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A quest for an epistemic reset in higher dimensional space

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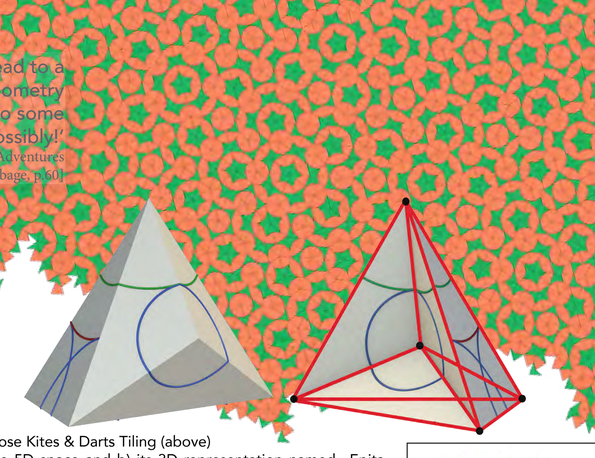
'It cannot help striking me that this extension of Algebra ought to lead to a further extension similar in nature, to the Geometry of three dimensions; & that again perhaps to a further extension into some unknown region & so on ad infinitum possibly!'
[Ada Lovelace in a letter to her tutor Augustus De Morgan, quoted in S. Padua, 2015: The Thrilling Adventures of Lovelace and Babbage, p.50]

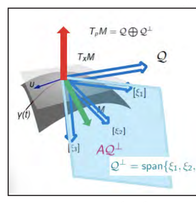


Brilliant mathematical concepts describe physical realities on all different levels, but classical mechanics, relativity, electrodynamics and quantum mechanics are epistemologically separated by different mathematical notations and space concepts: while ART resides in 4-dimensional curved Minkowski space-time, QM requires at least 6 dimensions for one particle in phase space.
— Let's assume, the diverging concepts do not require another formula but rather an epistemic reset which enables reinterpretations of physical phenomena from an unifying point of view:
In this regard visualizations of symmetries of physical phenomena in a higher-dimensional geometrical framework shall be useful.
The here suggested visualization model aims to unite features of different concepts such as Riemannian manifolds and Hilbert spaces, as a discrete dynamic hyper-Euclidean vector space.
It is derived from the geometry of the 5-dimensional space based on the 3D representations of the Penrose kites and darts tiling, which exhibits the principles of the Poincaré homology sphere. This is the space which works like a machine as required by group theory and may serve as a toy model for the universe as suggested by H. Poincaré.
The poster presents some cosmological structures therein such as Lorentzian symmetries in the 4D Kepler model on the large sale as well as for the depiction of 'time'. On the other hand it shows on the quantum level how the same 4-simplices which are constituting the higher-dimensional space, are serving as unitary representation of the Lorentz group in spin foam models for quantum gravity.
— Maybe finally only the assumption of a higher dimensional space-time reality is need?

The advantage of 5D geometries for describing physical phenomena was a.o. strongly recommended by H. Poincaré and L. de Broglie; c.f S. in Lie's theory of continuous transformations the sphere is „situated" in 5-dimensions, Klein's Klein quadratic set ~ of a 5-dimensional projective space P, -- with 24 x 7-fold symmetry which reflects the symmetries of the epita-dodecahedron.The recently developed hyper-Euclidean geometry derived form Penrose tiles (known as a 2D slice of 5-dimensional space) may be useful in testing algebraic structures visually by means of 3D animated geometry which provides an esthetic access to symmetries: it may be regarded as an extension of Mackay's generalized crystallography -- a universal tessellation by means of a fundamental polyhedron which confines Poincaré's idea of a homology sphere homeomorph to a 3 sphere;... or triangles cradled by the *wet nurse of becoming* in Platos *ideal world*? Following Descartes idea to regard the a world as relations and interaction of spaces ---the visualizations below may provide new hints on how Relativity theory with the concept of the simultaneity of events in SRT and ART's gravity in a „curved space time" can be embedded into this higher dimensional framework which exhibits the required symmetries and characteristics of phase space in QM.

'An immediate induction shows us that the classification of algebraic surfaces and the theory of their birational transformations are intimately connected with the classification of real closed hypersurfaces in the space of five dimensions from the point of view of Analysis situs.'
[H. Poincaré (1899),J Rend. Circ. Mat. Palermo, p.19]

Fig.1 a) The Penrose Kites & Darts Tiling (above) as 2D slice of the 5D space and b) its 3D representation named „Epitahedra E±", to be regarded as a manifold of the hyperplane C, it may be considered as a „frozen form" of edges on a hyperplane in movement (c.f. c) picture right, Grassmanian vectors, C.Woo et al. 2014) c.f. „amplituhedron" and as a simplex; the concave epitahedron (E-), here depicted with the matching Schlegel diagram of a 5-simplex (image left in red lines) and overlaid (image: RQ)



'To fathom the problem of matter and its atomic structure, it is without any doubt necessary to place systematically a view point in the Universe with 5-dimensions which seems more fruitful then the one of Mr. Weyl.' [L. de Broglie (1927), L'univers à cinq dimensions et la mécanique ondulatoire. J. Phys. Radium, 1927, 8 (2), pp.65-73. transl. RQ]

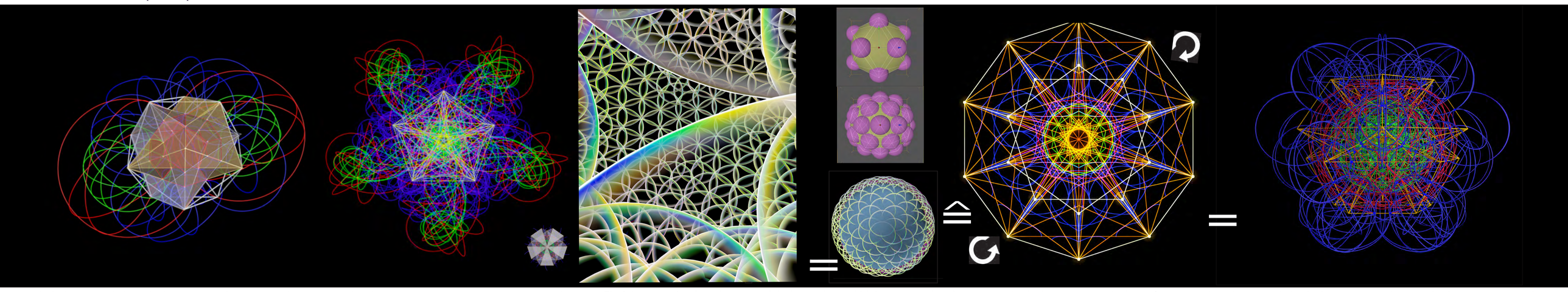


Fig.2 The „entangled epitahedra" picture: a 3D animation depicts two polyhedra E+ (Fig.1 left) in a Boolean intersection, while their faces are in rotation such that the circular decoration (the same like on the Penrose Kites & Darts" with an additional smaller green circle) evolves into complex circles which remind us of pictures of atoms and also of the "entangled photon" picture. Please note that thee dodecahedron appears naturally in the center as inner shape. [image: RQ; Quantum Cinema,

Fig.3 The image captures the traces of a rotation of five epitahedra E+ arranged as a star; around the inner 5-fold axis which appears as big white pentagon. It reminds us on de Broglie's <mécanique ondulatoire> in his 5-dim. univers (1927) (image: RQ; Quantum Cinema, 2012)

Fig.4 A camera view from the center of the hypersphere, which is constructed from an icosahedron (image above right); the sphere ads more and more spheres on the surface (middle picture) until finally on each point of the sphere the radius of a small sphere on the big sphere's surface (S2) contains the center point of the neighbor sphere, which shows only the „upper halve": hemispheres with „imaginary" radius and imaginary motion (moved by factor √-1) for the depiction of time. [image: RQ, 2012 cf.:film, https://vimeo.com/52020495]

Fig.5 Epita-dodecahedron:Twelve epitahedra E+ forming a 4-dimensional dodecahedron,— since it becomes among other polyhedral structures another dodecahedron in the center (also: 26 countable subspaces, also to be considered as 27 dimensional,— therefor a visualization of Jordan algebra could be possible). On this object the principle of counter-rotation of pentagons as described by W. Threlfall and H. Seifert (1931) is applied on the epita-dodecahedron; different evolving symmetries as foretold by H.

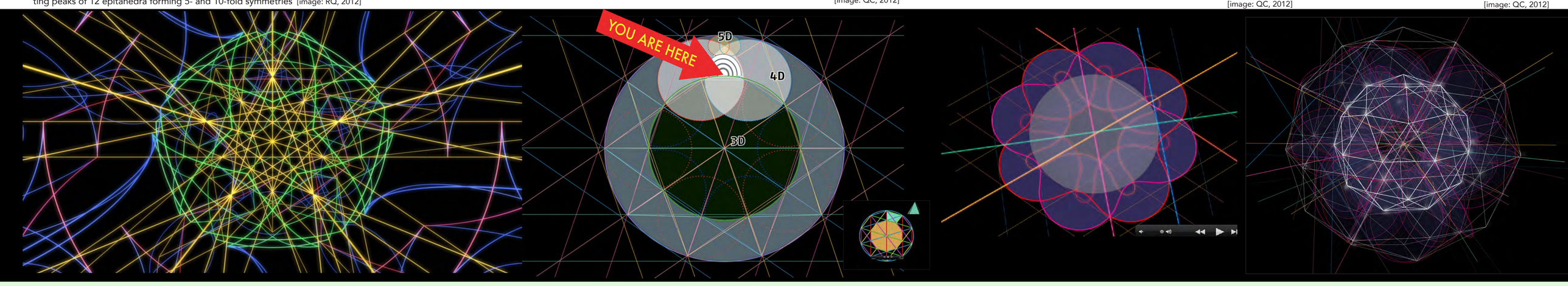
Fig.6 Epita-dodecahedron in rotation The principle of counter-rotation of 12 epitahedra forming the infinite 5-dimensional space, which works like a machine; the circles in different colors indicate again the rotation of the faces and the circular decoration which becomes a complex knot of circles; thus the dodecahedron becomes spherical. [image: RQ, QC2012]

Fig.7 (below) The center of the epita-dodecahedron; one of (infinitely?) many pictures of the twisted intersecting peaks of 12 epitahedra forming 5- and 10-fold symmetries [image: RQ, 2012]

Fig.8 (below) The 6-dimensional space model derived from the Ammann bars, the pentagrid, dual of Penrose Patterns; here quasicrystallography is applied on the world of perception: is depicted on the 2D representation of IR3, a big circle and two small circles on its surface. [image: QC, 2012]

Fig.9 (below) A 2D slice of Fig. 10: a geometrical test for the evolution of waves forming 8-fold symmetries for the depiction of <standing waves> as expressed in the Schrödinger equation [image: QC, 2012]

Fig.10 (below) In the same framework a dicrete 6D space model for one particle in the „quantum world" [image: QC, 2012]



Planetary Systems - spherical in 4D

'Hence, one might regard mechanics as a geometry in four dimensions and analytical mechanics as an extension of geometrical analysis'
[orig.: „Ainsi, on peut regarder la mécanique comme une géométrie à quatre dimensions, et l'analyse méchanique comme une extension de l'analyse géometrique" J.L. Lagrange,(1797).Théorie des fonctions analytiques, p. 337]

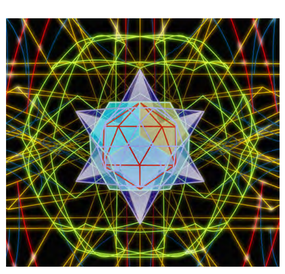


Fig.11 Center of the epita-dodecahedron with overlapping Platonic and Archimedian solids reflecting J. Keplers mysterium cosmographicum, who used a dodecahedron for the position of the earth (1586) [image: QC, 2012]

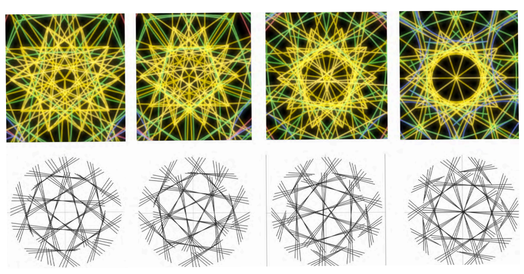


Fig.12 shows patterns of symmetries which appear form the counter-movements of epitahedra in the center of the epita-dodecahedron; they seem to correspond to computer calulations of Venus-Sun views [screenshot, Video RQ. „Epita-dodecahedron (2016, imaginary.org.), + H. Warm, 2011, p.216]

How would a perfectly spherical 5D geometry hold for elliptical orbits?
Recently J. Baez discussed J. Moser and J. Göransson's parametrization of time — by introducing two kinds of time, t for the movement and t' for the observed time —, which leads to circular planetary motions in 4 dimensional sphere, which appear only as ellipses when these circles are projected down to 3-dimensional space.

Jesper Göransson showed how parametrizing time by means of a system of Runge–Lenz vectors leads to conservation of energy :
$(t'-t)^2 + r.\ r' = 1$
This is the formula for a sphere in 4 dimensions.

The Runge–Lenz vector is also responsible for the hamonic oscillation; the intrinsic orbital momentum is similar to the quantum spin with factor √3/2. The relation 3:2 relates also to the 5- or 10-fold symmetry of 5D space, where for positive energy states SO(4) symmetry
E > 0: S0 (3, 1) the Lorentz-group-identity element appears -- just as in SRT & SO(4) symmetry of the hydrogen atom.

In SRT the heavenly sphere is the sphere inherits a conformal structure from the Minkowski metric on IR4 where it can be identified with CP1 where the Lorentz group acts as conformal transformations of
[J.Baez (2001), CP1 and Lorentzian Geometry,online]
it holds for both classical and quantum mechanics as unifying factor which connects QM with cosmology.

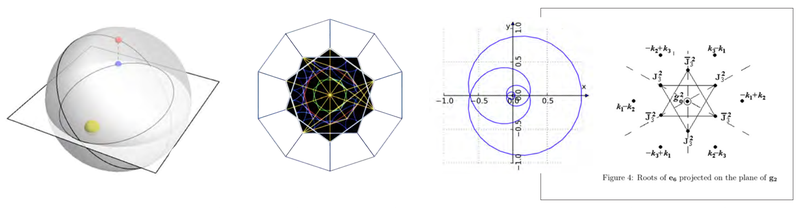


Fig.12 a) Elliptical motion on the great sphere projected on a plane [animation G. Egan] b) Principle of counter-rotation in the epita-dodecahedron c) Poinsot spiral visualizing inverse square law d) Roots of symmetry of E6 projected on G2 [A.Marrani & P. Truini (2014),arXiv:1403.5120v2]

As a philosophical consequence I would suggest to use Noether's theorem, which exhibits a 6-dimensional group of symmetries (c.f. E6 which are depicted as two intersecting triangles) to recognize the same 6-dimensional space in which the 4-dimensional celestial mechanics is embedded should gain epistemic notion: for the conservation of energy basically 6-dimensional systems are required on each level of reality; it holds for both classical and quantum mechanics as unifying factor which connects QM with cosmology.

Gravity obeys the inverse square law which can be depicted as Poinsot spirals which represent the polar equations; similar to the logarithmic spirals which you see in Fig. X for visualizing gravity in T. Kaluza's unification. Moreover it can be used for depicting Coulomb's law which is equivalent with Newtons law, despite positive and negative charges, which are already expressed be the two mirroring spirals. — Therefore from a geometrical point of view there is no discrepancy between Descartes vortex theory, GRT and QM.

'I ... hold that in the physical world nothing else takes place but this variation [of the curvature of space]'
[W. K. Clifford (1870, Mathematical Papers]

Electro-magnetism &„Space-Time"

'There is no absolute space and we only perceive relative movements.'
[H. Poincaré, Science et l'Hypothese, 1900, pp. 458-459]

H.Poincaré explained the principle of relativity by eliminating Newton's absolute space, rest and motion (1902, p. 100) in his popular book Science and Hypothesis (Poincaré,1902) which kept A. Einstein "spellbound for weeks" (M.Solovine, p. XiV).Together with H.A. Lorentz's contraction principle which he took as real, Einstein achieved the mind-blogging Relativity theory which keeps scientists still guessing of how it can be unified with quantum mechanics (QM).
Based on aether theories W.Voigt (1887), H.A. Lorentz (1895, 1899, 1904), J. Larmor (1897, 1900) and H. Poincaré (1905) developed Lorentz-transformations, the symmetries of the Maxwell equations, which proved electro-dynamic equations are invariant under transformations. They van be visualized in the 5D framework which allows to understand T. Kaluza's unification of gravity and eletro-magnetism with the "aid of an 5th dimension". [T. Kaluza (1921), Sitzber.Prss. 966–972]
The 5--dimensional <hyper-Euclidean geometry> exhibits a dual form:

1. a manifold of triangles - also to be considered as Riemannian — arranged in a quasi-crystalline shape derived form the pentagrid with dynamic „bundles" of parallels into 5 directions (cf. PG 5). Here H. Weyl's gauge theory can be applied on the points of intersection.
2. the hyperbolic shape as hyper-sphere; interacting hemispheres on a 2 sphere, conform with S3.
   In a 3D representation Lorentzian manifold can be regarded as dynamic space composition of 4-simplices (Fig.1) forming the tangent space (c.f. gray toric region in Fig.8). There the electro-dynamic process can be depicted in a quaternionic form (Fig. X.

In the framework of SRT the Maxwell equations have the same form in all inertial frames of reference.
As A. Conway pointed out in L.Silberstein's quaternionic relativity theory (1912) that a distinction between time and space is missing and therefor the quaternionic theory is 'more in touch with real phenomena'. Moreover, there There has been some question in the literature about whether special relativity handles rotations correctly: quaternionic rotation is over two points while usually relativistic 4-vectors are depicted at one point of origin.

A. Einstein recognized the consequences of general covariance in GRT.
'Thereby time and space lose the last remnant of physical reality..' [A.Einstein, 1916, Ann. d. Phys. 49: 769–822.]

-- Recall Minkowski's 1908 lecture „M.H.!…":
He first considered to use Hamilton's quaternions, but then decided to install a worldpostulate with a „certain freedom".
This may also allow to connect arbitrary points in space to form a world-line. Minkowski's gift was to realize the dream of his pupil (the „lazy dog") to ride on a ray of light. Recall: relativity theory was first criticized as „signal theory" (G. v. Gleich, 1929) and the amalgam of time and space was ironically said to be inspired by H.G. Wells rather then Palágy (Gilbert, 1916) — but finally it turned out to be the perfect tool for Global Positioning technologies. (GPS) See Fig. X left

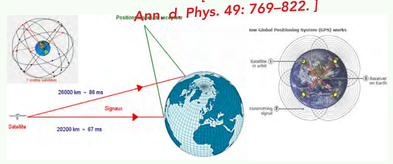


Fig.13 The principle of GPS a connection of arbitray points in a pseudo 4-dim. system (Image: wikipedia/commons)

... and created a new one.

In the framework of general relativity the Maxwell equations or the Hilbert-Einstein field equations have the same form in arbitrary frames of reference. P.A.M. Dirac who constructed an influential relativistic wave equation (1928) described a procedure for eliminating weak equations by fixing the coordinate system in the theory of gravitation. (PAM Dirac, Phys. Rev. 1959) This so-called canonical quantization techniques for constrained Hamiltonian systems is used in canonical quantum gravity.

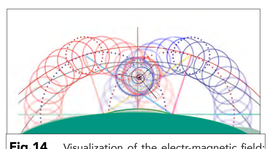


Fig.14 Visualization of the electr-magnetic field: a modification to the symmetry of the electromagnetic field when electrons accelerate away from one another implies that they emit (exchange) virtual photons. [Sketch, QC, 2012]

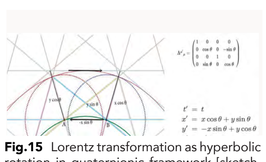


Fig.15 Lorentz transformation as hyperbolic rotation in quaternionic framework [sketch, RQ: 2013]

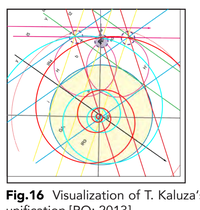


Fig.16 Visualization of T. Kaluza's unification [RQ: 2013]

Figure 14 meets Kleins's assumption hat electromagnetism results from a gravitational field that is "polarized" in the fifth dimension (O. Klein, 1926) depicted in a 2D model where the chiral lines which stand for gravity meet the straight lines (electricity) circles stand for magantic force and the 2 circles in 5D representing gauge group U(1)
O. Klein found that a part of the metric tensor of the fifth dimension behaves like the wave-function of an electrically charged particle which led to the development of QED.

The quaternionic form of SRT embedded into the 6-dimensional framework (Fig.8) depicts a truly 4-dimensional physical event of interchanging spaces fixed on a sphere, — the '1st symmetry breaking', depicted by a pair of quaternions in an imaginary movement √-1 which is denoted as „time".[cf. Hamilton (1853), Lectures on Quaternions,G.T. Fechner (1846) „The space has 4 dimensions"]

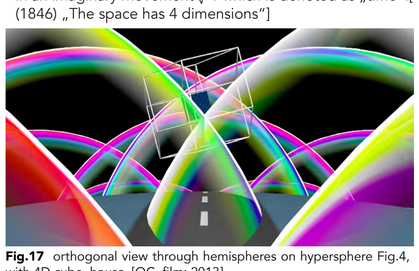


Fig.17 orthogonal view through hemispheres on hypersphere Fig.4, with 4D cube house [QC, film: 2013]

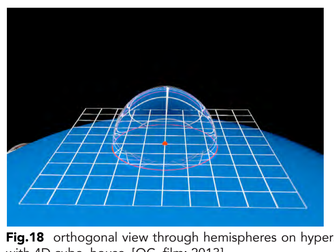


Fig.18 orthogonal view through hemispheres on hypersphere Fig.4, with 4D cube house [QC, film: 2013]

QUANTUM THEORIES

'We examine the question about a space geometry on Plack scale,(...) where quantum mechanics applied on space itself will lead to an discrete amount of quantum states [L. Smolin, The Life of the Cosmos 1997, p. 195]

Quantum mechanics is consistent with special relativity at an operational level but appears to violate it at an ontological level. In relativity, the objective properties of a system are dictated by the causal relations between events in spacetime. While quantum mechanical predictions comply with relativity, objective properties do not satisfy the same causal relations—the phenomenon known as non-locality in QM.
A higher-dimensional epistemology would suggest the idea of a super-position of spaces rather then of <particles /wave-bundles>; analoguous to quasicrystallography spaces of required symmetries for the description of <particles> by means algebraic operations such as 8- and 16 dimensional Clfifford algebra; then QM could be understood and even visualized as REAL permutations of space elements on the Planck scale.

This poposal connects with the famous debate between A. EInstein and de Sitter (1916-17) about 5-dimensional space as physical framework goes back to De Sitter considered a model, in which the spacetime geometry is that of a 4-dimensional hypersphere embedded in a 5-dimensional Euclidean space. about 5-dimensional space as physical model goes back to De Sitter who considered a spacetime geometry where a 4-dimensional hypersphere should be embedded in a 5-dimensional Euclidean space.

Even before the formulation of GRT by A. Einstein (1916) a splitting of five-dimensional spacetime (5D) into the Einstein equations and Maxwell equations in a six-vector and a four-vector was first discovered by G. Nordström (1914) who showed that a unified treatment of the electromagnetic field and the gravitational field is possible if one understands four-dimensional space-time as surface laid through a five-dimensional world.
Also unifying attempts between Relativity theory and gravity in 5 dimensions by T. Kaluza (1921), de Broglie (1927) and O. Klein (1926)
Later attempts by A. Einstein & W. Mayer (1931) and A. Einstein & P. Bergmann (1938) W. Pauli (1933) and P. Jordan (1941) as well as GUTs with symmetry groups SO(5) and SO (10) (c.f.H. Georgi and S. Glashow (1974) which goes comform with the 5-dimensioal space-remained unsatisfying. -- Why?
In fact in 1927 Bohr found the problem solved by O. Klein and his QUANTUM THEORY AND FIVE-DIMENSIONAL RELATIVITY THEORY [Orig. in Z. Phys. 37 (1926) 895].
Fritz London proposed to interpret spin in O. KLEIN's five-dimensional quantum theory, by identifying the canonical conjugate of its fifth dimension with the spin angular momentum. [F. London,"Über eine Deutungsmöglichkeit der Kleinschen fünfdimensionalen Welt," Die Naturwissenschaften, 15 (1927), 15-16]
This interpretation is quite interesting because it leads to the picture of the two entangled epitahedra instead of two overlapping circles, -- which looks like this:

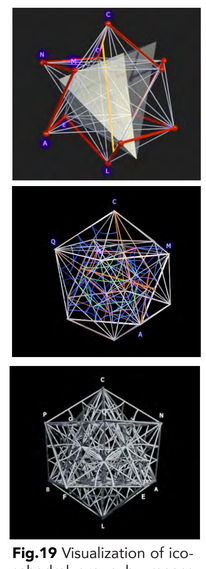


Fig.19 Visualization of icosahedral group by means of E* performing a Hamiltonian path inside the icosahedron. The image below shows a complex intersection of all circular decorations, from permutations of PSL (2, 11)
[Image: QC, 2012]

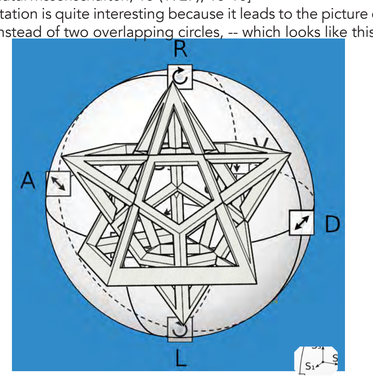


Fig.20 *Poincaré sphere*, illustrates the space of possible polarizations of electromagnetic waves and their corpuscular behavior. The sphere is drawn with three great circles, labels for six basic polarization H (linear horizontal), V (linear vertical), D (linear diagonal), A (linear antidiagonal), R (right-hand circular) and L (left-hand circular) and images of the polarization vectors for each where the (i, 0). The coordinate system of Stokes vectors with components S^1, S^2 and S^3(right below) is replaced by an entangled E+ structure in the center of the sphere which seems to express the Stokes relations (whose coefficient was chosen by Stokes so that they sum up to zero) which is the Pauli sigma x,y, z. By means of this 3D representation of spinors (left), the visualization of three different inter-locked spaces appears as Hyper-Euclidian image.
[image : RQ & Poincaré sphere, (image source: wikipedia creat. commons]

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You are invited to share omments here:

'It is wrong always, everywhere, and for anyone, to believe anything upon insufficient evidence.'
*[**W. K. Clifford** (1879), The Ethics of Belief]*