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MULTI RESPONSE OPTIMIZATION APPLICATION ON A MANUFACTURING FACTORY

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Abstract- The purpose of real-life problems is often to be able to find less expensive and more effective ways of production without compromising product quality because companies must provide competitive advantage to maintain existence. In order to improve quality, design of experiment techniques is employed. RSM is a widely used technique thanks to its minimum number of experiment requirement. Hence it is used especially with continuous solution spaces and high-cost experimentations. Moreover, in most cases there is more than one response that firms must optimize simultaneously. For instance companies want to reduce the costs while improving product quality. Decision making is more difficult when conflicting objectives exist. For this reason multi response optimization is an important field to study. In this study, optimization of a manufacturing problem with two responses was carried out by the application of response surface methodology (RSM) and desirability function.

Key Words- Multi-Response Optimization, Desirability Function, Response Surface Methodology, MRSM

1. INTRODUCTION

Customers tend to purchase quality products, timely and appropriate prices. In the face of the world's growing needs, firms must provide competitive advantages to maintain existence. Therefore, the purpose of real-life problems is often to be able to find less expensive and more effective ways of production without compromising on product quality. Companies and experts always try to achieve this goal by using statistical and mathematical optimization techniques. Optimization occurs in three different ways. Due to the nature of real life problems Nominal-the-best (N-type) is the most commonly used approach. The goal in N-type optimization is to achieve a certain target value within a predetermined specification for quality characteristic. Other types of optimizations are Smaller-the-better (S-type) and Larger-the-better (L-type). The aims of L-type and S-type approaches are to determine the optimal parameter levels to reach the greatest or smallest value of quality characteristics respectively.

In practice, the values of the parameters may be continuous as well as discrete. While the objectives and process parameters are permanent, the solution space often does not have a linear structure, so the first order models are not enough to identify the objective function curvatures and to create solutions. Thus second order models are needed.

Second-order models have the ability to show, how to behave the quality characteristics of interest on a surface and are capable of determining the best parameter levels. When continuous process variables exist, RSM is an effective method to use with second order models based on statistical and mathematical techniques. Because of this property RSM is widely used in real life problems. Response surface methods have been used in applications such as product quality optimization [1; 3; 7; 8; 12], quality control [19], process optimization [16; 17; 18], ergonomic designs [9], structural reliability [10], and multidisciplinary design optimization problems [4; 11; 13].

2. RESPONSE SURFACE METHODOLOGY (RSM)

Response surface methodology (RSM) is a collection of mathematical and statistical techniques that are useful for modeling and analysis in applications where a response of interest is influenced by several variables and the objective is to optimize this response [6].

RSM also has important applications in the design, development, and formulation of new products, as well as in improvement of existing product designs. RSM, first developed by Box and Wilson in 1951 has been successfully utilized in many industries for the design and improvement of systems where efficient design characteristics are sought [14]. Central Composite Design is the most widely used design technique in second order response surface models, thanks to provide scanning experimental region by a minimum number of experiments and rotatibility feature. In general, Response surface method consists of three phases [15];

Phase 1. Development of an experimental framework
Phase 2. Create response functions – predict the parameters of the functions
Phase 3. Optimization

In the first phase, feature of the objective function to be used is determined and the appropriate experimental design is prepared which provides the ability to retain information necessary for the optimization of the problem and modeling of the objective function. In the second phase, response function which best expresses the data obtained from applied design is generated and the objective function coefficients are predicted. In the last phase, optimum parameter levels determined to obtain the optimal value of the objective function are created in the light of the experiment results.

3. DESIRABILITY FUNCTION

Problems in multi response form have more than one response to a given situation. There are various techniques to optimize multi response problems. One of the most used methods to solve multi response surface problems is the desirability function. Because, optimization of all responses simultaneously is possible by combining them into a single objective function, which basically represents the relationship of all responses that are to be optimized [5].

A desirability function, $D(Y)$, is typically a (weighted) geometric mean of n individual desirability functions, $d_i(y_i)$, one for each element, y_i of Y. Each $d_i(y_i)$ value is converted from associated response y_i and scaled to be between 0 and 1. With a value of zero indicating unacceptable quality and 1 point out that the quality of associated response is optimal. A general form of mathematical relationship of responses with desirability function is as follows;

$$\max D(Y) = (d_1(y_1)^{k_1} \times d_2(y_2)^{k_2} \times \ldots \times d_n(y_n)^{k_n})^{\frac{1}{\sum_i k_i}} \tag{1}$$

y_i denote the determined value of response i, $d_i(y_i)$ is the converted desirability value of i'th response and k_i represent the relative importance of response i compared to others. If all responses have the same importance, then $D(Y)$ become a geometric mean of all n transformed responses without weights. Overall desirability value can only be close to 1 if all of the responses are close to their optimal values, because $D(Y)$ is a geometric mean of the $d_i(y_i)'s$. Likewise, $D(Y)$ will be small if any of the $d_i(y_i)'s$ are sufficiently close to zero. In consequence, to optimize responses simultaneously, one seeks to find values of x to maximize $D(Y)$ [2].

While optimization occurs in three different ways, desirability functions can be determined for any three kinds of questions. Aksezer stated that, weighted linear transformations are flexible in determining the risk associated with deviations from desired response levels. Because of the responses that optimized in this problem are L-type, a larger-the-better desirability function and transformation from Aksezer's study are as follows [5]:

If the response of interest is a kind of maximization problem, then the proposed individual larger-the-better desirability function is

$$d_i(y_i) = \begin{cases} 0 & y_i < LSL \\ \left(\frac{y_i - LSL}{USL - LSL}\right)^s & LSL \le y_i \le USL \\ 1 & y_i > USL \end{cases} \tag{2}$$

where LSL and USL are the lower and upper specification limits of the associated response y_i. The weight exponent s specifies the form of the response within the range of interest. With this desirability function USL automatically becomes the desired maximum value. It is the practical upper bound which any value above this would not improve the response.

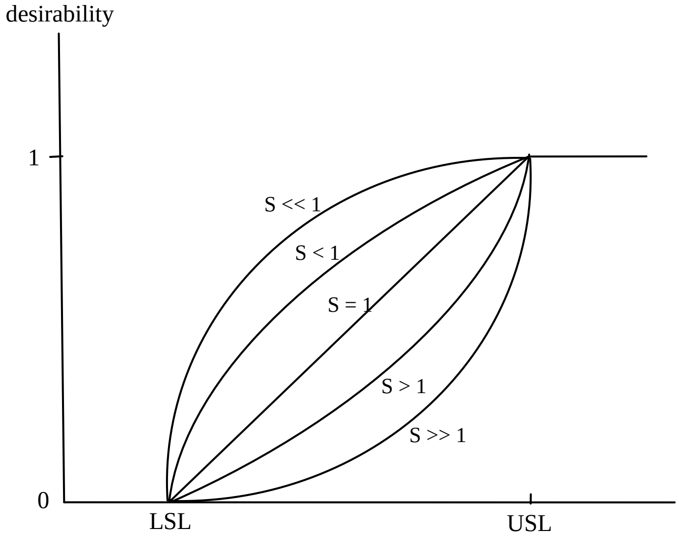

Figure 1. Larger-The-Better Desirability Function

It can clearly be observed from the shape of individual desirability function for various settings of its corresponding parameters. For example; for user specified value $s = 1$ the desirability function increases linearly, for $s < 1$ the function is convex, and for $s > 1$ the function is concave. Note that weight s provide greater flexibility in assigning the individual desirability within the range of interest. While these weight coefficient denote the desired trend of the response within itself, importance coefficient of each response, $k_i's$, associates the priority sequence of all responses so that a comparison between them is possible [5].

4. A CASE STUDY ON A MANUFACTURING FACTORY

4.1. Development of Experimental Framework

There are two responses in the problem. A central composite design application is carried out by taking three factors into account which affect responses. The design consists of a total of 20 experiments of which 6 center points, 8 factorial points, and 6 axial points. Design matrix and experimental results are given in Table 1. Zeros indicate center points, -1 and 1 specify factorial points, -1,6818 and 1,6818 state axial points. Real values of the design factors are not given due to the principle of company information security. Therefore, the design matrix is expressed in encoded values.

Table 1. Design matrix and values of quality characteristics

Std Order	A	B	C	Response 1 Run 1	Response 1 Run 2	Response 2 Run 1	Response 2 Run 2
1	-1	-1	-1	205,577	206,251	109,611	111,017
2	1	-1	-1	201,066	197,119	104,875	111,415
3	-1	1	-1	210,313	208,396	118,438	118,150
4	1	1	-1	200,728	203,660	120,211	117,166
5	-1	-1	1	198,134	196,330	110,400	112,681
6	1	-1	1	193,285	197,232	105,047	104,138
7	-1	1	1	198,134	201,630	107,581	108,709
8	1	1	1	199,938	202,419	109,385	113,332
9	-1,6818	0	0	202,645	205,238	106,115	114,460
10	1,6818	0	0	201,517	201,179	112,317	114,911
11	0	-1,6818	0	199,036	202,870	106,453	108,709
12	0	1,6818	0	203,321	203,209	115,588	110,626
13	0	0	-1,6818	201,292	201,630	112,543	115,926
14	0	0	1,6818	197,457	195,621	98,334	102,324
15	0	0	0	188,925	191,668	105,036	106,341
16	0	0	0	189,653	190,074	108,483	104,049
17	0	0	0	193,172	186,068	105,777	109,273
18	0	0	0	187,647	189,730	107,130	105,438
19	0	0	0	190,127	186,444	106,994	103,995
20	0	0	0	188,436	189,789	104,536	106,115

4.2. Create Response Functions – Predict the parameters of the functions

Using the response data in Table 1, prediction functions for each response were generated via Design-Expert (www.statease.com). Table 2 and Table 3 show ANOVA analyses for both responses. Since the model p values are less than 0.05, both models suggested are significant according to a 95% confidence interval. Equation (3) and equation (4) are the quadratic surface functions for the Response 1 and Response 2, respectively.

Table 2. ANOVA Results for Response 1

Source	***Sum of Squares***	***df***	***Mean Square***	***F Value***	***p-value Prob > F***	***Statement***
Model	1520,0993	7	217,1570	53,5151	< 0.0001	significant
A	52,9826	1	52,9826	13,0568	0.0010	
B	52,8689	1	52,8689	13,0288	0.0010	
C	143,3053	1	143,3053	35,3155	< 0.0001	
AB	2,1477	1	2,1477	0,5086	0.4812	
AC	44,2558	1	44,2558	10,4809	0.0029	
BC	1,0282	1	1,0282	0,2435	0.6253	
A^2	600,3382	1	600,3382	147,9445	< 0.0001	
B^2	551,5400	1	551,5400	135,9189	< 0.0001	
C^2	309,1852	1	309,1852	76,1941	< 0.0001	
Residual	129,8516	32	4,0579			
Lack of Fit	39,9243	7	5,7035	1,5856	0.1858	not significant
Pure Error	89,9272	25	3,5971			
Cor Total	1649,9508	39				

Table 3. ANOVA Results for Response 2

Source	*Sum of Squares*	*df*	*Mean Square*	*F Value*	*p-value Prob > F*	*Statement*
Model	745,0482	9	82,7831	12,0770	< 0.0001	significant
A	0,0011	1	0,0011	0,0002	0.9901	
B	142,4436	1	142,4436	20,7807	< 0.0001	
C	273,1928	1	273,1928	39,8554	< 0.0001	
AB	40,4814	1	40,4814	5,9057	0.0213	
AC	0,9604	1	0,9604	0,1401	0.7108	
BC	57,4034	1	57,4034	8,3744	0.0070	
A^2	158,2770	1	158,2770	23,0906	< 0.0001	
B^2	90,8405	1	90,8405	13,2525	0.0010	
C^2	13,8276	1	13,8276	2,0173	0.1658	
Residual	205,6381	30	6,8546			
Lack of Fit	70,0010	5	14,0002	2,5805	0.0515	not significant
Pure Error	135,6371	25	5,4255			
Cor Total	950,6863	39				

$$f_1(x) = 9582{,}91581 - 24{,}08191 * A - 2{,}32636 * B - 1{,}19363 * C + 0{,}0014546 * AC + 0{,}017069 * A^2 + 0{,}0015275 * B^2 + 0{,}000643318 * C^2 \tag{3}$$

$$f_2(x) = 5497{,}98285 - 13{,}2668 * A - 2{,}07842 * B - 0{,}33764 * C + 0{,}00181776 * AB - 0{,}000496055 * BC + 0{,}00850717 * A^2 + 0{,}000595901 * B^2 \tag{4}$$

4.3. Optimization

Firstly responses are optimized individually. Maximum value for Response 1 is calculated 237,117. In this case, factors A and C stay at their minimum level, while factor B sets at maximum level. Maximization for Response 2 has resulted in 136,909. In this instance, factors A and B set at their maximum level and C minimum level (Figure 2). Maximum value of overall desirability function for this two responses obtained was 0,87. As a result of interviews with company officials, it is understood that simultaneous optimization results provide the requirements. Therefore, the best parameter levels are determined for the process parameters as coded values. In this case factors A and C set at their minimum level (-1,6818) while factor B stays maximum level (1,6818) whereas Response 1 and Response 2 get the values of 237 and 127 respectively.

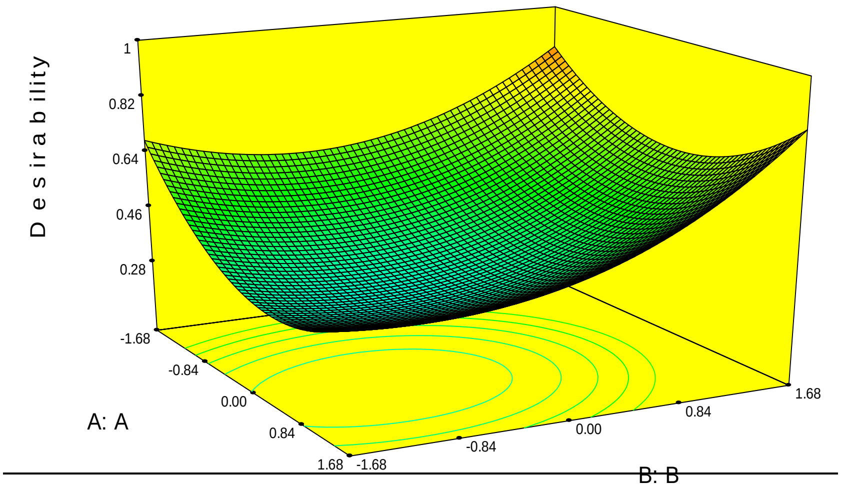

Figure 2. 3D Response Surface Plot for Desirability Function, Actual factor "C" at "-1.6818" level

5. CONCLUSIONS

The application of response surface methodology (RSM) for modeling and optimizing a manufacturing process was discussed. Central composite rotatable design was used to design an experimental plan for modeling the effects of three factors on two responses. A total of 40 experiments including center points were conducted. Results that obtained via designed experiments have entered software package design expert. Response functions were generated and coefficients were predicted by using experimental data. Finally the responses were optimized simultaneously thanks to desirability function. An overall desirability function value of 0,87 obtained. Optimization results were evaluated with company officials. Thanks to this project, company found a way to improve the quality of related products without incurring extra cost. The results show that RSM can be successfully applied to model and optimize real life problems. As future research, we suggest to apply RSM for modeling and optimization of other products of company simultaneously to improve overall product quality.

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