Article

The Cost of Overcoming the Zero Lower-Bound: A Welfare Analysis †

Gerhard Rösl 1, Franz Seitz 2,* and Karl-Heinz Tödter 3

1 Faculty of Economics, Technical University of Applied Sciences (OTH) Regensburg, 93053 Regensburg, Germany
2 Faculty of Economics, Technical University of Applied Sciences (OTH) Amberg-Weiden, 92637 Weiden, Germany
3 Formerly Research Centre, Deutsche Bundesbank, 60431 Frankfurt am Main, Germany
* Correspondence: f.seitz@oth-aw.de
† The views in this article are not necessarily those of the Deutsche Bundesbank.

Received: 5 March 2019; Accepted: 25 June 2019; Published: 4 July 2019

Abstract: To broaden the operational scope of monetary policy, several authors suggest cash abolition as an appropriate means of breaking through the zero lower-bound. We argue that the welfare costs of bypassing the zero lower-bound by getting rid of cash entirely are analytically equivalent to negative interest rates on cash holdings. Using a money-in-the-utility-function model, we measure in two ways the welfare loss consumers as money holders would be forced to bear once the zero lower-bound is broken: in terms of the amount needed to compensate consumers (compensated variation), and as excess burden (deadweight loss) imposed on the economy as a whole. We calibrated the model for the euro area and for Germany. Our findings suggest that the welfare losses of negative interest rates incurred by consumers as holders of cash and transaction balances (M3) are large and enduring, notably if implemented in the current low-interest rate environment.

Keywords: zero lower-bound; negative interest rates; money in utility; welfare loss; compensating variation; deadweight loss

JEL Classification: E41; E21; E58; I3

1. Introduction

Although cashless payments have gained importance over time, banknotes and coins are still the most frequently used means of payments at the point-of-sale in almost all countries (Bagnall et al. 2016; Esselink and Hernández 2017). Recently, however, more and more claims were put forward to restrict the use of cash, such as upper-limits and reporting obligations for cash payments, withdrawal of high banknote denominations, or even plain abolition of cash (see, e.g., Agarwal and Kimball 2015; Rogoff 2016; Sands 2016).

A relatively new argument put forward is that cash restricts the effectiveness of monetary policy because it creates an asymmetry. While central banks can freely increase nominal interest rates in times of an overheating economy, in times of an economic crisis, however, the opposite is not possible (Agarwal and Kimball 2015; Rogoff 2016). This is due to the fact that money holders are able to evade negative interest rates on bank deposits by holding cash and thus creating a lower-bound for the risk-free short-term interest rate. Apart from adjustment and conversion costs, this leads to the well-known “zero lower-bound” problem. Some authors argue that interest rates of −5% down to −10%
would be needed at times to avoid deflation and overcome deep crises (e.g., Buiter and Rahbari 2015; Agarwal and Kimball 2019). 1

Measured by the Harmonized Index of Consumer Prices (HICP), there was no deflation in the euro area after the financial and global crisis of 2008/09, though some euro member countries (notably Greece) experienced short spells of price declines. Nonetheless, the already expansionary monetary policy of the European Central Bank (ECB) was further eased in the wake of the escalating public debt crisis in the euro area in 2010/11 and the euro crisis in 2012/13 which led to a massive quantitative easing (QE) and low, zero, and negative interest rates. This policy was assessed to have positive output and price effects (Bundesbank 2016, pp. 29–43; Priftis and Vogel 2017; Papadamou et al. 2019). However, it is also heavily criticized as having been unsuccessful or even counterproductive due to negative side effects ((Bartzsch et al. 2013, chp. 6); Bundesbank 2016, pp. 44–52; Michail 2018; Corsetti et al. 2019).

There are obviously many micro- and macro-economic advantages of using cash. These range from its stabilizing role in financial crises and data protection to overview and control of spending, payments inclusion and privacy, as well as competitive aspects in the payments landscape (Krüger and Seitz 2017). Consequently, even if it is agreed that central banks should be able to use the interest rate as a monetary policy instrument beyond the lower-bound, there should be clarity about all consequences of abolishing cash. Most advocates of cash abolition almost axiomatically argue that cash is an inefficient and costly means of payment. Repelling of cash would, therefore, be equivalent to an increase in economic efficiency (Van Hove 2016). However, we are not aware of any study on the welfare consequences for consumers and money holders. The present paper closes this gap and quantifies the welfare losses in terms of “monetary disadvantages” for the euro area as well as in Germany. In a comprehensive cost-benefit analysis of a potential cashless society, this neglected cost factor should be seriously taken into consideration.

The paper is structured as follows. Against the background of the potential benefits of cash, Section 2 discusses alternatives to the abolition of cash, still allowing central banks to break through the lower-bound. In Section 3, we used a “money-in-the-utility-function” (MIU) model to estimate the welfare costs of negative interest rates on cash holdings for households and the economy in the euro area and in Germany, thereby shedding light on the social costs of potential cash abolition. Section 4 analyses the welfare consequences of negative interest rates on a broader range of monetary aggregates. Section 5 summarizes and is the concluding part.

2. Breaking through the Lower-Bound

2.1. Cash Abolition

Since MasterCard declared the “war on cash” in 2005 (Adams 2006), time and again a cashless society was predicted, and a lot of suggestions were made in order to impede or limit the use of banknotes and coins. As mentioned in the introduction, these proposals didn’t only come from the financial industry. In 2010 the European Commission published a recommendation in which it stated that there should be no restrictions with regard to euro banknotes and coins as legal tender in the euro area (EU-Commission 2010). In January 2017, however, the EU-Commission changed its attitude and put forward proposals to limit the use of cash (EU-Commission 2017).

It is a declared goal of the advocates of cash abolition to enable central banks to significantly break through the effective lower interest rate bound. Recent years have shown that this bound is not exactly at zero but rather at around 50 to 100 basis points below zero, due to “carry costs of cash” associated with storage, insurance, and general risks. The interest rate barriers vary among countries, depending on the different types and structures of banks and investors (see for instance Switzerland

---

1 A more traditional case against cash is that it would help to fight corruption, tax evasion, drug trade, and terrorist financing (Rogoff 2016; Sands 2016). See Krüger and Seitz (2017, chp. 7.1) for a critical assessment. On the motivation of cash usage from a shopper’s perspective see Wakamori and Welte (2017).
in comparison to the euro area). The Canadian central bank estimates that its national lower-bound lies between −0.25% and −0.75%, cf. Witmer and Yang (2016). However, Bech and Malkhozov (2016), emphasise that during a long-lasting phase of low interest rates, such an interest rate bound will increase over time because economic agents would adapt to the new situation and realize more cash-cost effective innovations.

2.2. Negative Interest Rates on Cash

Complete abolition of cash to break through the lower-bound would be a radical socio-economic experiment, almost like an “open-heart surgery” of an economy. A less radical intervention would be the introduction of interest-bearing cash. Such a policy would be less risky because it could be reversed easily at any time, in contrast to cash abolition, which is practically irreversible. With negative interest rates on cash, the lower-bound could be broken, cash as a means of payment and store of value would still be available and money holders would retain the freedom of choice.

Our study regards negative interest rates on cash as being equivalent to abolishing cash. In order to get a deeper understanding of this equivalence, consider the following three scenarios as a thought experiment: In scenario (a) non-interest-bearing cash exists alongside with interest-bearing bank deposits. Consequently, in a deep economic crisis, the central bank can reduce interest rates to zero, but not more than that, this is because bank deposit owners will exchange their book money holdings for cash in order to avoid negative interest rates. In the case of cash abolition (scenario (b)), money holders would be forced to hold all money as bank deposits and pay electronically. The central bank will now be able to move interest rates further down into negative territory because substituting bank deposits for cash is no longer an option. However, if there would be a way (scenario (c)) that cash holders could earn or pay interest on cash (see Sections 2.2.1–2.2.3), money holders could again be forced to accept negative interest rates. The difference to scenario (b) is that cash remains in circulation and negative interest rates on all types of money could nevertheless be imposed.

Let the amount of cash in circulation (M0) be €1 billion (bn) and bank deposits (D) €10 billion, summing-up to a total money stock of €11 billion. This broadly corresponds to cash in circulation and bank deposits owned by non-banks (M3, excluding cash) in the Euro area at the end of 2015. Further, assume that the interest rate on bank deposits is $i = 3\%$ and the central bank intends to bring it down to $i = -3\%$ in order to fight a severe economic downturn. In scenario (a) with the zero lower-bound in effect, the interest rate is reduced to $i = 0\%$ and money holders would lose €300 million (3% of D) interest income per year. In scenario (b), cash is abolished, the zero-bound is ineffective, and the revenue loss money holders now have to bear amounts to €660 million, consisting of €330 million foregone interest income from the whole money stock (€11 billion) and €330 million negative interest payments. Especially the latter should exert incentives to increase spending and boost demand in the economy. Exactly the same losses (and incentives to spend) would materialize in scenario (c) where cash remains a means of payment alongside bank deposits, as long as cash and bank deposits are subject to the same negative interest rates. In such a scenario, substituting bank deposits for cash is ineffective to avoid negative interest rates on bank deposits.

Negative interest rates on cash could be implemented in different ways. We provide a brief overview of three main variants discussed in the literature (Buitrer 2009).

2.2.1. Depreciative Money (Schwundgeld)

The idea of introducing depreciative money traces back to Gesell (1911, 1949). In order to efficiently fight against cash hoarding, Gesell suggested as early as 1911, that stamps should be stuck to banknotes on a weekly basis in order to maintain the status of a legitimate means of payment (“stamp scrip”). Those stamps had to be bought by money holders and can be viewed as fee or as negative interest rates on cash. A less complicated variant is the so called “table money”—cash with a printed table on it which shows the decreasing value over time. Nowadays, cash holding fees could be quite easily implemented directly on the banknotes by computer chips. Irving Fisher (1933) and especially...
Keynes (1936) found the idea of “Schwundgeld” quite attractive because they believed that it would enable the central bank to move the economy out of the liquidity trap. Depreciative money was put into circulation in the past on various occasions and places. In the form of “regional money”, it is still in existence in some countries. These unofficial means of payments circulate predominantly in Germany, Austria, and Switzerland, but also in some other European countries privately issued regional currencies can be found. However, it comes with no surprise that the cumulated face value of all regional currencies in Germany does not exceed €1 million. Amongst other reasons, this clearly has something to do with its high costs and being cumbersome to use.

2.2.2. The Flexible Exchange Rate between Cash and Bank Deposits

A relatively easy way of paying (negative or positive) interest rates on banknotes was outlined by Agarwal and Kimball (2015) following a suggestion by Bui\text{t}er (2009). Their proposal is based on the idea to separate the monetary functions of cash as a unit of account and a means of payment, by introducing a flexible exchange rate between cash and (electronic) bank deposits. In this scheme, paper money remains a medium of exchange, whereas book money becomes the central unit of account. This mechanism is implemented by a time-varying deposit fee on cash, which has to be paid by commercial banks at the “cash window” of the central bank. By this means, the central bank can at its choice implement an administered, but in principle a flexible, conversion rate between cash and deposits which implies either a positive or negative interest rate on cash. If the interest rate on cash is intended to become more negative, the cash fee would simply be raised and, as a consequence, paper money would depreciate against deposits. However, solely commercial banks would be directly charged with the cash fee and private cash owners would be affected only indirectly, and to an extent, commercial banks would hand over these costs to their customers.

2.2.3. Taxes on Cash

A further possibility for implementing a negative interest rate on cash would be levying a tax on cash. This tax would only to be effective if someone buys a product and pays in cash. If instead the product is paid by using bank deposits, the tax could be avoided legally (Bui\text{t}er 2009). This scheme is easy to handle and would impose a negative interest rate on cash, provided the central bank has the right to change the tax rate in a variable fashion.

3. Welfare Losses Due to Negative Interest Rates on Cash

As demonstrated above, the lower-bound can be broken by the abolition of cash, or equivalently, the introduction of negative interest rates on cash alike. But what are the economic costs the society has to bear resulting from such measures? Social welfare costs of negative interest rates are primarily caused by a loss in consumer surplus related to cash demand.

3.1. Sidrauski Model with Interest Rates on Cash

Drawing on Lucas (2000) and Rösl (2006), as an analytical framework, we used a Sidrauski model, augmented by interest bearing cash. The strict concave periodical utility (\( u \)) of a representative

---

2 For example, the “Miracle of Wörgl”, (Miracle of Wörgl 1932), was started during the Great Depression on 31 July 1932 with the issuing of ‘Certified Compensation Bills’, a form of local “Schwundgeld”. This was an application of the monetary theories of Silvio Gesell by the town’s then-mayor Michael Unterguggenberger. The experiment increased local employment and several local government projects could be completed, seeming to defy the depression in the country. Despite attracting great interest at the time, including the French Premier Edouard Daladier and Irving Fisher, the experiment was terminated by the Austrian central bank on 1 September 1933.

3 Cf. Rösl (2006); for an international perspective see Harper (1948), Timberlake (1987), and Warner (2014).

4 Agarwal and Kimball (2015) speak of a crawling peg. If this exchange rate would be a free market rate it is not clear beforehand whether it would settle eventually above or below par.

5 In this section money only consists of cash. However, Section 5 considers broader monetary aggregates.
household depends on the consumption of (real) goods \((c)\) and the use of money services being proportional to real cash holdings \((m)\).\(^6\) The household maximizes its discounted lifetime utility \((U)\):

\[
U = \int_{t=0}^{\infty} u(c_t, m_t) e^{-\theta t} dt
\]

with time index \(t\) and the discount rate \(\theta\). The household possesses real assets \((A)\) in the form of real capital \((K)\) and cash \((M)\), deflated by the price level \((P)\):

\[
A_t = K_t + M_t/P_t.
\]

It draws real income from labour \((wN)\), capital \((rK)\), and lump-sum transfers \((X)\) from the government:\(^7\)

\[
w_tN_t + r_tK_t + X_t = C_t - s_tM_t/P_t + dA_t/dt
\]

where, \(w\) is the real-wage rate, \(N\) is the number of household members, \(r\) is the real return on capital, and \(K\) is the real capital stock.\(^8\) This income is used for consumption, \(C\), and for financing the devaluation of the real money holdings due to negative (nominal) interest rates \((s < 0)\) on cash \((sM/P)\). The difference between income and expenditures for goods and money holdings represents the change in the households’ real assets \((dA/dt)\). In real per capita terms (lower case letters), the budget constraint \((3)\) reads:

\[
w_t + r_tk_t + x_t = c_t - s_tm_t + (dA_t/dt)/N_t.
\]

The change in real household’s assets \((2)\) per capita is thus:

\[
(dA_t/dt)/N_t = dk_t/dt + nk_t + dm_t/dt + n_tmt + n_tm_t,
\]

with \(n\) as the growth rate of the population, assuming a fixed number of households.

Using the real per capita assets \([a = k + m]\), we can rewrite the budget constraint \((4)\) as a differential equation for the real per capita assets of the household:

\[
da_t/dt = [(r_t - s_t)a_t + w_t + x_t] - [c_t + zm_t].
\]

It increases if real income exceeds real expenditures per capita. The latter comprises the sum of consumption of goods \((c)\) and consumption of money services \((zm)\), \(z\) being user costs of money \((r + \pi - s)\). If money earns or costs no direct interest \((s = 0)\) the user costs of cash are equal to nominal capital market interest rates representing the opportunity costs of holding cash, which in turn comprise the real capital market interest rate \((r)\) plus the inflation rate \((\pi)\):

\[
z_{0,t} = r_t + \pi_t.
\]

Also taking into account interest rates earned or to be paid on cash \((s)\) we get:

\[
z_t = r_t + \pi_t - s_t = z_{0,t} - s_t.
\]


\(^7\) This corresponds to the recycling of seigniorage from cash issuance (including transfer of real resources to the money producers owing to negative interest rates on nominal money holdings).

\(^8\) To keep things simple, the (capital market) interest rate \(r\) is assumed exogenous, although an endogenous interest rate (by introducing a production function) would not substantially alter the model results. For similar reasons, depreciation of the capital stock is not taken into consideration either.
If cash earns direct interest ($s > 0$) user costs of cash are reduced, whereas negative direct interest rates ($s < 0$) add to money holding costs accordingly.

The initial wealth can be positive or negative. However, at the “end” of the planning horizon ($t \rightarrow \infty$) it shall not have a negative value (no-Ponzi-game condition): $\lim_{t \rightarrow \infty}(a_t e^{-\theta t}) \geq 0$. The Hamiltonian associated with this maximization problem is:

$$H = \{u(c_t, m_t) + \lambda_t (da_t/dt)e^{-\theta t}\}.$$ (9)

The multiplier ($\lambda$) is the shadow price of the change in assets. The first-order conditions for a maximum are:

$$du_t/dc_t \equiv u_c(c_t, m_t) = \lambda_t,$$ (10)

$$du_t/dm_t \equiv u_m(c_t, m_t) = \lambda_t z_t,$$ (11)

$$d\lambda_t/dt = \theta \lambda_t - (r_t - n_t) \lambda_t,$$ (12)

$$\lim_{t \rightarrow \infty}[\lambda_t a_t e^{-\theta t}] = 0.$$ (13)

Condition (10) implies that the marginal utility of consumption of goods equals the shadow price of income. According to (11), the marginal utility of money services is equivalent to the user costs of money, weighted by the shadow price of income. If this would not be the case, a change in money holdings could increase lifetime utility. (12) is the dynamic equation of motion for the shadow price of the household’s wealth. Finally, (13) is the transversality condition. It links the assessment of the final situation to the change in the shadow price. This condition implies that the optimizing household, at the end of its planning horizon, will either have fully consumed its wealth or at least will have reduced it to an extent to which it does not generate any additional utility in the present. Conditions (10) and (11) combine to:

$$u_m/u_c = z_t,$$ (14)

saying that at the optimum the marginal rate of substitution between money and goods is equivalent to the user costs of money. The higher the real interest rate ($r$) and the higher the effective inflation rate ($\pi - s$) the “more expensive” the consumption of money services becomes in relation to the consumption of goods.

The model has a number of interesting implications: (10) and (12) together result in the well-known Keynes–Ramsey rule without any reference to monetary variables:

$$(d\lambda/dt)/u_c = n_t + \theta - r_t.$$ (15)

In the long-run, the equilibrium condition $d\lambda_t/dt = 0$ holds and hence, from (15) we get a modified form of the Golden Rule, $r_t = \theta + n_t$, i.e., the real (capital market) interest rate is the sum of the (subjective) discount rate and the rate of population growth. Furthermore, the equilibrium growth rate of real money per capita is also zero, $\mu_t - \pi_t - n_t = 0$, with $\mu_t$ as the growth rate of (nominal) money (M). This implies that in the long-run equilibrium, the inflation rate is determined by money growth per capita only: $\pi_t = \mu_t - n_t$. Thus, inflation is a purely monetary phenomenon in the sense of Friedman (1969).

Assuming that the production costs of cash are negligibly small, we get $u_m = 0$, and according to (11) $z_t = r_t + \pi_t - s_t = 0$ holds in the long run. If the direct interest rate on cash is zero ($s = 0$), we get the well-known Friedman rule which states that in the social optimum the nominal (capital) market interest rate is the sum of the real (capital) market interest rate and the inflation rate. Generally, at a positive real (capital market) interest rate the central bank should therefore aim for a corresponding deflation.

---

9 Blanchard and Fischer (1989); Romer (2019, chp. 2).
(corrected by the direct interest rate on cash, $s$) in order to realize a nominal (capital) market interest rate of zero. This result, however, can also be reached by means of paying positive direct interest rates on cash equivalent to nominal capital market rates ($s = r + \pi > 0$). Of course, an appropriate combination of deflation and positive interest rates on cash can also be chosen in order to meet the requirements of the Friedman rule. Negative inflation rates (deflation) and positive interest payments on money, however, are the exact opposite of what central banks currently aim for.

In order to estimate the welfare costs of negative interest rates on cash quantitatively, we posit the logarithmic utility function:

$$u_t = (1 - \alpha) \ln(c_t) + \alpha \ln(m_t),$$  \hspace{1cm} (16)

with parameter $\alpha$ denoting the relative preference for the consumption of cash services. From (14) we obtain real money demand:

$$m_t = \beta c_t/z_t, \beta = \alpha/(1 - \alpha)$$  \hspace{1cm} (17)

as a function of the real consumption of goods and the user costs of money. Money demand increases proportionally to the consumption of goods and decreases proportionally to the user costs of money. The velocity of money with respect to the consumption of goods is:

$$v_t = c_t/m_t = z_t/\beta.$$  \hspace{1cm} (18)

If no interest rate is paid on cash ($s = 0$), we can rewrite (18) as:

$$\tilde{v}_t = z_{0,t}/\beta.$$  \hspace{1cm} (19)

Therefore, the velocity of money can also be expressed as:

$$v_t = \tilde{v}_t(1 - s_t/z_{0,t}).$$  \hspace{1cm} (20)

This equation shows to what extent the velocity of money reacts to direct interest payments on money and how the interest factor affects money management of households.

Money demand (17) is a hyperbolic function of $z$ with an interest elasticity of $-1$. In equilibrium the opportunity cost of money

$$k_t = z_t m_t = \beta c_t$$  \hspace{1cm} (21)

is independent of interest rates. Nonetheless, negative interest rates on money cause welfare losses.

To be specific, an increase in the user costs of money ($z > z_0$), for instance by means of negative interest rates on cash, causes a loss in consumer surplus (CS) for consumers by:

$$CS_t = \int_{z_0}^{z} m_t(z_t)dz_t = \beta c_t \ln(z_t/z_{0,t}).$$  \hspace{1cm} (22)

This loss in consumer surplus comes along with additional revenues (SE) by the central bank and, finally, by the government, because negative interest rates on cash are equivalent to a tax on money:

$$SE_t = (z_t - z_{0,t})m_t = \beta c_t (z_t - z_{0,t})/z_t.$$  \hspace{1cm} (23)

These revenues could be used by the government to finance spending or to cut taxes. They would (at least in principle) be welfare enhancing and would partially compensate for the loss in consumer surplus. According to this logic, society suffers a welfare loss only by the difference between the loss of
consumer surplus and the additional revenues obtained by the government.\textsuperscript{10} This difference is the deadweight loss or excess burden of negative interest rates on money:\textsuperscript{11}

\[ \text{DWL}_t = \text{CS}_t - \text{SE}_t. \tag{24} \]

Compensated variation (CV) and equivalent variation (EV) are alternative measures of welfare losses closely related to consumer surplus (Johansson 1991). Substituting (17) into (16) yields the indirect utility function:

\[ \bar{u}_t = \kappa + \ln(c_t) - \alpha \ln(z_t), \tag{25} \]

where, \( c_t \) is the optimum consumption level and \( \kappa = \alpha \ln(\beta) \) is a constant.

To what extent does the consumption level have to be raised to compensate the private household for the loss in utility generated by an increase in the user costs of money? As a first step, we determined the compensating level of consumption if \( z_0 \) increases to \( z_t \). Equating the maximum utility level (25) before \((c_0,t, z_0)\) and after that change \((c_X,t, z_t)\) and solving for \( c_X \) yields:

\[ c_{X,t} = c_{0,t}(z_{t}/z_{0,t})^{\alpha}. \tag{26} \]

The compensation for the loss in utility resulting from money with negative interest rates has to be the higher, the larger the relative user costs of money \((z/z_0)\) and the higher the preference for money services (\( \alpha \)) are. Therefore, a lower nominal interest rate, i.e., lower foregone interest income on alternatives to holding money (opportunity costs of money, \( z_0 \)), implies an increasing compensation.

The difference between the consumption level before the increase in the user costs of money \((c_0,t)\) and the compensating consumption level \((c_X)\) is:

\[ \text{CV}_t = c_{0,t} - c_{X,t} = c_{0,t}\left[1 - (z_{t}/z_{0,t})^{\alpha}\right]. \tag{27} \]

CV measures the loss in consumption possibilities due to negative interest rates on cash. It is the amount the consumer needs as compensation in terms of goods to reach the original level of utility. Similarly, equivalent variation (EV),

\[ \text{EV}_t = (c_{0,t} - c_{X,t})(c_{0,t}/c_{X,t}) = c_{0,t}\left[(z_{t}/z_{0,t})^{\alpha} - 1\right] \tag{28} \]

measures the amount the consumer would be willing to pay at most to avoid the loss in utility caused by negative interest rates. In other words: If money holding costs increase, the consumer would at most pay \( \text{EV} \) in advance to avoid the loss, and he would at least demand the amount \( \text{CV} \) as a compensation in case the loss already occurred.

For sufficiently small \( s \), the three concepts of welfare losses of negative interest rates on money yield approximately the same result\textsuperscript{12}:

\[ |\text{CS}| \approx \text{CV} \approx \text{EV} \approx \alpha \frac{c_0}{s/z_0}. \tag{29} \]

3.2. Quantification of the Welfare Losses of Negative Interest Rates on Cash Holdings

The Sidrauski model allows for quantifying the costs of breaking through the zero-interest-rate-bound by means of negative interest rates on cash (or cash abolition for that matter). To calculate the welfare losses in the case of negative interest rates, we need an estimate of the

\textsuperscript{10} Whether or not such considerations are also appropriate for actions taken by a democratically non-legitimized central bank as the ECB is an open question, see Rösl and Tödter (2015, p. 43).

\textsuperscript{11} If the money demand function is linear, the excess burden is equivalent to the so called “Harberger triangle”.

\textsuperscript{12} For a more detailed treatment see Rösl et al. (2017).
relative preference for the consumption of cash services ($\alpha$). On a macroeconomic level, the money demand function (17) reads:

$$M_0_t = \beta C_t / z_t, \quad (30)$$

where, $M_0$ is cash in circulation and $C$ is nominal private consumption. Equation (30) suggests the following estimate of the preference parameter:

$$\hat{\alpha} = \frac{z M_0}{C + z M_0}. \quad (31)$$

The relative preference for monetary services is equal to the expenditure share of the consumption of monetary services. The higher the opportunity costs of cash holdings ($zM_0$), the higher are the revealed preferences of consumers for the use of monetary services.

To estimate the welfare losses of negative interest on cash, we used data for the euro area and Germany (see Appendix B). Our calculations are based on the year 2015. For comparison purposes, results for 2005, before the periods of different crises, are shown as well. Between 2005 and 2015, cash in circulation in the euro area had increased by 7% per year, while nominal private consumption increased on average by 1.9% only. Although cash in circulation had doubled during this period, the opportunity costs of cash holdings had only risen from €32 bn to €44 bn, while in Germany they remained nearly unchanged. Using (Equation (31)), we get an estimate for the preference parameter $\alpha$ of 0.75% (0.52%) for 2015 for the euro area (Germany). Thus, the user costs of cash are between 0.5% and 0.75% of consumption expenditures. In 2005, they stood at 0.66% in the euro area as well as in Germany.

To measure the opportunity costs of cash ($z$), we refer to the yield on total financial assets of private households (Bundesbank 2015 and Appendix B). Figure 1 shows the evolution of this yield in real terms for Germany from 1999 to 2015. The average real yield over this period was 2.4% p.a. (std. deviation 2.9%).

![Figure 1. Real yield (%) of financial assets of private households in Germany.](image)

In 2005, the real yield was at 4%. In the course of the global financial crisis, it fell sharply and has subsequently recovered. On average, in 2015 it was at 3.4%, despite near-zero inflation. However, this figure is distorted upwards by an outlier in the first quarter (5.4%). Therefore, and because money demand reacts with a lag to interest rate changes, we used the average yield of 2014, which amounts
to 2.6%. 20% of private households’ wealth consists of cash and overnight deposits. Taking this into account, the real return on the interest-bearing components of the financial assets is 3.3%. If the increase in consumer prices is also taken into account, the nominal return on financial assets (without cash and overnight deposits) is about $z_0 = 3.5\%$ p.a., compared to 6% in 2005. This reduction is similar to the decline of the yield on (10-year) government bonds.

Unfortunately, there are no comparable data on real returns on financial assets of private households in the euro area. Therefore, we assume that the difference between the nominal yield of financial assets in the euro area relative to Germany is the same as between long-term yields of government bonds. This results in a euro area yield on financial assets in 2015 of $z_0 = 4.2\%$, compared to 6.1% in 2005.

Table 1 shows the estimated welfare losses from the introduction of negative interest rates on cash holdings ($s < 0$). The negative interest rate is reduced in several steps from $s = 0$ to $s = -0.10$. As a result, cash demand is reduced, and its velocity of circulation increases. This forced economization of cash holdings leads to welfare losses. Already at an interest rate of $s = -3\%$, the loss in consumer surplus (CS) in the euro area is estimated at €24 bn. After subtracting (the assumed) additional government revenues, the annual deadweight loss (DWL) still comes to more than €5 bn.

### Table 1. Welfare losses of negative interest rates on cash holdings in the euro area and in Germany, 2015.

<table>
<thead>
<tr>
<th>Interest Rate on Cash</th>
<th>s Unit</th>
<th>0</th>
<th>-0.01</th>
<th>-0.02</th>
<th>-0.03</th>
<th>-0.04</th>
<th>-0.05</th>
<th>-0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euro Area</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>User cost of cash</td>
<td>$z$</td>
<td>rate 0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.14</td>
</tr>
<tr>
<td>Cash demand</td>
<td>$M_0$</td>
<td>€ bn. 1049</td>
<td>846</td>
<td>708</td>
<td>609</td>
<td>535</td>
<td>476</td>
<td>308</td>
</tr>
<tr>
<td>Velocity of circulation *)</td>
<td>$v$</td>
<td>1.0</td>
<td>1.2</td>
<td>1.5</td>
<td>1.7</td>
<td>2.0</td>
<td>2.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Loss in consumer surplus</td>
<td>$CS$</td>
<td>€ bn. 0</td>
<td>9.4</td>
<td>17.1</td>
<td>23.7</td>
<td>29.4</td>
<td>34.4</td>
<td>53.4</td>
</tr>
<tr>
<td>Additional gov. revenues</td>
<td>$SE$</td>
<td>€ bn. 0</td>
<td>8.5</td>
<td>14.2</td>
<td>18.3</td>
<td>21.4</td>
<td>23.8</td>
<td>30.8</td>
</tr>
<tr>
<td>Deadweight loss</td>
<td>$DWL$</td>
<td>€ bn. 0</td>
<td>0.9</td>
<td>3.0</td>
<td>5.4</td>
<td>8.0</td>
<td>10.6</td>
<td>22.6</td>
</tr>
<tr>
<td>Compensating cons. level</td>
<td>$C_{X}$</td>
<td>€ bn. 5743</td>
<td>5752</td>
<td>5760</td>
<td>5767</td>
<td>5772</td>
<td>5777</td>
<td>5796</td>
</tr>
<tr>
<td>Equivalent variation</td>
<td>$EV$</td>
<td>€ bn. 0</td>
<td>-9.3</td>
<td>-17.0</td>
<td>-23.5</td>
<td>-29.1</td>
<td>-34.1</td>
<td>-52.8</td>
</tr>
<tr>
<td>Compensating variation</td>
<td>$CV$</td>
<td>€ bn. 0</td>
<td>-9.3</td>
<td>-17.0</td>
<td>-23.6</td>
<td>-29.2</td>
<td>-34.3</td>
<td>-53.3</td>
</tr>
<tr>
<td>per capita</td>
<td>$cvp$</td>
<td>€ 0</td>
<td>-28</td>
<td>-1</td>
<td>-70</td>
<td>-87</td>
<td>-102</td>
<td>-158</td>
</tr>
<tr>
<td>as a share in cons.</td>
<td>$cvc$</td>
<td>% 0</td>
<td>-0.16</td>
<td>-0.30</td>
<td>-0.41</td>
<td>-0.51</td>
<td>-0.60</td>
<td>-0.93</td>
</tr>
<tr>
<td>as a share in GDP</td>
<td>$cvb$</td>
<td>% 0</td>
<td>-0.09</td>
<td>-0.16</td>
<td>-0.23</td>
<td>-0.28</td>
<td>-0.33</td>
<td>-0.51</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>User costs of cash</td>
<td>$z$</td>
<td>rate 0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.14</td>
</tr>
<tr>
<td>Cash demand</td>
<td>$M_0$</td>
<td>€ bn. 244</td>
<td>190</td>
<td>156</td>
<td>132</td>
<td>114</td>
<td>101</td>
<td>64</td>
</tr>
<tr>
<td>Velocity of circulation *)</td>
<td>$v$</td>
<td>1.0</td>
<td>1.3</td>
<td>1.6</td>
<td>1.9</td>
<td>2.1</td>
<td>2.4</td>
<td>3.8</td>
</tr>
<tr>
<td>Loss in consumer surplus</td>
<td>$CS$</td>
<td>€ bn. 0</td>
<td>2.1</td>
<td>3.9</td>
<td>5.3</td>
<td>6.5</td>
<td>7.6</td>
<td>11.6</td>
</tr>
<tr>
<td>Add. gov. revenues</td>
<td>$SE$</td>
<td>€ bn. 0</td>
<td>1.9</td>
<td>3.1</td>
<td>4.0</td>
<td>4.6</td>
<td>5.1</td>
<td>6.4</td>
</tr>
<tr>
<td>Deadweight loss</td>
<td>$DWL$</td>
<td>€ bn. 0</td>
<td>0.2</td>
<td>0.8</td>
<td>1.3</td>
<td>2.0</td>
<td>2.6</td>
<td>5.2</td>
</tr>
<tr>
<td>Compensating cons. level</td>
<td>$C_{X}$</td>
<td>€ bn. 1636</td>
<td>1638</td>
<td>1640</td>
<td>1641</td>
<td>1642</td>
<td>1644</td>
<td>1648</td>
</tr>
<tr>
<td>Equivalent variation</td>
<td>$EV$</td>
<td>€ bn. 0</td>
<td>-2.1</td>
<td>-3.8</td>
<td>-5.3</td>
<td>-6.5</td>
<td>-7.5</td>
<td>-11.5</td>
</tr>
<tr>
<td>Compensating variation</td>
<td>$CV$</td>
<td>€ bn. 0</td>
<td>-2.1</td>
<td>-3.9</td>
<td>-5.3</td>
<td>-6.5</td>
<td>-7.6</td>
<td>-11.6</td>
</tr>
<tr>
<td>per capita</td>
<td>$cvp$</td>
<td>€ 0</td>
<td>-26</td>
<td>-47</td>
<td>-64</td>
<td>-79</td>
<td>-92</td>
<td>-141</td>
</tr>
<tr>
<td>as a share in cons.</td>
<td>$cvc$</td>
<td>% 0</td>
<td>-0.13</td>
<td>-0.24</td>
<td>-0.32</td>
<td>-0.40</td>
<td>-0.46</td>
<td>-0.71</td>
</tr>
<tr>
<td>as a share in GDP</td>
<td>$cvb$</td>
<td>% 0</td>
<td>-0.07</td>
<td>-0.13</td>
<td>-0.17</td>
<td>-0.21</td>
<td>-0.25</td>
<td>-0.38</td>
</tr>
</tbody>
</table>

Source: Table A1, own calculations; *) relative velocity of circulation, see Equation (19).

Compensating private households in the euro area for the welfare loss (CV) of an interest rate of −3% on cash holdings requires €24 bn per year. At $s = -1\%$ the figure drops to €9 bn and at $s = -5\%$ it increases to €34 bn. These welfare losses are between 0.20% and 0.60% of private consumption or between 0.10% and 0.30% of GDP. Per capita, the burden in the euro area ranges between €30 and €100.

---

13 In 2017, the real yield was only at 1.4%. 
per year. In Germany, the welfare losses of consumers at negative interest rates on cash holdings of \((-1\%, -3\%, -5\%)\) are \(\mathcal{E}(2.1, 5.3, 7.6 \text{ bn})\). The deadweight loss for the German economy amounts to \(\mathcal{E}(0.2, 1.3, 2.6 \text{ bn})\). Per capita and relative to GDP or private consumption, the numbers in Germany are somewhat lower than those in the euro area. The conclusions remain qualitatively unchanged if re-calculated for 2016 or 2017.

3.3. Sensitivity Analysis

How sensitive are the welfare losses to variations in specific assumptions of our model? The logarithmic utility function (16) implies an elasticity of substitution between goods and monetary services of one. However, empirical estimates of substitution elasticities show a wide range of values (see, e.g., Havranek et al. 2015). To get an idea of how different values affect our results, we assume in the following an alternative utility function with constant relative risk aversion (CRRA, see Appendix A). This functional form implies an elasticity of substitution \((\sigma = 1/\rho)\) which is constant, but different from one.

The CRRA utility function leads to the following money demand function:

\[
m_t = \lambda c_t z_t^{-1/\rho},
\]

where \(\lambda\) is a constant. The relation of money demand functions (32) for two periods \((t, \tau)\) provides a point estimate for the elasticity of substitution:

\[
\hat{\sigma} = \frac{\ln(\nu_t/\nu_\tau)}{\ln(z_t/z_\tau)}.
\]

The more the velocity of circulation of cash responds to changes in the user costs of cash, the higher the elasticity of substitution. With data for 2005 and 2015 (Appendix B), we get a value of \(\hat{\sigma} = 1.29 (0.61)\) for the euro area (Germany).

Table 2 illustrates for a negative interest rate of \(s = -3\%\) how the welfare costs (CV) change when the elasticity of substitution \((\sigma)\) varies. At \(\sigma = 0.5\), the welfare loss reduces from \(\mathcal{E}24 \text{ bn}\) to \(\mathcal{E}14 \text{ bn}\), whereas at \(\sigma = 2\), it increases to \(\mathcal{E}36 \text{ bn}\).

<table>
<thead>
<tr>
<th>Elasticity of substitution</th>
<th>CV</th>
<th>cvc</th>
<th>cvb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>1.00</td>
<td>-24</td>
<td>-0.41</td>
</tr>
<tr>
<td>2.00</td>
<td>-36</td>
<td>-0.63</td>
<td>-0.35</td>
</tr>
<tr>
<td>0.50</td>
<td>-16</td>
<td>-0.27</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preference parameter</th>
<th>CV</th>
<th>cvc</th>
<th>cvb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.075</td>
<td>-24</td>
<td>-0.41</td>
</tr>
<tr>
<td>1.00</td>
<td>-31</td>
<td>-0.54</td>
<td>-0.30</td>
</tr>
<tr>
<td>6.00</td>
<td>-18</td>
<td>-0.31</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nominal yield of financial wealth</th>
<th>CV</th>
<th>cvc</th>
<th>cvb</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.16</td>
<td>2.00</td>
<td>-40</td>
<td>-0.69</td>
</tr>
</tbody>
</table>

Source: own calculations. CV = compensating variation, cvc (cvb) = share in consumption (GDP).

If the preference parameter \((\alpha)\) is lowered from 0.75% to 0.50% or raised to 1.0%, the welfare losses are \(\mathcal{E}16 \text{ bn}\) and \(\mathcal{E}31 \text{ bn}\), respectively. Increasing the yield of financial assets \((z_0)\) to 6% or decreasing it to 2% results in welfare losses of \(\mathcal{E}18 \text{ bn}\) and \(\mathcal{E}40 \text{ bn}\), respectively.
4. Negative Interest Rates on Broader Monetary Aggregates

So far, we have analysed the welfare loss solely for negative interest on cash holdings. However, it is not the primary aim of negative interest on cash to generate government revenues. The main purpose of such a measure is to overcome the effective lower-bound and to introduce negative interest rates on a wide range of deposits. In this section, we examined the welfare losses if not only cash is subject to negative interest rates, but all other assets included in the monetary aggregates M1 and M3.

The components of broader monetary aggregates fulfil the medium of exchange function to a different extent. They provide monetary services and are therefore included in the utility function of the representative household. These near-money assets \( m_i \) are close, but not perfect substitutes. Therefore, we do not introduce them in an aggregated way but consider each component separately. The utility function of the Sidrauski model reads as:

\[
 u_t = (1 - W) \ln(c_t) + \sum_{i=1}^{k} \alpha_i \ln(m_{i,t}). \tag{34}
\]

The parameter \( \alpha_i \) is the relative preference of the monetary component \( i (=1, \ldots, k) \) and \( 1 - W \) is the relative preference for the consumption of goods, with \( W = \sum_{i=1}^{k} \alpha_i \) as the preference rate for the consumption of monetary services of the respective monetary aggregate (M1, M3). The demand for monetary asset \( m_i \) equals:

\[
 m_{i,t} = \frac{\alpha_i}{1 - W} \frac{c_t}{z_{i,t}}, \quad i = 1 \ldots k. \tag{35}
\]

This leads to the following estimate of the preference parameters:

\[
 \hat{\alpha}_i = \frac{z_i M_i}{C + \sum_{i=1}^{k} z_i M_i}. \tag{36}
\]

As in (8), the user cost of a monetary component is the interest differential between the nominal yield of financial assets \( (z_0 = r + \pi) \) and its own rate \( (s_i) \):

\[
 z_{i,t} = r_t + \pi_t - s_{i,t}. \tag{37}
\]

If monetary policy is altered, the own rates of all monetary components, \( i \), change to \( z_i + \Delta s_i \), and the compensating consumption level becomes:

\[
 c_{x,t} = c_t \prod_{i=1}^{k} \left[ (z_{i,t} + \Delta s_{i,t}) / z_{i,t} \right]^{\alpha_i}. \tag{38}
\]

The monetary aggregates of the euro area is comprised of the liquid assets, especially deposits, of domestic non-banks issued by domestic Monetary Financial Institutions (MFIs). The monetary aggregate M1 comprises currency in circulation and overnight deposits (OD). M3 additionally includes deposits with an agreed maturity of up to 2 years, deposits redeemable at notice of up to 3 months, repurchase agreements, money market fund shares/papers, and debt securities issued with a maturity of up to 2 years. For the sake of simplicity, we denote these assets in the following as term deposits and savings deposits (TS).

The table in Appendix B shows the monetary aggregates for the euro area and for Germany from 2005–2015. It is evident that M0, M1, and M3 grew more strongly than private consumption and GDP during this period. Cash in circulation in the euro area (Germany) rose at a rate of 7.0% (5.5%) on average. The respective rates for M1 and M3 are 6.7% (8.7%) and 4.3% (5.2%). Nominal private consumption, however, grew only at an average rate of 1.9% (2.1%). The nominal GDP growth was slightly stronger at 2.1% (2.8%).
Again, we used the nominal yield on financial assets to capture the opportunity costs of money. As own rate of overnight deposits, we used the overnight interest rate ($s_2$), whereas the interest rate on deposits with an agreed maturity of between 1 and 2 years ($s_3$) represents the own rate of the other parts.\textsuperscript{14}

Appendix B shows that in 2015, the opportunity costs of holding M1 (see (14)) in the euro area (Germany) were €267 bn (68). For M3, these figures were €413 bn (93). This amounts to roughly ten times the user costs of cash. The relative preference for monetary services of M3 components in 2015 is $W = 6.7\%$ (5.4\%) for the euro area (Germany). This represents only a marginal change compared to 2005 (6.5\% and 5.3\%, respectively).

We calculated the welfare losses only in the case of a reduction of the interest rate for all monetary components by 3 percentage points. Thus, the user costs of the assets included in the monetary aggregates M1 and M3 increase by $\Delta s_i = 0.03$. Table 3 shows the associated welfare losses for 2005 and 2015 in the euro area (EMU) and Germany (GY).

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMU</td>
<td>GY</td>
</tr>
<tr>
<td><strong>Cash M0</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compensating Variation</td>
<td>CV € bn.</td>
<td>−13</td>
</tr>
<tr>
<td>share in GDP</td>
<td>cvb %</td>
<td>−0.15</td>
</tr>
<tr>
<td><strong>Monetary aggregate M1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compensating variation</td>
<td>CV € bn.</td>
<td>−81</td>
</tr>
<tr>
<td>share in GDP</td>
<td>cvb %</td>
<td>−0.95</td>
</tr>
<tr>
<td><strong>Monetary aggregate M3</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deadweight loss</td>
<td>DWL *) € bn.</td>
<td>50</td>
</tr>
<tr>
<td>per capita</td>
<td>€</td>
<td>149</td>
</tr>
<tr>
<td>share in GDP</td>
<td>%</td>
<td>0.59</td>
</tr>
<tr>
<td>Compensating consumption</td>
<td>CV € bn.</td>
<td>4921</td>
</tr>
<tr>
<td>per capita</td>
<td>€</td>
<td>−156</td>
</tr>
<tr>
<td>share in consumption</td>
<td>cvp %</td>
<td>−463</td>
</tr>
<tr>
<td>share in GDP</td>
<td>cvc %</td>
<td>−3.27</td>
</tr>
<tr>
<td>share in GDP</td>
<td>cvb %</td>
<td>−1.84</td>
</tr>
</tbody>
</table>

Source: Appendix B and own calculations. *) Approximatively calculated as (CV)-SE.

If solely cash in circulation is subject to a negative interest rate of 3\%, the welfare loss (CV) of consumers in the euro area was €24 bn in 2015, as already shown in Table 1. This amount increases to €144 bn if the interest rate reduction applies to all components of M1. Moreover, if the own rates of all components of M3 are reduced by three percentage points, welfare losses rise to €228 bn per year, which is 4\% of private consumption or 2.2\% of GDP. Per capita, the welfare loss in the euro area amounts to €676. This is about 50 percent higher than the loss of €463, which would have resulted in 2005 when the overall level of interest rates was higher. Taking additional government revenues into account, the resulting deadweight loss (DWL) amounts to €62 bn per year, corresponding to 0.59\% of GDP or €183 per capita.

For Germany, the welfare loss (CV) with respect to M3 amounts to €59 bn which equals 3.6\% of private consumption and 2\% of GDP. On a per-capita basis, a decrease in interest rates by 3 percentage points of all assets included in M3 would result in a loss of €720 per year, i.e., about two euros per day. The deadweight loss (DWL) is €18 bn per year, which is 0.58\% of GDP or €213 per capita.

\textsuperscript{14} This leads to a small underestimation of the opportunity costs of the corresponding monetary asset, as interest rates on savings deposits are lower. The same holds for overnight deposits.
Figure 2 summarizes the welfare losses (CV) of an interest rate cut by 3 percentage points for all M3 components in Germany and the euro area.

Figure 2. Welfare losses (CV) for Germany and the euro area in € billion.

5. Summary and Conclusions

In many countries, the population is sceptical towards attempts to abolish cash. However, it is no coincidence that proposals for cash abolition are put forward in times of crisis because its proponents believe that negative interest rates would be needed most by central banks in this situation to break through the lower-bound on interest rates in order to counter deflation risks and to stimulate the economy.

We argue that the same goal could be pursued by implementing a regime with negative interest rates on cash holdings. Contrary to cash abolition, such a measure would be reversible, and the advantages of cash could be maintained. However, as our analysis has also shown, the welfare losses of negative interest rates on cash alone are likely to be significant. Measured as compensated variation (CV), an interest rate of −3% on cash holdings would create a welfare loss of around €24 bn per year for consumers in the euro area. Negative interest rates on cash eliminate the lower-bound on interest rates and thus enable central banks to impose negative interest rates on a much wider range of bank deposits. If interest rates on all components of M3 were reduced by 3 percentage points, we estimate welfare losses for consumers in the euro area of €228 bn, equivalent to 2% of GDP or €700 per capita. Even in terms of net welfare, the annual deadweight loss of €200 per capita would be sizable.

Our micro-founded and empirically-calibrated model calculations of the welfare effects of negative interest rates on money holdings are based on strong, yet plausible, simplifying assumptions. There are certainly limitations to the partial equilibrium approach pursued. This requires being careful in drawing far-reaching conclusions. However, a full-blown DSGE model was beyond the scope of our paper. In a simple and comprehensive way, the MIU-model addresses our main research question and captures the effects of negative interest rates on cash and broader monetary aggregates on household welfare. Moreover, the computed welfare losses have proved fairly robust against various parameter variations.
We conclude that negative interest rates on cash (or even worse: cash abolition) to break through the (effective) zero lower-bound should only be taken under consideration after careful evaluation of the welfare consequences. Supporters of such a socio-economic large-scale experiment need to establish, by way of theoretical and empirical analyses that the benefits they postulate, better crisis management of central banks. Containment of illegal cash-based activities are sustainably and significantly larger than the associated private and social welfare losses estimated in this paper. Thereby, due consideration should also be given to the economic and social advantages of cash (Krüger and Seitz 2017).

**Author Contributions:** The paper is a genuine collaborative work whose parts cannot be assigned to individual authors.

**Funding:** No funding.

**Acknowledgments:** We thank M. Krüger, M. Rupprecht, G. Ziebarth as well as seminar participants at the Deutsche Bundesbank, the ROME network, the 2018 meeting of the Radein seminar, Aktionskreis: Stabiles Geld, and three anonymous referees for their valuable comments.

**Conflicts of Interest:** We hereby declare that there is no conflict of interest by any of the authors.

### Appendix A Sidrauski Model with CRRA Utility Function

A utility function with constant relative risk aversion (CRRA) reads as:

$$u_t = \left[ (1 - a) c_t^{1-\rho} + a m_t^{1-\rho} \right] / (1 - \rho), \quad (A1)$$

where $a$ is the so-called distribution parameter and $\sigma = 1/\rho$ is the elasticity of substitution between the consumption of goods and monetary services. If $a = \rho = 1$, (A1) corresponds to the logarithmic utility function (16). In line with (17), the resulting money demand function is:

$$m_t = \lambda c_t z_t^{-1/\rho}, \quad (A2)$$

with $\lambda = (a/(1 - a))^{1/\rho}$ as a scaling parameter. The essential difference to (17) is that the elasticity of money demand with respect to changes in the user costs of money (in absolute terms) is $\sigma$ rather than 1. Substituting (A2) into (A1) yields:

$$u_t = c_t^{1-\rho} \left[ (1 - a) + a \lambda z_t^{-1-(1-\rho)/\rho} \right] / (1 - \rho) = u_t(c_t, z_t). \quad (A3)$$

To get the resulting compensating consumption level, we set $u_t(c_{0,t}, z_{0,t}) = u_t(c_{X,t}, z_t)$:

$$c_{X,t} = c_{0,t} \left[ (1 - a) + a \lambda z_t^{-1-(1-\rho)/\rho} \right]^{1/(1-\rho)} / \left[ (1 - a) + a \lambda z_t^{-1-(1-\rho)/\rho} \right]. \quad (A4)$$

(42) together with data for the elasticity of substitution, private consumption ($C_t$) and money ($M_t$) yields the following estimate for the distribution parameter $a$:

$$a = \frac{z_t M_t^\rho}{C_t^\rho + z_t M_t^\rho}. \quad (A5)$$

(31) is a special case of (43) for $\sigma = \rho = 1$. 

---

Additional notes and references may be included here if the context of the text requires them.
### Appendix B Data Basis

#### Table A1. Monetary aggregates, user costs of money, and preference parameters.

<table>
<thead>
<tr>
<th>Period</th>
<th>2005</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMU</td>
<td>GY</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>C</td>
<td>€ bn.</td>
</tr>
<tr>
<td>Gross Domestic Product</td>
<td>GDP</td>
<td>€ bn.</td>
</tr>
<tr>
<td><strong>Monetary Aggregates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash in circulation</td>
<td>M0</td>
<td>€ bn.</td>
</tr>
<tr>
<td>plus overnight deposits</td>
<td>M1</td>
<td>€ bn.</td>
</tr>
<tr>
<td>plus term and savings deposits</td>
<td>M3</td>
<td>€ bn.</td>
</tr>
<tr>
<td><strong>Average growth rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private Consumption</td>
<td>C</td>
<td>%</td>
</tr>
<tr>
<td>Gross Domestic Product</td>
<td>GDP</td>
<td>%</td>
</tr>
<tr>
<td>Cash in circulation</td>
<td>M0</td>
<td>%</td>
</tr>
<tr>
<td>plus overnight deposits</td>
<td>M1</td>
<td>%</td>
</tr>
<tr>
<td>plus term and savings deposits</td>
<td>M3</td>
<td>%</td>
</tr>
<tr>
<td><strong>Interest rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government bond yield *)</td>
<td></td>
<td>%</td>
</tr>
<tr>
<td>Real return on financial wealth **)</td>
<td></td>
<td>%</td>
</tr>
<tr>
<td>Nominal return of financial wealth +)</td>
<td>z</td>
<td>%</td>
</tr>
<tr>
<td>Opportunity costs of overnight deposits</td>
<td>z-s2</td>
<td>%</td>
</tr>
<tr>
<td>Opportunity costs of term deposits ++</td>
<td>z-s3</td>
<td>%</td>
</tr>
<tr>
<td><strong>User costs of money</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash in circulation</td>
<td>k1</td>
<td>€ bn.</td>
</tr>
<tr>
<td>plus overnight deposits</td>
<td>k2</td>
<td>€ bn.</td>
</tr>
<tr>
<td>plus term and savings deposits</td>
<td>k3</td>
<td>€ bn.</td>
</tr>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash in circulation</td>
<td>α1</td>
<td>%</td>
</tr>
<tr>
<td>Overnight deposits</td>
<td>α2</td>
<td>%</td>
</tr>
<tr>
<td>Term and savings deposits</td>
<td>α3</td>
<td>%</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>W</td>
<td>%</td>
</tr>
</tbody>
</table>

Sources: Deutsche Bundesbank, European Central Bank. Bundesbank (2015) and own calculations. *) with term to maturity of 9 to 10 years; **) without cash and overnight deposits; real return for 2005 and 2015. +) equal to opportunity costs of cash, as s1 ≡ 0. ++ rate on deposits of private households with agreed maturity of 1 to 2 years.

---

**Note on the cash data for Germany:** We use the officially reported cash figures for Germany (see, for example, the figures in the Monthly Report of the Deutsche Bundesbank, Statistics Section, p. 11*). These figures do not correspond to actual cash in circulation in Germany and also not to the cash issues of the Deutsche Bundesbank. Rather, it is a purely numerical quantity, calculated as the German capital share multiplied by the total euro cash issue. For attempts to estimate cash circulation in Germany, see Bartzsch et al. (2013).

**Note on the return on financial assets:** At the end of 2015, the financial assets of households in Germany amounted to around €5300 billion. The nominal rate of return on households’ financial assets determines the opportunity costs of using cash. The same applies to the user costs of the remaining parts of M1 and M3. The data on the real rate of return on households’ financial assets in Germany in Table A1 are based on a detailed study by Bundesbank (2015). The asset components considered are cash and sight deposits, term and savings deposits (including short-term notes), shares, investment funds, bonds, claims against insurance companies, and others.
References

Harper, Joel W. 1948. Scrip and Other Forms of Local Money. Ph.D. dissertation, University of Chicago, Chicago, IL, USA.


Miracle of Wörgl. 1932. Available online: https://en.wikipedia.org/wiki/W%C3%B6rgl#The_W%C3%B6rgl_Experiment (accessed on 5 March 2019).


