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A Preference Model for Supplier Selection Based on Hesitant Fuzzy Sets

Zhexuan Zhou [1], Yajie Dou [1,*], Tianjun Liao [2] and Yuejin Tan [1]

[1] College of System Engineering, National University of Defense Technology, Changsha 410073, China;
    zhouzhexuan16@nudt.edu.cn (Z.Z.); yjtan@nudt.edu.cn (Y.T.)
[2] State Key Laboratory of Complex System Simulation, Beijing Institute of System Engineering,
    10 An Xiang Bei Li Road., Beijing, 100101, China; liaotianjun@gmail.com
* Correspondence: douyajie@nudt.edu.cn

Abstract: The supplier selection problem is a widespread concern in the modern commercial economy. Ranking suppliers involves many factors and poses significant difficulties for decision makers. Supplier selection is a multi-criteria and multi-objective problem, which leads to decision makers forming their own preferences. In addition, there are both quantifiable and non-quantifiable attributes related to their preferences. To solve this problem, this paper presents a preference model based on hesitant fuzzy sets (HFS) to select suppliers. The cost and service quality of suppliers are the main considerations in the proposed model. HFS with interactive and multi-criteria decision making are used to evaluate the non-quantifiable attributes of service quality, which include competitive display, qualification ability, suitability and competitiveness of solutions, and relational fitness and dynamics. Finally, a numerical example of supplier selection for a high-end equipment manufacturer is provided to illustrate the applicability of the proposed model. The preferences of a decision maker are then analyzed by altering preference parameters.

Keywords: supplier selection; equipment manufacturer; preference relationship; hesitant fuzzy set; score function; TODIM

1. Introduction

Supplier selection is a hot issue in many industries. It is an important part of the procurement process, which is strongly related to the core competitiveness of enterprises [1]. The choice of a supplier is directly related to the costs of a business, quality of a product, and customer response time. With continuing manufacturing development, supplier demand is steadily increasing. As a result, the competition between suppliers is becoming increasingly intense. Decision makers often consider various factors, such as cost, product quality, lead time, and risk appetite, when selecting a supplier. Considering these indexes, the problem of supplier selection is a multi-objective and multi-criteria decision-making (MCDM) problem.

Significant research has been performed on supplier selection. AM Groot [2] defined an economic order quantity (EOQ) model for ordering goods and Disktion [3] summarized 23 criteria for supplier selection for procurement. Some of these criteria are related to the quality, cost, and delivery of products. For modelling supplier selection, several scholars have researched multi-objective decision-making and MCDM methods [4,5]. Birsen et al. [5] used a multi-objective model to assess supplier quality, cost, and product lead time. Buffa and Jackson [6] primarily focused on supplier cost, product quality, and services for multi-objective planning. Ghodsypour and O'Brien [7] used the analytic hierarchy process (AHP) to find the largest integrated value of a candidate supplier, then ranked suppliers based on that value. James [8] improved supply chain management by using a new integrated manufacturing enterprise system based on parking and forwarding algorithms, RFID, and a Markov decision process.

Supplier selection is an MCDM problem. Dae-Ho Byun [9] used the AHP to analyze suppliers in the Korean automobile industry. Tam and Tummala [10] considered several criteria for supplier selection for communication devices. Recently, supplier selection has seen a few new developments. Tadić et al. [11] used a fuzzy version of the technique for order of preference by similarity to ideal solution (TOPSIS) to rank medical device suppliers. Yu et al. [12] built a model to select a supplier for closed-loop logistics systems using hesitant fuzzy sets. Zhou [13] evaluated supplier performance using triangular fuzzy information. Gupta et al. [14] solved the supplier selection problem using a modified intuitionistic fuzzy superiority and inferiority ranking approach. Cao et al. [15] used an integrated multi-criterion decision-making method to evaluate green suppliers. Bohner and Minner [16] analyzed supplier selection in terms of risk of failure, quantity, and business volume discounts. Wan et al. [17] presented a novel hybrid MCDM method with two-level criteria in an interval 2-tuple linguistic environment to find superior suppliers. A hybrid model with AHP and TOPSIS was presented to select the most appropriate tomography equipment [18]. Zhao [19] proposed a hybrid MCDM Method for sustainability to selected green supplier of thermal power equipment. An Integrated MCDM Model was used to select mining method in presence of uncertainty with FAHP and FTOPSIS [20]. A fuzzy ISM (Fuzzy Interpretive Structural Modelling) with AHP and fuzzy PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) was used to build the interdependence of supply chain complexity management [21].

However, in the above studies on supplier selection, either supplier-explicit criteria attributes were considered or non-quantifiable attributes were considered, but not both. To date, there has been no research that considers combining quantifiable attributes and non-quantifiable attributes within an objective. Furthermore, decision makers were only considered in the design of the model and their psychological decision-making behavior was not considered. However, in a real-world supplier selection process, there may be hesitation or personal preferences for certain attributes based on individual reasoning. To better reflect these influences, this paper proposes a multi-objective preference model using hesitant fuzzy sets (HFS). Additionally, the service quality of suppliers is evaluated using interactive and multi-criteria decision making (TODIM), which better represents real-world decision-making processes. This was the major research motivation for this study.

Zadeh first proposed the fuzzy set in 1965 [22] and used it to solve multiple attribute decision-making problems in 1970 [23]. Since then, many researchers have explored the use of fuzzy sets in decision making. Recently, Torra [24] proposed an important form of fuzzy sets called HFS. Xu and Xia [25] then gave the HFS a mathematical form and defined related operations. Farhadinia [26,27] proposed a series of scoring functions and a ranking method for hesitant fuzzy numbers. HFS play an important role in decision making. Zhang and Xu [28] proposed a novel measuring function for hesitant fuzzy numbers and used the function in TODIM. Zhang and Xu also used HFS in a qualitative flexible multiple criteria method with a signed distance-based comparison method [29] and TOPSIS with incomplete weight information [30]. Novel distance and similarity measures have also been used for clustering analysis in a hesitant fuzzy environment [31]. In another study, Ai et al. [32] developed dependent hesitant fuzzy aggregation operators to apply to multiple attribute group decision making. Tang [33] solved the green supplier selection problem with hesitant fuzzy information. Based on the above studies, one can see that HFS are good tools to describe non-quantitative decision-making properties, which better represent real-world decision-making processes. Therefore, this paper proposes a preference model with multiple objectives that integrates HFS and TODIM.

The primary focus of this study was building a preference decision-making model and solving the supplier selection problem using HFS. In the proposed model, the cost and service quality of suppliers are considered to reflect decision maker preferences. To evaluate the service quality of a supplier efficiently, HFS are used to describe the evaluations of experts. During service quality evaluation, a hesitant fuzzy number score function is analyzed and TODIM is employed to evaluate the individual

behavior of decision makers. Finally, a numerical supplier selection example is provided to illustrate the advantages of the proposed method.

The remainder of this paper is organized as follows. Section 2 presents the preference model for supplier selection considering cost and service quality. Section 3 contains an evaluation of supplier service quality using HFS with TODIM. Section 4 presents a case study on high-end equipment supplier selection. Section 5 contains analysis of the influence of different preferences on decision making. Section 6 presents a final discussion and summary of this study.

2. Preference Models for Supplier Selection

The topic of supplier selection has been extensively studied by researchers. In existing supplier selection methods, most of the factors considered are quantifiable (e.g., cost). Quantitative and non-quantitative factors are rarely considered simultaneously. This study considers both the minimum cost of suppliers and service quality of suppliers. Therefore, the following multi-objective model (Model-1) is established, with the goal of finding both the smallest cost and best quality of service. Let C_i and Q_i represent the cost and service quality, respectively, of suppliers.

Model-1

$$\begin{cases} \min C_i \\ \max Q_i \end{cases} \tag{1}$$

Multi-objective problem-solving processes are complex and typically contain various obstacles. In real-world situations, decision-making results tend to be accompanied by decision-maker preferences. These preferences play a pivotal role in decision-making. This is an issue that previous multi-objective models for supplier selection have not adequately solved. Based on this consideration, this paper presents a preference model for supplier selection. To make the preference objective consistent, Equation (2) is used to standardize the data.

$$\begin{cases} C_i' = \frac{\min C_i}{C_i} \\ Q_i' = \frac{Q_i}{\max Q_i} \end{cases} \tag{2}$$

From the above formula, C_i' and Q_i' are the standardized cost and service quality, respectively. They are considered to be better when they have larger values. The preference parameter τ is introduced and Model-1 is transformed into Model-2.

Model-2

$$\max V_i = \tau C_i' + (1 - \tau) Q_i' \tag{3}$$

where V_i is the ranking value of suppliers, and $\tau \in [0, 1]$. When $1 \geq \tau > 0.5$, the decision maker has a cost preference and pursues the lowest possible cost, even to the detriment of service quality. When $\tau = 0.5$, the decision maker is neutral and when $0 \leq \tau < 0.5$, the decision maker has a service preference and pursues the highest quality, regardless of the cost.

HFS are introduced to describe non-quantifiable service quality. Attributes of service quality evaluation for suppliers were used in [34], which included competitive display a_1, qualification ability a_2, suitability and competitiveness of solutions a_3, and relational fitness and dynamics a_4. The attributes of supplier service quality are listed in Table 1. The factor of "risk" is one of the important roles in manufacturing activity [35], and especially when selecting suppliers. In this model, the risk is included in the Supplier Service Quality Attribute (suitability and competitiveness of solutions a_3).

Table 1. Supplier Service Quality Attribute Description.

Attributes	Description
Competitive display a_1	Personnel, business processes, technology, business certification, innovation records
Qualification ability a_2	Financial stability, infrastructure and resources, management systems, and service stability
The suitability and competitiveness of Solutions a_3	Solutions, service provision, risk sharing, terms, human resource requirements
Relational fitness and dynamic a_4	Team culture, team management, relative importance, existing achievements

3. Supplier Service Quality Evaluation Based on HFS with TODIM

3.1. Hesitant Fuzzy Sets

3.1.1. Preliminaries

Quality of service is typically not accurately quantified. In this study, HFS are used to describe the service quality of a supplier. In real-world decision-making processes, people typically use hesitant fuzzy information to represent preference evaluations. Recently, Xia and Xu [25] defined HFS that include the membership and non-membership degrees of several different values.

Definition 1 ([25]). *HFS can be defined as*

$$H = \{< x, h_A(x) > | x \in X\} \tag{4}$$

where $h_H(x)$ takes on real values in the range [0, 1]. *This value represents the degree of possibility that element x belongs to X in set A. Xia and Xu* [25] *defined $h_H(x)$ as a hesitant fuzzy number.*

Definition 2 ([24,25]). *According to the definition of a hesitant fuzzy number, the following operation can be derived. Suppose that there are three hesitant fuzzy numbers h_1, h_2, and h_3. Then, the following is true:*

(1)　$h_1 \oplus h_2 = H\{(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)|\gamma_1 \in h_1, \gamma_2 \in h_2\}$
(2)　$h_1 \otimes h_2 = H\{\gamma_1\gamma_2|\gamma_1 \in h_1, \gamma_2 \in h_2\}$
(3)　$\lambda h = H\{1 - (1 - \gamma)^\lambda|\gamma \in h\}(\lambda > 0)$
(4)　$h^\lambda = H\{\gamma^\lambda|\gamma \in h\}(\lambda > 0)$
(5)　$h^C = H\{1 - \gamma|\gamma \in h\}$

Hypothesis 1 ([36]). *To calculate the distance between two hesitant fuzzy numbers or compare them conveniently, the hesitant number with fewer elements should be extended until all hesitant numbers have the same number of elements. If the decision maker is pursuing risk, then the largest element of hesitation should be added to all corresponding elements until the two hesitant fuzzy numbers are equal, and* vice versa.

3.1.2. Hesitant Fuzzy Number Score Function Analysis

In the process of hesitant fuzzy multi-attribute decision-making, ranking hesitant numbers is a very important step. Xia and Xu [25] first defined a scoring function for hesitant fuzzy numbers. Farhadinia [26] later proposed an improved hesitant fuzzy scoring function. Recently, Zhang [28] proposed a novel hesitant fuzzy scoring function that only requires one step to rank hesitant fuzzy numbers. However, this method requires analysis of parameters. Therefore, a geometric hesitant fuzzy scoring function is analyzed and considered in conjunction with a mathematical geometric mean.

Definition 3 ([27]). *If there is a hesitant fuzzy number $h = \{\gamma^1, \gamma^2, ..., \gamma^n\}$, then a geometric hesitant fuzzy scoring function $S(h)$ can be defined as*

$$S(h) = \sqrt[n]{\prod_{i=1}^{n} \gamma^i} = (\gamma^1 \times \gamma^2, ..., \times \gamma^n)^{1/n} \tag{5}$$

From this geometric scoring function, the following properties can be derived:

Property 1. *For any hesitant fuzzy number $h = \{\gamma^1, \gamma^2, ..., \gamma^n\}$, its geometric hesitant fuzzy scoring function value $S(h) \in [0, 1]$.*

Proof. Let $\gamma^+ = \max\{\gamma^\lambda | \lambda = 1, 2, ..., n\}$ and $\gamma^- = \min\{\gamma^\lambda | \lambda = 1, 2, ..., n\}$. Because

$$\gamma^\lambda \in [0, 1] (\lambda = 1, 2, ..., n),$$

$$S(h) = \sqrt[n]{\prod_{i=1}^{n} \gamma^i} = (\gamma^1 \times \gamma^2, ..., \times \gamma^n)^{1/n} \leq (\gamma^+ \times \gamma^+, ..., \times \gamma^+)^{1/n} = \sqrt[n]{(\gamma^+)^n} = \gamma^+ \leq 1$$

and

$$S(h) = \sqrt[n]{\prod_{i=1}^{n} \gamma^i} = (\gamma^1 \times \gamma^2, ..., \times \gamma^n)^{1/n} \leq (\gamma^- \times \gamma^-, ..., \times \gamma^-)^{1/n}$$

$$= \sqrt[n]{(\gamma^-)^n} = \gamma^- \geq 0.$$

Therefore, $0 \leq S(h) \leq 1$.

Property 2. *If $h_1 = \{\gamma_1^1, \gamma_1^2, ..., \gamma_1^n\}$ and $h_2 = \{\gamma_2^1, \gamma_2^2, ..., \gamma_2^n\}$ are both hesitant fuzzy numbers, their internal elements are arranged in ascending order and their elements are equal. Therefore, if $\gamma_1^1 \leq \gamma_2^1$, $\gamma_1^2 \leq \gamma_2^2$, ... , and $\gamma_1^n \leq \gamma_2^n$, then:*

$$S(h_1) \leq S(h_2).$$

Proof. Because $\gamma_1^n \leq \gamma_2^n$, it follows that

$$\sqrt[n]{\prod_{i=1}^{n} \gamma_1^i} = (\gamma_1^1 \times \gamma_1^2, ..., \times \gamma_1^n)^{1/n} \leq \sqrt[n]{\prod_{i=1}^{n} \gamma_2^i} = (\gamma_2^1 \times \gamma_2^2, ..., \times \gamma_2^n)^{1/n}.$$

Therefore, $S(h_1) \leq S(h_2)$.

For this geometric hesitant fuzzy scoring function, a hesitant fuzzy number ranking method is described as follows:

Definition 4. *If there are two hesitant fuzzy numbers h_1 and h_2, according to (5), a new ranking method can be defined as follows:*

(1)   *If $S(h_1) > S(h_2)$, then h_1 is superior to h_2 and $h_1 \succ h_2$.*
(2)   *If $S(h_1) = S(h_2)$, then h_1 is equal to h_2 and $h_1 \sim h_2$.*
(3)   *If $S(h_1) < S(h_2)$, then h_1 is inferior to h_2 and $h_1 \prec h_2$.*

To illustrate the usability of the proposed ranking method, the following example is presented [37]: There are three hesitant numbers

$$h_1 = \{0.3, 0.5\}, h_2 = \{0.4\}, \text{ and } h_2 = \{0.2, 0.4, 0.6\}.$$

Different ranking methods are used to rank them.

According to the geometric scoring function (5), $S(h_1) = 0.3873$, $S(h_2) = 0.4000$, and $S(h_3) = 0.3634$. Therefore,

$$h_2 \succ h_1 \succ h_3.$$

The comparison results of the ranking methods are listed in Table 2.

Table 2. Results of the different ranking methods.

Ranking Methods	Score Value	Order
Xia and Xu [25]	$S(h_1) = S(h_2) = S(h_3) = 0.4$	$h_1 \sim h_2 \sim h_3$
Farhadinia [26]	$S(h_1) = 0.467, S(h_2) = 0.4, S(h_3) = 0.467$	$h_1 \sim h_3 \succ h_2$
Liao and Xu [38]	$S(h_1) = S(h_2) = S(h_3) = 0.4$ $v(h_1) = 0.1, v(h_2) = 0, v(h_3) = 0.1633$	$h_2 \succ h_1 \succ h_3$
Zhang ($\delta = 0.1$) [28]	$S(h_1) = 0.3886, S(h_2) = 0.4, S(h_3) = 0.3672$	$h_2 \succ h_1 \succ h_3$
This study	$S(h_1) = 0.3873, S(h_2) = 0.4, S(h_3) = 0.3634$	$h_2 \succ h_1 \succ h_3$

It can be seen from Table 2 that the ranking method of Xia and Xu [25] cannot distinguish between the three hesitant fuzzy numbers. The method from Farhadinia [26] cannot distinguish between h_1 and h_2. The results of the geometric hesitant fuzzy scoring function are the same as those of the methods from Liao and Xu [38], and Zhang [28]. However, the method from Liao and Xu [38] is computationally complex and the method from Zhang [28] requires analysis of the parameter δ. Therefore, the geometric hesitant fuzzy scoring function is superior to the other ranking methods.

3.2. TODIM

The psychological factors of decision makers often play a major role in the process of multi-attribute decision-making. The proposed method selects suppliers by incorporating the preferences of decision makers in the selection process. By using TODIM, the proposed model considers the mentality of decision makers in the evaluation of supplier service quality. Additionally, a hesitant fuzzy number is used to describe the evaluation value, which results in a more objective description of supplier service quality.

Assume that there are m candidate suppliers and n attributes. A decision matrix is then constructed as $(a_{ij})_{m \times n}$, where a_i are the attributes of suppliers and x_i are candidate suppliers. Let w be the weight of the attributes and a_m be the attribute with the largest weight. Then, let a_m be a reference attribute, where the reference weights of the other attributes for a_m are:

$$w_{jm} = \frac{w_j}{w_m}, j = 1, 2, ..., n \tag{6}$$

Then, the perceived value function of supplier x_i to x_k can be defined as

$$\vartheta(x_i, x_k) = \sum_{j=1}^{n} \varphi(x_i, x_k), i, k = 1, 2, ..., m \tag{7}$$

where

$$\varphi(x_i, x_k) = \begin{cases} \sqrt{w_{jm} d(h_{ij}, h_{kj}) / \sum_{j=1}^{n} w_{jm}}, & if \ S(h_{ij}) > S(h_{kj}) \\ 0, & if \ S(h_{ij}) = S(h_{kj}) \\ -\frac{1}{\theta} \sqrt{(\sum_{j=1}^{n} w_{jm}) d(h_{ij}, h_{kj}) / w_{jm}}, & if \ S(h_{ij}) < S(h_{kj}) \end{cases} \tag{8}$$

Here, θ represents the loss avoidance coefficient and

$$d(h_{ij}, h_{kj}) = \sqrt{\frac{1}{n} \sum_{\lambda=1}^{n} (\gamma_{ij}^{\lambda} - \gamma_{kj}^{\lambda})^2} \tag{9}$$

is the Euclidean distance from h_{ij} to h_{kj}.

In (8), there are three possibilities: $S(h_{ij}) > S(h_{kj})$ is beneficial, $S(h_{ij}) = S(h_{kj})$ is neutral, or $S(h_{ij}) < S(h_{kj})$ represents a loss.

Therefore, a general perceived value function for the service quality of supplier $x_i (i = 1, 2, ..., m)$ can be defined as

$$\Phi(x_i) = \frac{\sum\limits_{k=1}^{m} \vartheta(x_i, x_k) - \min_i \{\sum\limits_{k=1}^{m} \vartheta(x_i, x_k)\}}{\max_i \{\sum\limits_{k=1}^{m} \vartheta(x_i, x_k)\} - \min_i \{\sum\limits_{k=1}^{m} \vartheta(x_i, x_k)\}}, \quad i = 1, 2, ..., m. \tag{10}$$

The steps for supplier service quality evaluation based on HFS with TODIM can then be defined as follows:

Step 1: HFS are used to describe the evaluation matrix $(a_{ij})_{m \times n}$ of supplier service quality.

Step 2: The relationship between the pros and cons of each pair of attributes can be calculated using the geometric hesitant fuzzy scoring function (5).

Step 3: Equation (8) is used to calculate the perceived value $\vartheta(x_i, x_k)$ of every supplier.

Step 4: Then, the generalized perceived value $\Phi(x_i)$ for supplier service quality can be calculated using Equation (10).

4. Case Study

With continued economic development, the selection of suppliers has become an important issue in the manufacturing industry. This study analyzes supplier selection based on both cost and quality of service. In a world of intelligent systems, intelligent manufacturing plays a major role and the core pillar of manufacturing is a wide range of high-end equipment. There are three major facets of the high-end equipment manufacturing industry: First, there is significant technological content, including performance of knowledge, technology intensity, and multi-disciplinary technology. Second, high-end value chains with high value-added features. Third, its position at the core of the industry chain and its level of development, which determines the overall competitiveness of the industry chain. Therefore, the selection of high-end equipment manufacturing suppliers is very important.

In this case, study, there are four high-end equipment part suppliers and a high-end equipment manufacturer that wants to select the best supplier. To select the right supplier for the high-end equipment manufacturing industry, a multi-object model is built to analyze the suppliers in terms of cost and service quality. The costs of the suppliers are listed in Table 3. The attributes of supplier service quality [34] include competitive display a_1, qualification ability a_2, suitability and competitiveness of solutions a_3, and relational fitness and dynamics a_4. The attributes of supplier service quality are described in detail in Table 1. The weight vector for the attributes is (0.2, 0.1, 0.4, 0.3), which is consistent with research by Gad J. Selig [34]. Considering the preferences and knowledge of the decision-maker, a hesitant fuzzy number is used to describe the evaluation values for the four suppliers under each attribute. The constructed decision matrix is presented in Table 4.

Table 3. Cost of suppliers.

Cost	C_1	C_2	C_3	C_4
	1000	1500	2000	1200

Table 4. Hesitant fuzzy decision matrix.

	a_1	a_2	a_3	a_4
x_1	H{0.55,0.6}	H{0.6,0.7}	H{0.5,0.7,0.75}	H{0.6,0.8,0.85}
x_2	H{0.5,0.7,0.8}	H{0.4,0.6}	H{0.5,0.8}	H{0.7,0.8}
x_3	H{0.55,0.7}	H{0.65,0.8}	H{0.7,0.8}	H{0.6,0.7,0.9}
x_4	H{0.6,0.8}	H{0.7,0.9}	H{0.5,0.6,0.8}	H{0.8,0.9}

In Table 4, H{0.55,0.6} indicates that the decision maker has two opinions regarding supplier x_1 under the attribute a_1 and that the value may be 0.55 or 0.6. The other data follows the same pattern.

From Equation (5), each hesitant fuzzy number scoring value can be calculated, and each supplier can be compared to the other suppliers under each attribute. Let S represent a supplier being superior and I represent a supplier being inferior. For example, the hesitant fuzzy number scoring value of supplier x_1 under attribute a_1 is $S_{11} = 0.574$ and the scoring value of x_2 is $S_{21} = 0.6542$. Therefore, $S_{11} < S_{21}$, meaning this relationship can be described by I. The score values were in Table 5 and the relationships listed in Table 6 were calculated from the score values.

Table 5. Scoring values of the hesitant fuzzy numbers.

	a_1	a_2	a_3	a_4
x_1	$S_{11} = 0.574$	$S_{12} = 0.6481$	$S_{13} = 0.6403$	$S_{14} = 0.7417$
x_2	$S_{21} = 0.6542$	$S_{22} = 0.4899$	$S_{23} = 0.6325$	$S_{24} = 0.7486$
x_3	$S_{31} = 0.6205$	$S_{32} = 0.7211$	$S_{33} = 0.7483$	$S_{34} = 0.7230$
x_4	$S_{41} = 0.6928$	$S_{42} = 0.7937$	$S_{43} = 0.6214$	$S_{44} = 0.8485$

Table 6. The relationships "superior-inferior" of suppliers.

	x_1/x_2	x_1/x_2	x_1/x_4	x_2/x_3	x_2/x_4	x_3/x_4
a_1	I	I	I	S	I	I
a_2	S	I	I	I	I	I
a_3	S	I	S	I	S	S
a_4	I	I	I	S	I	I

To calculate the distances between the hesitant fuzzy numbers in Table 4, the hesitant fuzzy numbers can be extended according to Hypothesis 1. Therefore, the previous decision matrix is translated into the new decision matrix presented in Table 7.

Table 7. The extended hesitant fuzzy matrix.

	a_1	a_2	a_3	a_4
x_1	H{0.55,0.55,0.6}	H{0.6,0.6,0.7}	H{0.5,0.7,0.75}	H{0.6,0.8,0.85}
x_2	H{0.5,0.7,0.8}	H{0.4,0.4,0.6}	H{0.5,0.5,0.8}	H{0.7,0.7,0.8}
x_3	H{0.55,0.55,0.7}	H{0.65,0.65,0.8}	H{0.7,0.7,0.8}	H{0.6,0.7,0.9}
x_4	H{0.6,0.6,0.8}	H{0.7,0.7,0.9}	H{0.5,0.6,0.8}	H{0.8,0.8,0.9}

Then, the perceived value function $\vartheta(x_i, x_k)$ of supplier $x_i (i = 1, 2, 3, 4)$ to supplier $x_k (k = 1, 2, 3, 4)$ under the same attribute a_j can be calculated using Equation (7). Here, the loss avoidance coefficient is $\theta = 1$ [39]. The results of these operations can be found in Table 8.

Table 8. The perceived values function value of each supplier.

	a_1					a_2			
	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
x_1	0	−0.8579	−0.5373	−0.7825	x_1	0	0.1316	−0.8409	−1.1892
x_2	0.1716	0	0.147	−0.6389	x_2	−1.3161	0	−1.5314	−1.7321
x_3	0.1075	−0.7349	0	−0.5946	x_3	0.0841	0.1531	0	−0.8409
x_4	0.1565	0.1278	0.1189	0	x_4	0.1189	0.1732	0.0841	0

	a_3					a_4			
	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
x_1	0	0.2182	−0.5455	0.1607	x_1	0	−0.5373	−0.4639	−0.664
x_2	−0.5455	0	−0.6389	−0.3799	x_2	0.1612	0	0.1565	−0.5774
x_3	0.2182	0.2556	0	0.2272	x_3	0.1392	−0.5217	0	−0.656
x_4	−0.4017	0.152	0.5681	0	x_4	0.1992	0.1732	0.1968	0

The perceived value function values for each supplier under all attributes can be integrated using Equation (8). The results are listed in Table 9.

Table 9. The general perceived value function value for suppliers' service quality.

	x_1	x_2	x_3	x_4
x_1	0	−1.0454	−2.3876	−2.475
x_2	−1.5288	0	−1.8668	−3.3283
x_3	0.549	−0.8479	0	−1.8643
x_4	0.0729	0.6262	0.9679	0

The general perceived value functions $\Phi(x_i)$ for service quality can be obtained using Equation (10):

$$\Phi(x_1) = 0.0972, \Phi(x_2) = 0, \Phi(x_3) = 0.5435, \Phi(x_4) = 1 .$$

Through the above steps, the general perception function values of supplier quality of service are calculated. When considering the psychological factors of decision makers, this result more closely describes the service quality of suppliers, which provides an important basis for supplier selection. The best supplier will then be selected based on the proposed preference model for multi-objective decision making.

The cost is standardized using Equation (2):

$$C'_1 = 1, C'_2 = 0.667, C'_2 = 0.5, C'_4 = 0.8333$$

The supplier service qualities are determined by

$$Q'_1 = \Phi(x_1) = 0.0972, Q'_2 = \Phi(x_2) = 0, Q'_3 \Phi(x_3) = 0.5435, Q'_4 = \Phi(x_4) = 1 .$$

Here, the decision maker has a cost preference, so the preference parameter is set to $\tau = 0.7$. According to Equation (3), the final value for each supplier is calculated as follows:

$$V_1 = 0.7291, V_2 = 0.4669, V_3 = 0.5131, V_4 = 0.8833.$$

The final ranking of the four suppliers is $V_4 \succ V_1 \succ V_3 \succ V_2$.

Through the proposed preference model, the cost and service quality of suppliers has been considered. When the decision maker has a cost preference, supplier x_4 ranks highest and supplier x_2 ranks the lowest. Therefore, supplier x_4 is the best selection for the high-end equipment manufacturer.

$$V_4 \succ V_1 \succ V_3 \succ V_2$$

5. Preference Analysis

Preference relationships are widely used in multi-attribute decision-making problems. During real-world decision-making processes, the result is not determined entirely from the decision-making model and most of the decision-making results are determined by the personal preferences of the decision maker. In particular, in the selection process for suppliers, many attributes are considered and many goals are optimized. As a result, the preferences of decision makers play a vital role. This study considers supplier cost and service quality, builds a preference model, and introduces the preference relation parameter $\tau \in [0, 1]$. The example in Section 4 analyzed the influence of differing preferences of the decision maker on Model-2. The results of analysis are listed in Table 10.

Table 10. Supplier preference analysis.

Preference ø	x_1	x_2	x_3	x_4	Order
$\tau = 0.1$	0.1875	0.0667	0.5392	0.9833	$x_4 > x_3 > x_1 > x_2$
$\tau = 0.3$	0.3681	0.2001	0.5305	0.9500	$x_4 > x_3 > x_1 > x_2$
$\tau = 0.5$	0.5486	0.3335	0.5218	0.9167	$x_4 > x_1 > x_3 > x_2$
$\tau = 0.7$	0.7292	0.4669	0.5131	0.8833	$x_4 > x_1 > x_3 > x_2$
$\tau = 0.9$	0.9097	0.6003	0.5044	0.8500	$x_1 > x_4 > x_2 > x_3$

From Table 10, the ranking of suppliers is different based on the preferences (τ) of the decision maker. Specifically, when the decision maker has a service quality preference ($\tau < 0.5$), supplier x_4 is the best and x_2 is the worst. When the decision maker has a cost preference ($\tau > 0.5$), supplier x_4 or x_1 is the best and x_2 or x_3 is the worst. The ranking of suppliers also differs based on the preference degrees of decision makers, as shown in Figure 1.
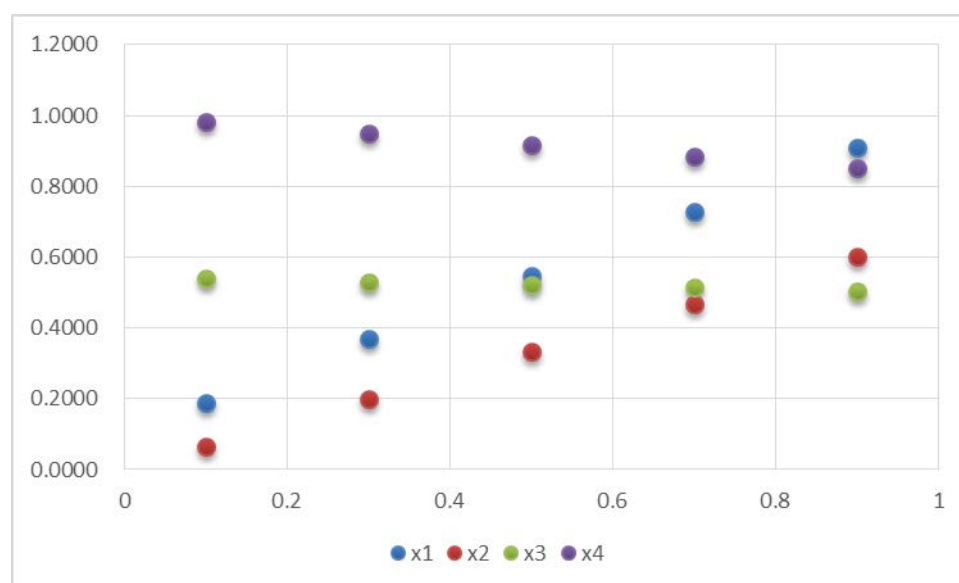

Figure 1. Ranking of suppliers with different preference degrees.

6. Discussion and Conclusions

Supplier selection has been the subject of significant research. However, most previous studies aimed at quantifying supplier selection as an optimization target and did not pay significant attention to combining non-quantifiable and quantifiable modeling processes. Furthermore, for the service quality evaluation of suppliers, most previous studies did not consider the psychological behavior of experts. To solve these problems, a preference model for supplier selection based on HFS was proposed in this paper. The proposed model analyzes the cost and service quality of suppliers while

considering the preferences of decision makers. For the non-quantifiable quality of service, HFS are used to describe the evaluations of experts. In this service quality assessment process, TODIM is used to represent the individual psychological behaviors of experts. Using the proposed model, suppliers can be ranked based on the differing preferences of decision makers to determine the best supplier. Finally, a numerical example of supplier selection for high-end equipment manufacturing was presented to verify the proposed model.

In the illustrative example of high-end equipment manufacturer supplier selection, cost and service quality were considered. Some significant results of this case study can be summarized as follows:

(1) The proposed model considers the preferences and behavior of the decision maker. As a result, their needs are more accurately reflected based on their preference for cost or service quality.

(2) The supplier service quality assessment considers the hesitation of expert assessments and psychological behavior. The supplier service assessment based on HFS and TODIM is an important input for the proposed preference model.

The supplier selection problem is of paramount importance in the modern commercial economy. However, improving supplier ranking involves many factors and poses difficulties for decision makers. The main purpose of the proposed model for supplier selection is to accurately represent the preferences of decision makers. Furthermore, supplier selection primarily considers cost and service quality, where service quality includes many attributes. Therefore, supplier selection is an important and challenging problem.

To solve the problems outlined above, a preference model based on HFS was proposed to select a supplier and a case study was presented to verify the effectiveness of the proposed model. However, the attributes for supplier service quality evaluation were simplified to some extent. Therefore, as a next step, more complex attributes could be obtained to evaluate supplier selection using a more comprehensive preference model.

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