Article

Quantum Information: What Is It All About?

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Abstract: This paper answers Bell's question: What does quantum information refer to? It is about quantum properties represented by subspaces of the quantum Hilbert space, or their projectors, to which standard (Kolmogorov) probabilities can be assigned by using a projective decomposition of the identity (PDI or framework) as a quantum sample space. The single framework rule of consistent histories prevents paradoxes or contradictions. When only one framework is employed, classical (Shannon) information theory can be imported unchanged into the quantum domain. A particular case is the macroscopic world of classical physics whose quantum description needs only a single quasiclassical framework. Nontrivial issues unique to quantum information, those with no classical analog, arise when aspects of two or more incompatible frameworks are compared.

1. Introduction

A serious study of the relationship between quantum information and quantum foundations needs to address Bell's rather disparaging question, "Quantum information ... about what?" found in the third section of his polemic against the role of measurement in standard (textbook) quantum mechanics [1]. The basic issue has to do with quantum ontology, "beables" in Bell's language. I believe a satisfactory answer to Bell's question is available, indeed was already available (in a somewhat preliminary form) at the time he was writing. (If he was aware of it, Bell did not mention it in any of his publications.) Further developments have occurred since, and I have found this approach to be of some value in addressing some of the foundational issues which have come up during my own research on quantum information. So I hope the remarks which follow may assist others who find the textbook (both quantum and quantum information) presentations confusing or inadequate, and are looking for something better.

Here is a summary of the remainder of this paper. The discussion begins in Section 2 by asking Bell's question about *classical* (Shannon) information: what is *it* all about? That theory works very well in the world of macroscopic objects and properties. Hence if classical physics is fundamentally quantum mechanical, as I and many others believe, and if Shannon's approach is, as a consequence, quantum information theory applied to the domain of macroscopic phenomena, we are already half way to answering Bell's question. The other half requires extending Shannon's ideas into the microscopic domain where classical physics fails and quantum theory is essential. This is possible, Section 3, using a consistent formulation of standard (Kolmogorov) probability theory applied to the quantum domain. Current quantum textbooks do not provide this, though their discussion of measurements, Section 4, gives some useful hints. The basic approach in Section 3 follows von Neumann: Hilbert subspaces, or their projectors, represent quantum properties, and a projective decompositions of the identity (PDI) provides a quantum sample space. By not following Birkhoff and von Neumman, but instead using a simplified form of quantum logic, Section 5, one has, in the "single framework rule" of consistent histories, a means of escaping the well-known paradoxes that inhabit the quantum foundations swamp. Section 6 argues that when quantum theory is equipped with

(standard!) probabilities, quantum information theory is identical to Shannon's theory in the domain of macroscopic (classical) physics, as one might have expected, since only a single quasiclassical quantum framework (PDI) is needed for a quantum mechanical description. However, classical information theory also applies, unchanged, in the microscopic quantum domain if only a *single framework* is needed. Section 7 provides a perspective on the highly nontrivial problems that are unique to quantum information and lack any simple classical analog: they arise when one wants to *compare* (not combine!) two or more *incompatible* frameworks applied to a particular situation.

2. Classical Information Theory

Let us start by asking Bell's question about *classical* information theory, the discipline which Shannon started. What is it all about? If you open any book on the subject, you will soon learn that it is all about probabilities, and information measures expressed in terms of probabilities. So we need to ask: probabilities of what? Standard (Kolmogorov) probability theory, the sort employed in classical information theory, begins with a *sample space* of mutually exclusive possibilities, like the six faces of a die. Next an *event algebra* made up of subsets of elements from the sample space, to which one assigns *probabilities*, nonnegative numbers between 0 and 1, satisfying certain *additivity* conditions.

The simplest situation, quite adequate for the following discussion, is a sample space with a finite number n of mutually exclusive possibilities, let them be labeled with an index j between 1 and n (or 0 and $n-1$ if you're a computer scientist). The event algebra consists of all 2^n subsets (including the empty set) of elements from this sample space. Then for probabilities choose a collection of n nonzero real numbers p_j lying between 0 and 1, which sum to 1. The probability of an element S in the event algebra is the sum of the p_j for j in S.

The mutually exclusive possibilities might be distinct letters of an alphabet used to send messages through a communication channel, and in the actual physical world each letter will be represented by some unique *physical property*(s) that identifies it and distinguishes it from the other letters of the alphabet. One way to visualize this is to think of a classical phase space Γ in which each point γ represents the precise state of a mechanical system, and a particular letter of the alphabet, say F, is represented by some collection of points in Γ, the set of points where the property corresponding to F is true, and where the corresponding *indicator function* $F(\gamma)$ takes the value 1, whereas for all other γ, $F(\gamma) = 0$. The different indicator functions associated with letters of the alphabet then split the phase space up into tiles, regions in which a particular indicator function for a particular letter is equal to 1, and indicators for the other letters are all equal to zero. If this tiling does not cover the entire phase space, simply add another letter to the alphabet, call it "NONE", and let its indicator be 1 on the remaining points, and 0 elsewhere. In this manner, one can map the abstract notion of an alphabet of mutually exclusive letters onto a collection of mutually exclusive physical properties, one and only one of which will be true at any given time, because the point in phase space representing the actual state of the mechanical system will be located in just one of the nonoverlapping tiles. Given the sample space of tiles and some way of assigning probabilities, we have a setup to which the ideas of classical information theory can be applied, with a fairly clear answer to the question of what the information is all about.

In summary, classical information theory is all about probabilities, and in any specific application, say to signals coming over an optical fiber, the probabilities are about, or make reference to, physical events or properties of physical systems.

3. Quantum Probabilities

If we want quantum information theory to look something like Shannon's theory, the first task is to identify a quantum sample spaces of mutually-exclusive properties to which probabilities can be assigned. The task will be simplest if these quantum probabilities obey the same rules as their classical counterparts. In particular, since Shannon's theory employs expressions like $p_j \log(p_j)$, it would

be nice if the quantum probabilities were nonnegative real numbers, in contrast to the negative quasiprobabilities sometimes encountered in discussions of quantum foundations.

Can we identify a plausible sample space which relative to the quantum Hilbert space plays a similar role to a tiling of a classical phase space? (In what follows, I will assume that the quantum Hilbert space is a finite-dimensional complex vector space with an inner product. Thus, all subspaces are closed, and we can ignore certain mathematical subtleties needed for a precise discussion of infinite-dimensional spaces.) A useful beginning is suggested by the quantum textbook approach to probabilities given by the Born rule. Let A be an observable, a Hermitian operator on the quantum Hilbert space, and let

$$A = \sum_j a_j P_j \tag{1}$$

be its spectral representation: the a_j are its eigenvalues and the P_j are projectors, orthogonal projection operators, which form a *projective decomposition of the identity I* (PDI):

$$I = \sum_j P_j; \quad P_j = P_j^\dagger; \quad P_j P_k = \delta_{jk} P_j. \tag{2}$$

If the eigenvalue a_j is nondegenerate and $|\phi_j\rangle$ is the corresponding eigenvector, then

$$P_j = |\phi_j\rangle\langle\phi_j| = [\phi_j], \tag{3}$$

where $[\phi]$ is a convenient abbreviation for the Dirac dyad $|\phi\rangle\langle\phi|$.

According to the textbooks, given a normalized ket $|\psi\rangle$, the probability that when A is measured the outcome is a_j, is given by the Born rule:

$$p_j = \Pr(a_j) = \Pr(P_j) = \langle\psi|P_j|\psi\rangle = |\langle\psi|\phi_j\rangle|^2, \tag{4}$$

where the final equality applies only when P_j is the rank one projector in (3). Now a measurement of A will yield just one eigenvalue, not many, so these eigenvalues correspond to the mutually-exclusive properties P_j in the PDI used in (4). The idea that a quantum property should be associated with a subspace of the Hilbert space, or the corresponding projector, goes back at least to von Neumann, see Section III.5 of his oft-cited (but little read) book [2].

The projector P_j has eigenvalues 0 and 1, so it resembles an indicator function on the classical phase space. In fact, a PDI divides up the Hilbert space into a set of mutually exclusive subspaces—$P_j P_k = 0$ for $j \neq k$—somewhat like a tiling of the classical phase space, whereas $I = \sum_j P_j$ tells us this tiling is complete: no part of the Hilbert space has been left out. Thus, the PDI is a plausible candidate for a quantum sample space. The event algebra will then consist of the projectors in the PDI along with other projectors formed from their sums, including I, along with the zero operator. The result is a commutative Boolean algebra. We already have one scheme, (4), for assigning probabilities to elements of the PDI, and thus, by additivity, to all the projectors in the event algebra. In particular, for $j \neq k$,

$$\Pr(P_j \text{ OR } P_k) = \Pr(P_j) + \Pr(P_k) = \langle\psi|(P_j + P_k)|\psi\rangle, \tag{5}$$

and similarly for sums of three or more distinct projectors.

In summary, this looks like a plausible beginning for a theory of *quantum information*: use a PDI on the Hilbert space as a sample space; then assign probabilities to the individual projectors. Not necessarily using (4), for it is only a particular example, but by some scheme which yields nonnegative real numbers adding to 1. Indeed, this strategy works very well, and I believe it covers all legitimate uses of (standard) probability theory in quantum mechanics, at least for a Hilbert space of finite dimension.

4. Quantum Measurements

There is, of course, more to be said, and it can be motivated by noting that a carefully written quantum textbook is likely to assign the probability p_j not to the *microscopic property* of the measured system, represented by P_j, but instead to the *macroscopic measurement outcome*, the *pointer position* in the picturesque, albeit archaic, language of quantum foundations. However, in the above presentation, it looks as if the probability is assigned *directly* to the microscopic property. Was this a mistake? Not if one believes, as I do, that a properly constructed and calibrated apparatus designed to measure some quantum observable can actually do what it was designed to do. Furthermore, if there is a one-to-one correspondence between prior properties and later pointer positions, the probability p_j will be the same for both.

In support of my belief that quantum measurements measure something, I note that this is assumed by my colleagues who do experiments at accelerator laboratories. They think that when they detect a fast muon emerging from an energetic collision, there really was a fast muon that approached and triggered their detector. Are they being naive? I do not think so. In passing, I note that these colleagues do not seem to worry about the "collapse" of the muon wavefunction produced by its interaction with the detector; they are less interested in what happened to the muon after it left their measuring device, and more interested in knowing what it was doing before it arrived there.

In addition, the notion that outcome j corresponds to the earlier property P_j can in certain cases be tested by *preparing* a particle which has the property P_j (see Section IV C of [3] on the topic of preparation), sending it into the measurement apparatus, and seeing whether the result is that the pointer points to j. Given that the apparatus has been tested and calibrated in this way, is not the experimenter justified in thinking that the particle *had* the property indicated by the pointer in a run in which the particle was *not* prepared in one of the P_j states? Justified or not, this is how many of my colleagues who carry out experiments do interpret things, and if they did not it would be difficult to draw interesting conclusions from their data. Quantum physics can hardly be called an *experimental* science if experiments designed to reveal prior microscopic properties do not actually do so! For additional details on the topic of what quantum measurements measure, including POVM and weak measurements, see [3].

There is, to be sure, a conceptual difficulty lurking in the background if we assume that measurements reveal prior microscopic properties. A hint is provided by the (correct) statement in textbooks that the x and z components of spin angular momentum, S_x and S_z, of a spin-half particle *cannot be measured simultaneously*. True, but what principle lies behind this? If we assume that experimenters really do understand something about what their devices measure, their inability to carry out such a simultaneous measurement might plausibly be explained by the fact that *there is nothing there to be measured.* Even very skilled experimenters cannot measure what is not there; indeed, this could be one thing that distinguishes them from less capable colleagues.

The Hilbert space of a spin-half particle is two-dimensional, and while it contains two subspaces corresponding to $S_x = \pm 1/2$ (in units of $\hbar$), and another two corresponding to $S_z = \pm 1/2$, there is no subspace which can plausibly be associated with, to take an example, "$S_x = +1/2$ AND $S_z = -1/2$". Hence if we assume that quantum measurements measure microscopic properties represented by subspaces of the quantum Hilbert space (or their projectors), we have a ready explanation for what lies behind the assertion that S_x and S_z cannot both be measured simultaneously. This is one way in which quantum mechanics is very different from classical mechanics.

5. Incompatible Properties

5.1. Issues of Logic

The absence of a Hilbert subspace corresponding to "$S_x = +1/2$ AND $S_z = -1/2$" reflects an important difference between the logic of indicator functions on the classical phase space and quantum projectors on the Hilbert space. One analogy has already been noted: the indicator $F(\gamma)$ for

a classical property F takes one of two values, 0 and 1, while a quantum projector P has eigenvalues that are either 0 or 1. In addition, the negation "NOT F" of a classical property has an indicator function $I(\gamma) - F(\gamma)$, where $I(\gamma)$ is the function which is equal to 1 everywhere on the phase space. Similarly, the negation "NOT P" of a quantum projector P is the projector $I - P$, with I the quantum identity operator. However, the analogy begins to break down when we consider the conjunction "F AND G" of two classical properties: the property which is true if and only if both F and G are true. It corresponds to the intersection of the two subsets of phase space points associated with F and G, and its indicator is the product $F(\gamma)G(\gamma)$ of the two indicators. So we might expect that the conjunction "P AND Q" of two quantum properties P and Q would be represented by the product PQ. Indeed, this is the case *if* the projectors P and Q *commute*, $PQ = QP$, in which case PQ is again a projector. However, if PQ is *not* equal to QP, then neither product is a projector, and it is not obvious how to define "P AND Q".

The point can be illustrated using S_x and S_z for a spin-half particle. The projectors representing $S_x = +1/2$ and $-1/2$ are $[x^+] = |x^+\rangle\langle x^+|$ and $[x^-]$, where $|x^+\rangle$ and $|x^-\rangle$ are the eigenvectors corresponding to $S_x = +1/2$ and $-1/2$. Since $\langle x^+|x^-\rangle = 0$ (distinct eigenvalues means the eigenvectors are orthogonal) $[x^+][x^-] = [x^-][x^+] = 0$. Thus, these projectors commute, and the property "$S_x = +1/2$ AND $S_x = -1/2$" is represented by the zero operator on the Hilbert space: the property that is always false and thus never occurs. Also $[x^+] + [x^-] = I$ so these two mutually-exclusive properties constitute a PDI, a quantum sample space. Likewise the projectors $[z^+]$ and $[z^-]$ that correspond to $S_z = +1/2$ and $-1/2$ form a PDI.

However, neither $[x^+]$ nor $[x^-]$ commutes with either $[z^+]$ or $[z^-]$, so we cannot assign a quantum property to "$S_x = +1/2$ AND $S_z = -1/2$" by taking the product of the projectors. Again, this is consistent with the idea that the reason a simultaneous measurement of S_x and S_z is impossible is that there is nothing there to be measured.

5.2. Compatible and Incompatible

Thus, one way, perhaps the most essential way, quantum physics differs from classical physics is that *projectors representing different quantum properties need not commute*. We will say that the projectors P and Q are *compatible* provided $PQ = QP$, and *incompatible* if $PQ \neq QP$. Likewise a PDI $\{P_j\}$ and another PDI $\{Q_k\}$ are *compatible* if every projector in one commutes with every projector in the other: $P_jQ_k = Q_kP_j$ for every j and k. Otherwise, they are *incompatible*. In the compatible case, there is a *common refinement* consisting of all products of the form $P_jQ_k = Q_kP_j$, and every property in the event algebra associated with $\{P_j\}$ or with $\{Q_k\}$ is also in the event algebra associated with this refinement. Hence a very central issue in quantum foundations, and also for quantum information theory if one wants to use PDI's as sample spaces, is what to do when quantum projectors do *not* commute with each other. There have been various approaches.

Von Neumann was well aware of this problem, and together with Birkhoff invented *quantum logic* [4] to deal with it. In the case of a spin-half particle, quantum logic says that "$S_x = +1/2$ AND $S_z = -1/2$" is the property represented by the zero operator; that is, it is meaningful, but it is always false. This means its negation "$S_x = -1/2$ OR $S_z = +1/2$" is always true. Think about it: is that reasonable? If you continue to try and apply ordinary logical reasoning in this situation, you will soon end up in difficulty; see Section 4.6 of [5] for details. To prevent paradoxes, Birkhoff and von Neumann modified some of the rules of ordinary logic. Alas, their quantum logic requires a revision of the rules of ordinary (propositional) logic so radical that no one (known to me) has succeeded in using it to think in a useful way about what is going on in the quantum world. Maybe we physicists are just too stupid, and will have to wait for the day when clever quantum robots with intelligence vastly superior to ours can use quantum logic to resolve the quantum mysteries. However, if they succeed, will they be able to (or even want to) explain it to us?

A second approach to the incompatibility problem is employed in quantum textbooks and is also widespread in the quantum foundations community. Instead of talking about the quantum properties revealed by measurements, discussion is limited to measurement *outcomes*, the pointer positions that

are part of the macroscopic world where classical physics is an adequate approximation to quantum physics, and noncommutation can be ignored for all practical purposes. (More in Section 6 below.) I call this the "black box" approach to quantum foundations. One starts with the *preparation* of a microscopic quantum state using a *macroscopic* apparatus, and then a later *measurement* of the state using another *macroscopic* apparatus, and what lies in between—well, that is inside the black box, and we will say as little as possible about it. A quantum $|\psi\rangle$? That is just a symbolic way of representing the preparation procedure. A PDI $\{P_j\}$? That is nothing but a mathematical tool for calculating the probabilities of measurement outcomes. The black box approach has the advantage that it avoids the problem of noncommuting quantum projectors. Its disadvantage is that it provides no way of understanding in physical terms what is going on at the microscopic level inside the box.

A third approach was popularized by Bell and his followers: replace the *noncommuting* Hilbert space projectors with *commuting* hidden variables. In essence, assume that in some way classical physics applies at the microscopic level. However, if, as I believe, noncommutation of projectors and PDI's marks the frontier between classical and quantum physics, one should not be surprised that an approach which is fundamentally classical—assumes a classical sample space, as is evident from the way the mysterious symbol λ is employed in formulas—results in the famous Bell inequality that disagrees with both quantum mechanical calculations and experimental results. (Nonlocal influences can be ignored, since they do not exist; see [6].)

5.3. The Single Framework Rule

The solution to the incompatibility problem that I favor can be viewed as a lowbrow form of quantum logic, one that a physicist like me can actually make use of. Its essential idea is that as long as one is dealing with a *single* PDI the rules of classical reasoning and classical probability theory can be applied unaltered in the quantum domain. So let us do that. If two PDI's are compatible, there is a PDI which is a common refinement. So let us use it. However, if two PDI's are incompatible, combining them will lead to nonsense. So do not do it. These ideas have been worked out in considerable detail in the *consistent histories* (CH) interpretation of quantum mechanics, where the prohibition against combining incompatible PDI's is known as the *single framework rule*. Here, the term *framework* is used either for a PDI or the associated event algebra, and the single framework rule prohibits combining incompatible PDI's. The difference between CH and quantum logic can be illustrated using the example "$S_x = +1/2$ AND $S_z = -1/2$" discussed earlier. In quantum logic, this is meaningful but false, while in CH it is meaningless, neither true nor false. The negation of a false statement is a true statement, so quantum logic has to say something about it. However, the negation of a meaningless statement is equally meaningless, allowing CH to remain silent. See [7] for more details.

In order to discuss the *time development* of quantum systems, a similar approach can be used (the "histories" part of consistent histories). Once again probabilities are assigned using PDI's as sample spaces, but in this case on an extended Hilbert space of histories [8]. In addition, in order to assign probabilities to a family of histories (a PDI on the history sample space) using an extension of the Born rule, it is necessary to impose certain *consistency* conditions (the "consistent" part of consistent histories), if this family is to constitute an acceptable framework, so the single framework rule is extended to incorporate the consistency conditions. For a short introduction to the CH interpretation of quantum mechanics, see [9]. Various conceptual difficulties are discussed in [7], whereas [10] gives a fairly thorough discussion of the ontology (Hilbert subspaces as "beables"). Finally, reference [5] is a standard reference with lots of details.

One aspect of the CH approach has raised a lot of objections, so it deserves a comment. In a given situation, it may be possible to describe what is going on using various different but incompatible frameworks, so the question arises: "What is the *right* framework to use?" The right answer is that this is the wrong question to ask in the quantum domain. In classical mechanics, the state of a mechanical system at a particular instant of time can be exactly specified by a single point in its phase space, the intersection of all properties (sets of points) which are "true" at that instant. This is consistent

with the idea, which I have elsewhere called *unicity* (Section 27.3 of [5]), that at every instant of time there is a single unique "state of the universe" which, even if we do not know what it is, determines all physical properties. What might be its quantum counterpart? A "wavefunction of the universe"? If there really is something of that sort, it is likely to be a horrible, uninterpretable superposition of different pointer positions at the end of a measurement, or some other form of Schrödinger cat. The corresponding projector will then not commute with properties that might resemble something in the ordinary macroscopic world, and the single framework rule will then prevent discussing the world of everyday experience. I do not see any way in which a single quantum state could plausibly represent the "true state of the world", and I believe unicity must be abandoned in the transition from classical to quantum physics.

In practice, the choice of which framework to use will depend on the problem one is interested in. Consider, for example, a situation in which a spin-half particle is prepared in an eigenstate of S_x, say $S_x = +1/2$, before being sent through a magnetic field-free region (so its spin direction will not change) into an S_z measuring device. The outcome of the measurement will be either $S_z = +1/2$ or $S_z = -1/2$; let us assume the latter. This means we can say that S_z was $-1/2$ just before the measurement took place. However, is it possible that the particle had both $S_x = +1/2$ (because it was prepared in this state) and $S_z = -1/2$ (the value measured later) *at the same time*, just before the measurement was made? This makes no sense, as the properties are incompatible. There is one framework in which at the intermediate time $S_x = +1/2$, reflecting its earlier preparation, and a different, incompatible framework in which at the intermediate time $S_z = -1/2$, reflecting the outcome of the later measurement. These frameworks cannot be combined, and each has its own uses. If we are concerned about whether S_x was perturbed (say by a stray magnetic field), then the S_x framework is helpful, while if we want to identify what the measurement measured, the S_z framework is helpful. In textbook quantum mechanics, only the S_x framework is employed. Nothing wrong with that, except that one cannot discuss in what way the measurement measures something, leaving the poor student rather confused.

This example suggests that the liberty to choose different frameworks is not as dangerous as it might at first appear. A particular choice yields some type of information, and a different choice may yield something different. By looking at a coffee cup from above you can tell if it contains some coffee, while to see if there is a crack in the bottom you need to look from below. The oddity about the quantum world is not that different views, different frameworks, are possible. Instead, it is that certain frameworks cannot be combined into a consistent quantum description, because they are incompatible. For another, less trivial, example of a case in which choosing alternative frameworks proved useful, see the end of Section 7.

6. Quantum Information Theory I

Once a proper *quantum* sample space, a PDI or framework, has been defined, standard (Kolmogorov) probability theory can be used, and this means that the whole machinery of classical (Shannon) probability theory can be imported, unchanged, into the quantum domain. However, the reasoning and the results are restricted to this *single framework*; in particular, they cannot be combined with the analysis carried out in a separate, incompatible framework. Probabilities associated with incompatible frameworks cannot be combined; paying attention to this this eliminates a lot of well-known quantum paradoxes (See Chapters 19 to 25 of [5]).

In particular, this provides a *quantum* justification for all the usual applications of classical information theory to macroscopic properties and their time development. The reason is that from a quantum perspective the classical mechanics of macroscopic objects can be discussed with quite adequate precision using a single *quasiclassical* quantum framework, in which ordinary macroscopic properties are represented by enormous subspaces—a dimension of 10 raised to the power 10^{16} should be counted as relatively small—whose projectors commute with one another for all practical purposes; and quantum dynamics, which is intrinsically stochastic, is well approximated by deterministic

classical dynamics. See [11]; Chapters 7, 17, 18 of [12]; Chapter 26 of [5]; and Section 4 of [10]. Consequently, we can immediately claim that all of classical information theory, all seventeen chapters of Cover and Thomas [13], or name your favorite reference, are a valid part of *quantum* information theory when it is applied to macroscopic properties and processes. In this domain, we understand quite well what *quantum* information is all about: its probabilities refer to quasiclassical properties and processes, all the things for which classical physics provides a satisfactory approximation to a more exact quantum description.

It is worth remarking, in passing, that using a quasiclassical framework provides a solution to the infamous *measurement problem* of quantum foundations: what to do with a wavefunction which is a coherent superposition of states in which the pointer points in two (or more) directions. While in the CH approach there is nothing inherently wrong with such a thing, it can be ignored if one wants to describe the usual macroscopic outcomes of laboratory experiments. Use a quasiclassical framework, and the problems represented by Schrödinger's cat are absent—and, by the single framework rule, they are excluded from the description.

In addition, Shannon's theory can be employed, unchanged, in situations in which some or all of the properties being discussed are *microscopic, quantum properties*, provided the discussion is restricted to a *single framework*. This includes what I have elsewhere [14] referred to as the *second measurement problem*: inferring from the measurement outcome (the pointer position) something about the earlier microscopic state of the system being measured. It can be analyzed in a manner which demonstrates that my colleagues who carry out experiments at accelerator laboratories are not being foolish when they assert that a fast muon has triggered their detector. The measurement apparatus is, in effect, an information channel leading from microscopic quantum properties at the input to macroscopic quantum properties (pointer positions) at the output.

7. Quantum Information Theory II

Does this mean that *all* problems of *quantum* information can be reduced to problems of *classical* information? No, not at all, but it does provide some insight into the nature of the additional problems which are unique to quantum information, and what is needed to attack them. These problems, and there are a vast number, all have to do with *comparing* (but not *combining*!) situations involving *incompatible frameworks*. But how can this be if a strict application of the single framework rule is needed to avoid falling into nonsensical paradoxes? The answer will emerge from considering some examples, starting with that of a noisy quantum channel.

Consider a one-qubit memoryless quantum channel whose input and output is a two-dimensional Hilbert space, the quantum analog of a classical one-bit channel. The classical channel is characterized by two real parameters: the probability that a 0 entering the channel will emerge as a 1, and the probability that a 1 entering the channel will emerge as a 0. If both are zero, the channel is perfect, noiseless. I like to visualize a perfect one-qubit quantum channel as a pipe through which a spin-half particle is propelled in such a way that its spin is left unchanged. If it enters with $S_x = +1/2$ it exits with $S_x = +1/2$, if it enters with $S_z = -1/2$ it exits with $S_z = -1/2$, and so forth. Of course, on any particular run the particle can only have a well-defined spin angular momentum in a particular direction; e.g., it can be prepared in such a state, and when it comes out only one component of its spin angular momentum can be measured. So to *test* whether the channel is perfect, it is necessary to carry out many repeated measurements. This by itself is no different from a classical channel, where repeated measurements are needed to estimate the probabilities of a bit flip when a signal passes through the channel. However, in the quantum case, the probabilities that S_z gets flipped, either from $+1/2$ to $-1/2$, or from $-1/2$ to $+1/2$, can be very different from those for S_x, so repeated measurements need to be carried out using different components of the spin angular momentum. The single framework rule does not prohibit a discussion of both S_x and of S_z *provided* these refer to *different* runs of the experiment. There is no problem in supposing that in one run $S_z = -1/2$, on the next run $S_x = +1/2$,

and so forth. Of course, one has to assume that the channel continues to behave in the same way, at least in a probabilistic sense, during successive runs, but the same is true for a classical channel.

Suppose Joe has built what he claims is a perfect channel, but we want to test it. This is straightforward for a 1-bit classical channel: send in a series of 0s and 1s, and see if what emerges from the channel is the same as what was sent in. A one-qubit quantum channel is more complicated. If we test it using a sequence of states in which $S_z = +1/2$ or $-1/2$, and what emerges is the same as what went in, this is not sufficient, as it could very well be the case that if one sends in $S_x = +1/2$ it will emerge with S_x either $+1/2$ or $-1/2$ in a completely random fashion, uncorrelated with the input. So we have to check something in addition to S_z. Does this mean we have to carry out experiments with $S_w = +1/2$ and $-1/2$ for *every* possible spin component w? That would take a lot of time, and is not necessary. It suffices to check both $S_z = \pm 1/2$ and $S_x = \pm 1/2$. This result is far from obvious, and to derive it one must use principles of quantum mechanics which have no classical analog. Quantum information theorists need not fear unemployment; we will be kept busy for a long time.

As another example, consider teleportation, often presented as an instance of the mysterious and almost magical way in which quantum mechanics goes beyond classical physics. A standard textbook presentation of a protocol to teleport one qubit, e.g., Section 1.3.7 of [15], consists in applying unitary time evolution to an initial quantum state, followed by a measurement which collapses it. The measurement has four possible outcomes, and the result is communicated from A to B through two uses of a perfect one-bit *classical* channel. The end result of the protocol is a quantum state transmitted unchanged from A to B; in effect, a perfect one-qubit *quantum* channel. The student will certainly learn something by working through the formulas in the textbook, but this is of limited value in developing an intuition about microscopic quantum processes. My own approach [16] to understanding teleportation employs two incompatible frameworks. One framework shows how information about S_x is transmitted from Alice to Bob with the assistance of one use of the classical channel, and the other how S_z information is transmitted with the help of the other use of the classical channel. Similar ideas (but without referring to frameworks) will be found in [17,18]. This way of "opening the black box" should, I think, assist students in gaining a better intuition for microscopic quantum processes, and I hope it will become more widespread in the quantum information community, where research, or at least its publication, is still dominated by the "shut up and calculate" mentality encouraged by textbooks.

The preceding example could be easily dismissed in that it did not lead (directly, at least) to any new results in quantum information: the original teleportation protocol [19] appeared fourteen years in advance of my analysis. Hence it may be worth mentioning another example. A student and I were trying to understand Shor's algorithm for factoring numbers, which ends with a quantum Fourier transform followed by measurements of each of the qubits in the standard basis $|0\rangle, |1\rangle$ basis ($|z^+\rangle, |z^-\rangle$ for a spin-half particle). We noted that if you suppose that the final measurement reveals a property that the qubit possessed *before* the measurement, there is a way of looking at the problem that leads to an alternative and simpler way to carry out the algorithm [20]. Our perspective required using a framework incompatible with that employed in the standard textbook approach: unitary time development right up to the moment when measurement "collapses" the wavefunction—which, when done properly, leads to the same final answer. I was pleased that Nielsen and Chuang mentioned our work (Exercise 4.35 on p. 188, and see p. 246 of [15]), but disappointed in that they presented it as part of one more phenomenological principle, rather than as a way of gaining insight by using measurements outcomes to infer something about what happened earlier.

In my opinion, the discipline of quantum information could benefit from paying attention to the developments in quantum foundations mentioned above. If you open your favorite book on quantum information you will discover that measurements are quite firmly embedded in the discussion, and this in the manner of other textbooks in which measurements do not actually measure something, but instead enter as a primitive concept without further definition, a rule for carrying out calculations which requires no real physical understanding of processes at the microscopic quantum level. My guess

is that if quantum information texts were to provide a consistent discussion of microscopic properties and processes, it could lead to some new and interesting advances, and perhaps even some new insights into quantum foundations.

8. Conclusions

Bell's question, "Quantum information ... about what?" can be given a quite definite answer. It is about physical properties and processes, which in quantum theory are represented by subspaces of the quantum Hilbert space, and to which standard (Kolmogorov) probabilities can be assigned, using sample spaces constructed from projective decompositions of the identity operator (PDI's). The single framework rule of consistent histories forbids combining incompatible PDI's or frameworks, resulting in a consistent theory not troubled by unresolved quantum paradoxes. From a quantum perspective, classical (Shannon) information theory is the application of quantum information theory to the domain of macroscopic properties and processes, where a single quasiclassical quantum framework is sufficient for all practical purposes, and therefore quantum incompatibilities can be ignored. However, in addition, all the ideas of classical information, and in particular its probabilistic formulation, can be imported unchanged into the microscopic quantum domain, as long as one is considering only a *single* quantum framework.

That there are many distinct frameworks available in quantum theory, frameworks which cannot be combined but can be compared, represents the new frontier of information theory that is specifically *quantum*, where classical ideas no longer suffice. At this point, new, and sometimes very difficult, problems arise in the process of comparing (but not combining) different incompatible quantum frameworks. They have no analogs in classical information theory, and some of them are quite challenging. Progress in this domain might well benefit were textbooks to abandon their outdated "black box" approach to quantum theory, in which "measurement" is an undefined primitive and measurements do not actually measure anything, but are simply a calculational tool to collapse wavefunctions. It is past time to open the black box with tools that can consistently handle noncommuting projectors. Consistent histories provide one approach for doing this; if the reader can come up with something better, so much the better.

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