1. Development of empirical local patches

We developed empirical local patches using WSE data modeled with CaMa-Flood for 1980 to 2000. First, CaMa-Flood-modeled WSE values were converted into a spatial dependency weighting term. The spatial dependency weighting function was derived from the auto-correlation length, which was obtained from semi-variogram analysis. Then, we derived local patches that were defined through introduction of a threshold for the spatial dependency weightings. We explain the steps used to derive the empirical local patch in the following sections.

1.1. Transformation of data

The CaMa-Flood-modeled WSE data were transformed into a variable that exhibits less periodicity and has a distribution close to normal [1] for semi-variogram analysis, as the CaMa-Flood-modeled daily WSE did not follow a normal distribution according to the Kolmogorov-Smirnov, D’Agostino K-squared, Shapiro-Wilk, and Anderson-Darling statistical tests. Therefore, three transformation steps were implemented, each using the data output from the previous step, starting with CaMa-Flood-modeled daily WSE data. The three steps used for transformation of data were: 1) removing trends, 2) removing seasonality, and 3) standardizing.

Some hydrological time series may show trends in the time domain (e.g., global warming, deforestation, and land surface changes) that can be represented with a simple linear model. Hence, we calculated the intercept and gradient using the time series of CaMa-Flood-modeled WSE for 1980 to 2000, and then subtracted the trend represented by this line. A slight upward or downward trend is observable for each river pixel in the basin, with a magnitude of 10\(^{-4}\).

We removed seasonality to avoid exaggerating temporal dependence using Fourier transformation; a periodic function was fitted to the data based on the sum of the sine and cosine functions [2] at frequencies that are integer multiples of the annual cycle, as well as half- and quarter-year cycles. For each river pixel, the number of covariates was set to six to allow a good fit to the data (more covariates increases the flexibility of the function, enabling better fitting). The effect of seasonality was removed by subtracting the magnitude of changes caused by seasonality, calculated using the periodic function, for each day in the time period considered. Our Fourier transformation showed higher spectral densities near the frequency of the annual cycle (1/365 Hz). WSE data with the trend and seasonality removed were standardized for each model grid by subtracting the mean and dividing by the standard deviation of the time series. Standardizing enables comparison between locations with different magnitudes of WSE. Histograms of the transformed WSE data verify that transformation brought the CaMa-Flood-modeled WSE data much closer to a normal distribution with zero mean and unit standard deviation.

1.2. Semi-variogram analysis

A semi-variogram presents a spatial dependence structure with a one-dimensional spatially averaged semi-variance value, which was calculated using Equation 1. A theoretical semi-variogram will exhibit a monotonic increase with increasing lag distance from the ordinate of the appropriate shape until it reaches a constant maximum or asymptote, called a ‘sill’. A semi-variogram has several components. The ‘sill’ is defined as the semi-variance value where the gradient of the semi-variogram is zero. A gradient of zero indicates the limit of temporal dependence and is an indicator of the total variance between the ordinate and the surrounding area. The ‘range’ is the time required to reach this zero gradient. The positive intercept on the initiate is referred to as the ‘nugget’.
An experimental semi-variogram was calculated for each pixel using the average squared difference between each pair of values that are separated by a spatial distance lag. As described above, the experimental semi-variogram is calculated as follows:

\[ \gamma(h) = \frac{1}{2N} \sum_{i=1}^{N} (Z_i - Z_{i+h})^2, \]

where \( \gamma \) is the experimental semi-variance, \( h \) is the lag distance, \( N \) is the number of data points (here, number of days), and \( Z \) is the variable considered, which is standardized WSE in this case.

A semi-variogram model, selected from among conventional semi-variogram models including spherical, cubic, Gaussian, pentaspherical, sine hole effect, and exponential, was fitted to the data using a weighted least squares method with the Levenberg-Marquardt algorithm, which has been established as an optimal solution for parameter estimation of nonlinear functional models [3]. We identified the best performing semi-variogram model as the Gaussian model according to Akaike’s information criterion (AIC) values. We determined the sill, range, and nugget corresponding to the best-fitting semi-variogram using the weighted least squares method.

We fitted semi-variograms for upstream and downstream areas independently for each river pixel.

1.3. Empirical determination of the local patch

After determining the corresponding sill, range and nugget values of the best-fitting semi-variogram model, the experimental data were inverted to obtain a distribution with unity for the ordinate and zero outside of the range (auto-correlation length). The experimental semi-variance was converted to a weighting term representing the spatial dependency. Values close to 1 indicate high spatial dependency, while those close to zero show low spatial dependency. Furthermore, we allocated the greatest weight to common upstream river stems in the target pixel. We defined a threshold for the spatial dependency weight to derive the empirical local patch. In this study, we treated each river pixel independently and determined an empirical local patch for each pixel. We used a weighting threshold of 0.6 to derive these local patches, while considering 1) the effectiveness of assimilating observations with weights of less than 0.6 and 2) spurious errors caused by distant observations due to the limited ensemble size.

2. Selection of fixed local patch sizes

We performed preliminary simulations (90 days) on different fixed local patch sizes namely 7×7 pixels, 11×11 pixels, 15×15 pixels, 21×21 pixels, and 31×31 pixels to compare the normalized root mean

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**Figure S1.** a) spatial dependency weights and b) local patch for Kinshasa in Congo River. River pixels inside the local patch are shown in blue and other river pixels are shown in black.
square error (NRMSE) as in Figure S2. There, we found that the 7×7, 11×11, and 15×15 local patches behaved similarly while 21×21 local patch showed large fluctuations in NRMSE. In contrast 31×31 local patch showed large errors after 10 days of simulation. LETKF algorithm may become unstable due to spurious errors due to error covariance after the 10th day (NRMSE > 1.0). Therefore we selected 11×11 and 21×21 local patches for Fixed-Small and Fixed-Large experiments respectively.

3. Very large local patch assimilation

We performed a very large local patch (81 × 81 pixels) OSSE. Figure S3 shows the local patch size and the assimilated WSE for Kinshasa. The WSE values have very high errors after May,

Figure S3. a) Local patch with the number of SWOT observations (colors) and b) time series of WSE of Kinshasa. The Congo River network is shown in black. Red circle indicates the target pixel.
indicating that the assimilation became inefficient. The discharge is very large after around 75 days ($> 1 \times 10^{20}$) at Kinshasa. These spurious errors are due to error covariance between non-significantly correlated areas and the target pixels. Even though this large patch can utilize more observations (similar number of observations on the main stem) than the empirical local patch, it cannot filter out areas affected by error covariance from the assimilation. The empirical local patch only includes locations with significant correlations, particularly along the main stem at Kinshasa. Therefore, increasing conventional local patch size introduces more errors into the assimilation.