omega^2$) equal to r_0^2, then the factor before dx^2 becomes equal to $2m/r_0$, and it again remains to add the terms $\propto \tau^2$ to designate the assumed minimum and maximum.

In addition to these assumptions, let us also suppose that the time evolution of the metric is symmetric with respect to the bouncing instant $\tau = 0$. Then, in the notations of (1), we can present the functions $\beta(\tau)$ and $\gamma(\tau)$ as Taylor expansions with only even powers of τ,

$$\begin{aligned} \beta(\tau) &= \beta_0 + \frac{1}{2}\beta_2\tau^2 + \frac{1}{24}\beta_4\tau^4 + \frac{1}{720}\beta_6\tau^6 + \ldots, \\ \gamma(\tau) &= \gamma_0 + \frac{1}{2}\gamma_2\tau^2 + \frac{1}{24}\gamma_4\tau^4 + \frac{1}{720}\gamma_6\tau^6 + \ldots, \end{aligned} \tag{5}$$

where β_i, γ_i $(i = 0, 2, 4, 6, ...)$ are constants. Then, according to (4),

$$r_0 = e^{\beta_0}, \qquad 2m/r_0 = e^{2\gamma_0}, \qquad 2\bar{b}^2 = \beta_2/\beta_0, \qquad 2\bar{c}^2 = -\gamma_2/\gamma_0. \tag{6}$$

To explain the behavior (4) of the metric, we invoke the semiclassical approach, writing the Einstein equations as

$$G_\mu^\nu = -\varkappa\langle\, T_\mu^\nu\,\rangle, \qquad \varkappa = 8\pi G, \tag{7}$$

where the right-hand side (r.h.s.) represents a renormalized stress–energy tensor (SET) $\langle\, T_\mu^\nu\,\rangle$ of quantum fields, containing, in general, both local and nonlocal contributions.

In the general metric (1), the Einstein tensor G_μ^ν has the following nonzero components:

$$\begin{aligned} G_0^0 &= -\dot\beta(\dot\beta + 2\dot\gamma) - e^{-2\beta}, \\ G_1^1 &= -2\ddot\beta - 3\dot\beta^2 - e^{-2\beta}, \\ G_2^2 &= G_3^3 = -\ddot\gamma - \ddot\beta - \dot\gamma^2 - \dot\beta^2 - \dot\beta\dot\gamma. \end{aligned} \tag{8}$$

Substituting the Taylor expansions (5), we can explicitly present these components up to $O(\tau^2)$ as follows:

$$\begin{aligned} -G_0^0 &= \frac{1}{r_0^2}\Big(1 - \frac{\beta_2}{2\beta_0}\tau^2\Big) + \beta_2(\beta_2 + 2\gamma_2)\tau^2, \\ -G_1^1 &= \frac{1}{r_0^2}\Big(1 - \frac{\beta_2}{2\beta_0}\tau^2\Big) + 2\beta_2 + \beta_4\tau^2 + 3\beta_2^2\tau^2, \\ -G_2^2 &= \beta_2 + \gamma_2 + \frac{1}{2}(\beta_4 + \gamma_4)\tau^2 + (\beta_2^2 + \gamma_2^2 + \beta_2\gamma_2)\tau^2. \end{aligned} \tag{9}$$

2.2. *The Stress-Energy Tensor*

In agreement with the vast literature on quantum field theory in curved spacetimes, including the books by [34] (Section 6.2) and [35] (Section 12.2), the renormalized vacuum SET T_ν^μ of quantum fields may be presented as a linear combination of two tensors of geometric origin ${}^{(i)}H_{\mu\nu}$ $(i = 1, 2)$ (which can be obtained by variation of actions containing R^2 and $R_{\mu\nu}R^{\mu\nu}$, i.e., the Ricci scalar and tensor squared), with some phenomenological constants N_1, N_2, and two other contributions, ${}^{(c)}H_\nu^\mu$ and P_ν^μ:

$$\langle\, T_\nu^\mu\,\rangle = N_1\,{}^{(1)}H_\nu^\mu + N_2\,{}^{(2)}H_\nu^\mu + {}^{(c)}H_\nu^\mu + P_\nu^\mu, \tag{10}$$

where

$$
\begin{aligned}
{}^{(1)}H_\nu^\mu &\equiv 2RR_\nu^\mu - \frac{1}{2}\delta_\nu^\mu R^2 + 2\delta_\nu^\mu \Box R - 2\nabla_\nu \nabla^\mu R, \\
{}^{(2)}H_\nu^\mu &\equiv -2\nabla_\alpha \nabla_\nu R^{\alpha\mu} + \Box R_\nu^\mu + \frac{1}{2}\delta_\nu^\mu \Box R + 2R^{\mu\alpha}R_{\alpha\nu} - \frac{1}{2}\delta_\nu^\mu R^{\alpha\beta}R_{\alpha\beta},
\end{aligned} \tag{11}
$$

and $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$. The tensor ${}^{(c)}H_\nu^\mu$ is of local nature and depends on the spacetime topology and/or the boundary conditions (e.g., the Casimir effect [37,38]), while P_ν^μ is nonlocal, it depends on the particular quantum states of the constituent fields and, in particular, describes particle production in a nonstationary metric. Its nonlocal nature means that it is not a function of a spacetime point but depends, in general, on the whole history. Its calculation is rather a complex task and requires additional assumptions on quantum states of different fields. We temporarily assume that the contribution of P_ν^μ is small as compared to the other terms in (10) (at least under a suitable choice of quantum states) and try to justify this assumption in Section 4.

The components of the tensors ${}^{(i)}H_\nu^\mu$ (which turn out to be diagonal) can be easily calculated from the ansatz (1) with the Taylor expansions (5). At the very instant $\tau = 0$ (at bounce), they are

$$
\begin{aligned}
{}^{(1)}H_0^0 &= -\frac{2}{r_0^4} + 8\beta_2^2 + 8\beta_2\gamma_2 + 2\gamma_2^2, \\
{}^{(1)}H_1^1 &= -\frac{2}{r_0^4} - 32\beta_2^2 - 16\beta_2\gamma_2 - 6\gamma_2^2 - 8\beta_4 - 4\gamma_4, \\
{}^{(1)}H_2^2 &= \frac{2}{r_0^4} + \frac{12\beta_2}{r_0^2} - 24\beta_2^2 - 20\beta_2\gamma_2 - 10\gamma_2^2 - 8\beta_4 - 4\gamma_4, \\
{}^{(2)}H_0^0 &= -\frac{1}{r_0^4} + 3\beta_2^2 + 2\beta_2\gamma_2 + \gamma_2^2, \\
{}^{(2)}H_1^1 &= -\frac{1}{r_0^4} - 9\beta_2^2 - 6\beta_2\gamma_2 - 3\gamma_2^2 - 2\beta_4 - 2\gamma_4, \\
{}^{(2)}H_2^2 &= \frac{1}{r_0^4} + \frac{4\beta_2}{r_0^2} - 9\beta_2^2 - 6\beta_2\gamma_2 - 3\gamma_2^2 - 3\beta_4 - \gamma_4.
\end{aligned} \tag{12}
$$

As is made clear below, their higher orders in τ are unnecessary in our calculations.

What is known about the numerical coefficients N_1 and N_2 in (10)? According to the authors of [34,35], their values should be found from experiments or observations. The orders of magnitude of these coefficients may be roughly estimated by recalling that they appear in higher-derivative theories of gravity where the action has the form

$$
S \sim \int d^4x\sqrt{-g}(R/(2\varkappa) + N_1R^2 + N_2R_{\mu\nu}^2 + \ldots), \tag{13}
$$

the tensors ${}^{(1,2)}H_{\mu\nu}$ resulting from variation of the corresponding terms. The upper bounds on these parameters are $N_{1,2} \lesssim 10^{60}$ (see [39]), as follows from observations performed at very small curvatures, at which any possible effects of terms quadratic in the curvature are extremely weak. However, the factors $N_{1,2}$ may be estimated in another way if such theories of gravity are used to describe the early (inflationary) Universe with much larger curvatures, for example, $N_1 \sim 10^{10}$ [40–42]. For our purposes, we keep in mind this order of magnitude.

Concerning the Casimir contribution, there are arguments indicating that it must be much smaller than the contribution of ${}^{(i)}H_\mu^\nu$. If we consider, for instance, the static counterpart of the metric (1) with $e^\beta = r = r_0$, something treatable as a description of an infinitely long wormhole throat, we can use the result obtained in [43] for a conformally coupled massless scalar field, which reads for this geometry

$$ {}^{(c)}H^{\mu}_{\nu} = \frac{1}{2880\pi^2 r_0^4}\Big[2\,\mathrm{diag}(-1,-1,1,1)\ln(r_0/a_0) + \mathrm{diag}(0,0,-1,-1)\Big], \tag{14} $$

where a_0 is some fixed length to be determined by experiment. Note that the quantity (14) is obtained for a single massless scalar, and the total Casimir contribution must take into account all existing fields with different spins and masses, hence this contribution may be two or three orders of magnitude larger than (14).

On the other hand, for the same spacetime geometry,

$$ {}^{(1)}H^{\nu}_{\mu} = 2\,{}^{(2)}H^{\nu}_{\mu} = \frac{2}{r_0^4}\,\mathrm{diag}(-1,-1,1,1). \tag{15} $$

In what follows we assume that the bounce occurs on the scales of at least a few orders of magnitude larger than the Planck length $l_{\rm Pl}$ to provide the validity of the semiclassical approximation. As mentioned above, it is reasonable to assume a regular minimum at $r_0 \sim 10^5 l_{\rm Pl}$. On the other hand, comparing (14) and (15), we see that for ${}^{(c)}H_{\mu\nu}$ to be of the same order as ${}^{(i)}H_{\mu\nu}$, it would be necessary that $|\ln(r_0/a_0)| \simeq 10^4$, i.e., $a_0 \simeq r_0\,\mathrm{e}^{\pm 10^4} \simeq r_0 \cdot 10^{\pm 4342}$. Taking into account the order of r_0, we conclude that the length scale parameter a_0 must then be either many orders smaller than the Planck scale or many orders larger than the radius of the Universe, and both cases seem to be physically incredible.

Therefore, if N_1 and/or N_2 are of the order of unity or larger (as is assumed), the tensors ${}^{(i)}H_{\mu\nu}$ contribute much stronger to $\langle T^{\nu}_{\mu}\rangle$ in the Einstein equations (7) than ${}^{(c)}H_{\mu\nu}$, unless the uncertain length a_0 in (14) is unreasonably small or high, or the total number of fields is so large as to overcome the denominator which is $\sim 10^4$.

In our further consideration, we assume that ${}^{(c)}H^{\mu}_{\nu}$ can be neglected in our geometry (4) and take into account only the contributions ${}^{(i)}H^{\nu}_{\mu}$.

3. The Semiclassical Bounce

In this section, we consider the Einstein equations (7) with the SET (10), taking into account only the first two terms. Our task is to find out whether or not there are solutions consistent with the bouncing metric (4), and if it is the case, what are the requirements to the free parameters of the model that would justify the semiclassical nature of the equations. In the subsequent sections, we analyze the influence of other effects that could in principle destroy the model thus constructed: the nonlocal contribution to the SET (10) and the possible influence of matter surrounding the black hole and falling to its interior region.

For our purpose, we express G^{μ}_{ν} and ${}^{(i)}H^{\mu}_{\nu}$ in terms of the Taylor series coefficients in (5) and equate the coefficients at equal powers of τ on different sides of the resulting equations. Let us introduce, for convenience, the following dimensionless parameters:

$$ A = \varkappa r_0^{-2}, \qquad B_2 = \varkappa\beta_2, \qquad C_2 = \varkappa\gamma_2, \qquad B_4 = \varkappa^2\beta_4, \qquad C_4 = \varkappa^2\gamma_4, \ \text{ etc.} \tag{16} $$

Since $\varkappa \approx l_{\rm Pl}^2$ (the Planck length squared), it is evident that our system remains on the semiclassical scale only if all parameters (16) are much smaller than unity. Hence, in particular, the minimum spherical radius $r = r_0$, reached at bounce should be much larger than the Planck length. Other parameters that should be small are values of the derivatives $\ddot\beta, \ddot\gamma$, etc. close to the bounce.

An inspection shows that, in the approximation used, it is sufficient to consider the order $O(1)$ in the $\binom{0}{0}$ component of Equations (7) (or explicitly (9)), from which we find

$$A = N_1[-2A^2 + 2(2B_2 + C_2)^2] + N_2[-A^2 + (B_2 + C_2)^2 + 2B_2^2]. \tag{17}$$

The role of all other equations reduces to expressing the constants B_4, C_4, etc. in terms of A, B_2, C_2. Thus, we have a single equation for the three parameters A, B_2, C_2 of the bouncing geometry, along with the coefficients N_1, N_2. Therefore, we have a broad space of possible solutions.

As stated above, we must assume that r_0 is much larger than the Planck length $l_{\rm Pl} \sim \sqrt{\varkappa}$, from which it follows that $A \ll 1$, or $A = O(\varepsilon)$, ε being a small parameter. We can also make the natural assumptions $B_2 = O(\varepsilon)$ and $C_2 = O(\varepsilon)$, which means that $\ddot\beta$ and $\ddot\gamma$ are of the same order of magnitude as $1/r_0^2$. Then, since the r.h.s. of Equation (17) is $O(\varepsilon^2)$ while the left-hand side (l.h.s.) is $O(\varepsilon)$, to provide the equality, we must require that N_1 and/or N_2 should be large, of the order $O(1/\varepsilon)$.

The remaining Einstein equations $\binom{1}{1}$ and $\binom{2}{2}$ at $\tau = 0$ then show that B_4 and C_4 are of the order $O(\varepsilon^2)$ (see (11)), therefore, the fourth-order derivatives of β and γ are of a correct order of smallness with respect to the Planck scale (see (16)). Similar estimates are obtained for B_6, C_6, etc. if we analyze equations in the order $O(\tau^2)$, and so on. It can also be verified that the curvature invariants R, $R_{\mu\nu}R^{\mu\nu}$ and $\mathcal{K} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are small at bounce ($\tau = 0$) as compared to the Planck scale:

$$\begin{aligned}
&R = \frac{2}{r_0^2} + 4\beta_2 + 2\gamma_2 = O\Big(\frac{\varepsilon}{\varkappa}\Big),\\
&R_{\mu\nu}R^{\mu\nu} = \frac{2}{r_0^4} + \frac{4\beta_2}{r_0^2} + 6\beta_2^2 + 4\beta_2\gamma_2 + 2\gamma_2^2 = O\Big(\frac{\varepsilon^2}{\varkappa^2}\Big),\\
&\mathcal{K} = \frac{4}{r_0^4} + 8\beta_2^2 + 4\gamma_2^2 = O\Big(\frac{\varepsilon^2}{\varkappa^2}\Big).
\end{aligned} \tag{18}$$

Consider a numerical example for illustration. Assuming $N_1 = 0$, $N_2 = 10^{10}$, and $A = 10^{-10}$, a minimum radius r_0 is of 10^5 Planck lengths. Since, by construction (see (6) and (16)), $B_2 > 0$ and $C_2 < 0$, we can assume for convenience $B_2 + C_2 = 0$. As a result, from Equation (17) we find

$$B_2 = -C_2 = 10^{-10}.$$

If we substituting this into the $\binom{1}{1}$ and $\binom{2}{2}$ components of the Einstein equations at $\tau = 0$, with the expressions (8) and (12), we can obtain the values of B_4 and C_4:

$$B_4 = 3.5 \times 10^{-20}, \qquad C_4 = -8.5 \times 10^{-20}.$$

From the equations of order $O(\tau^2)$ one can then determine B_6, C_6, and so on.

One can recall that in spherically symmetric spacetimes, if the spherical radius $e^\beta = r$ has a regular minimum (it is a wormhole throat if the minimum is in an R-region and a bounce if it is in a T-region), then the SET must satisfy the condition $T_0^0 - T_1^1 < 0$ which means violation of the Null Energy Condition (see [2,44]). In our model, supposing a bounce at $\tau = 0$, we automatically obtain the inequality $T_0^0 - T_1^1 < 0$.

4. Nonlocal Contribution to the Vacuum SET

To estimate the contribution of the nonlocal term P_μ^ν in the SET (10), we rewrite the general metric (1) of a Kantowski–Sachs cosmology as

$$ds^2 = e^{2\alpha}d\eta^2 - e^{2\gamma}dx^2 - \mu^2 e^{2\beta}d\Omega^2, \tag{19}$$

where the time coordinate η is called "conformal time" and is defined by the condition $3\alpha(\eta) = 2\beta(\eta) + \gamma(\eta)$, it is convenient when dealing with quantum fields. The black hole under consideration is assumed to have a stellar or larger mass m_{Sch}, and $\mu = 2Gm_{\text{Sch}} \gtrsim 10^5$ cm = 1 km is the corresponding gravitational radius. Meanwhile, at bounce (say, at the time $\eta = 0$), in agreement with the previous section, we assume that the minimum radius $r_0 = \mu\,\mathrm{e}^{\beta(0)}$ is $\sim 10^5 l_{\text{Pl}} \sim 10^{-28}$ cm. We accordingly introduce the small parameter $\epsilon = r_0/\mu \lesssim 10^{-33}$ (quite different from the parameter ε from the previous section). Then, at times not too far from the bounce time we can write

$$\mathrm{e}^{2\alpha} = \epsilon(1 + a\eta^2), \qquad \mathrm{e}^{2\beta} = \epsilon^2(1 + b\eta^2), \qquad \mathrm{e}^{2\gamma} = \epsilon^{-1}(1 + c\eta^2). \tag{20}$$

where $3a = 2b + c$ according to the definition of η; we have $b > 0$ since e^{β} has a minimum and $c < 0$ since e^{γ} is at maximum at $\eta = 0$. The powers of ϵ in $\mathrm{e}^{2\beta}$ and $\mathrm{e}^{2\gamma}$ characterize the magnitudes of these metric coefficients when approaching to a would-be Schwarzschild singularity, or, which is the same, to the powers of r_0 in (4), while the factor in $\mathrm{e}^{2\alpha}$ then follows from the definition of conformal time. In other words, the metrics (4) and (1) with (20) coincide at $\eta = 0 \Longleftrightarrow \tau = 0$, while the coefficients a, b, c simply correspond to a maximum of g_{xx} and a minimum of $r = \mathrm{e}^{\beta}$.

Consider a quantum scalar field satisfying the equation $(\Box + M^2 + \xi R)\Phi = 0$ and its standard Fourier expansion:

$$\Phi = \mathcal{N}\,\mathrm{e}^{-\alpha} \int dk \sum_{lm} \mathrm{e}^{-ikx} Y_{lm}(\theta, \varphi) g_{klm}(\eta) c^{+}_{klm} + \text{h.c.}, \tag{21}$$

where $\mathcal{N}$ is a normalization factor, ξ is a coupling constant, c^{+}_{klm} is a creation operator, Y_{lm} are spherical functions, and each time-dependent mode function $g_{klm}(\eta) \equiv g$ obeys the equation obtained from the original Klein–Gordon-type scalar field equation by separation of variables:

$$\ddot{g} + \Omega^2 g = 0, \tag{22}$$

where the dots denote $d/d\eta$ and Ω is the effective frequency:

$$\Omega^2 = k^2\,\mathrm{e}^{2(\alpha-\gamma)} + \frac{l(l+1) + 2\xi}{\mu^2}\,\mathrm{e}^{2(\alpha-\beta)} + M^2\,\mathrm{e}^{2\alpha} + \frac{2\xi(\dot{\beta} - \dot{\gamma})^2}{3} + (6\xi - 1)(\ddot{\alpha} + \dot{\alpha}^2). \tag{23}$$

At the bounce time $\eta = 0$, we have, due to standard normalization, $|g| \sim \Omega^{-1/2}$ and

$$\Omega^2(0) = k^2\epsilon^2 + \frac{l(l+1) + 2\xi}{\mu^2\epsilon} + M^2\epsilon + (6\xi - 1)a. \tag{24}$$

Further on, to make our estimates, we adhere to a natural assumption, justified by much experience [34] (Section 3.5), that the most intensive particle production takes place at energies not too far from the curvature scale $\sim r_0^{-1}$. This energy is roughly of the same order of magnitude as the frequency $\bar{\Omega}(\tau)$ calculated in terms of proper cosmic time τ, which is related to our conformal time by $d\tau = \mathrm{e}^{\alpha} d\eta$. Therefore, our assumption means $\bar{\Omega} \sim 1/r_0$. Since $\mathrm{e}^{\alpha} \sim \sqrt{\epsilon}$, one has $\tau \sim \sqrt{\epsilon}\eta$, and, from the relation $\Omega\eta = \bar{\Omega}\tau$, we immediately obtain $\bar{\Omega} = \Omega/\sqrt{\epsilon}$, so that

$$\bar{\Omega}^2(0) = k^2\epsilon + \frac{l(l+1) + 2\xi}{\mu^2\epsilon^2} + M^2 + \frac{(6\xi - 1)a}{\epsilon}. \tag{25}$$

It is now of interest, at which values do the parameters of the model appreciably contribute to $\bar{\Omega}^2$ having the order $\sim r_0^{-2} = (\mu\epsilon)^{-2}$. These are:

$$k \sim \frac{1}{\mu\epsilon^{3/2}} \sim 10^{45}\,\text{cm}^{-1} \sim 10^{12}\,m_{\text{Pl}}; \qquad l, \xi \sim 1; \qquad M \sim \frac{1}{r_0}; \qquad a = \ddot{\alpha}(0) \sim \frac{\epsilon}{r_0^2}. \tag{26}$$

Apparently, momenta k strongly exceeding the Planckian value look quite meaningless, and we can conclude that, at reasonable (that is, sub-Planckian) values of k, their contributions to $\bar{\Omega}$ are negligibly small.

Note that the result $a \sim \epsilon/r_0^2$ can be obtained in another way using the relations

$$e^{2\alpha} = \epsilon^{-1}(1+\tau^2/r_0^2) = \epsilon^{-1}(1+a\eta^2), \qquad \tau \sim \sqrt{\epsilon}\eta.$$

A similar analysis leads to $b, c \sim \epsilon/r_0^2$. Furthermore, at small values of η, we can assume

$$\Omega \approx B + C\eta^2, \quad \text{where } B = \Omega(0) \sim \sqrt{\epsilon}/r_0, \qquad C/B \sim (a,b,c) \sim \epsilon/r_0^2. \tag{27}$$

The energy density of created particles may be estimated using the standard technique of Bogoliubov coefficients. For of bounce-type metrics similar to ours, the most important Bogoliubov coefficient β_{kl} can be accurately enough computed by using the formulas [45]

$$\beta_{kl} = \sqrt{\frac{I^-}{I^+}}\sinh\sqrt{I^-I^+}, \quad I^\pm \equiv \int_{\eta_1}^{\eta} g^\pm(\bar\eta)d\bar\eta, \quad g^\pm \equiv \frac{\dot\Omega}{2\Omega}\exp\left(\pm 2i\int_{\eta_1}^{\eta}\Omega(\bar\eta)d\bar\eta\right), \tag{28}$$

where η_1 is the initial time instant at which, by assumption, $\beta_{kl} = 0$ (that is, assuming that the field is in a vacuum state, without particles). Using Equation (27) and making the assumption $B\eta \lesssim O(1)$ (which means that η is not very far both from zero value and from η_1), we obtain

$$\int_{\eta_1}^{\eta}\Omega(\bar\eta)d\bar\eta \approx B\bar\eta + \frac{1}{3}C\bar\eta^3\Big|_{\eta_1}^{\eta} \approx B(\eta-\eta_1), \tag{29}$$

$$g^\pm(\eta) \approx \frac{C\eta}{B}\,\mathrm{e}^{\pm 2iB(\eta-\eta_1)} \sim \frac{\varepsilon\eta}{r_0^2}\,\mathrm{e}^{\pm 2iB(\eta-\eta_1)}. \tag{30}$$

Now, we are ready to estimate the integrals $I^\pm$ defined in (28) at times close to the bounce time $\eta = 0$:

$$\begin{aligned} I^\pm(\eta)\Big|_{\eta\to 0} &\sim \frac{\varepsilon}{r_0^2}\int_{\eta_1}^{0}\eta d\eta\,\mathrm{e}^{\pm 2iB(\eta-\eta_1)} = \frac{\varepsilon}{r_0^2}\,\mathrm{e}^{\mp 2iB\eta_1}\left[\frac{\mathrm{e}^{\pm 2iB\eta}}{4B^2}(1\mp 2iB\eta)\right]_{\eta_1}^{0} \\ &= \frac{1}{4}\left[\mathrm{e}^{\mp 2iB\eta_1} - 1 \pm 2iB\eta_1\right] \approx -\frac{1}{2}B^2\eta_1^2. \end{aligned} \tag{31}$$

Then, assuming $B\eta_1 \lesssim O(1)$, we arrive at

$$\beta_{kl} \sim I^- \sim -\frac{1}{2}B^2\eta_1^2, \qquad |\beta_{kl}^2| \sim \frac{1}{4}B^4\eta_1^4 \lesssim O(1). \tag{32}$$

Thus, the energy density of produced particles is

$$\rho_{\text{nonloc}} = \langle T_0^0\rangle \sim \frac{1}{8\pi}\int dk \sum_l (2l+1)\frac{e^{-4\alpha}}{\mu^2}\Omega|\beta_{kl}|^2 \sim \frac{10^5\sqrt{\epsilon}}{r_0^4} \sim \frac{10^{-11}}{r_0^4}, \tag{33}$$

where, for each factor in (33), we have taken the following approximate orders of magnitude, in accord with (26): (i) $\int dk \sim 2m_{\text{Pl}} = 10^5/r_0$ since we integrate from $-m_{\text{Pl}}$ to $+m_{\text{Pl}}$; (ii) $\sum_l(2l+1) \sim 10^2$, involving a few low multipolarities according to (25), (26) (since large multipolarities would mean too large mode energies contrary to our assumption that particle creation occurs most intensively at energies close to the curvature scale); (iii) $e^{-4\alpha}/\mu^2 \sim 1/r_0^2$; (iv) $\Omega \sim \sqrt{\epsilon}/r_0$; and (v) $|\beta_{kl}|^2 \sim 1$ as a very rough upper bound.

A comparison of the estimate (33) with that of the local energy density contribution from vacuum polarization obtained in the previous section and [33], $\rho_{\text{loc}} \sim 10^{10}r_0^{-4}$, leads to $\rho_{\text{nonloc}}/\rho_{\text{loc}} \sim 10^{-21}$, and this value is still smaller if we consider black holes heavier than the Sun. Even if one relaxes some of the requirements in (33) within a few orders of magnitude (for instance, including larger multipolarities), the smallness of the factor 10^{-21} would safely preserve our qualitative estimate. We conclude that the nonlocal contribution to the vacuum energy density due to particle production is negligibly small in the regime of semiclassical bounce, and a more accurate calculation including other physical fields of different spins can hardly change this estimate too strongly.

5. Matter Accretion into a Schwarzschild Black Hole

5.1. CMB Accretion

Black holes in the real Universe are surrounded by various kinds of matter: interstellar or intergalactic gas, dust and stellar matter if the black hole gravity destroys approaching stars. Depending on specific astrophysical circumstances, the ambient matter may form an accretion disk or experience spherical or close to spherical accretion. The falling matter crosses the horizon and should ultimately approach the black hole singularity, if the latter really exists. Conversely, if the theory predicts a bouncing region instead of a singularity, it is natural to ask: Will the gravity of the accreted matter strongly change the geometry of the bouncing region? Can it happen that this falling matter will destroy the bounce (whatever be its origin) and restore the singularity?

We try to answer this question for a Schwarzschild black hole with a semiclassical bounce described in [33] and in the previous sections. Thus, we assume that the spacetime metric is approximately Schwarzschild,

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{34}$$

everywhere except for a region close to bounce, that is, $r \lesssim nr_0$, where, say, $n \lesssim 10$, and r_0 is the minimum radius at bounce.

In this section, we consider spherical accretion of the kind of matter that exists anywhere in the Universe, the Cosmic Microwave Background (CMB). Thus, our calculation can correspond to an isolated Schwarzschild black hole in intergalactic space, surrounded by the CMB only, and the accretion consists in capture of CMB photons. It is thus a minimum possible environment of any black hole. At each point of the black hole's ambient space, there is a flow of photons to be captured: these are photons whose path gets into the so-called photon sphere with the radius $r_{\rm ph} = 3m$. Such photons may be considered as those forming a radiation flow with the SET

$$T^\nu_\mu = \Phi(r,t) k_\mu k^\nu, \qquad k_\mu k^\mu = 0, \tag{35}$$

where the null vector k^μ is, in a reasonable approximation, radially directed, so that

$$k^\mu = (\,{\rm e}^{-\gamma}, -{\rm e}^{\gamma}, 0, 0), \qquad k_\mu = (\,{\rm e}^{\gamma},\, {\rm e}^{-\gamma}, 0, 0), \tag{36}$$

where ${\rm e}^\gamma = \sqrt{1 - 2m/r}$. Then, the conservation law $\nabla_\nu T^\nu_\mu = 0$ in the metric (34) gives for $\Phi = \rho_{\rm flow}$ (the flow energy density)

$$\Phi(r,t) = \frac{\Phi_0}{r(r-2m)}, \qquad \Phi_0 = \text{const}. \tag{37}$$

The constant Φ_0 should be determined by the CMB energy density and the black hole mass, taking into account bending of photon paths in the black hole's gravitational field. Fortunately, there is no necessity to carry out such a computation anew: we can use, for example, the result obtained by Bisnovatyi-Kogan and Tsupko [46]. They showed that, if a source of radiation is located at $r = 10^4 m$ in Schwarzschild spacetime, then the black hole will capture radiation emitted inside a cone with an angular radius $\alpha \approx 0.0298^\circ \approx 5.203{\cdot}10^{-4}$. If the source radiates isotropically, then the fraction $\Delta(r)$ of the emitted radiation energy captured by the black hole will be equal to the part of the complete solid angle of 4π contained in the spot of $\pi\alpha^2$, that is,

$$\Delta(r) = \pi\alpha^2/(4\pi) = \alpha^2/4 \approx 6.768{\cdot}10^{-8} \ \text{ for } \ r = 10^4\, m. \tag{38}$$

At $r = 10^4 m$ or larger, the spacetime may be regarded as approximately flat; therefore, due to flux conservation, the fraction Δ should be proportional to r^{-2}. On the one hand, since the area of a sphere from which the flux is collected is $\propto m^2$, it should be also $\Delta \propto m^2$. As a result, we can write, using (38),

$$\Delta(r) \approx \frac{\Delta_0 m^2}{r^2}, \qquad \Delta_0 = \text{const} \quad \Rightarrow \quad \Delta_0 = \frac{\Delta(r) r^2}{m^2} \approx 6.678. \tag{39}$$

On the other hand, at such distances from the black hole, the CMB can be safely regarded as homogeneous and isotropic, and we can conclude that the accretion flow will have the energy density

$$T^0_0 \approx \frac{\Phi_0}{r^2} = \Delta(r)\rho_{\text{CMB}} = \frac{\Delta_0 m^2}{r^2}\rho_{\text{CMB}} \quad \Rightarrow \quad \Phi_0 = \Delta_0 m^2 \rho_{\text{CMB}}, \tag{40}$$

where the CMB density ρ_{CMB} is nowadays

$$\rho_{\text{CMB}} \approx 0.4{\cdot}10^{-12}\,\text{erg}\,\text{cm}^{-3} \approx 1.41{\cdot}10^{-128}\, l_{\text{Pl}}^{-4}, \tag{41}$$

where $\rho_{\text{Pl}} = l_{\text{Pl}}^{-4}$ is the Planck density.

Thus, we know the SET (35) with (37) and (40) in the external region of the black hole, but the quantity (37) diverges at the horizon $r = 2m$. This looks natural since in our static reference frame the radiation is infinitely blueshifted at the horizon, where this reference frame in no more valid. However, our purpose is to find out how this radiation behaves deeply beyond the horizon. To extend the expression (35) to $r < 2m$, let us transform it to the Kruskal coordinates valid at all r. To do that, it is convenient to use at $r > 2m$ the so-called tortoise radial coordinate

$$r_* = r + 2m \ln\left(\frac{r}{2m} - 1\right) \quad \Rightarrow \quad ds^2 = \left(1 - \frac{r}{2m}\right)(dt^2 - dr_*^2) - r^2 d\Omega^2 \tag{42}$$

(note that $r_* \to -\infty$ as $r \to 2m$). This coordinate belongs to the same static reference frame, hence the flow energy density $T^0_0 = \Phi$. However, the null vector k^μ is now, instead of (36),

$$k^\mu = (\,\mathrm{e}^{-\gamma}, -\mathrm{e}^{-\gamma}, 0, 0), \qquad k_\mu = (\,\mathrm{e}^{\gamma}, \mathrm{e}^{\gamma}, 0, 0), \tag{43}$$

where, as before, $\mathrm{e}^{\gamma} = \sqrt{1 - 2m/r}$, and the nonzero covariant SET components have the form

$$T_{00} = T_{01} = T_{10} = T_{11} = \Phi\,\mathrm{e}^{2\gamma} = \frac{\Phi_0}{r^2}, \tag{44}$$

convenient for the transformation.

The Kruskal coordinates R, T, in which the metric has the form

$$ds^2 = \frac{32m^3}{r}\,\mathrm{e}^{-r/(2m)}(dT^2 - dR^2) - r^2 d\Omega^2, \tag{45}$$

are related to r_*, t by

$$t = 2m \ln\frac{R+T}{R-T}, \qquad r_* = 2m \ln\frac{R^2 - T^2}{4}. \tag{46}$$

Using this, we transform $T_{\mu\nu}$ to the Kruskal coordinates and find the nonzero components

$$T_{TT} = T_{TR} = T_{RT} = T_{RR} = \frac{16\Phi_0 m^2}{r^2(R+T)^2}. \tag{47}$$

In (45) and (47), the horizon $r_* = -\infty \mapsto R^2 = T^2$ is a regular surface, the static region $r > 2m$ corresponds to $R^2 > T^2$, while at $r < 2m$ instead of the coordinates r^*, t or r, t we can introduce their counterparts x (analog of t) and τ (analog of r^*) by putting, for $T > R > 0$ (the upper quadrant in Kruskal's diagram),

$$R = e^{\tau/(4m)} \sinh \frac{x}{4m}, \qquad T = e^{\tau/(4m)} \cosh \frac{x}{4m}, \tag{48}$$

so that the metric acquires the Kantowski-Sachs form

$$ds^2 = \left(\frac{2m}{r} - 1\right)(d\tau^2 - dx^2) - r^2 d\Omega^2 = \left(\frac{2m}{r} - 1\right)^{-1} dr^2 - \left(\frac{2m}{r} - 1\right) dx^2 - r^2 d\Omega^2, \tag{49}$$

the two timelike coordinates r and τ being related by

$$\tau = r + 2m \ln \frac{2m - r}{2m}. \tag{50}$$

The horizon corresponds to $r = 2m$ or $\tau \to -\infty$, while the singularity $r = 0$ occurs at $\tau = 0$. Using (48), we transform the tensor (47) to the Kantowski–Sachs coordinates τ, x, obtaining

$$T_{\tau\tau} = T_{\tau x} = T_{x\tau} = T_{xx} = \frac{\Phi_0}{r^2}, \tag{51}$$

from which it follows that the energy density is

$$T^{\tau}_{\tau} = \rho_{\text{flow}} = \frac{\Phi_0}{r(2m - r)}. \tag{52}$$

We see that in the Kantowski–Sachs reference frame, in which the Schwarzschild metric looks very similar to its usual appearance in the static region, the expression for ρ_{flow} also looks very similar. It is the density in the same reference frame that was used for describing the bounce and can thus be compared with the vacuum polarization density $\rho_{\text{vac}} \sim 10^{10} r_0^{-4} \sim 10^{-10} \rho_{\text{Pl}}$ at bounce.

Assuming that the internal Schwarzschild metric (49) is the true metric up to $r \gtrsim r_0 \ll 2m$, using (40) and (41), we obtain for such small radii

$$\rho_{\text{flow}} \approx \frac{\Phi_0}{2mr} \approx \frac{\Delta_0 m \rho_{\text{CMB}}}{2r}, \tag{53}$$

and, since Δ_0 is of the order of unity, we conclude that the flow density at small radii is larger than ρ_{CMB} approximately by a factor of m/r. For a black hole of stellar mass, $m \sim 10^5$ cm and $r \sim r_0 \sim 10^5 l_{\text{Pl}}$, this factor is $\sim 10^{33}$, so that, with $\rho_{\text{vac}} \sim 10^{-10} \rho_{\text{Pl}}$ and recalling (41), we obtain $\rho_{\text{flow}}/\rho_{\text{vac}} \sim 10^{-85}$.

This ratio will certainly be larger for heavier black holes and for earlier epochs when ρ_{CMB} was larger by a factor of $(a_0/a)^4$, where a is the cosmological scale factor and a_0 its present value. Assuming the existence of supermassive black holes with $m \sim 10^9$ solar masses at scale factors $a \sim 10^{-3} a_0$ (that is, at $z \sim 1000$, close to the recombination time), the above ratio gains 21 orders of magnitude, resulting in $\rho_{\text{flow}}/\rho_{\text{vac}} \sim 10^{-64}$.

We conclude that CMB accretion cannot exert any influence on the model dynamics at small radii close to bounce or a would-be singularity inside a Schwarzschild black hole. Very probably, accretion of ambient matter can be much more important, and our next task is to estimate its impact.

5.2. Dust Accretion

Matter falling onto a black hole has in general the form of hot gas but close to the horizon this gas is nearly in a state of free fall [47] (Section 9.3), therefore the approximation of dust freely radially moving to the horizon looks quite adequate, and it is reasonable to assume that the same regime well describes its further motion in the T-region.

Thus, we consider the Schwarzschild spacetime with the metric (34) or, in terms of the tortoise coordinate r_*, (42). In this metric, we consider matter with the SET

$$T^{\nu}_{\mu} = \rho u_\mu u^\nu, \tag{54}$$

where the components of the 4-velocity vector u^μ for radial motion may be written, in terms of the radial coordinate r_*, in the form

$$u^\mu = (\,\mathrm{e}^{-\gamma}\sqrt{1+v^2}, -\,\mathrm{e}^{-\gamma}v, 0, 0), \qquad u_\mu = (\,\mathrm{e}^{\gamma}\sqrt{1+v^2}, \,\mathrm{e}^{\gamma}v, 0, 0), \tag{55}$$

where $v = \mathrm{e}^{-\gamma}dr_*/ds$ (s is proper time along the world line), so that $u_\mu u^\nu = 1$.

We assume a steady infalling flow, so that both ρ and u^μ in the R-region ($r > 2m$) depend on r only. Then, the conservation law $\nabla_\nu T^\nu_\mu$ has two nontrivial components:

$$\begin{aligned}
&(\rho v\sqrt{1+v^2})' = -(\rho v\sqrt{1+v^2})(2\beta' + 2\gamma'). \\
&(\rho v^2)' + \rho v^2(2\beta' + 2\gamma') + \rho\gamma' = 0,
\end{aligned} \tag{56}$$

where the prime denotes d/dr, $\mathrm{e}^\gamma = \sqrt{1-2m/r}$, $\mathrm{e}^\beta = r$. Solving these equations to find ρ and v as functions of r, we obtain[1]

$$\rho = \frac{K\,\mathrm{e}^{-2\beta}}{E\sqrt{E^2 - \mathrm{e}^{2\gamma}}} = \frac{K}{r^2E\sqrt{E^2 - 1 + 2m/r}}, \qquad E, K = \text{const}, \tag{57}$$

$$v^2 = E^2\,\mathrm{e}^{-2\gamma} - 1 = \frac{E^2 r}{r - 2m} - 1. \tag{58}$$

Recalling that dust particles move along geodesics, one can independently obtain v^2 from the geodesic equations which lead precisely to the expression (58), and the constant E has the meaning of conserved energy in the course of geodesic motion.

Now, our task is to follow the motion of the dust flow to the T-region. To do that, we again use the transformation (46), now for $T_{\mu\nu} = \rho u_\mu u_\nu$, and the result in the (R, T) coordinates is

$$\begin{aligned}
T_{TT} &= \frac{16m^2\rho(ER - T\sqrt{E^2 - \mathrm{e}^{2\gamma}})^2}{(R^2 - T^2)^2}, \\
T_{RT} &= \frac{16m^2\rho((R^2 + T^2)E\sqrt{E^2 - \mathrm{e}^{2\gamma}}) - RT(2E^2 - \mathrm{e}^{2\gamma})}{(R^2 - T^2)^2}, \\
T_{RR} &= \frac{16m^2\rho(ET - R\sqrt{E^2 - \mathrm{e}^{2\gamma}})^2}{(R^2 - T^2)^2}.
\end{aligned} \tag{59}$$

One can verify that these expressions lead to the correct expression for the SET trace, $T^\mu_\mu = \rho$. The expressions (59) are valid in both R- and T-regions, even though in the T-region ($r < 2m$) we have $\mathrm{e}^{2\gamma} < 0$, so this notation should be perceived as a symbolic one.

The next step is to use the transformation (48) to the metric (49), which results in

$$\begin{aligned}
T_{\tau\tau} &= \rho(E^2 - \mathrm{e}^{2\gamma}) = \rho(E^2 - 1 + 2m/r), \\
T_{\tau x} &= -\rho\frac{R^4 + T^4}{(T^2 - R^2)^2}, \\
T_{xx} &= \rho E^2.
\end{aligned} \tag{60}$$

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[1] Note that the expressions for ρ and v^2 in terms of β and γ are valid not only in the Schwarzschild metric but in any static, spherically symmetric metric written as $ds^2 = \mathrm{e}^{2\gamma(x)}(dt^2 - dx^2) - \mathrm{e}^{2\beta(x)}d\Omega^2$.

It is again easy to verify the correctness of these expressions by confirming that $T^\mu_\mu = \rho$, now in the metric (49) in terms of τ and x.

With (60), we find the following expression for the energy density of the dust flow in the T-region:

$$T^\tau_\tau = \frac{r}{2m - r} T_{\tau\tau} = \frac{K\sqrt{E^2 - 1 + 2m/r}}{Er(2m - r)}. \tag{61}$$

Let us estimate this quantity at $r \ll 2m$, assuming $E = 1$ (which corresponds to zero velocity of dust particles at infinity):

$$\rho_E = T^\tau_\tau = \frac{K}{\sqrt{2m} r^{3/2}}. \tag{62}$$

The constant K can be found if we know the dust density at some r in the R-region. To this end, we can recall that, according to [47] (page 324), under typical conditions the falling matter density is $\rho \simeq (6{\cdot}10^{-12}\ \mathrm{g/cm^3})(2m/r)^{3/2}$. Thus, say, at $r = 10m$ we obtain $\rho \sim 10^{-12}\ \mathrm{g/cm^3}$ which approximately equals $2{\cdot}10^{-106}\,\rho_{\rm Pl}$.[2] We thus have

$$\rho\Big|_{r=10m} = \frac{K}{\sqrt{2m}(10m)^{3/2}} \simeq 2{\cdot}10^{-106}\rho_{\rm Pl} \quad \Rightarrow \quad K \simeq m^2{\cdot}10^{-104}\,\rho_{\rm Pl}. \tag{63}$$

With this value of K, let us estimate the dust energy density ρ_E at the radius $r = r_0 = 10^5\, l_{\rm Pl}$, the supposed bounce radius. According to (62),

$$\rho_E\Big|_{r=10^5\, l_{\rm Pl}} \approx \frac{K}{\sqrt{2m} r^{3/2}} = \frac{10^{-104}}{\sqrt{2}} \left(\frac{m}{r}\right)^{3/2} \rho_{\rm Pl}. \tag{64}$$

For the black hole mass $m \approx m_\odot$, we have $(m/r)^{3/2} \approx 10^{50}$, so that

$$\rho_E\Big|_{r=10^5\, l_{\rm Pl}} \approx 10^{-52}\rho_{\rm Pl} \approx 10^{-42}\rho_{\rm vac} \tag{65}$$

if we assume $\rho_{\rm vac} \approx 10^{-10}\rho_{\rm Pl}$. We conclude that the influence of the accretion flow on the hypothetic semiclassical bounce is quite negligible. The situation does not change if we assume, say, the initial dust density five orders of magnitude larger and a supermassive black hole of $10^9 m_\odot$: we thus gain about 18 orders of magnitude in (65), and there remains a difference of 24 orders.

6. Conclusions

We have constructed a simple model [33] describing a possible geometry that can exist deeply inside a sufficiently large black hole at its sufficiently early stage of evolution, when the Hawking radiation is negligible due to its extremely low temperature, and one could not yet feel the influence of quantum entanglement phenomena. The model is semiclassical in nature and is governed by vacuum polarization leading to the emergence of quadratic curvature invariants in the effective action. We assumed that the free constants appearing at these invariants have values of the same order as in some well-known models of the inflationary universe, and showed that the corresponding terms in the effective Einstein equations lead to solutions in which the Schwarzschild singularity is replaced by a regular bounce, ultimately leading to a white hole.

Furthermore, we argued that other quantum effects such as the Casimir effect, caused by the spherical topology of a subspace in the Kantowski–Sachs cosmology inside the black hole, and particle production from vacuum caused by a nonstationary nature of the metric, make only negligible contributions to the total effective SET and therefore cannot destroy the bouncing geometry. The same was shown for possible classical phenomena that could interfere, namely accretion of different kinds

[2] $1\ \mathrm{g/cm^3} \approx 2{\cdot}10^{-94}\rho_{\rm Pl}$.

of matter and its further motion to the black hole interior. It can be said that, in a sense, our simple bouncing model is stable under both quantum and classical perturbations.

It would be of substantial interest to study how this model would be modified if Hawking radiation at its early stages is considered. Another subject of future studies can be concerned with using similar assumptions for black holes with charge and spin, where the nature of singularities is quite different and where Cauchy horizons take place. As mentioned in [31], according to the stability analysis of Kerr and Reissner-Nordström spacetimes, their Cauchy horizons are unstable under small perturbations, from which it follows that a generic black hole singularity must be null rather than spacelike as in the Schwarzschild metric, and the analysis of such singularities and their possible avoidance should be a promising field of research.

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