A Comprehensive Approach for Modelling Horizontal Diffuse Radiation, Direct Normal Irradiance and Total Tilted Solar Radiation Based on Global Radiation under Danish Climate Conditions

Zhiyong Tian, Bengt Perers, Simon Furbo, Jianhua Fan, Jie Deng* and Janne Dragsted

Department of Civil Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark; zhiytia@byg.dtu.dk or tianzy0913@163.com (Z.T.); beper@byg.dtu.dk (B.P.); sf@byg.dtu.dk (S.F.); jif@byg.dtu.dk (J.F.); jaa@byg.dtu.dk (J.D.)

* Correspondence: deng-jie2@163.com; Tel.: +45-45251889

Received: 17 April 2018; Accepted: 5 May 2018; Published: 22 May 2018

Abstract: A novel combined solar heating plant with flat plate collectors (FPC) and parabolic trough collectors (PTC) was constructed and put into operation in Taars, 30 km north of Aalborg, Denmark in August 2015. To assess the thermal performance of the solar heating plant, global radiation, direct normal irradiance (DNI) and total radiation on the tilted collector plane of the flat plate collector field were measured. To determine the accuracy of the measurements, the calculated solar radiations, including horizontal diffuse radiation, DNI and total tilted solar radiation with seven empirical models, were compared each month based on an hourly time step. In addition, the split of measured global radiation into diffuse and beam radiation based on a model developed by DTU (Technical University of Denmark) and the Reduced Reindl correlation model was investigated. A new method of combining empirical models, only based on measured global radiation, was proposed for estimating hourly total radiation on tilted surfaces. The results showed that the DTU model could be used to calculate diffuse radiation on the horizontal surface, and that the anisotropic models (Perez I and Perez II) were the most accurate for calculation of total radiation on tilted collector surfaces based only on global radiation under Danish climate conditions. The proposed method was used to determine reliable horizontal diffuse radiation, DNI and total tilted radiation with only the measurement of global radiation. Only a small difference compared to measured data, was found. The proposed method was cost-effective and needed fewer measurements to obtain reliable DNI and total radiation on the tilted plane. This method may be extended to other Nordic areas that have similar weather.

Keywords: Danish climate conditions; solar radiation models; horizontal diffuse radiation; direct normal irradiance (DNI); total radiation on the tilted surface

1. Introduction

Energy consumption in the building sector accounts for about 40% of society’s energy consumption in developed countries. Using renewable energy, especially solar energy, for heating and cooling in the building sector is a promising way to reduce the fossil energy consumption of buildings [1,2]. Solar thermal energy is one of the most commercial renewable energies in the building sector [3,4]. Large solar heating plants connected to district heating networks have been of great success in Europe, especially in Denmark. Most large scale solar heating plants in Europe, even worldwide, are constructed in Denmark. Denmark is the first and the only country with a mature commercial market for solar district heating plants. By the end of 2016, more than 1.3 million m² solar heating...
plants were in operation in Denmark [5]. Real-time solar radiation data is widely used as a basic input to control large-scale solar collector fields across thousands of square meters during their lifetime. Furthermore, accurate solar radiation data are very important for designing solar heating systems and estimating the thermal performance of solar district heating plants. Compared to global irradiance, the direct beam component shows much more variability in space and time. Global radiation split into beams and diffuse radiation on the collector plane is important for evaluation of the performance of different collector types and collector field designs as well. In the past, in most cases, inexpensive and inaccurate solar radiation sensors have been used to measure solar radiation on collector planes in solar district heating plants in Denmark. Few technicians onsite in solar heating plants have paid much attention to the accuracy of measurements about solar radiation. Poor solar radiation may result in the wrong control strategies for such large-scale solar collector fields, which can influence the cost-performance of solar heating plants significantly. A simple model for accurately modelling solar radiation is needed to double-check solar radiation measurements onsite, in a cost-effective and fast way.

1.1. State of the Art

Generally, climate stations measure global radiation and, only in rare cases, accurate DNI or diffuse solar radiation on the horizontal surface. Therefore, total irradiation on tilted surfaces and DNI in most cases is calculated using measured global irradiation by means of empirical models for general use. Shukla et al. [6] carried out a comparative study of isotropic and anisotropic sky models to estimate solar radiation incidence on tilted surfaces in India. Demain et al. [7] evaluated 14 empirical models to predict global radiation on inclined surfaces. A hybrid model from the coupling of three models under different sky conditions have been developed for Belgium. Khorasanizadeh et al. set up a new diffuse solar radiation model to determine the optimum tilt angle of surfaces in Tabass, Iran [8]. Marques Filho et al. carried out observational charaterisation and empirical modelling of global radiation, diffuse and direct solar radiation on surfaces in the city of Rio de Janeiro [9]. El Mghouchi et al. evaluated four empirical models to predict daily direct diffuse and global radiation in Tutuan city, north of Morocco [10]. Jakhrani et al. investigated the accuracy of different empirical models for calculating total solar radiation on tilted surfaces [11]. It was found that the isotopic model (Liu and Jordan model) was better for the prediction of solar energy radiation in cloudy weather conditions and could be used to calculate available solar radiation on tilted surfaces in overcast skies under Malaysian climate conditions. El-Sebaii et al. also calculated diffuse radiation on horizontal surfaces and total solar radiation on tilted surfaces using empirical models [12]. They [12] also found that the isotropic model (Liu and Jordan model) could be used to calculate total radiation on tilted surfaces with good accuracy in Jeddah, Saudi Arabia. Gopinathan investigated solar radiation on variously oriented sloping surfaces in Lesotho, South Africa, with the isotropic model [13]. Li et al. carried out estimation of daily global solar radiation in China [14]. Alyahya et al. analysed the new solar radiation Atlas for Saudi Arabia [15]. Bird et al. developed a simple solar spectral model for direct and diffuse irradiance on horizontal and tilted planes on the earth’s surface for cloudless atmospheres [16]. There have also been several studies on the prediction of solar radiation using machine learning and multivariable regression methods [17,18]. Despotovic et al. [19] investigated the accuracy of different empirical models in predicting total tilted solar radiation and diffuse horizontal solar radiation, respectively. Ineichen concluded that the Perez model is slightly better (in terms of RMSD) than other models in any case, even with synthetic data [20]. Gueymard et al. [21] carried out a comprehensive evaluation study of the performance of 140 separation models selected from the literature to predict direct normal irradiance from global horizontal irradiance. The evaluation was based on measured, high-quality, 1-min data of global horizontal irradiance and DNI at 54 research-class stations from seven continents. Only two models consistently delivered the best predictions over the arid, temperate and tropical zones and no model performed consistently well over the high-albedo zone. A comparative study of the impact of horizontal-to-tilted solar irradiance
conversion in modelling small PV array performance was presented in [22]. A neural network model was employed to predict daily direct solar radiation in [23]. Frydrychowicz-Jastrzębska et al. compared selected isotropic and anisotropic mathematical models to calculate the distribution of solar radiation on the photovoltaic module plane with any spatial orientation for Poland [24].

Mubarak et al. [25] compared five empirical models for PV applications. The authors concluded that the models of Hay and Davies and Reindl are recommended to estimate tilted irradiance for south-facing modules in regions with mainly cloudy conditions and when albedo measurements are not available. The Hay and Davies model is useful for vertical surfaces (e.g., facades and glazing), whereas the Perez model is recommended for sunny sites and when albedo measurements are available. Lee et al. [26] investigated solar radiation models to estimate direct normal irradiance for Korea. The Reindl-2 model was selected as the best among the evaluated ten existing models for Korea. Different conclusions can be drawn for different locations. Using previous empirical models to convert global solar radiation data for general use in high latitude areas, such as Denmark, may not give highly accurate results. Furthermore, limited literature was found on the analysis and prediction of total tilted solar radiation and DNI at high latitudes. A novel combined solar heating plant with a 4039 m$^2$ parabolic trough collector field and a 5960 m$^2$ flat plate collector field in Taars was put into operation in August 2015 [27]. To evaluate the thermal performance of the plant and the accuracy of the calculated solar radiation, total tilted and horizontal solar radiation was measured in the collector field. In addition, a weather station was in operation close to the solar collector fields to ensure that the pyranometers in the plant had correct values to reduce systemic errors and to measure direct normal irradiance (DNI).

Diffuse radiation influences the thermal performance of the flat plate collector field. In this study, diffuse horizontal radiation was estimated using the RR model (Reduced Reindl correlation model) [28] and the DTU (Technical University of Denmark) model [29]. The DTU model was developed based on measurements from 2006–2010 at a climate station at DTU [29] and was used in this paper to calculate diffuse radiation on the horizontal surface with only global radiation as an input. The RR model was developed by Reindl in 1990 for general use to calculate diffuse radiation on the horizontal surface with only global radiation as an input [30]. These two models were compared to the measured data from the Taars plant.

When diffuse radiation on the horizontal surface has been calculated, direct radiation on the same surface can be derived by the subtraction of beam radiation from global radiation. DNI can then be determined by dividing by the cosine of the zenith angle indirectly. The last two steps for direct radiation are exact numerical conversions without calculation error.

1.2. Scope and Objective

The novel contributions of this paper are as follows: (1) A solar radiation model developed for the Danish climate conditions in the Nordic area was validated; (2) The measured data were from a pilot solar heating plant, not a laboratory, which is more practical and the whole chain of calculations, up to the long term performance of the solar collectors, can be validated; and (3) DNI, diffuse radiation on the horizontal surface and total radiation on the tilted surface during the whole year were analysed.

This paper validated the performance of the DTU model for the derivation of horizontal diffuse irradiance and beam radiation based on more widely available global horizontal irradiance data under Danish climate conditions. One isotropic model and four anisotropic models for the calculation of total tilted radiation were also investigated. The difference between measured solar radiation and modelled solar radiation estimated by the empirical formulas under Danish climate conditions, including DNI, diffuse horizontal radiation and total tilted solar radiation, were shown. MBE, RMSE, MAPE and RPE were used to assess the feasibility of the investigated empirical models under Danish climate conditions. Calculated total tilted radiation only based on global horizontal radiation and on both global horizontal radiation and beam radiation were discussed, which could provide a new method to calculate total tilted radiation with less measurements under Danish climate conditions and may be
extended to other Nordic areas that have similar weather. The combined method to calculate total tilted solar radiation could be a useful tool for design large-scale solar district heating plants, which have been of great success in Denmark.

The aim of this article was to develop a solar radiation model to predict DNI and total tilted solar radiation accurately for solar thermal systems under Danish climate conditions.

The structure of the article is summarised as follows: Section 1 is the introduction; Section 2 is the introduction of the measurements; Sections 3 and 4 present the method and detailed empirical models used in this study; Section 5 shows the validation of the empirical models; Section 6 shows the predicted DNI and total tilted solar radiation of the selected models; Section 7 is the discussion; and Section 8 is the conclusion.

2. Data Collection and Location Description

As is shown in Figure 1, Denmark has six solar radiation zones with different yearly global radiation, around 1000–1200 kWh/m². The Taars plant is located in the first solar radiation zone, in the northern part of Denmark [31–33]. Figure 2 illustrates the locations of the weather station and the pyranometers in the flat plate collector field. The weather station is next to a solar heating plant. There are several pyranometers to measure global solar radiation and total radiation on the tilted plane of the flat plate collectors in the middle of flat plate collector field (Figure 1). The latitude of Taars is 57.39°N and the longitude is 10.11°E.

As is shown in Figures 2 and 3, four south facing pyranometers with a tilt 50° were installed on the top of a flat plate collector plane in the middle of the flat plate collector field [35]. Three Apogee Pyranometer SP-110 are used as backup sensors to double check the measured total radiation, as is shown in the Figure 3 left. Two of the pyranometers to measure solar radiation on the horizontal surface and solar radiation on the tilted collector plane were Kipp & Zonen SMP11 (see Figure 3), which are used in this study. DNI was measured with a PMO6-CC pyrheliometer with the sun tracking platform Sunscanner SC1 in the weather station next to the solar heating plant (see Figures 2 and 4). Tables 1 and 2 show the technical specifications of the Kipp & Zonen SMP11 pyranometer and PMO6-CC pyrheliometer [36,37]. The measurements, including global radiation, total tilted solar radiation and DNI, were recorded in 2 minutes intervals from the middle of August 2015.

![Figure 1. Location of Taars in Denmark [34].](image-url)
As is shown in Figures 2 and 3, four south facing pyranometers with a tilt 50° were installed on the flat plate collectors in the middle of flat plate collector field. The weather station is next to the northern part of Denmark, around 57.39° N and the longitude is 10.11° E. The latitude of Taars is 1000 kWh/m². The structure of this article is summarized in Section 5.

**Figure 2.** Location of the weather station and pyranometers (PTC: parabolic trough collector, FPC: flat plate collector) [27].

**Table 1.** Specifications of Kipp & Zonen SMP11 pyranometer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral range (50% points)</td>
<td>285 to 2800 nm</td>
</tr>
<tr>
<td>Response time (63%)</td>
<td>&lt;0.7 s</td>
</tr>
<tr>
<td>Response time (95%)</td>
<td>&lt;2 s</td>
</tr>
<tr>
<td>Zero offset A</td>
<td>&lt;7 W/m²</td>
</tr>
<tr>
<td>Zero offset B</td>
<td>&lt;2 W/m²</td>
</tr>
<tr>
<td>Directional response (up to 80° with 1000 W/m² beam)</td>
<td>&lt;10 W/m²</td>
</tr>
<tr>
<td>Temperature dependence of sensitivity (−20 °C to +50 °C)</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Analogue output (−V version)</td>
<td>0 to 1 V</td>
</tr>
<tr>
<td>Analogue output (−A version)</td>
<td>4 to 20 mA</td>
</tr>
</tbody>
</table>
As is shown in Figures 2 and 3, four south facing pyranometers with a tilt 50° were installed on the top of a flat plate collector plane in the middle of the flat plate collector field [35]. Three Apogee Pyranometer SP-110 are used as backup sensors to double check the measured total radiation, as is shown in the Figure 3 left. Two of the pyranometers to measure solar radiation on the horizontal surface and solar radiation on the tilted collector plane were Kipp & Zonen SMP11 (see Figure 3), which are used in this study. DNI was measured with a PMO6-CC pyrheliometer with the sun tracking platform Sunscanner SC1 in the weather station next to the solar heating plant (see Figures 2 and 4). Tables 1 and 2 show the technical specifications of the Kipp & Zonen SMP11 pyranometer and PMO6-CC pyrheliometer [36,37]. The measurements, including global radiation, total tilted solar radiation and DNI, were recorded in 2 minutes interval from the middle of August 2015.

Figure 3. The pyranometers in the middle of the flat plate collector field. (a) Weather station; (b) the weather sensors including pyrheliometer; and (c) used pyrheliometer and sunscanner.

Table 2. Specifications of PMO6-CC pyrheliometer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>80 × 80 × 230 mm</td>
</tr>
<tr>
<td>Mass</td>
<td>2.15 kg</td>
</tr>
<tr>
<td>Field of view (full angle)</td>
<td>5°</td>
</tr>
<tr>
<td>Slope angle</td>
<td>1°</td>
</tr>
<tr>
<td>Range</td>
<td>up to 1400 W/m² (or custom design available)</td>
</tr>
<tr>
<td>Traceability to WRR</td>
<td>&lt;0.1%</td>
</tr>
<tr>
<td>Operating temperature range</td>
<td>−25 °C to +50 °C</td>
</tr>
</tbody>
</table>

3. Methodology

Figure 5a gives a schematic illustration of this study. Firstly, the DTU model and RR models were used to calculate horizontal diffuse radiation based on measured global radiation. Five calculation models for total radiation on tilted surfaces for general use were investigated: one isotropic model and four anisotropic models. Circumsolar diffuse and horizon-brightening components on the tilted surfaces were taken into consideration in the anisotropic models, but not in the isotropic model. Calculated total tilted solar radiation of the empirical models based on measured global radiation and DNI was derived. Measured total tilted radiation was used to evaluate the suitability of the empirical models for measuring total tilted radiation in Danish conditions (see the validation cycle in Figure 5a), as elaborated on in Section 5.
4. Empirical Models

DNI, global radiation and total tilted solar radiation on the top of a 50° tilted, south facing collector were measured with a high time resolution of 2 min. Hourly mean values were calculated based on the measured values. Calculated solar radiation in this study was based on the mean data of a 1 h time step. Both the DTU model and the RR model were used to calculate diffuse radiation on the horizontal surface. Five other empirical models (one isotropic model and four anisotropic models) were used to calculate total solar radiation on the tilted surface. Ground reflectance or albedo was assumed to be 0.1. This value was a reasonable estimation of ground reflectance when shadows between collector rows in the solar heating plant were considered.

4.1. Measured Horizontal Diffuse Radiation

Diffuse radiation on the horizontal surface was not measured directly in the Taars plant. However, diffuse radiation can be derived accurately as the difference between total radiation and beam radiation. Measured beam radiation was calculated by measured DNI and solar zenith angle. Then, the selected empirical models based on calculated diffuse radiation and beam radiation were employed to calculate total tilted radiation (Figure 5b), as described further in Section 6. Calculated total tilted radiation using the DTU model and the investigated empirical models (Perez models) only based on global radiation showed good agreement with the measured values from September 2015 to August 2016 (Figure 5b). DNI calculated by the DTU model also had good agreement with measured DNI. In summary, the proposed method to calculate total tilted solar radiation only based on measured global horizontal radiation (red flow chart) is a new, simple and cost-effective approach to obtain accurate total tilted solar radiation for Danish conditions, as measured global radiation data is always available from climate stations. Furthermore, DNI and diffuse radiation measurements are relatively costly both in terms of equipment and manpower. Accurate long-term data for these variables are seldom available in most cases. Therefore, accurate calculated DNI, diffuse radiation and total
tilted radiation based only on measured global radiation using the method proposed in this paper is very valuable.

4. Empirical Models

DNI, global radiation and total tilted solar radiation on the top of a $50^\circ$ tilted, south facing collector were measured with a high time resolution of 2 min. Hourly mean values were calculated based on the measured values. Calculated solar radiation in this study was based on the mean data of a 1 h time step. Both the DTU model and the RR model were used to calculate diffuse radiation on the horizontal surface. Five other empirical models (one isotropic model and four anisotropic models) were used to calculate total solar radiation on the tilted surface. Ground reflectance or albedo was assumed to be 0.1. This value was a reasonable estimation of ground reflectance when shadows between collector rows in the solar heating plant were considered.

4.1. Measured Horizontal Diffuse Radiation

Diffuse radiation on the horizontal surface was not measured directly in the Taars plant. However, diffuse radiation can be derived accurately as the difference between total radiation and beam radiation. Measured beam radiation was calculated by measured DNI and solar zenith angle using Equation (1). Measured diffuse radiation on the horizontal surface was determined as the difference between the measured global radiation and beam radiation components, indirectly, using Equation (2).

\[ G_b = \text{DNI} \times \cos \theta_z \]  
\[ G_d = G - G_b \]

4.2. Modelled Horizontal Diffuse Radiation

(1) DTU model

Dragsted et al. measured and analysed solar radiation from a climate station at the Technical University of Denmark from 2006 to 2010, and developed an empirical model to calculate horizontal diffuse radiation from global radiation on the horizontal surface for Danish climate conditions [29]. The empirical model is as follows in Equations (3)–(7):

\[ K_T = \frac{G}{G_0} \]  
\[ \frac{G_d}{G} = -0.60921K_T^3 + 1.9982K_T^2 - 0.2787K_T + 1, \quad 0.00 \leq K_T < 0.29 \]  
\[ \frac{G_d}{G} = 3.99K_T^3 - 7.1469K_T^2 + 2.3996K_T + 0.746, \quad 0.29 \leq K_T < 0.72 \]  
\[ \frac{G_d}{G} = 288.63K_T^4 - 625.26K_T^3 + 448.06K_T^2 - 105.84K_T, \quad 0.72 \leq K_T < 0.80 \]  
\[ \frac{G_d}{G} = 65.89K_T^4 - 210.69K_T^3 + 222.91K_T^2 - 77.203K_T, \quad 0.80 \leq K_T < 1.20 \]

(2) Reduced Reindl correlation model

The Reduced Reindl correlation model is based on the relationships developed by Reindl et al. [30]. The Reduced Reindl model uses clearness index and solar altitude angle to estimate diffuse radiation on the horizontal surface. The correlation is given by Equations (8)–(10):

\[ \frac{G_d}{G} = 1.020 - 0.254K_T + 0.0123 \sin \alpha, \quad 0 \leq K_T \leq 0.3, \quad G_d/G \leq 1.0 \]  
\[ \frac{G_d}{G} = 1.400 - 1.794K_T + 0.177 \sin \alpha, \quad 0.3 < K_T < 0.78, \quad 0.1 \leq G_d/G \leq 0.97 \]  
\[ \frac{G_d}{G} = 0.486K_T - 0.182 \sin \alpha, \quad 0.78 \leq K_T, \quad 0.1 \leq G_d/G \]
4.3. Modelled Total Tilted Solar Radiation

(1) Isotropic model

The typical isotropic model was developed by Liu and Jordan (Liu–Jordan model; Equations (11) and (12)) [38] and has been used widely in recent decades. The isotropic model assumes that diffuse radiation is uniformly distributed over the complete sky dome and that reflection on the ground is diffuse.

\[ R_b = \frac{\cos \theta}{\cos \theta_2} \]  
\[ G_T = G_b R_b + G_d \left( \frac{1 + \cos \beta}{2} \right) + G_{\rho_g} \left( \frac{1 - \cos \beta}{2} \right) \]  

(11), (12)

(2) Anisotropic model

(a) Hay and Davies model (HD model)

The Hay and Davies model (Equations (13) and (14)) accounts for both circumsolar and isotropic diffuse radiation [39,40]. Horizon brightening is not taken into account. There is an increased intensity of diffuse radiation in the area around the sun (circumsolar diffuse radiation). An anisotropy index \( A_i \) is introduced in the HD model to weight the amount of circumsolar diffuse radiation. The anisotropy index is used to quantify a portion of the diffuse radiation treated as circumsolar, with the remaining portion of diffuse radiation assumed isotropic. The circumsolar component is assumed to be from the sun’s position.

\[ A_i = \frac{G_b}{G_0} \]  
\[ G_T = (G_b + G_d A_i) R_b + G_d (1 - A_i) \left( \frac{1 + \cos \beta}{2} \right) + G_{\rho_g} \left( \frac{1 - \cos \beta}{2} \right) \]  

(13), (14)

(b) Hay, Davies, Klucher and Reindl model (HDKR model)

A horizon brightening diffuse term was added to the HD model by Reindl et al. in the HDKR model [39]. Horizon brightening is combined with the isotropic diffuse term and the magnitude is named by a modulating factor \( \sqrt{G_b/G_0} \), as is shown in the Equation (15).

\[ G_T = (G_b + G_d A_i) R_b + G_d (1 - A_i) \left( \frac{1 + \cos \beta}{2} \right) \left( 1 + \sqrt{\frac{G_b}{G_0}} \sin^3 \left( \frac{\beta}{2} \right) \right) + G_{\rho_g} \left( \frac{1 - \cos \beta}{2} \right) \]  

(15)

(c) Perez I model

Compared to the other models described, the Perez model is more computationally intensive and represents a more detailed analysis of isotropic diffuse, circumsolar and horizon brightening radiation by using empirically derived coefficients [41]. Perez et al. developed the model accounting for circumsolar, horizon brightening and isotropic diffuse radiation with an empirically derived “reduced brightness coefficient” [42] in 1988. This is called the Perez I model, and is given by Equations (16)–(21). The coefficients of the Perez I model are listed in Table 3.

\[ G_T = G_b R_b + G_d \left[ (1 - F_1) \left( \frac{1 + \cos \beta}{2} \right) + F_1 \left( \frac{a/c}{c} \right) + F_2 \sin \beta \right] + G_{\rho_g} \left( \frac{1 - \cos \beta}{2} \right) \]  

(16)

\[ a/c = \frac{\max[0, \cos \theta]}{\max[\cos 85, \cos \theta_2]} \]  

(17)

\[ \varepsilon = \frac{\left[ 1 + \frac{G_N}{G_d} + 1.041 \theta_2 \right]^3}{\left[ 1 + 1.041 \theta_2 \right]^3} \]  

(18)

\[ \Delta = \frac{G_d}{G_0} \]  

(19)
\[ F_1 = f_{11}(\varepsilon) + f_{12}(\varepsilon) \cdot \Delta + f_{13}(\varepsilon) \cdot \theta_z \]  
(20)

\[ F_2 = f_{21}(\varepsilon) + f_{22}(\varepsilon) \cdot \Delta + f_{23}(\varepsilon) \cdot \theta_z \]  
(21)

Table 3. The coefficients of the Perez I model

<table>
<thead>
<tr>
<th>ε Bin</th>
<th>Upper Limit for ε</th>
<th>Cases (%)</th>
<th>( f_{11} )</th>
<th>( f_{12} )</th>
<th>( f_{13} )</th>
<th>( f_{21} )</th>
<th>( f_{22} )</th>
<th>( f_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.065</td>
<td>13.6</td>
<td>−0.196</td>
<td>1.084</td>
<td>−0.006</td>
<td>−0.114</td>
<td>0.18</td>
<td>−0.019</td>
</tr>
<tr>
<td>2</td>
<td>1.23</td>
<td>5.6</td>
<td>0.236</td>
<td>0.519</td>
<td>−0.18</td>
<td>0.011</td>
<td>0.2</td>
<td>−0.038</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>7.52</td>
<td>0.454</td>
<td>0.321</td>
<td>−0.255</td>
<td>0.072</td>
<td>−0.098</td>
<td>−0.046</td>
</tr>
<tr>
<td>4</td>
<td>1.95</td>
<td>8.87</td>
<td>0.866</td>
<td>−0.381</td>
<td>−0.375</td>
<td>0.203</td>
<td>−0.403</td>
<td>−0.049</td>
</tr>
<tr>
<td>5</td>
<td>2.8</td>
<td>13.17</td>
<td>1.026</td>
<td>−0.711</td>
<td>−0.426</td>
<td>0.273</td>
<td>−0.602</td>
<td>−0.061</td>
</tr>
<tr>
<td>6</td>
<td>4.5</td>
<td>21.45</td>
<td>0.978</td>
<td>−0.986</td>
<td>−0.35</td>
<td>0.28</td>
<td>−0.915</td>
<td>−0.024</td>
</tr>
<tr>
<td>7</td>
<td>6.2</td>
<td>16.06</td>
<td>0.748</td>
<td>−0.913</td>
<td>−0.236</td>
<td>0.173</td>
<td>−1.045</td>
<td>0.065</td>
</tr>
<tr>
<td>8</td>
<td>−</td>
<td>13.73</td>
<td>0.318</td>
<td>−0.757</td>
<td>0.103</td>
<td>0.062</td>
<td>−1.698</td>
<td>0.236</td>
</tr>
</tbody>
</table>

(d) Perez II model

The Perez II model has the same formulation as the Perez I model \[43\]. Both models differ only in the \( F_1 \) and \( F_2 \) coefficients. The method for calculating the detailed parameters \( a, c, F_1 \) and \( F_2 \) in the Perez I and Perez II models can be found in Equations (17)–(21). The coefficients of the Perez II model are shown in Table 4.

Table 4. The coefficients of the Perez II model.

<table>
<thead>
<tr>
<th>ε Bin</th>
<th>( f_{11} )</th>
<th>( f_{12} )</th>
<th>( f_{13} )</th>
<th>( f_{21} )</th>
<th>( f_{22} )</th>
<th>( f_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−0.00831</td>
<td>0.58773</td>
<td>−0.06206</td>
<td>−0.05960</td>
<td>0.07212</td>
<td>−0.02202</td>
</tr>
<tr>
<td>2</td>
<td>0.12999</td>
<td>0.68260</td>
<td>−0.15138</td>
<td>−0.01893</td>
<td>0.06597</td>
<td>−0.02887</td>
</tr>
<tr>
<td>3</td>
<td>0.32970</td>
<td>0.48687</td>
<td>−0.22110</td>
<td>−0.055414</td>
<td>−0.06396</td>
<td>−0.02605</td>
</tr>
<tr>
<td>4</td>
<td>0.56821</td>
<td>0.18745</td>
<td>−0.29513</td>
<td>−0.10886</td>
<td>−0.15192</td>
<td>−0.01398</td>
</tr>
<tr>
<td>5</td>
<td>0.87303</td>
<td>−0.39204</td>
<td>−0.36162</td>
<td>0.22556</td>
<td>−0.46204</td>
<td>−0.00124</td>
</tr>
<tr>
<td>6</td>
<td>1.13261</td>
<td>−1.23673</td>
<td>−0.41185</td>
<td>0.28778</td>
<td>−0.82304</td>
<td>0.05587</td>
</tr>
<tr>
<td>7</td>
<td>1.06016</td>
<td>−1.59999</td>
<td>−0.35892</td>
<td>0.26421</td>
<td>−1.12723</td>
<td>0.13107</td>
</tr>
<tr>
<td>8</td>
<td>0.67775</td>
<td>−0.32726</td>
<td>−0.25043</td>
<td>0.26421</td>
<td>−1.37650</td>
<td>0.25062</td>
</tr>
</tbody>
</table>

5. Diffuse Radiation and Total Tilted Radiation

5.1. Diffuse Radiation on the Horizontal Surface

Measured diffuse radiation and calculated diffuse radiation based on the DTU and RR models only using measured global radiation is shown in Figure 6. Yearly measured diffuse radiation and calculated diffuse radiation according to the DTU model was 524 and 510 kWh/m\(^2\), respectively. Yearly calculated diffuse radiation according to the RR model was 494 kWh/m\(^2\). Monthly calculated results according to the RR model were 6% lower than the measured values on average in Taars. Diffuse radiation calculated by the DTU model was closer to the measured values than the RR model. The difference between measured and simulated diffuse radiation according to the DTU model was about 3% on average. It may be concluded that the DTU model is more suitable for Danish climate conditions compared to the other universal and classic empirical model.

5.2. Total Radiation on the Tilted Surface Based on Global Radiation and Beam Radiation

Calculated total radiation on the tilted surface by use of the isotropic and anisotropic models based on measured total horizontal radiation and measured beam radiation from September 2015 to August 2016 is shown in Figure 7 together with measured values. The surface is facing south with a tilt of 50°.
The calculated monthly total tilted radiation levels according to the isotropic model were much lower than the measured values in Figure 7 compared to the anisotropic models. The measured yearly total radiation was 1170 kWh/m². The monthly total tilted solar radiation in November 2015, December 2015 and January 2016, was around 20 kWh/m². Contrary to the conclusions derived under Saudi Arabian and Malaysian weather conditions in past studies [11,12], in the present study, the anisotropic models were better than the isotropic model under Danish climate conditions. For the four anisotropic models, the calculated total tilted radiation according to the Perez II model and the Perez I model gave results closest to the sum of measured values, with only average differences of 1–2%, which is similar to the results reported by Andersen et al. [44].

5.3. Comparison of the Different Models

Measured data from the Taars solar heating plant were used to evaluate the models. Four statistical error parameters were introduced to evaluate the monthly results from September 2015 to August 2016 in Figures 6 and 7 to determine the accuracy of the models for Danish climate conditions.

(1) MBE, mean bias error

\[
MBE = \frac{1}{k} \sum_{i=1}^{k} \left( \frac{C_{\text{Calculated}}^i}{C_{\text{Measured}}^i} \right)
\]
(2) RMSE, root mean square error
\[
\text{RMSE} = \left( \frac{1}{k} \sum_{i=1}^{k} \left( G_{\text{Calculated}}^i - G_{\text{Measured}}^i \right)^2 \right)^{1/2}
\] (23)

(3) MAPE, mean absolute percentage error
\[
\text{MAPE} = \frac{1}{k} \sum_{i=1}^{k} \left| \frac{G_{\text{Calculated}}^i - G_{\text{Measured}}^i}{G_{\text{Measured}}^i} \right|
\] (24)

(4) RPE, relative percentage error
\[
\text{RPE} = \frac{\sum_{i=1}^{k} (G_{\text{Calculated}}^i - G_{\text{Measured}}^i)}{\sum_{i=1}^{k} G_{\text{Measured}}^i}
\] (25)

Comparisons between measured values and calculated values of diffuse radiation on the horizontal surface and total radiation on the tilted surface are shown in Tables 5 and 6, respectively. The lower the MBE and RMSE are, the better the agreement between the measured and calculated values. For MBE, a positive value means an overestimation of the calculated values and a negative value means an underestimation of the calculated values. A drawback of MBE is that one positive value in one calculation step may cancel a negative value in another calculation step. RMSE is always positive. MAPE is positive and a low MAPE means the model is accurate. A negative RPE means the proposed model slightly underestimates radiation. Table 5 shows that the DTU model is more accurate than the RR model for Danish conditions based on the four investigated criteria. The RMSE of the DTU model and the RR model were 2 and 3 kWh/m², respectively. The RMSE and MAPE of the Perez models were much lower than other models, as shown in Table 6. It can be concluded that the anisotropic models (Perez II model and Perez I model) were the most accurate among the investigated empirical models and the most suitable for calculations of total tilted radiation under Danish climate conditions.

**Table 5.** Measured and calculated MBE (kWh/m²), RMSE (kWh/m²), MAPE (%) and RPE (%) for diffuse horizontal radiation.

<table>
<thead>
<tr>
<th>Items</th>
<th>DTU Model</th>
<th>RR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBE</td>
<td>−1.3</td>
<td>−2.5</td>
</tr>
<tr>
<td>RMSE</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>MAPE</td>
<td>3.5%</td>
<td>8.1%</td>
</tr>
<tr>
<td>RPE</td>
<td>−2.9%</td>
<td>−5.7%</td>
</tr>
</tbody>
</table>

**Table 6.** Measured and calculated MBE (kWh/m²), RMSE (kWh/m²), MAPE (%) and RPE (%) for monthly total tilted radiation.

<table>
<thead>
<tr>
<th>Items</th>
<th>Perez II Model</th>
<th>Perez I Model</th>
<th>HDKR Model</th>
<th>HD Model</th>
<th>Isotropic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBE</td>
<td>−2.4</td>
<td>−3.4</td>
<td>−8.4</td>
<td>−10.0</td>
<td>−18.6</td>
</tr>
<tr>
<td>RMSE</td>
<td>2.0</td>
<td>2.6</td>
<td>4.9</td>
<td>5.8</td>
<td>10.0</td>
</tr>
<tr>
<td>MAPE</td>
<td>2.1%</td>
<td>2.8%</td>
<td>5.7%</td>
<td>5.9%</td>
<td>12.0%</td>
</tr>
<tr>
<td>RPE</td>
<td>−1.2%</td>
<td>−1.8%</td>
<td>−4.3%</td>
<td>−5.2%</td>
<td>−9.7%</td>
</tr>
</tbody>
</table>
6. DNI and Total Tilted Radiation Only Based on Global Radiation

6.1. Measured and Calculated DNI Only Based on Global Radiation

Global radiation data is available from climate stations at the Danish Meteorological Institute (DMI). DNI is not measured at climate stations and only seldom in solar heating plants in Denmark. Moreover, DNI is a very important design parameter for concentrating solar collectors, such as the parabolic trough collectors in Taars. As was shown in Sections 5.1 and 5.3, diffuse radiation calculated by the DTU model was more accurate than the RR empirical model under Danish conditions. Using Equations (1) and (2), diffuse radiation calculated by the DTU model was used to calculate DNI or beam radiation. Therefore, the DTU model was used as the optimal model in this section to predict DNI. Figure 8 shows monthly calculated DNI (DTU) and measured DNI from September 2015 to August 2016. The calculated yearly DNI (997 kWh/m²) was about 1% larger than measured yearly DNI (990 kWh/m²) for the period from September 2015 to August 2016, which was within measuring accuracy. Monthly DNI can be higher than 100 kWh/m² in the summer season, as is shown in Figure 8.

![Figure 8. Calculated DNI (DTU) and measured DNI in the Taars solar heating plant.](image)

6.2. Measured and Calculated Total Tilted Radiation Only Based on Global Radiation

As mentioned, normally global radiation from the Danish Meteorological Institute is available. Total radiation on collector surfaces is measured at most solar heating plants but with a poor accuracy in Denmark. Using the DTU model and Equations (1) and (2), calculated diffuse radiation and beam radiation could be obtained based on measured global radiation on the horizontal surface. In addition, because the isotropic model could be used easily and widely and the anisotropic models (Perez II model and Perez I model) were closest to the measured values, as described in Sections 5.2 and 5.3, the isotropic model and the anisotropic models (Perez II and Perez I) were selected to calculate total radiation on the tilted surface based on calculated diffuse radiation and calculated beam radiation from the DTU model, which was calculated only from measured global radiation. The calculated total radiation on the tilted surface using the isotropic model (1070 kWh/m²) was 8% lower than the measured one (1170 kWh/m²) from September 2015 to August 2016 (Figure 9a). The calculated total tilted radiation by the Perez I model (1160 kWh/m²) and Perez II model (1169 kWh/m²) was less than 1% different than the yearly measured total radiation (Figure 9b,c). Both of the Perez models had the best agreement with the measurements of the investigated three empirical models.
The isotropic model and the anisotropic models (Perez II and Perez I) were selected to measure solar radiation data for planning solar collector fields based on available horizontal global radiation. The proposed method could be employed to check measured total radiation on tilted flat plate collector surfaces in good agreement with yearly measurements of the investigated three empirical models elsewhere.

As mentioned, normally global radiation from the Danish Meteorological Institute is available. Using the DTU model and equation, because the isotropic model could be used easily and widely and the anisotropic model was higher than 8 kWh/m². From the above results, it was found that the DTU model described Danish conditions very accurately. Furthermore, the calculated total radiation could be obtained based on measured global radiation on tilted surfaces from the DTU and Perez I model; (DTU and Perez II model).

Calculated Total Tilted Radiation Only Based on measured global radiation under the isotropic model and the anisotropic model (Perez II and Perez I) models could be used to predict total radiation on tilted surfaces as shown in Figure 9. Measured monthly tilted total radiation and calculated tilted total radiation based on calculated diffuse radiation and beam radiation (September 2015–August 2016: (a) DTU and Isotropic model; (b) DTU and Perez I model; (c) DTU and Perez II model).
Figure 10 shows the daily measured total tilted solar radiation as a function of the modelled total tilted solar radiation from the DTU and Perez II model. The trend in Figure 10 demonstrates good agreement between the daily measured and modelled data. It was also found that maximum daily total tilted solar radiation could be higher than 8 kWh/m². These results are in good agreement with conclusions presented elsewhere [44]. From the above results, it was found that the DTU model together with the Perez II and I models could be used to predict total radiation on tilted surfaces based only on measured global radiation under Danish conditions very accurately. Furthermore, the proposed models could be employed to check measured total radiation on tilted flat plate collector planes in solar district heating plants in Denmark. The proposed method could also be used to derive solar radiation data for planning solar collector fields based on available horizontal global radiation measurements.

![Graph showing daily measured total tilted solar radiation as a function of modelled total tilted solar radiation from DTU & Perez II model.](image)

**Figure 10.** Daily measured total tilted solar radiation as a function of modelled total tilted solar radiation from the DTU and Perez II model.

7. Discussion

In general, the anisotropic sky models (Hay and Davies, Reindl, and Perez) provide comparable estimates of the total radiation on a tilted surface and are recommended for general use [45]. The Hay and Davies and the Reindl models are computationally simple when compared to the Perez model. The isotropic sky model under-predicts total radiation on a tilted surface and is not recommended for general use. The HD and Reindl models were recommended in the mentioned references [25,26]. The HD model has also been selected to predict total tilted solar radiation in Greece [45]. The Perez models were the best models under Danish climate conditions in this study, which aligns with other past research [46,47].

8. Conclusions

Measured and calculated monthly horizontal diffuse solar radiation and total tilted solar radiation from September 2015 to August 2016 (a full year) in a demonstration solar district heating plant in Denmark were analysed in this study using an hourly time step. The DTU model, developed for the calculation of horizontal diffuse radiation in Danish climate conditions, was evaluated and validated
using the measured data. Calculated monthly DNI based on the DTU model with only measured global radiation as an input was also investigated with good agreement with measurements. Furthermore, one isotropic model and four anisotropic models for general use were investigated for the calculation of total monthly radiation on the tilted surface under Danish climate conditions. From these results, the following conclusions can be drawn:

(1) It was found that the DTU model could be used for the calculation of diffuse radiation on the horizontal surface or DNI in Denmark with better accuracy than the other classic empirical model.

(2) Anisotropic models could be used to calculate total radiation on tilted surfaces with better accuracy than the isotropic model under Danish conditions.

(3) The Perez models together with the DTU model could be a suitable new method to determine total radiation on tilted surfaces and double-check real-time measured solar radiation for Danish solar heating plants. The only input for this method was global radiation measurement.

(4) Yearly global radiation and DNI was around 1000 kWh/m$^2$ and total tilted solar radiation was around 1200 kWh/m$^2$ in this study.

The proposed method was simple, cost-effective and gave relatively accurate measurements of total tilted radiation under Danish conditions.

**Author Contributions:** Zhiyong Tian set up the model, carried out the calculations and wrote the original article; Bengt Perers has contribution on the literature search and data collection. Simon Furbo has contribution to the writing and data collection; Jianhua Fan has contribution to the writing and data collection; Jie Deng provided assistance to set up the model; Janne Dragsted contributed to the modelling

**Acknowledgments:** Special thanks are expressed to AalborgCSP A/S (Andreas Zourellis, Jan Holst Rothman, Steffen Rovsing Møller and Per Aasted) for the information provided. The first author acknowledges financial support from the China Scholarship Council for PhD study at the Technical University of Denmark (No.201506120074). This work was also a part of an EUDP project financed by the Danish Energy Agency and the IEA-SHC Task 55 “Towards the Integration of Large SHC Systems into DHC Networks”.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNI</td>
<td>Direct normal irradiance, W/m$^2$</td>
</tr>
<tr>
<td>MBE</td>
<td>Mean bias error, kWh/m$^2$</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean square error, kWh/m$^2$</td>
</tr>
<tr>
<td>MAPE</td>
<td>Mean absolute percentage error</td>
</tr>
<tr>
<td>RPE</td>
<td>Relative percentage error</td>
</tr>
<tr>
<td>DTU</td>
<td>Technical University of Denmark</td>
</tr>
<tr>
<td>RR model</td>
<td>Reduced Reindl correlation model</td>
</tr>
<tr>
<td>PTC</td>
<td>parabolic trough collector</td>
</tr>
<tr>
<td>FPC</td>
<td>flat plate collector</td>
</tr>
</tbody>
</table>

**Nomenclature**

- $R_b$: The ratio of beam radiation on the tilted surface to that on a horizontal surface at any time
- $A_i$: Anisotropy index
- $k$: Number of calculated values
- $i$: Every calculated value
- $G$: Mean total radiation on the horizontal surface, W/m$^2$
- $G_d$: Mean diffuse radiation on the horizontal surface, W/m$^2$
- $G_0$: Mean extraterrestrial radiation on the horizontal surface, W/m$^2$
- $G_T$: Mean total radiation on the tilted surface, W/m$^2$
- $G_b$: Mean beam radiation on the horizontal surface, W/m$^2$
- $G_N$: Mean direct normal beam radiation, W/m$^2$
- $K_T$: Clearness index
- $a/c$: Weighted circumsolar solid angle
Calculated solar radiation, kWh/m²
Measured solar radiation, kWh/m²
Reduced brightness coefficient (circumsolar)
Reduced brightness coefficient (horizon brightening)

Reduced brightness coefficient (circumsolar)
Reduced brightness coefficient (horizon brightening)

Slope
Zenith angle
Incident angle
Solar altitude angle
Diffuse reflectance for the total solar radiation
Sky clearness parameter
Sky brightness parameter

References


29. Dragsted, J.; Furbo, S. *Solar Radiation and Thermal Performance of Solar Collectors for Denmark; DTU Civil Engineering Reports, No. R-275; Department of Civil Engineering, Technical University of Denmark: Copenhagen, Denmark, 2012.*


© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).