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Editorial

Physical and Mathematical Fluid Mechanics

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Abstract: Fluid mechanics has emerged as a basic concept for nearly every field of technology. Despite there being a well-developed mathematical theory and available commercial software codes, the computation of solutions of the governing equations of motion is still challenging, especially due to the nonlinearity involved, and there are still open questions regarding the underlying physics of fluid flow, especially with respect to the continuum hypothesis and thermodynamic local equilibrium. The aim of this Special Issue is to reference recent advances in the field of fluid mechanics both in terms of developing sophisticated mathematical methods for finding solutions of the equations of motion, on the one hand, and on novel approaches to the physical modelling beyond the continuum hypothesis and thermodynamic local equilibrium, on the other.

Keywords: analytical and numerical methods; variational calculus; deterministic and stochastic approaches; incompressible and compressible flow; continuum hypothesis; advanced mathematical methods

1. Introduction

Fluid Mechanics has a long history, going back at least to the era of ancient Greece, when Archimedes [1] investigated fluid statics and buoyancy and formulated his famous law, known now as Archimedes' principle, which was published in his work, "On Floating Bodies"—generally considered to be the first major work on fluid mechanics. Later, E. Torricelli and B. Pascal identified the pressure as a decisive physical quantity [2,3], while I. Newton [4] discovered the viscosity as another physical phenomenon of basic importance, which was later explored by J. L. M. Poiseuille and G. Hagen.

Mathematical fluid dynamics was first introduced by D. Bernoulli [5] and developed further by the mathematicians d'Alembert, Lagrange, Laplace, and Poisson, resulting in the well-known potential flow theory, being nowadays an essential topic in standard fluid dynamics text books [6–9]. Despite the obvious advantage of making various flow problems more tractable, the approach is restricted to inviscid and irrotational flows. A consistent mathematical treatment of viscosity by C.–L. Navier and G. G. Stokes led to the well-known Navier–Stokes equation, which, together with the continuity equation, continues to play the role of the essential field equation in fluid mechanics to this day. Initially, solutions of the Navier–Stokes equation could only be obtained for simple flow geometries until L. Prandtl discovered the mathematical singular boundary layer character of flows with high Reynolds numbers in the vicinity of rigid walls [10]. Prandtl's boundary layer theory and its advancement by T. von Kármán was a keystone both in a mathematical and a physical sense. Another branch of research is related to the formation of chaotic turbulent flow structures due to the nonlinearity of the Navier–Stokes equation, beginning with the early studies of O. Reynolds [11] and later advanced by G. I. Taylor [12] and A. Kolmogorov [13].

Considerable progress in solving the Navier–Stokes equation has been made since the middle of the 20th century, thanks to the availability of computers and the development of efficient numerical methods. Following this, computational fluid dynamics (CFD) has emerged as an essential investigative tool in nearly every field of technology. Despite there being a well-developed mathematical theory and available commercial software codes, the computation of solutions of the governing equations of motion is still challenging, especially due to the nonlinearity involved, giving motivation for further research related to the mathematical and physical foundations.

2. Overview of this Special Issue

Seven articles are published in the issue—four research articles, two reviews, and one technical report, covering a wide range of topics and methodical approaches.

In their research article [14], the formation of coherent vortex structures in a turbulent flow is analysed by direct numerical simulations, followed by image processing techniques and statistical analysis in order to identify and quantify streak characteristics of the flow. Motivated by the aim to complete our knowledge about and the understanding of vortices, the authors compare their findings to three standard vortex models, showing that they all give reasonably close results, and providing a deeper understanding of the interrelationships among different vortex models.

The basic mechanisms underpinning infiltration and drainage of water in soils and the role of viscosity is considered by Germann [15], introducing the basics of Newtonian shear flow in permeable media, presenting experimental applications and exploring the relationships of Newtonian shear flow with Darcy's law, Forchheimer's, and Richards' equations. An extension of the model to the transport of solutes and particles is finally presented.

Acoustic traveling waves in dual-phase media, such as a fluid in a porous solid, are investigated by Jordan [16], utilising the Rubin–Rosenau–Gottlieb theory of generalised continua. Exact and asymptotic expressions for linear and nonlinear poroacoustic waveforms are obtained. Numerical simulations are also presented, where von Neumann–Richtmyer "artificial" viscosity is used to derive an exact kink-type solution to the poroacoustic piston problem, and possible experimental tests of the findings are noted.

As a basic problem with respect to agricultural water resources, the turbulent flow in open channels is studied by [17], who derive a mathematical expression for the characteristic point location of depth average velocity in channels with flat or concave boundaries, particularly rectangular and semi-circular channels. For validation of the analytical model, experiments are carried out through comparison of velocity and discharge.

In their review article, [18] retrace alternative formulations of the Navier-Stokes equation based on potential fields, ranging from the classical potential theory to recent developments in this evergreen research field. The focus is centred on two major approaches which are diametrically opposed in their origin: (i) the Clebsch transformation originally applies to inviscid flow ($Re \rightarrow \infty$), while (ii) the classical complex variable method utilising Airy's stress function applies to Stokes' flow ($Re \rightarrow 0$). It is shown how both methods have been generalised by successive advancements and finally applied to the full Navier-Stokes equation, requiring the extension of the complex variable method to a tensor potential method. Basic questions relating to the existence and gauge freedoms of the potential fields and the satisfaction of the boundary conditions required for closure are addressed; with respect to (i), the properties of self-adjointness and Galilean invariance are of particular interest.

While most research in fluid mechanics is based on the continuum hypothesis, the stochastic variational description, based on the Lagrangian equations of motion in terms of material path lines instead of a field description, has proven to be a remarkable alternative to the classical theoretical, deterministic field approach. An obvious advantage of this approach is that it is very close to classical Newtonian mechanics, where the Lagrange formalism has been successfully established, allowing adoption of many of its features. It also closely refers to kinetic models in statistical physics. Cruzeiro [19] presents a selective review about this research field, regarding the velocity solving the deterministic Navier–Stokes equation as a mean time derivative taken over stochastic Lagrangian paths and obtaining the equations of motion as critical points of an associated stochastic action functional, involving the kinetic energy computed over random paths. Different related probabilistic methods are discussed.

Finally, the technical report by [20] analyses the damage characteristics and mechanisms of a disastrous groundwater inrush that occurred at the Luotuoshan coal mine on 1 March 2010, and gives a detailed overview about this incident in which 32 people lost their lives. The authors see a serious need for improvement in the timely detection of groundwater intrusion and its rapid rectification.

3. Conclusions

The seven publication contributions to the special edition cover a wide range of topics, provide valuable results, and point out open questions and possible future work. [14]'s analysis of the characteristic dimensions of streaky structures and vortices motivates the suggestion of straightforward hypotheses concerning the average width of streaks and the average distance between adjacent streaks, their development from the inner turbulent region to the outer region, the spanwise vortex density, and the coexistence of three different vortex structures as their contribution to improve the understanding of the mechanics of coherent structures in turbulent flows. [15] revealed novel aspects associated with Newton's infiltration that were not considerable in previous approaches to preferential flows, and state that the analytical expressions are amenable to mathematical procedures, such as kinematic wave theory, and their theoretical combinations may lead to new and solid hypotheses calling for experimental testing. Future work on the poroacoustic RRG theory is outlined by [16], who suggests the use of homogenisation methods in problems wherein the coefficients vary with position. Other possible extensions include the poroacoustic generalisation toward power-law fluids. Follow-on work might also include the study of poroacoustic signalling problems involving sinusoidal and/or shock input signals, as well as problems in which changes in entropy and temperature are taken into account. [17] consider an extension of future experimental studies of flow in open channels with regard to wall roughness to be very useful, especially with respect to the transition from smooth channels to vegetation-covered channels. Based on a detailed analysis and discourse, the two different potential approaches considered by [18] can be explained in the light of their different origins. Despite the very positive stage of development of both methods, some open questions remain, for instance, whether a general and all-encompassing potential approach exists, reducing to both the Clebsch and the tensor potential approach as special cases. The search for this "missing link" between two conceptually different approaches represents another future research topic of general interest. An extremely attractive further development of the tensor potential method would be the mapping of the entire problem to a matrix-algebra framework based on quaternions or Dirac matrices with the goal of developing highly efficient methods of solution. Having demonstrated the benefits of probabilistic methods for the study of the deterministic Navier–Stokes equation, Cruzeiro [19] envisages the development of novel numerical methods in the future. Finally, the tragic incident reported by Cui et al. [20] shows the need to detect and prevent such incidents in time with improved prediction models. Mathematical fluid mechanics can make a valuable contribution to this.

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