On the Effectiveness of the Abatement Policy Mix: A Case Study of China’s Energy-Intensive Sectors

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Abstract: To achieve carbon emissions control targets, policymakers often need a basket of policies to account for the complexity of abatement. The instruments in the policy mix are often interconnected. It is of great importance to study how different abatement policies perform in practice—in other words, to evaluate the effectiveness of the abatement policy mix. This paper builds a multisector partial equilibrium model and then studies the policy effectiveness using data from two energy-intensive sectors in China, namely, the iron and steel sector and the cement sector. The results show clear evidence that these policies interact, and the policy mix is not a simple aggregation but rather differs across sectors, which leads to fundamentally different scenarios in terms of energy savings, emissions reductions and production behaviors. Energy-savings subsidies can increase production and profit with a lower equilibrium level of carbon prices, whereas output-based rebating of allowances reduces production and is associated with higher carbon prices.

Keywords: abatement policy mix; emissions trading; energy-intensive sectors; output-based rebates; subsidies

1. Introduction

Climate change has become a major global issue, and reducing carbon emissions is considered to be necessary for human welfare. The Paris agreement which entered into force in November 2016 has strengthened the global response to the threat of climate change, and the participating countries across the world have outlined their Intended Nationally Determined Contributions (INDCs) (The Paris Agreement. http://unfccc.int/paris_agreement/items/9485.php). Effectively achieving low carbon emissions, however, is much more complicated and often requires a combination of various kinds of efforts. Using a single policy instrument, even the most advanced system, such as the European Union (EU) Emissions Trading System (EU ETS), has clear disadvantages. The EU ETS covers only approximately 40% of the sources of emissions in the EU, and the design of the system has often been criticized. Additional abatement policies (i.e., energy efficiency standards, carbon taxes) can be adopted to form a policy mix to ensure effective emissions reductions.

China, the biggest user of fossil fuel-based energy in the world and the largest source of carbon dioxide (CO2) emissions, has been actively engaged in reducing CO2 emissions in recent years. Jointly with the second-largest source of carbon-based emissions, the United States, China has pledged that the peak of its CO2 emissions will occur no later than 2030. It is necessary and urgent for the Chinese government to design an effective policy mix to achieve such an ambitious goal. Noticeably,
China has already established seven emissions-trading pilot programs and aims to introduce a nationwide emissions trading system (ETS) in 2017. Meanwhile, other supplementary policies, such as energy-savings subsidies and laws mandating the use of renewable energy, either have already been implemented or will be implemented.

It is perhaps easier to comment on the effectiveness of a single policy because a policy mix can be more complicated, as different policies can interact with one another. Take subsidies, for example; Fredriksson shows that subsidies for pollution abatement are inefficient when a Pigovian pollution tax is available [1]. He also explains the use of pollution abatement subsidies in environmental policy as a primary tool for redistribution. The equilibrium level of a subsidy depends on the elasticity of the subsidy in terms of pollution abatement and lobby group membership. Its effectiveness does not involve a simple aggregation of the impacts from each individual policy.

Two interrelated policies may be mutually complementary to increase effectiveness, but they can also be substitutional, which leads to lower aggregate impacts. For example, previous studies, such as Böhringer and Keller, suggest that other abatement policies in the EU ETS may not bring further emissions reductions [2]. Instead, they may reduce carbon prices and cause economic distortion.

It is therefore important to study the interrelationship of policy instruments within a policy mix and evaluate their joint effectiveness. In addition, Dechezlepretre and Sato present an ex-post evaluation and discuss the impact of environmental regulations with a combination of different instruments (i.e., carbon price and compensations) on productivity, employment, trade, industry location and innovation [3]. They note that a reasonable compensation level is important. In the EU ETS, overcompensation to secure profits of regulated firms in energy intensive industries can result in paradoxically high levels of subsidies. Policies such as these may have no direct environmental benefits but instead generate negative effects on the cost-effective measures.

Some recent studies address this question based on the ETS framework. Lecuyer and Quirion analyze overlapping policy instruments for the same emissions sources [4]. It can be socially beneficial to implement an additional abatement policy instrument (i.e., renewable energy subsidy) when carbon prices drop to zero or when a carbon tax is politically difficult to implement. Fischer and Newell estimate the impact on carbon emissions reductions from different abatement policies, and they suggest that a policy mix combining abatement, learning, and R&D is more effective than a single policy instrument [5].

Policies for encouraging emissions reductions have to be consistent with economic incentives in order to be effective. A general concern at the early stage of the EU ETS was how much negative impact would be created for the covered industries by increasing carbon abatement costs. Thus, a free allocation or payback mechanism was considered to offset the rising costs. Arlinghaus presents a review of the literature on the ex-post empirical evaluations of the impacts of carbon prices on indicators of competitiveness [6]. She notes that although most of the studies found that carbon pricing could result in emissions abatement, the studies fail to measure any economically meaningful competitiveness effects as a consequence of these policies. Additionally, she suggests that to ensure that these results hold in the absence of this measure, future research could focus on comparing the effects of carbon prices on firms paying the full rate of the carbon price to firms that are exempt or pay a rebate. In addition, this future research could be important in estimating competitiveness effects among different sectors considering sectoral heterogeneity. Demailly and Quirion analyze the impacts of grandfathering and output-based allocation (OBA) on emissions reduction, firm competitiveness, and carbon leakage [7]. They find that an optimal free allocation share exists, and it would be beneficial to firm competitiveness. In another study, Demailly and Quirion show that a reasonable free allocation in the EU ETS can increase firm competitiveness without hurting their profitability [8]. The results show that both OBA and emissions-based allocation (EBA) are helpful. Meunier et al., on the contrary, find that OBA is not the best option when global carbon taxation and border adjustment policies are absent [9]. They suggest that the best choice for the EU would be to combine these two allocation methods.
The situation may be even more complicated when multiple emissions reduction policies are applied to multiple sectors. In other words, the impacts of a policy mix can vary across industries. For example, the objective of energy-savings subsidies is to offset losses in profit, whereas the objective of ETS is to reduce emissions through a cap-and-trade system. As a result, their impacts on different industries can vary significantly. A series of studies have noted that the adoption of output-based rebating in an emissions trading scheme plays important roles in affecting sectoral competitiveness [8–13]. Quantitative assessments of the impact of unilateral emissions abatement policies on energy intensive industries, such as emissions trading, carbon taxes and border carbon adjustments, can also be found in some recent studies [2,8,14–17]. Evaluating multipolicy effects in multisector setups is, to a large extent, missing. Top-down approaches, such as the computable general equilibrium (CGE) models, are often used in multi-sector studies and are widely used when studying the economic effects of policies on given countries or specific areas. Goulder, for example, uses the dynamic CGE model to simulate the impact of carbon tax policy on the economy in the United States [18]; Carbone and Rivers consider the measurement of competitiveness due to climate policies in computable general equilibrium (CGE) models and discuss the weaknesses of the existing approach [19]. CGE models also have been shown to be useful in the assessment of the impact of the EU 20 energy and climate package (European Union (EU), 2020 Climate & Energy Package. https://ec.europa.eu/clima/policies/strategies/2020_en). These models, however, have noticeable problems, for example, the underlying mechanism is less clear, and it is hard to adapt the models to a particular sector.

Models for multipolicy, multicountry analyses have been developed; for example, Branger and Quirion use a stochastic model to rank different policies according to their expected total social costs [20]. They found that a tax is preferred to caps, and relative caps are preferred to fixed caps in the US and emerging countries, whereas a fixed cap is a better choice over a relative cap in Europe and Japan. In terms of the policy impacts in single sector studies, a partial equilibrium model is often used, such as in Lecuyer and Quirion [4], and Demailly and Quirion [7]. The only problem with these existing partial equilibrium models is that they are applied mostly to a single sector, and are thus unable to address sectoral differences.

This paper first builds a multisector model following the framework in Lecuyer and Quirion [4]. Instead of focusing on the impact of overlapping instruments to cover the same emission sources in one sector, we are focusing on a heterogeneous multisector case. Specifically, we extend the single sector partial equilibrium framework to a multisector model to investigate the diverse responses of different sectors, as well as the inter-sectoral interaction, under the given abatement policy mix. The other unique features of our model are summarized as follows:

1. This study aims to investigate the potential interaction of different policies from the perspective of the social optimum. The emission targets in accordance with equilibrium carbon price levels are derived by social welfare maximization, consisting of more than one sector, which are linked through the carbon market. Relaxing the constant emissions reduction target but holding the social optimum in our analysis can contribute to the literature in terms of understanding the interaction effects on policy mix design and evaluation.

2. Energy-savings subsidies and output-based rebates have been introduced as additional policy options together with an emissions trading scheme in our model. In addition, the analytical solutions have been derived from a two-sector case. We explain how and why the sectoral responses to the same policy can vary substantially by comparing the analytical solutions. In addition, we also present some inconsistent conclusions on the effectiveness of policy mixes from previous studies under the social welfare maximization. For example, we show that emissions reductions in one sector can decrease with energy-savings subsidies. We believe that the findings on policy mix distortion among sectors can help improve the efficiency of the design of emissions trading schemes, as well as complementary policy options.
Based on the results of our extended multisector partial equilibrium model, we evaluate the following cases using empirical data from China: the effectiveness of multiple reduction policies on one sector, different responses from multiple sectors to a single policy, and the interactions and effects of multiple policies in a multisector environment. We further investigate the impact on social welfare and total emissions, which leads to relevant policy discussions. The rest of this paper proceeds as follows: Section 2 builds the multisector partial equilibrium model; Section 3 describes our case study and results for two energy intensive sectors in China; and Section 4 concludes.

2. Analytical Framework

In this paper, we assume that three policy instruments are available, namely, ETS with a full auction for allowances, ETS together with energy-savings subsidies, and ETS with output-based rebating (OB). We examine three combinations of these policy instruments (Table 1) in our multiple policy analysis. To simplify the analysis, we further assume that transactions occur among sectors rather than at the firm level (in other words, each of the sectors has one representative firm). All of the sectors are price takers (perfect competition) in the allowance market, and their trading behavior cannot affect carbon prices.

Two policy mixes have been considered in our analysis. For energy-savings subsidies, they have already been launched for the energy intensive sectors in China as a part of the 11th Five Year Plan [21], which was developed by the government to incentivize sectors to reduce energy consumption and achieve energy savings targets. The subsidy level is set as a certain monetary refund per unit of energy savings (per ton of standard coal). This subsidy may still exist even after the implementation of ETS. For output-based rebates, they have been partially adopted as part of the allowance allocation in the power sector in China’s planned national Emission Trading Scheme. This rebating policy is also being considered for implementation in other sectors covered by the national ETS.

The unit and description of variables and parameters used in the model are listed in Table 2.

Table 1. Description of the three cases.

<table>
<thead>
<tr>
<th>Cases</th>
<th>ETS Auction</th>
<th>Energy-Savings Subsidies</th>
<th>Output-Based Rebates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Multi_ES</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Multi_OB</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2. Variables and parameters used in the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit *</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_j )</td>
<td>CNY/t output</td>
<td>Price of the per unit product produced by the sector ( j )</td>
</tr>
<tr>
<td>( q_j )</td>
<td>million t output</td>
<td>Output level of sector ( j )</td>
</tr>
<tr>
<td>( \mu_j )</td>
<td>N/A</td>
<td>Intercept of the demand curve of sector ( j )</td>
</tr>
<tr>
<td>( \sigma_j )</td>
<td>N/A</td>
<td>Slope of the demand curve of sector ( j )</td>
</tr>
<tr>
<td>( c_j )</td>
<td>CNY/t output</td>
<td>Unit cost of production of sector ( j )</td>
</tr>
<tr>
<td>( a_j )</td>
<td>million t CO(_2)</td>
<td>CO(_2) abatement of sector ( j )</td>
</tr>
<tr>
<td>( \alpha_j )</td>
<td>N/A</td>
<td>Coefficient of the abatement cost curve of sector ( j )</td>
</tr>
<tr>
<td>( \beta_j )</td>
<td>N/A</td>
<td>Coefficient of the abatement cost curve of sector ( j )</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>CNY/t CO(_2)</td>
<td>CO(_2) price</td>
</tr>
<tr>
<td>( \tau_j )</td>
<td>t CO(_2)/t output</td>
<td>CO(_2) intensity</td>
</tr>
<tr>
<td>( \varsigma )</td>
<td>million tce</td>
<td>Energy savings</td>
</tr>
<tr>
<td>( \varphi'_j )</td>
<td>CNY/tce</td>
<td>Unit energy-savings subsidy</td>
</tr>
<tr>
<td>( \psi_j )</td>
<td>CNY/t CO(_2)</td>
<td>Unit CO(_2) abatement subsidy (converted from energy-savings subsidy)</td>
</tr>
<tr>
<td>( f )</td>
<td>N/A</td>
<td>Adjustment coefficient between energy savings and CO(_2) abatement</td>
</tr>
<tr>
<td>( \omega_j )</td>
<td>-</td>
<td>Per-unit rebating rate (range from 0–100%)</td>
</tr>
</tbody>
</table>

* CNY is the abbreviation of Chinese currency unit (Yuan); t denotes ton.
2.1. Base Case

Assume that there are completely heterogeneous \( J \) energy-intensive sectors, whose demand functions, production cost functions, and emissions reduction cost functions are all independent. The demand function for sector \( j \) can be written as

\[
p_j = \mu_j - \sigma_j q_j \quad \text{for} \quad j \in J
\]

where \( p_j \) is price, \( q_j \) is output, and \( \mu_j \) and \( \sigma_j \) are the intercept and slope of the demand curve, respectively. In addition, the demand uncertainty proposed by Lecuyer and Quirion [4] is not included as we are focusing on the diverse sectoral responses to the given abatement policy mix (We appreciate the comments given by an anonymous reviewer that note this issue and think this can be our next step to further improve the model) and for simplicity.

The demand function for sector \( j \) can be written as \( C_j(q_j) = c_j q_j \). Following Lecuyer and Quirion [4], it takes a linear form. Here, \( c_j \) is the marginal cost of production, and we further assume \( C_j'(q) > 0 \).

The abatement cost curve is in a quadratic form (Meunier et al. [9])

\[
AC_j(a) = a_i a_j + \beta_j a_j^2
\]

where \( a_j \) is the level of carbon abatement in sector \( j \), and \( a_i \) and \( \beta_j \) are the coefficients.

The cost of the carbon permit in sector \( j \) is \( PC_j \), which can be written as

\[
PC_j(q_j, a_j, \phi) = \phi(\tau_j q_j - a_j)
\]

where \( \phi \) is the carbon price, which is unified for all sectors. \( \tau_j \) is the carbon intensity of sector \( j \) before carbon emissions reduction.

The profit function of sector \( j \) can be expressed as

\[
\Pi_j = p_j \cdot q_j - AC_j(a_j) - C_j(q_j) - PC_j(q_j, a_j, \phi)
\]

The negative impacts of emissions on society (the negative externality) are included in the social welfare function through carbon prices. Total net CO\(_2\) emissions can be represented as

\[
E = \sum_j (\tau_j q_j - a_j)
\]

The cost of purchasing emissions permits is supposed to be paid back to society. Therefore, the social welfare function can be expressed as

\[
W(\phi) = \sum_j (CS_j(q_j) + \Pi_j - D_j(q_j, a_j) + PC_j(q_j, a_j, \phi))
\]

where \( CS_j(q_j) \) is the consumer surplus in sector \( j \), represented as \( CS_j(q_j) = \frac{1}{2} \sigma_j q_j^2 \); \( D_j(q_j, a_j) = \varepsilon(\tau_j q_j - a_j) \) is the loss function that resulted from the sectoral CO\(_2\) emissions, which depicts the social damage from carbon emissions; and \( \varepsilon \) is the unit damage coefficient. The environmental loss function is regularly used to depict the environmental damage from harmful emissions, such as sulfur dioxide (SO\(_2\)) and nitrogen oxides (NO\(_X\)), in classical environmental economic models (i.e., Nordhaus [22]). Unlike other pollutants, carbon emissions have a cumulative effect. In other words, the damage from CO\(_2\) is determined not by annual emissions but by the total amount accumulated in the atmosphere, and Nordhaus relates cumulative emissions to changes in CO\(_2\) concentrations and in temperature for the damage function [22]. The function form proposed by Nordhaus [22] can not be used in our sectoral-level partial equilibrium analysis [22].

We refer to Lecuyer and Quirion [4] by using a linear damage function in the model, in which the model only accounts for the damage that results from the increment CO\(_2\) emissions without considering the CO\(_2\) stock in the atmosphere. There are two advantages in using this function; first, the emissions control target is generally set to the incremental level of emissions rather than stocks.
Second, the linear function simplifies the environmental damage of emissions and thus allows us to address multisector issues analytically.

After substitution, the social welfare function can be written as

\[ W(\phi) = \sum_{j \in J} \left( \frac{1}{2} \sigma_j(q_j)^2 + (\mu_j - \sigma_j a_j) \cdot q_j - (\alpha_j a_j + \beta_j a_j^2) - c_j a_j - \phi (\tau_j a_j - a_j) - \epsilon (\tau_j a_j - a_j) + \phi (\tau_j a_j - a_j) \right) \]  

(7)

It should be noted that, in the welfare function (7), the costs of the carbon permit cancel each other out after substitution, for we assume that the cost of purchasing the CO\(_2\) permit will be paid back or come from society. That is, the CO\(_2\) permit purchasing/selling cost has no influence on the total welfare.

2.2. Extension 1: Multi_ES (Auction Plus Energy-Savings Subsidies)

In addition to the ETS with a full auction in the base case, policymakers can also give sector \( j \) subsidies to achieve their energy savings \( s_j \), which results in \( ES_j(s_j) = \psi_j' \cdot s_j \), and the unit of the energy-savings subsidy is \( \psi_j' \), which is exogenously given in our model.

In most energy-intensive sectors, energy savings \( s_j \) and carbon emissions reductions \( a_j \) are, to a large extent, equivalent (with an adjustment coefficient \( f \)) because saving fossil fuel-based energy (different forms of fossil fuel-based energy, such as coal, oil, and natural gas, can be converted to standard coal and thus are comparable) is the main option for these sectors to reduce emissions. Their relationship satisfies the following condition:

\[ a_j = f \cdot s_j \]  

(8)

The energy-savings subsidy function can be rewritten as

\[ ES_j(a_j) = \psi_j' \cdot s_j = \frac{\psi_j'}{f \epsilon} \cdot a_j = \psi_j \cdot a_j \]  

(9)

where \( \psi_j \) is the unit of the energy-savings subsidy after adjustment. The demand function, abatement cost function, and production cost function are the same as in the base case. It is worth to note that the discussion here is essentially short-term oriented. In the long-term, fuel switching and moving to a low-carbon production process are possible solutions to emission. For example, Arens et al. study the low-carbon pathways in Germany and suggest that the German steel sector is highly unlikely to meet its climate target regardless of what production pathway they choose [23]. An alternative way is to develop new carbon free production process, such as switching to hydrogen or using carbon free electricity.

The profit function of sector \( j \) is

\[ \Pi_j(p_j, q_j, a_j, \phi) = p_j \cdot q_j - AC_j(a_j) - C_j(q_j) - PC_j(q_j, a_j, \phi) + ES_j(a_j) \]  

(10)

Energy-savings subsidies are supposed to come from society. Therefore, the social welfare function under the “Multi_ES” case can be written as

\[ W(\phi) = \sum_{j} (CS_j(q_j) + \Pi_j - D_j(q_j, a_j) + PC_j(a_j, q_j, \phi) - ES_j(a_j)) \]  

(11)

2.3. Extension 2: Multi_OB (Auction Plus OB)

In this case, output-based allowance rebating (OB) is combined with the ETS and an auction. Output-based rebating has already been implemented in New Zealand and California [9,15]. Furthermore, Canada has put into practice an output-based pricing system for industrial facilities that emit above a certain threshold, with an opt-in capability for smaller facilities with emissions that are below the threshold. The pan-Canadian approach to pricing carbon pollution provides jurisdictions the flexibility to implement an explicit price-based system, for example, a hybrid approach composed
of a carbon levy, or an output-based pricing system such as in Alberta [24], Environment and Climate Change Canada [25].

Output-based allowance rebating is a subsidy in the form of rebated allowances associated with output. Each sector receives a given number of allowances per unit of output. Assume that the per-unit rebating rate is \( ob_j \) (the range of \( ob_j \) can be set from 0 to 1), and the free allowances permit that sector \( j \) obtains is \( \tau_j q_j \cdot ob_j \). Because the permit can be sold in the ETS, it could be seen as a part of the income of these sectors (output incentive mechanism). The additional income in sector \( j \) due to permits returned is \( \tau_j q_j \cdot ob_j \cdot \phi \). The profit in sector \( j \) can be revised to

\[
\Pi_j(p_j, q_j, a_j, \phi) = p_j q_j - AC_j(a_j) - C_j(q_j) - PC_j(q_j, a_j, \phi) + \tau_j q_j \cdot ob_j \cdot \phi
\]  

(12)

Each sector maximizes its profit by choosing an optimal level of production and emissions reductions. Again, all of the permits returned to sectors are supposed to come from society, so the social welfare function can be expressed as

\[
W(\phi) = \sum_j (CS_j(q_j) + \Pi_j - D_j(q_j, a_j) + PC_j(a_j, q_j, \phi) - \tau_j q_j \cdot ob_j \cdot \phi)
\]  

(13)

### 2.4. Solving the Model

It is worth noting that the model is essentially static in nature. We assume that the ETS auction system is set to optimize the total social welfare. Therefore, the first thing to do is to find an optimal carbon price level \( \phi^* \) that maximizes the total social welfare (it varies in different cases). Given the optimal carbon price level, each sector set its production \( q^* \) and abatement level \( a^* \) to maximize profit. The equilibrium level of carbon emissions can be written as

\[
E^* = \sum_j (\tau_j q^*_j - a^*_j)
\]  

(14)

where \( a^* \) and \( q^* \) are functions of \( \phi^* \) and vary in different cases. The equilibrium solution of these factors is given in the Appendix A.1–A.3. Table 3 summarizes the analytical solutions to the model among the three cases. For the equilibrium carbon price, the solution is based on the coverage of two sectors.

<table>
<thead>
<tr>
<th>Cases</th>
<th>( q^*_j )</th>
<th>( a^*_j )</th>
</tr>
</thead>
</table>
| Base Case   | \[
\frac{2\alpha_1\sigma_2(\beta_1 + \beta_2)}{2\alpha_1(\beta_1 + \beta_2) + \beta_1\beta_2(\tau_1 + \tau_2)}
\]  
\[
+ (-\mu_2 + c_2 + 2\varepsilon_2)\tau_2\sigma_1\beta_1\beta_2
\]
\[
+ (-\mu_1 + c_1 + 2\varepsilon_1)\tau_1\beta_1\beta_2
\]
\[
\frac{\mu_2 - c_2 - \phi \tau_1}{2\alpha_1}
\]  
\[
\frac{\phi - a_j}{2\beta_1}
\]  
| Multi_ES    | \[
\frac{2\alpha_1\sigma_2(\beta_1 + \beta_2)}{2\alpha_1(\beta_1 + \beta_2) + \beta_1\beta_2(\tau_1 + \tau_2)}
\]  
\[
+ (-\mu_2 + c_2 + 2\varepsilon_2)\tau_2\sigma_1\beta_1\beta_2
\]
\[
+ (-\mu_1 + c_1 + 2\varepsilon_1)\tau_1\beta_1\beta_2
\]
\[
\frac{\mu_1 - c_1 - \phi \tau_1}{2\alpha_1}
\]  
\[
\frac{\phi + \psi - a_j}{2\beta_1}
\]  
| Multi_OB    | \[
\frac{2\alpha_1\sigma_2(\beta_1 + \beta_2)}{2\alpha_1(\beta_1 + \beta_2) + \beta_1\beta_2(\tau_1 + \tau_2)}
\]  
\[
+ (-\mu_2 + c_2 + 2\varepsilon_2)(1 - ob_2)\tau_2\sigma_1\beta_1\beta_2
\]
\[
+ (-\mu_1 + c_1 + 2\varepsilon_1)(1 - ob_1)\tau_1\beta_1\beta_2
\]
\[
\frac{\mu_j - c_j}{2\alpha_1}
\]  
\[
\frac{\phi - a_j}{2\beta_1}
\]  

In the Multi_ES case, it can be seen that \( \phi^* \) is inversely proportional to the subsidy level, \( \psi \), when compared to that in the base case. From the perspective of production, an incentive to the output can be observed after the introduction of the energy-savings subsidies. Furthermore, the output from the other sector will also increase, even if the subsidy is only given to one sector (given \( \phi^* \) will also be decreased when the subsidies are introduced to only one sector).
From the perspective of emissions reductions, the emissions reductions in the sector with the energy-savings subsidy depend on the changes in the equilibrium carbon price level $\phi^*$, as well as the energy subsidy level $\psi_j$. Specifically, for the sector with the energy-savings subsidy, given $E_{BC} - E_{ES} = \frac{(\beta_j \sigma_j + \sigma_j)}{2(\beta_j + \beta_j)}(\phi_j^* - \phi_{BC}^* - \sigma_j \psi_j)$, if $\psi_j > \left( \frac{\beta_j \sigma_j}{\sigma_j} + 1 \right)(\phi_{BC}^* - \phi_{ES}^*)$, an increase in emissions reductions within the sector can be observed, otherwise its emissions reductions will decrease. However, the emissions reductions of the other sector will decrease due to the fall of the equilibrium carbon price level.

In the Multi_OB case, the solution form of the equilibrium carbon price level is more complicated with the introduction of output-based allowance rebating. The equilibrium carbon price will increase with the level of $ob$, which ranges from 0 to 1. From the perspective of production, it is obvious that the increase in the $ob$ rate will raise output levels when the carbon price level remains unchanged, although the carbon price is also related to the $ob$ rate. Specifically, given $\bar{q}_{iob}^* - \bar{q}_{iBC}^* = -\frac{\phi_{iob}^* (1 - \sigma_j)}{\phi_{BC}^*}$, this output of this sector increases if $\phi_{BC}^* > \phi_{ob}^* (1 - \sigma_j)$, otherwise its output will be decreased. Furthermore, the output of the other sector will decrease even if the $ob$ is only given to one sector (given $\phi^*$ will be increased with the introduction of $ob$).

One could argue that this is contrary to the initial design of output-based rebating, which is applied to compensate for losses in sectoral competitiveness that result from implementing ETS and can provide incentives for sectoral output [7,8]. The output incentive effects of output-based rebating make sense if we look at the analytical solution of $\bar{q}_j^*$ under the Multi_OB case. In our model, each $ob$ rate corresponds to an equilibrium carbon price level, and the increase in the equilibrium carbon price level can offset the output incentive that resulted from the increasing $ob$ rate and even make sectoral performance worse after implementation of output-based rebating.

From the perspective of emissions reductions, the emissions reductions in the sector with an output-based rebate depends on the changes in the equilibrium carbon price level $\phi^*$, as well as the $ob$ rate. Specifically, for the sector with $ob$, given $E_{BC} - E_{ob} = \frac{(\sigma_j + \beta_j \sigma_j)}{2(\beta_j + \beta_j)}(\phi^*_j - \sigma_j \phi_{BC}^* + \sigma_j \phi_{ob}^*)$, if $\phi_{BC}^* < \frac{\sigma_j + \beta_j \sigma_j}{\sigma_j + \beta_j \sigma_j} \phi_{ob}^*$, an increase in emissions reductions within the sector can be observed. However, the emissions reductions of the other sector will increase due to the increase in the equilibrium carbon price level.

In addition, one concern about the analysis above is that the assumption of one representative firm, as well as the demand of sectoral outputs are set as independent, may generate unrealistic predictions. In Appendix A.5–A.6, we presented the theoretical analysis on the situations of: (1) relax the assumption of one representative firm and allow more than one firm in each sector; and (2) consider the correlation of demands across sectors. Under the condition of welfare maximization, although the form of analytical solutions to the alternative models vary from the basic model, for example, $\phi^*$ is now a function of firms’ numbers across two sectors (Appendix A.5). It is therefore including sector size into the model, though the general indication when comparing three cases within each model setup do not change.

3. Case Study of Two Energy-Intensive Sectors in China

3.1. Data and Parameters

Two energy-intensive industries—the iron and steel industry (IS) and the cement industry (CM)—are used as examples to illustrate the effectiveness of the multiple abatement policies in China. Abatement cost functions are estimated by the GTAP (Global Trade Analysis Project) model [26,27]. Cui et al. establish an interprovincial emissions trading model and adopt the abatement cost functions of the different sectors of China to study the cost-effectiveness of implementing ETS [26]. Fan and Wang establish an interprovincial emissions trading model, which distinguishes between trade and non-trade sectors to explore the effective coverage of sectors in the ETS in China [27].
To establish an initial equilibrium for policy analysis, the production costs of the two sectors are calibrated with the demand function and output prices in 2010. The demand functions are estimated with a linear regression relating the real prices of sectoral output quantities of IS and CM sectors in China from 2005 to 2010. This estimation method has been commonly adopted in previous studies [9,28]. According to the “Interim Measures for Financial Funds of Energy-Saving Technology” issued by the Ministry of Finance of the People’s Republic of China and National Development and Reform Commission [21], the energy-savings subsidy is approximately 200 CNY/tce (29.99 USD/tce) (CNY refers to Chinese Renminbi. 100 CNY is worth approximately 15 US dollars or 13 Euros, and tce is ton of standard coal equivalent) or CNY 81.30 (USD 12.18) per ton of CO₂ (the conversion is based on the IPCC [29], which sets the emissions factor of standard coal at 2.46 tons of CO₂ per tce). In our calculation, the level of the energy-savings subsidy changes from 50 CNY/tce to 200 CNY/tce (which is equal to 20.33~81.30 CNY/tCO₂ by dividing the CO₂ emissions factor of standard coal). The ob rate is set from 0.05 to 0.2, and a wider range of ob from 0 to 0.9 is also considered. Those results are, however, available upon request. The environmental damage per unit is set at CNY 650 per ton of CO₂ (USD 97.45). We use a higher unit of environmental damage than that in Lecuyer and Quirion [4], who used a span from 10 to 30 Euro/tCO₂, for two reasons: (1) China faces greater pressure to reduce CO₂ emissions, and the Chinese government has set up an ambitious target of 2030 as the peak for emissions; and (2) we also take into consideration the synergistic effects of environmental loss. Detailed parameters and their sources in each sector are given in Table 4.

Table 4. Parameter settings (based on 2010).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>IS</th>
<th>CM</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients of Demand Curve</td>
<td>µ_j</td>
<td>8213.35</td>
<td>Estimated with demand and price data from 2006-2010; data came from the National Bureau of Statistics [30]</td>
</tr>
<tr>
<td></td>
<td>σ_j</td>
<td>5.29</td>
<td></td>
</tr>
<tr>
<td>Coefficients of CO₂ abatement cost curve</td>
<td>α_j</td>
<td>-65.98</td>
<td>Derived from the GTAP model</td>
</tr>
<tr>
<td></td>
<td>β_j</td>
<td>1.99</td>
<td></td>
</tr>
<tr>
<td>Initial carbon intensity (t CO₂/t)</td>
<td>τ_j</td>
<td>1.68</td>
<td>Referring to Li and Zhu [31], Xu et al. [32], and CO₂ emissions due to chemical reactions in the process of production are not considered</td>
</tr>
<tr>
<td>Per-unit damage coefficient of CO₂ emission (CNY/t CO₂)</td>
<td>ε</td>
<td>650</td>
<td>Partially referring to Lecuyer and Quirion [4]</td>
</tr>
</tbody>
</table>

3.2. Results and Analysis

3.2.1. Base Case

Table 5 reports the results of the base case. Here, we only consider the implementation of ETS without any other complementary policies. Both sectors will bear losses in production and profit. The CM sector suffers more production losses than the IS sector, and meanwhile, the CM sector will also reduce more emissions than the IS sector. The carbon price under social welfare maximization is 107.22 CNY/tCO₂, which resulted in a 12.03% decrease in CO₂ emission among the two sectors. In addition, a sensitivity analysis of the demand and abatement parameters of the two sectors in the base case is presented in Figure A1 in the Appendix A.4.

Table 5. Calculation results for the base case.

<table>
<thead>
<tr>
<th>Compare to the Base Year (Year 2010)</th>
<th>IS</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changes in Production</td>
<td>-2.67%</td>
<td>-8.77%</td>
</tr>
<tr>
<td>Changes in Profit</td>
<td>-5.09%</td>
<td>-15.38%</td>
</tr>
<tr>
<td>Emissions Reduced</td>
<td>-6.72%</td>
<td>-17.57%</td>
</tr>
<tr>
<td>Total Emissions Reduced</td>
<td>-12.03%</td>
<td></td>
</tr>
<tr>
<td>Changes in Welfare</td>
<td>-5.39%</td>
<td></td>
</tr>
</tbody>
</table>
Results of the impacts on both sectors from two policy combinations, namely, Multi_ES and Multi_OB (see Table 1), are reported in the following sections. Our objective is to show the relative effects; thus, all results (except the carbon price, which is given in actual values) are shown as percentage changes compared to the base case (Table 4). In addition to the identical policies in each sector, our simulation also considers policy discrimination, in other words, when a supporting policy may be given to only one sector whereas another sector is given nothing.

3.2.2. Multi_ES Case

We assume that the energy-savings subsidies are given to both sectors, ranging from 50 CNY/tce (7.50 USD/tce) to 200 CNY/tce (29.99 USD/tce). Given the model solutions (see the Appendix A.2), the equilibrium values of production $q^*_s$, profit $\pi^*_s$, carbon price $\phi^*_s$, emissions ($\tau q^*_s - a_s$), and welfare $W^*_s$ for each sector can be calculated. These values are then compared to those in the base case (e.g., $p_{es}/p_{pc} - 1$), and the percentage changes are reported graphically in Figures 1–3.

**Figure 1.** Sectoral performance under the multi_ES case.

**Figure 2.** Emissions abatement under the multi_ES. Note: carbon prices are indexed to the right axis. Emission reduced by $\ast$ refers to the changes in CO$_2$ abatement, or $a_j$ in our model.
In general, giving energy-savings subsidies can boost production and increase profits in both sectors. However, these effects are more obvious in the CM sector than in the IS sector. For example, when both sectors are given a subsidy of 200 CNY/tce (29.99 USD/tce), the CM sector can earn an additional (relative to the base case) 9% profit, whereas the same subsidy can increase profit by only slightly more than 2% in the IS industry. The same general pattern remains when subsidies are given to only one sector (policy discrimination), although the CM sector is more sensitive to subsidies than the IS sector for all three policies.

Subsidies can affect sectoral abatements, carbon prices, and total emissions. We can see in Figure 2 that carbon prices can fall quite significantly, from 107.22 CNY/tCO₂ (16.07 USD/tCO₂) to 61.30 CNY/tCO₂ (9.19 USD/tCO₂), when subsidies (200 CNY/tce) are given to both sectors. The price impacts are smaller when policy discrimination is present. As a consequence of increasing production, total emissions will increase, though not significantly (1.38%, 0.48%, and 0.89%, with identical subsidies for both sectors).

Emissions abatements, however, have shown different patterns in the two sectors. When subsidies are given to both sectors, emissions reductions decrease more in the CM sector than in the IS sector. There is a relative increase in emissions of more than 20% (less abatement efforts) in the CM sector, which is caused by increasing production (see Figure 1), whereas the same level of subsidy led to a much lower increase in emissions in the IS sector.

The two sectors respond differently when policy discrimination is present. With the solution in the multi_ES case (See details in the Appendix A.2), the production is also inversely and proportionally related to the carbon price. The carbon price will be lower with subsidies provided to both sectors than provided to a single sector (See Figure 2). If subsidies were given to the IS sector, it would not only increase the emissions reductions of this sector but also incentivize production in the CM sector (due to the decreased carbon price level from the subsidy provided to the IS sector). Furthermore, the abatement level of each sector is related to carbon price and subsidy. The sector without subsidies tends to emit more, and emissions abatement does not have a clear response to the subsidy that only the sector receives. For example, according to our analysis in Section 2.4, with subsidy levels set as 150 CNY/tce: (1) if the subsidy is only given to the IS sector, we have \( \psi_j - \left( \beta_j \gamma^2 \sigma_j + 1 \right) (\phi^*_BC - \phi^*_ES) = 36.14 > 0 \), so the emissions reductions will be increased in the IS sector; and (2) if the subsidy is only giving to the CM sector, we have \( \psi_j > \left( \beta_j \gamma^2 \sigma_j + 1 \right) (\phi^*_BC - \phi^*_ES) = -1.70 < 0 \), so the emissions reductions will show a trend of a slight decrease in the CM sector. In general, this policy discrimination will reduce emissions reductions in the IS sector significantly.

Social welfare can be improved by increasing the level of energy-savings subsidies. With the energy-savings subsidy of 200 CNY/tce (29.99 USD/tce) social welfare improves by 4.77%, 1.66%, and 3.11%, respectively, compared to the results in the base case, which has only ETS but no energy-savings subsidy. Because of the positive effects on sectoral output and profit, energy-saving subsidies can be implemented as a complementary policy option to compensate for losses in competitiveness in the sectors covered by the ETS. The performance is more sensitive to changes in the energy-savings subsidies in the CM sector than in the IS sector. Moreover, according to the negative relationship between the equilibrium carbon price level and the energy-savings subsidy level, subsidies and ETS are substitutional in this sense, and policymakers can use subsidies as an effective tool to intervene in the carbon market.
We examine a set up similar to the one in the previous section by first looking at an identical rebating rate in both sectors and then studying the results of policy discrimination. We assume that the impact of the ob rate changes from 0 to 0.2 (A wider range of ob from 0 to 0.9 is also considered. The associated equilibrium carbon price can increase to over CNY 500/t CO₂ (USD 74.96/t CO₂), which is unrealistic and thus not reported. Those results are, however, available upon request). Following the equilibrium conditions specified in the Appendix A.3, Figure 4 shows sectoral performance under different ob rate levels, Figure 5 is the emissions abatement in the two sectors, and Figure 6 shows the changes in welfare.

### Figure 3. Welfare changes under the multi_ES case.

**3.2.3. Multi_OB Case**

This section evaluates the impact of adding output-based allowance rebating to the existing ETS. We examine a set up similar to the one in the previous section by first looking at an identical rebating rate in both sectors and then studying the results of policy discrimination.

![Figure 3](image-url)  
**Figure 3. Welfare changes under the multi_ES case.**

...
With an identical ob rate in both sectors, the equilibrium carbon price increases as much as 76% when the ob rate rises to 0.2. In addition, sectoral performance (both production and output) decreases compared to the results in the base case, while emissions reductions in both sectors increase a great deal with the increase in the ob rate. One could argue that this is contrary to the initial design of output-based rebating, which is applied to compensate for losses in sectoral competitiveness that result from implementing ETS and can, to a great extent, provide incentives for sectoral output [7,8]. The output incentive effects of output-based rebating make sense if we look at the analytical solution of sectoral production \( q^*_j \) under the multi_OB case in the Appendix A.3, in which the production level increases with the increase in the ob rate if we keep other parameters unchanged. In addition, in our model, as the ob rate ranges from 0–1, it can be inferred that the equilibrium carbon price level will increase with the introduction of ob. It should be acknowledged that the results, to a large extent, depend on the parameter values that we adopted for demand and abatement cost curves of the sector. In our case study, the results show that the significant increase in the equilibrium carbon price level has largely offset the output incentive that resulted from the increasing ob rate and even makes the sectoral performance worse after the implementation of output-based rebating.

**Figure 5.** Emissions abatements under the multi_OB case. Note: carbon prices are indexed to the right axis. Emission reduced by ** refers to the changes in CO₂ abatement, or \( a_j \) in our model.

**Figure 6.** Welfare change under the multi_OB case.
With discriminatory output-based rebating, the results also show that the IS and CM sectors respond differently to the policy. First, if output-based rebating is applied in the IS sector, performance trends in both sectors become worse. In addition, performance in the CM sector can be stimulated if it alone receives output-based rebating. For example, according to our analysis in Section 2.4, where \( ob = 0.15 \), we have the following: (1) if the \( ob \) is only given to the IS sector, we have \( \phi_{BC}^* - \phi_{ob}^* (1 - \phi_j) = -24.13 < 0 \), which means that the production of the sector will decrease; meanwhile, the emissions reductions in the sector will increase due to \( \phi_{BC}^* - \phi_{ob}^* (1 - \phi_j) = -35.73 \); and (2) if the \( ob \) is only given to the CM sector, we have \( \phi_{BC}^* - \phi_{ob}^* (1 - \phi_j) = 10.47 > 0 \), which means that the production of the sector will increase, meanwhile the emissions reductions in the sector will decrease due to \( \phi_{BC}^* - \phi_{ob}^* (1 - \phi_j) = 4.73 > 0 \). Second, welfare can be improved if output-based rebating is applied only to the CM sector; otherwise, social welfare trends downward, regardless of whether only the IS sector receives the rebates or both sectors receive identical rebates.

3.2.4. Cross Comparison of the Cases

Energy-savings subsidies and output-based rebating are two different policy options that supplement ETS. Here, to facilitate a cross comparison, we present the magnitude of the changes in the results when the policy is changed to a certain level (the energy-savings subsidy increases from 100 CNY/tce(15.00 USD/tce) to 200 CNY/tce(29.99 USD/tce), and the \( ob \) rate increases from 0.1 to 0.2—thereby, both levels are doubled) to present the diversified effects of the two policies on the two sectors, as well as compare the sensitivities of the sectoral response to the changes in the two policies. In addition, the change in production in one sector is calculated as \( p = \frac{p_{es} - p_{ob}}{\tilde{\sigma}_j - \beta \tilde{\sigma}_j^2} \). Second, welfare can be improved if output-based rebating is applied only to the CM sector; otherwise, social welfare trends downward, regardless of whether only the IS sector receives the rebates or both sectors receive identical rebates.

### Table 6. Cross-comparison between the energy-savings subsidy and output-based rebating cases.

<table>
<thead>
<tr>
<th></th>
<th>ES (Change from 100 CNY/tce to 200 CNY/tce)</th>
<th>OB Rate (Change from 0.1 to 0.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Identical Only IS Only CM</td>
<td>Identical Only IS Only CM</td>
</tr>
<tr>
<td>Production-IS</td>
<td>+ (*)</td>
<td>– (*)</td>
</tr>
<tr>
<td>Production-CM</td>
<td>+ (**)</td>
<td>– (**)</td>
</tr>
<tr>
<td>Profit-IS</td>
<td>+ (**)</td>
<td>– (**)</td>
</tr>
<tr>
<td>Profit-CM</td>
<td>+ (**)</td>
<td>– (**)</td>
</tr>
<tr>
<td>Emissions reduced by IS</td>
<td>– (**)</td>
<td>+ (****)</td>
</tr>
<tr>
<td>Emissions reduced by CM</td>
<td>– (****)</td>
<td>+ (****)</td>
</tr>
<tr>
<td>Total emissions</td>
<td>+ (*)</td>
<td>– (**)</td>
</tr>
<tr>
<td>Welfare</td>
<td>+ (**)</td>
<td>– (**)</td>
</tr>
</tbody>
</table>

Notes: * 0–1%; ** 1%–5%; *** 5%–10%; **** > 10%. – means there is a negative impact, and + means there is a positive impact. Emission reduced by ** refers to the changes in CO2 abatement, or \( a_j \) in our model.

Energy-savings subsidies can offset losses in production and profit in subsidized sectors because of the implementation of ETS, and this does not have a notable impact on total net CO2 emissions in both sectors, whether subsidized or not. As we have discussed above, output-based rebating seems to have more complex impacts on the covered sectors. However, performance in the IS sector is less sensitive to changes in policy levels than performance in the CM sector, while emissions reductions in both sectors are sensitive to changes in policy levels.

From the original intention of policy design, an energy-savings subsidy is to encourage more energy savings, and the output-based rebate is to give some compensation for a higher level of production. These are two different compensation mechanisms. Energy-savings subsidies would be affected more by the abatement cost of the sectors, whereas the output-based rebates would be affected more by the demand function. Given that sectors differ in abatement costs and face different demand curves, we would expect to see a clear heterogeneity of policy impacts across sectors.
4. Discussion and Conclusions

A multisector partial equilibrium model is established to analyze the impacts of multiple CO₂ abatement policies (energy-savings subsidies and OBA) in two sectors on the basis of ETS (auction). Empirical data on China’s iron and steel sector and cement sector are used to illustrate the impacts of various policy mixes relative to the auction-based ETS (base case). Specifically, we are interested in the impacts on production decisions, carbon prices, emissions reductions, and social welfare.

It is clear that these two policy mixes have distinctively different effects on the two sectors. Energy-savings subsidies play a substitution role for carbon prices because they encourage reductions in energy use and reduce equilibrium carbon prices. We find that even when a subsidy is given only to one sector, both sectors increase their production/profit. Total emissions levels increase because fewer efforts are made by these sectors to reduce emissions (except when subsidies are given only to the IS sector, which can increase abatement). Moreover, the equilibrium results show that the CM sector is more sensitive to the energy-savings subsidies. Carbon prices fall in response to incremental energy-savings subsidies. The results are based on the assumption that these two policies are set up independently, whereas they may be considered jointly by the policymakers in reality. In a scenario that the subsidy scheme is included in the cap of ETS, price may not necessarily fall. The policy independence assumption and its associated price impacts might be a bit too strong, it is nonetheless suggesting that energy saving subsidies are supplemental to ETS and failure to incorporate it to the ETS can weaken the pricing signal to carbon markets.

The policy mix with output-based rebating (in addition to auction-based ETS) of allowances, however, shows clearly different patterns. If identical policies are applied to both sectors, then the carbon prices increase significantly, both sectors lower production/profit, and consequently, the goal of lower emissions is achieved. Total social welfare, however, is reduced. In addition, the responses show a totally different trend with discriminatory output-based rebating: Almost identical patterns are seen when OBA is applied to only the IS sector, but when it is applied only to the CM sector, the production and profit increase, and total emissions, carbon prices, and social welfare are all slightly higher. Moreover, it should be noted that the findings in the IS and CM sectors may not applicable to other sectors, as each sector has its own special features in terms of demand and abatement.

Given that energy-savings subsidies are independent from the ETS and play a substitutive role, whereas output-based rebating is designed to compensate for the loss of competitiveness by the firms covered by the ETS, it is not surprising to see that the effect of subsidies is intuitively clearer. The role of output-based rebating is, on the contrary, more complicated. Policymakers should bear these effects in mind when additional policies are introduced along with the ETS. China has recently announced the National Emission Trading Scheme (Market Construction Plan of National Carbon Emission Trading (in Chinese). http://www.ndrc.gov.cn/gzdtt/201712/W020171220577386656660.pdf), which has been viewed as a significant policy movement to achieve the committed emission target in the Paris Agreement. The national platform means larger scale of participation in the system and the effectiveness of multiple policy mix is more important to the policymakers. Although it is hard to accommodate all policies (such as resource efficiency regulations) into our model, the results do confirm different roles of each policy instrument in the mix and provide important information to the policy makers.

It should be acknowledged that several limitations exist in our study. First, this paper shows that two policy combinations can have clearly different impacts on sectoral performance, emissions, carbon prices, and social welfare. Results may differ when more policy instruments are included, though the general implication should stand. Second, the model provides a simplified partial equilibrium solution with only two energy intensive sectors, which are assumed to be independence from each other. A more general framework that allows linkages across sectors may be more informative, though a simple extension in Appendix A5–A8 shows that the general conclusion remains.

Third, it is worth noting that the presence of market power will potentially affect the results of our model. For example, Hahn’s model suggests that participants in the ETS with market power could
manipulate the price to make the allocated permit price differ from that in the market equilibrium and then exercise their influence over the compliance costs in the ETS [33]. A simple extension with more than one firms in each sector is discussed briefly in Appendix A.5, which shows general consistency of our model, though a more advanced model that allows firms with market power maybe interesting for future research.

In addition, according to Wang et al., the eco-efficiency indicators of cement industry in China are still at a low level [34]. And the resource cycle rates in the iron and steel and cement sectors are expected to rise in the future. How to link the factors of resource cycling to the emission trading scheme design, as well as taking the material flow of the products in energy intensive sectors into consideration, will be potential directions in our future study.

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Author Contributions: Dayong Zhang and Lei Zhu are responsible for the idea and the writing of the paper. And Liu Xin and Yuan Li are responsible for the modeling of the idea, in which they have equal contribution to the paper.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Appendix A.1. Analytical Solution to the Base Case

In this section, we take two sectors as the example and present the detailed solution of the base case. Each sector maximizes its profit by optimizing production and emissions reductions, and the first-order conditions of Equation (4) are

$$\begin{align*}
\frac{\partial \Pi_j}{\partial q_j} &= \mu_j - 2\sigma_j q_j - c_j - \phi \tau_j = 0 \\
\frac{\partial \Pi_j}{\partial a_j} &= -a_j - 2\beta_j a_j + \phi = 0
\end{align*}$$

The results are

$$\begin{align*}
q_j^* &= \frac{\mu_j - c_j - \phi \tau_j}{2\sigma_j} \\
q_j^* &= \frac{\mu_j - a_j - \phi \tau_j}{2\beta_j}
\end{align*}$$

Social welfare could be represented as

$$W(\phi) = \sum_j (C_j(p_j) + \Pi_j - D_j(q_j, a_j) + PC_j(a_j, q_j, \phi))$$

$$W(\phi) = \sum_{j \in J} \left( \frac{1}{2} \sigma_j (q_j^*)^2 + (\mu_j - \sigma_j q_j^*) \cdot q_j^* - (a_j q_j^* - a_j^*) - \epsilon (\tau_j q_j^* - a_j^*) + \phi (\tau_j q_j^* - a_j^*) \right)$$

Policymakers optimize the carbon price $\phi$ by maximizing social welfare. We assume that there are two sectors in the ETS, and we use the subscript 1 to represent values in the IS sector and subscript 2 to represent those in the CM sector. Social welfare can be rewritten as
Appendix A.3. Analytical Solution to the Multi_OB Case

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emissions reductions. The first-order conditions based on Equation (12) are

\( W(\phi) = (\mu_1 - c_1 - \epsilon \tau_1) \frac{\mu_1 - c_1 - \phi \tau_1}{2\sigma_1} - \frac{1}{2} \sigma_1 (\mu_1 - c_1 - \phi \tau_1)^2 \)

\[ + (\epsilon - \alpha_1) \frac{\phi - \alpha_1}{2\beta_1} - \beta_1 (\frac{\phi - \alpha_1}{2\beta_1})^2 \]

\[ + (\mu_2 - \epsilon \tau_2) \frac{\mu_2 - c_2 - \phi \tau_2}{2\sigma_2} - \frac{1}{2} \sigma_2 (\mu_2 - c_2 - \phi \tau_2)^2 \]

\[ + (\epsilon - \alpha_2) \frac{\phi - \alpha_2}{2\beta_2} - \beta_2 (\frac{\phi - \alpha_2}{2\beta_2})^2 \]

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \)

\[ \phi^* = \frac{2\epsilon \sigma_1 \sigma_2 (\beta_1 + \beta_2) + (-\mu_2 + c_2 + 2\epsilon \tau_2) \tau_2 \sigma_2 \beta_1 \beta_2}{2\sigma_1 \sigma_2 (\beta_1 + \beta_2) + \beta_1 \beta_2 (\sigma_2 \tau_1^2 + \sigma_1 \tau_2^2)} \]

Appendix A.2. Analytical Solution to the Multi_ES Case

In this section, we take two sectors as the example and present the detailed solution of the multi_ES case. Each sector maximizes its profit by optimizing production and emissions reductions, and the first-order conditions of Equation (10) are

\[ \begin{align*}
\frac{\partial \Pi_j}{\partial a_j} &= \mu_j - 2\sigma_j a_j - c_j - \phi \tau_j = 0 \\
\frac{\partial \Pi_j}{\partial q_j} &= -a_j - 2\beta_j a_j + \phi + \psi_j = 0 
\end{align*} \]

The solutions are

\[ \begin{align*}
q_j^* &= \frac{\mu_j - c_j - \phi \tau_j}{2\sigma_j} \\
a_j^* &= \frac{\phi + \psi_j - a_j}{2\beta_j}
\end{align*} \]

If we assume that the energy-savings subsidy comes from society and that there are two sectors in the ETS, the social welfare equation can be rewritten as

\[ W(\phi) = \sum_{j \in J} \left( \frac{1}{2} \sigma_j (q_j^*)^2 + (\mu_j - \sigma_j a_j^*) \cdot q_j^* - (a_j a_j^* + \beta_j (a_j^*)^2) - c_j q_j^* \right) \]

\[ = (\mu_1 - c_1 - \epsilon \tau_1) \frac{\mu_1 - c_1 - \phi \tau_1}{2\sigma_1} - \frac{1}{2} \sigma_1 (\mu_1 - c_1 - \phi \tau_1)^2 \]

\[ + (\epsilon - \alpha_1) \frac{\phi + \psi_1 - a_1}{2\beta_1} - \beta_1 (\frac{\phi + \psi_1 - a_1}{2\beta_1})^2 \]

\[ + (\mu_2 - \epsilon \tau_2) \frac{\mu_2 - c_2 - \phi \tau_2}{2\sigma_2} - \frac{1}{2} \sigma_2 (\mu_2 - c_2 - \phi \tau_2)^2 \]

\[ + (\epsilon - \alpha_2) \frac{\phi + \psi_2 - a_2}{2\beta_2} - \beta_2 (\frac{\phi + \psi_2 - a_2}{2\beta_2})^2 \]

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \)

\[ \phi^* = \frac{2\epsilon \sigma_1 \sigma_2 [(\epsilon - \psi_2) \beta_1 + (\epsilon - \psi_1) \beta_2] + (-\mu_2 + c_2 + 2\epsilon \tau_2) \tau_2 \beta_1 \beta_2 \sigma_1}{2\sigma_1 \sigma_2 (\beta_1 + \beta_2) + \beta_1 \beta_2 (\sigma_2 \tau_1^2 + \sigma_1 \tau_2^2)} \]

Appendix A.3. Analytical Solution to the Multi_OB Case

In this section, we take two sectors as the example and present the detailed solution of the multi_OB case. Each sector maximizes its profit by choosing an optimal level of production and emissions reductions. The first-order conditions based on Equation (12) are
production and profit of the CM sector are more sensitive to the changes in the demand parameters not only within the sector but also across the sectors.

Appendix A.4. Sensitivity Analysis of Model Parameters

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \)

\[
\phi^* = \frac{2\sigma_1 \sigma_2 (\beta_1 + \beta_2) + (-\mu_2 + c_2 + 2\epsilon_2)(1 - ob_2)\tau_2 \sigma_1 \beta_1 \beta_2}{2\sigma_1 \sigma_2 (\beta_1 + \beta_2) + (1 - ob_1)^2 \beta_1 \beta_2 \sigma_1^2 + (1 - ob_2)^2 \beta_1 \beta_2 \sigma_2^2}
\]

Social welfare can be written as

\[
W(\phi) = \sum_j (CS_j(p_j) + \Pi_j - D_j(q_j, a_j) + PC_j(a_j, q_j, \phi) - \tau_j q_j \cdot ob_j \cdot \phi)
\]

Figure A1 shows a sensitivity analysis of the demand and abatement parameters of the two sectors in the base case in Section 3.2.1. Each parameter is set to change by 10% (either increase or decrease). Production and profit are only sensitive to their own demand parameters; meanwhile, the production and profit of the CM sector are more sensitive to the changes in the demand parameters in the IS sector. In addition, the emissions abatement in each sector is not only sensitive to its own demand and abatement parameters but also to the parameters of the other sector. In general, the demand parameters adopted in the model can have a significant impact on the values of the results, not only within the sector but also across the sectors.
Appendix A.5. Model Extension: More Than One Firms in Each of the Sectors

In this section, we will discuss the first extension of our model provided in the manuscript, that is, relax the assumption of one representative firm and allow more than one firm in each sector.

One concern about the analysis above is that the assumption of one representative firm may generate unrealistic predictions. This section will relax this assumption following Demailly and Quirion [7] and allow more than one firm in each sector. Assuming there are \( n_j \) homogeneous firms in sector \( j \), whose demand function can be written as:

\[
\bar{p}_j = \bar{\mu}_j - \bar{c}_j \cdot \bar{q}_j \text{ for } j \in J
\]  

(A1)

where \( \bar{p}_j \) is price, \( \bar{q}_j \) is total output in sector \( j \), we have \( \bar{q}_j = \sum_{i=1}^{n_j} q_{ji} \), \( q_{ji} \) is the output of firm \( i \) in sector \( j \), and \( \bar{\mu}_j \) and \( \bar{c}_j \) are the intercept and slope of the demand curve, respectively.

The cost function of firm \( i \) in sector \( j \) is \( C_{ji}(q_{ji}) = \bar{\mu}_j a_{ji} + \bar{c}_j a_{ji}^2 \). The abatement cost curve can be written as \( A_{Cji}(a_{ji}) = \bar{\epsilon} a_{ji} \). The cost of carbon permits in sector \( j \) can be expressed as \( PC_{ji}(q_{ji}, a_{ji}, \phi) = \phi(\bar{\mu}_j q_{ji} - a_{ji}) \).

(1) In the base case, the profit function of firm \( i \) in sector \( j \) can be written as

\[
\Pi_{ji} = \bar{p}_j \cdot q_{ji} - A_{Cji}(a_{ji}) - C_{ji}(q_{ji}) - PC_{ji}(q_{ji}, a_{ji}, \phi)
\]  

(A2)

Total net CO\(_2\) emissions are \( E = \sum_j (\tau_j \bar{q}_j - \bar{a}_j) \), where \( \bar{a}_j = \sum_{i=1}^{n_j} a_{ji} \). The social welfare function is therefore:

\[
W(\phi) = \sum_j \{CS_j(\bar{q}_j) + \sum_{i=1}^{n_j} (\Pi_{ji} - D_{ji}(q_{ji}, a_{ji}) + PC_{ji}(q_{ji}, a_{ji}, \phi))\}
\]

where \( CS_j(\bar{q}_j) \) is the consumer surplus in sector \( j \), represented as \( CS_j(\bar{q}_j) = \frac{1}{2} \bar{\epsilon}^2 \bar{q}_j^2 \); \( D_{ji}(q_{ji}, a_{ji}) = \epsilon(\bar{\mu}_j q_{ji} - a_{ji}) \) is the loss function that results from the sectoral CO\(_2\) emissions, which depicts the social damage from carbon emissions; and \( \epsilon \) is the unit damage coefficient.

(2) In the Multi_ES case, the energy-saving subsidy function of firm \( i \) in sector \( j \) can be written as \( ES_{ji}(a_{ji}) = \psi_j \cdot a_{ji} \), and the profit function is:

\[
\Pi_{ji} = \bar{p}_j \cdot q_{ji} - A_{Cji}(a_{ji}) - C_{ji}(q_{ji}) - PC_{ji}(q_{ji}, a_{ji}, \phi) + ES_{ji}(a_{ji})
\]  

(A3)
The social welfare function can be expressed as:

\[ W(\phi) = \sum_j [CS_j(\bar{q}_j) + \sum_{i=1}^{n_j} (\Pi^E_{ij} - D_{ij}(q_{ij}, a_{ij}) + PC_{ij}(q_{ij}, a_{ij}, \phi) - ES_{ij}(a_{ij}))] \]

(3) In the Multi_OB case, the additional income of firm \( i \) in sector \( j \) due to premits returned is \( \tau_j q_{ij} \cdot obj \cdot \phi \). The profit of firm \( i \) in sector \( j \) can be revised to

\[ \Pi_{ij} = \bar{p}_j \cdot q_{ij} - AC_{ij}(a_{ij}) - C_{ij}(q_{ij}) - PC_{ij}(q_{ij}, a_{ij}, \phi) + \bar{\tau}_j q_{ij} \cdot obj \cdot \phi \]  \hspace{1cm} (A4)

The social welfare function can be expressed as

\[ W(\phi) = \sum_j [CS_j(\bar{q}_j) + \sum_{i=1}^{n_j} (\Pi_{ij} - D_{ij}(q_{ij}, a_{ij}) + PC_{ij}(q_{ij}, a_{ij}, \phi) - \tau_j q_{ij} \cdot obj \cdot \phi)] \]

Assume that there are two sectors in the ETS, the equilibrium solution to the model can be summarized in Table A1 (solution to the model is given in Appendix A.7).

**Table A1. Summary of the analytical solutions to the extended model.**

<table>
<thead>
<tr>
<th>Cases</th>
<th>GE*</th>
<th>( \bar{q}_j )</th>
<th>( \bar{a}_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>( 2\bar{\theta}_1 \bar{\theta}_2 ) [ \binom{n_1^2 \epsilon \eta_1 + n_1^2 \epsilon_1}{-n_1^2 \epsilon_1 + n_1 \epsilon_1} \bar{v}_1 \bar{v}_2(n_2 + 1)^2 ] [ + \binom{n_2^2 \epsilon_2 + n_2 \epsilon_2}{n_2^2 \epsilon_2 + n_2 \epsilon_2} \bar{v}_3 \bar{v}_1(n_1 + 1)^2 ] [ + \epsilon(n_1 + 1)^2(n_2 + 1)^2 \bar{v}_3 \bar{v}_2(n_1 \bar{\theta}_2 + n_2 \bar{\theta}_1) ]</td>
<td>[ n_1(\bar{\theta}_1 - \bar{\epsilon}_1 - \phi \bar{v}_1) ] [ (n_1 + 1)^2 \eta_1 ]</td>
<td>[ n_1(\phi \bar{\theta}_1 - \bar{v}_3 - \phi \bar{v}_2) ] [ (n_1 + 1)^2 \eta_1 ]</td>
</tr>
<tr>
<td>Multi_ES</td>
<td>( 2\bar{\theta}_1 \bar{\theta}_2 ) [ \binom{n_1^2 \epsilon \eta_1 + n_1^2 \epsilon_1}{-n_1^2 \epsilon_1 + n_1 \epsilon_1} \bar{v}_1 \bar{v}_2(n_2 + 1)^2 ] [ + \binom{n_2^2 \epsilon_2 + n_2 \epsilon_2}{n_2^2 \epsilon_2 + n_2 \epsilon_2} \bar{v}_3 \bar{v}_1(n_1 + 1)^2 ] [ + (n_1 + 1)^2(n_2 + 1)^2 \bar{v}_3 \bar{v}_2(\bar{\theta}_2 n_1 + \bar{\theta}_1 n_2) ]</td>
<td>[ n_1(\bar{\theta}_1 - \bar{\epsilon}_1 - \phi \bar{v}_1) ] [ (n_1 + 1)^2 \eta_1 ]</td>
<td>[ n_1(\phi \bar{\theta}_1 - \bar{v}_3 - \phi \bar{v}_2) ] [ (n_1 + 1)^2 \eta_1 ]</td>
</tr>
<tr>
<td>Multi_OB</td>
<td>( 2\bar{\theta}_1 \bar{\theta}_2 ) [ \binom{n_1^2 \epsilon \eta_1 + n_1^2 \epsilon_1}{-n_1^2 \epsilon_1 + n_1 \epsilon_1} \bar{v}_1 \bar{v}_2(n_2 + 1)^2 ] [ + \binom{n_2^2 \epsilon_2 + n_2 \epsilon_2}{n_2^2 \epsilon_2 + n_2 \epsilon_2} \bar{v}_3 \bar{v}_1(n_1 + 1)^2 ] [ + (n_1 + 1)^2(n_2 + 1)^2 \bar{v}_3 \bar{v}_2(\bar{\theta}_2 n_1 + \bar{\theta}_1 n_2) ]</td>
<td>[ n_1(\bar{\theta}_1 - \bar{\epsilon}_1 - \phi \bar{v}_1) ] [ (n_1 + 1)^2 \eta_1 ]</td>
<td>[ n_1(\phi \bar{\theta}_1 - \bar{v}_3 - \phi \bar{v}_2) ] [ (n_1 + 1)^2 \eta_1 ]</td>
</tr>
</tbody>
</table>

With more than one firms considered in each sector, the general conclusions remain unchanged.

In the base case, we have \( \lim_{n_1 \to \infty} \phi^* = \epsilon \), which means the equilibrium carbon price should equal \( n_1 \to \infty \) and \( n_2 \to \infty \).
to the unit damage coefficient under the situation of perfect competition. And it is worth to mention that, the equilibrium prices of the product in sector \( j \) (\( \tilde{p}_j^* \)) under the three cases (Base, Multi_ES, and Multi_OB) can be expressed as \( \frac{\mu_j + n_j \phi j}{n_j + 1} \), \( \frac{\mu_j + n_j \phi j (1 - ob_j)}{n_j + 1} \), and \( \frac{\mu_j + n_j \phi j (1 - ob_j) \tilde{y}_j}{n_j + 1} \), respectively. And the number of the firms in each of the sector will determine to what extent the emission costs will be passed on to consumers in the cases of Base and Multi_ES, which is the same to the conclusions in Demailly and Quirion [7]. In the Multi_OB case, we have \( \tilde{p}_j^{ob} - \tilde{p}_j^{BC} = \frac{n_j \phi_{bc} (1 - ob_j) - n_j \phi_{bc}^* \tilde{y}_j}{n_j + 1} \), the price of this sector is higher than the base case if \( \phi_{bc}^* > \phi_{bc}^* (1 - ob_j) \). Such condition hold even when \( n_j \to \infty \).


In this section, we will discuss the second extension of our model provided in the manuscript, that is, consider the correlation of demands across sectors.

The demand of sectoral outputs is set as independent in previous analysis, and the correlation of demands across sectors will be considered in this section since the demands for the products in the energy intensive sectors may to some extent depend on each other (e.g., the changes in demand for the products in iron and steel and cements sectors are often synchronized in the construction sector). The demand function is revised to consider the effects of crossing sector linkages:

\[
p_j = \mu_j - \sigma_j q_j - \sum_{k \neq j} \sigma_{jk} q_k \text{ for } j, k \in J
\]  

where \( \sigma_{jk} \) is the slope parameter to describe the cross-effect of the demand change in sector \( k \) on sector \( j \).

1. In the base case, with the consideration of the effects of crossing demands among sectors, the output of sectors covered by the ETS can be written as \( Q^* = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \Sigma^{-1} \cdot F \), where \( \Sigma = \begin{bmatrix} 2\sigma_1 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & 2\sigma_2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & 2\sigma_N \end{bmatrix} \) (i.e., \( \Sigma_{kj} = \begin{cases} \sigma_{kj}, & k \neq j \\ 2\sigma_k, & k = j \end{cases} \)).

2. In the Multi_ES case, the derivation of outputs among sectors are the same to that in the base case.

3. In the Multi_OB case, the output of sectors can be written as \( Q^* = \Sigma^{-1} \cdot F_{OB} \), where \( F_{OB} = \begin{bmatrix} \mu_1 - c_1 - \phi \tau_1 \\ \mu_2 - c_2 - \phi \tau_2 \\ \vdots \\ \mu_N - c_N - \phi \tau_N \end{bmatrix} \).

Table A2 summarizes the analytical solutions to the model among the three cases with two sectors considered.
Table A2. Summary of the analytical solutions to the model extension (The linkage of demands across sectors).

<table>
<thead>
<tr>
<th>Cases</th>
<th>( q_j^* )</th>
<th>( a_j^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>[ q_1^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td>( a_j^* = \frac{\sigma - \alpha_j}{2\rho_j} )</td>
</tr>
<tr>
<td></td>
<td>[ q_2^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td></td>
</tr>
<tr>
<td>Multi_ES</td>
<td>[ q_1^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td>( a_j^* = \frac{\sigma - \alpha_j}{2\rho_j} )</td>
</tr>
<tr>
<td></td>
<td>[ q_2^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td></td>
</tr>
<tr>
<td>Multi_OB</td>
<td>[ q_1^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td>( a_j^* = \frac{\sigma - \alpha_j}{2\rho_j} )</td>
</tr>
<tr>
<td></td>
<td>[ q_2^* = \frac{2\sigma (\mu_1 - \phi_1 \phi \tau \eta_1 - \phi_1 \phi \tau \eta_2)}{4\tau \eta_2 - \phi_2 \phi \tau \eta_1 - \phi_2 \phi \tau \eta_2} ]</td>
<td></td>
</tr>
</tbody>
</table>

Note, the equilibrium carbon price levels among the three cases are:

1. Base case:
   \[
   \phi^* = \frac{2\rho_j \beta_1 \beta_2}{c_1 (c_1 \tau_2 - 2c_1 \tau_1)^2 + c_2 (c_2 \tau_1 - 2c_1 \tau_2)^2} \left( \frac{2(c_1 + c_2) \epsilon c_1 \tau_2 - 2c_1 \tau_1 (c_1 \tau_2 - 2c_1 \tau_1)}{(\beta_1 + \beta_2) \epsilon c_1 \tau_2 - 2c_1 \tau_1} \right)^{2} 
   \]

2. Multi ES:
   \[
   \phi^* = \frac{2\rho_j \beta_1 \beta_2}{c_1 (c_1 \tau_2 - 2c_1 \tau_1)^2 + c_2 (c_2 \tau_1 - 2c_1 \tau_2)^2} \left( \frac{2(c_1 + c_2) \epsilon c_1 \tau_2 - 2c_1 \tau_1 (c_1 \tau_2 - 2c_1 \tau_1)}{(\beta_1 + \beta_2) \epsilon c_1 \tau_2 - 2c_1 \tau_1} \right)^{2} 
   \]

3. Multi OB:
   \[
   \phi^* = \frac{2\rho_j \beta_1 \beta_2}{c_1 (c_1 \tau_2 (1 - \phi \eta_1 - \phi \eta_2)) + c_2 (c_2 \tau_1 (1 - \phi \eta_1 - \phi \eta_2))} \left( \frac{2(c_1 + c_2) \epsilon c_1 \tau_2 (1 - \phi \eta_1 - \phi \eta_2)}{(\beta_1 + \beta_2) \epsilon c_1 \tau_2 (1 - \phi \eta_1 - \phi \eta_2)} \right)^{2} 
   \]
Here the general conclusions of the main model remain unchanged.

Appendix A.7. Analytical Solution to Model Extension: More Than One Firms in Each of the Sectors

(1) Analytical Solution to the Base Case

Each firm in sector \( j \) maximizes its profit by optimizing production and emissions reductions, and the first-order conditions of Equation (16) are:

\[
\begin{align*}
\frac{\partial W_j}{\partial q_{ji}} &= \tilde{\mu}_j - \tilde{\sigma}_j \sum_{i=1}^{n_j} q_{ji} - \tilde{c}_j - \Phi \tilde{\tau}_j = 0 \\
\frac{\partial W_j}{\partial a_{ji}} &= -\tilde{\alpha}_j - 2\tilde{\beta}_ja_{ji} + \phi = 0
\end{align*}
\]

for \( i = 1, 2, \cdots, n_j, j \in \mathbb{J} \)

Solving the \( n_j \) sets of equations simultaneously, the results are:

\[
\begin{align*}
q_{ji}^* &= \frac{\tilde{\mu}_j - \tilde{c}_j - \Phi \tilde{\tau}_j}{(n_j + 1)\tilde{\sigma}_j} \\
a_{ji}^* &= \frac{\phi - \tilde{\alpha}_j}{2\tilde{\beta}_j}
\end{align*}
\]

for \( i = 1, 2, \cdots, n_j, j \in \mathbb{J} \)

So the total output and emissions of sector \( j \) are:

\[
\begin{align*}
\tilde{q}_j^* &= \frac{n_j(\tilde{\mu}_j - \tilde{c}_j - \Phi \tilde{\tau}_j)}{(n_j + 1)\tilde{\sigma}_j} \\
\tilde{\alpha}_j^* &= \frac{n_j(\phi - \tilde{\alpha}_j)}{2\tilde{\beta}_j}
\end{align*}
\]

for \( j \in \mathbb{J} \)

And the equilibrium price of product in sector \( j \) is \( \tilde{p}_j = \frac{\tilde{\mu}_j + n_j\tilde{\alpha}_j + n_j \Phi \tilde{\tau}_j}{n_j + 1} \).

Social welfare could be represented as:

\[
W(\phi) = \sum_j \left[ CS_j(\tilde{q}_j^*) + \frac{\bar{n}_j}{\bar{n}_i} \left( \Pi_{ji} - D_{ji}(q_{ji}, a_{ji}) + PC_{ji}(q_{ji}, a_{ji}, \phi) \right) \right]
\]

\[
W(\phi) = \sum_{j \in \mathbb{J}} \left( \frac{1}{2}\tilde{\sigma}_j(\tilde{q}_j^*)^2 + \frac{n_j}{\bar{n}_i} \left( \tilde{p}_j^* q_{ji}^* - (\tilde{\alpha}_j a_{ji}^* + \tilde{\beta}_j(a_{ji}^*)^2 - \tilde{c}_j a_{ji}^* - \Phi(\tilde{\tau}_j q_{ji}^* - a_{ji}^*)) - \epsilon(\tilde{\tau}_j q_{ji}^* - a_{ji}^*) + \phi(\tilde{\tau}_j q_{ji}^* - a_{ji}^*) \right) \right)
\]

Policymakers optimize the carbon price \( \phi \) by maximizing social welfare. Here also assume that there are two sectors in the ETS, and we use the subscript 1 to represent values in the IS sector and subscript 2 to represent those in the CM sector. Social welfare can be rewritten as:

\[
W(\phi) = \frac{n_1^2(\tilde{\mu}_1 - \tilde{c}_1 - \tilde{\tau}_1 \phi)}{2(n_1 + 1)^2 \tilde{\sigma}_1} + \frac{n_1(\tilde{\mu}_1 + n_1 \tilde{\alpha}_1 + n_1 \tilde{\tau}_1 \phi)(\tilde{\mu}_1 - \tilde{c}_1 - \tilde{\tau}_1 \phi)}{(n_1 + 1)^2 \tilde{\sigma}_1} - \frac{n_1(\tilde{\mu}_1 - \tilde{c}_1 - \Phi \tilde{\tau}_1)}{4\tilde{\beta}_1} \\
+ \frac{\epsilon - \tilde{\alpha}_1}{2n_1} \frac{n_1(\phi - \tilde{\alpha}_1)}{\tilde{\sigma}_1} - \frac{\epsilon \tilde{\tau}_1 + \tilde{\alpha}_1}{(n_1 + 1)\tilde{\sigma}_1} - \frac{n_1(\tilde{\mu}_1 - \tilde{c}_1 - \Phi \tilde{\tau}_1)}{4\tilde{\beta}_1} \\
+ \frac{n_2^2(\tilde{\mu}_2 - \tilde{c}_2 - \tilde{\tau}_2 \phi)^2}{2(n_2 + 1)^2 \tilde{\sigma}_2} + \frac{n_2(\tilde{\mu}_2 + n_2 \tilde{\alpha}_2 + n_2 \tilde{\tau}_2 \phi)(\tilde{\mu}_2 - \tilde{c}_2 - \tilde{\tau}_2 \phi)}{(n_2 + 1)^2 \tilde{\sigma}_2} - \frac{n_2(\phi - \tilde{\alpha}_2)^2}{4\tilde{\beta}_2} \\
+ \frac{\epsilon - \tilde{\alpha}_2}{2n_2} \frac{n_2(\phi - \tilde{\alpha}_2)}{\tilde{\sigma}_2} - \frac{\epsilon \tilde{\tau}_2 + \tilde{\alpha}_2}{(n_2 + 1)\tilde{\sigma}_2} - \frac{n_2(\tilde{\mu}_2 - \tilde{c}_2 - \Phi \tilde{\tau}_2)}{4\tilde{\beta}_2}
\]
We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \):

\[
\phi^* = \frac{2\tilde{\beta}_1\tilde{\beta}_2[(n_1^2\varepsilon_1 + n_1\varepsilon_1 + n_1\tilde{c}_1 - n_1\tilde{\mu}_1)(n_1 + 1)^2 + (n_2^2\varepsilon_2 + n_2\varepsilon_2 + n_2\tilde{c}_2 - n_2\tilde{\mu}_2)(n_1 + 1)^2]}{2\beta_1\beta_2[n_1^2\tilde{c}_2(n_1 + 1)^2 + n_2^2\tilde{c}_2(n_1 + 1)^2] + (n_1 + 1)^2\tilde{c}_1\tilde{\sigma}_2(\tilde{\beta}_2\gamma_1 + \tilde{\beta}_1n_2)}
\]

(2) Analytical Solution to the Multi_ES Case

Each firm in sector \( j \) maximizes its profit by optimizing production and emissions reductions, and the first-order conditions of Equation (17) are:

\[
\begin{aligned}
\frac{\partial W_j}{\partial q_{ji}} &= \tilde{\mu}_j - \tilde{c}_j - \tilde{\sigma}_j = 0 \\
\frac{\partial W_j}{\partial a_{ji}} &= -\tilde{\alpha}_j - 2\tilde{\beta}_ja_{ji} + \phi + \psi_j = 0
\end{aligned}
\]

for \( i = 1, 2; j \in J \).

Solving the \( n_j \) sets of equations simultaneously, the results are:

\[
\begin{aligned}
q_{ji}^* &= \frac{\tilde{\mu}_j - \tilde{c}_j - \phi_{ji}}{(n_j + 1)\tilde{\sigma}_j} \\
a_{ji}^* &= \frac{\phi - \tilde{\alpha}_j + \psi_j}{\tilde{\beta}_j}
\end{aligned}
\]

for \( i = 1, 2; j \in J \).

So the total output and emissions of sector \( j \) are:

\[
\begin{aligned}
\tilde{q}_j^* &= \frac{n_j(\tilde{\mu}_j - \tilde{c}_j - \phi_{ji})}{(n_j + 1)\tilde{\sigma}_j} \\
\tilde{a}_j^* &= \frac{n_j(\phi - \tilde{\alpha}_j + \psi_j)}{\tilde{\beta}_j}
\end{aligned}
\]

for \( j \in J \).

And the equilibrium price of product in sector \( j \) is \( \tilde{p}_j^* = \frac{\tilde{\mu}_j + n_j\tilde{c}_j + n_j\phi_{ji}}{n_j + 1} \).

Here also assuming the energy-savings subsidy comes from society and that there are two sectors in the ETS, the social welfare equation can be rewritten as:

\[
W(\phi) = \frac{n_1^2(\tilde{\mu}_1 - \tilde{c}_1 - \tilde{\tau}_1\phi)^2}{2(n_1 + 1)^2\tilde{\sigma}_1} + \frac{n_1(\tilde{\mu}_1 + n_1\tilde{c}_1 + n_1\tilde{\tau}_1\phi)(\tilde{\mu}_1 - \tilde{c}_1 - \tilde{\tau}_1\phi)}{(n_1 + 1)^2\tilde{\sigma}_1} + \frac{n_1(\phi - \tilde{\alpha}_1 + \psi_1)^2}{4\tilde{\beta}_1}
\]

\[\begin{aligned}
+ \frac{n_2^2(\tilde{\mu}_2 - \tilde{c}_2 - \tilde{\tau}_2\phi)^2}{2(n_2 + 1)^2\tilde{\sigma}_2} + \frac{n_2(\tilde{\mu}_2 + n_2\tilde{c}_2 + n_2\tilde{\tau}_2\phi)(\tilde{\mu}_2 - \tilde{c}_2 - \tilde{\tau}_2\phi)}{(n_2 + 1)^2\tilde{\sigma}_2} + \frac{n_2(\phi - \tilde{\alpha}_2 + \psi_2)^2}{4\tilde{\beta}_2}
\end{aligned}\]

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \):

\[
\phi^* = \frac{2\tilde{\beta}_1\tilde{\beta}_2[(n_1^2\varepsilon_1 + n_1\varepsilon_1 + n_1\tilde{c}_1 - n_1\tilde{\mu}_1)(n_2 + 1)^2 + (n_2^2\varepsilon_2 + n_2\varepsilon_2 + n_2\tilde{c}_2 - n_2\tilde{\mu}_2)(n_1 + 1)^2]}{2\beta_1\beta_2[n_1^2\tilde{c}_2(n_2 + 1)^2 + n_2^2\tilde{c}_2(n_1 + 1)^2] + (n_1 + 1)^2\tilde{c}_1\tilde{\sigma}_2(\tilde{\beta}_2\gamma_1 + \tilde{\beta}_1n_2) + (n_1 + 1)^2\tilde{c}_1\tilde{\sigma}_2(\tilde{\beta}_2\gamma_1 + \tilde{\beta}_1n_2)}
\]
(3) **Analytical Solution to the Multi_OB case**

Each firm in sector \( j \) maximizes its profit by optimizing production and emissions reductions, and the first-order conditions of Equation (18) are:

\[
\begin{align*}
\frac{\partial \Pi_i}{\partial q_{ji}} &= \bar{\mu} - \bar{c} - \sum_{i=1}^{n_j} q_{ji} - \bar{c} - \phi(1 - \bar{o}) \bar{e}_j = 0 \\
\frac{\partial \Pi_i}{\partial a_{ji}} &= -\bar{\alpha}_j - 2\bar{\beta}_j a_{ji} + \phi = 0 \\
\end{align*}
\]

for \( i = 1, 2, \ldots, n_j, j \in J \)

Solving the \( n_j \) sets of equations simultaneously, the results are:

\[
\begin{align*}
q_{ji}^* &= \frac{\bar{\mu}_j - \bar{c}_j - \phi(1 - \bar{o}) \bar{e}_j}{(n_j + 1) \bar{a}_j} \\
a_{ji}^* &= \frac{\phi - \bar{a}_j}{2\bar{\beta}_j} \\
\end{align*}
\]

for \( i = 1, 2, \ldots, n_j, j \in J \)

So the total output and emissions of sector \( j \) are:

\[
\begin{align*}
\tilde{q}_j &= \sum_{i=1}^{n_j} q_{ji}^* = \frac{n_j(\bar{\mu}_j - \bar{c}_j - \phi(1 - \bar{o}) \bar{e}_j)}{(n_j + 1) \bar{a}_j} \\
\tilde{a}_j &= \frac{n_j(\phi - \bar{a}_j)}{2\bar{\beta}_j} \\
\end{align*}
\]

for \( j \in J \)

And the equilibrium price of product in sector \( j \) is \( \tilde{p}_j^* = \frac{\bar{\mu}_j + n_j \bar{c}_j + n_j \phi(1 - \bar{o}) \bar{e}_j}{n_j + 1} \).

If we assume there are two sectors in the ETS, the social welfare equation can be rewritten as:

\[
W(\phi) = \frac{n_1^2(\bar{\mu}_1 - \bar{c}_1 - \bar{e} \phi(1 - \bar{o}_1))^2}{2(n_1 + 1)^2 \bar{c}_1} + \frac{n_2^2(\bar{\mu}_2 - \bar{c}_2 - \bar{e} \phi(1 - \bar{o}_2))^2}{2(n_2 + 1)^2 \bar{c}_2} + \frac{n_2(n_2 \bar{c}_2 + n_2 \bar{e} \phi(1 - \bar{o}_2)(\bar{\mu}_2 - \bar{c}_2 - \bar{e} \phi(1 - \bar{o}_2)))}{(n_2 + 1)^2 \bar{c}_2}
\]

\[
- (\bar{e}_2 + \bar{c}_2) \frac{n_2(\bar{\mu}_2 - \bar{c}_2 - \phi(1 - \bar{o}_2) \bar{e}_2)}{(n_2 + 1) \bar{c}_2} + (\epsilon - \bar{\alpha}_2) \frac{n_2(\phi - \bar{\alpha}_2)}{2\bar{\beta}_2} - \frac{n_2(\phi - \bar{\alpha}_2)^2}{4\bar{\beta}_2}
\]

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \):

\[
\phi^* = \frac{2\bar{\beta}_1 \bar{\beta}_2 \left[ (n_2^2 \bar{e} \bar{c}_1 + n_1 \bar{e} \bar{e}_1 + n_2 \bar{e} \bar{c}_2 - n_1 \bar{\mu}_1)(1 - \bar{o}_1) \bar{e}_2(n_2 + 1)^2 \\
+ (n_2^2 \bar{e} \bar{c}_1 + n_2 \bar{e} \bar{c}_2 - n_2 \bar{\mu}_2)(1 - \bar{o}_2) \bar{e}_1(n_1 + 1)^2 \\
+ (n_1 + 1)^2(\bar{e}_1 \bar{c}_2 + n_1 \bar{\mu}_2 + n_2 \bar{\mu}_1)(n_2 + 1)^2 \\
+ (n_1 + 1)^2(\bar{e}_2 \bar{c}_1 + n_1 \bar{\mu}_1 + n_2 \bar{\mu}_2)(n_2 + 1)^2 \right]}{2\bar{\beta}_1 \bar{\beta}_2 [n_1^2 \bar{e} \bar{e}_1(n_2 + 1)^2 + n_2^2 \bar{e} \bar{c}_2(n_2 + 1)^2 + n_2^2 \bar{e} \bar{e}_2(n_2 + 1)^2]}
\]

**Appendix A.8. Analytical Solution to Model Extension: The Linkage of Demands across Sectors**

(1) **Analytical Solution to the Base Case**

Taken Equation (19) to replace the demand function, the first-order conditions of Equation (4) are:

\[
\begin{align*}
\frac{\partial \Pi_i}{\partial q_{ij}} &= \mu_j - \sum_{k \neq j} \sigma_{jk} q_k - 2\sigma_j q_j - c_j - \phi \tau_j = 0 \\
\frac{\partial \Pi_i}{\partial a_{ij}} &= -a_j - 2\bar{\beta}_j a_j + \phi = 0 \\
\end{align*}
\]

for \( j, k \in J \)
So the emissions of sector \(j\) is:

\[
a_j^* = \frac{\phi - a_j}{2\beta_j}
\]

The first-order condition on output can be rewritten as:

\[
2\sigma_j q_j + \sum_{k \neq j} \sigma_{jk} q_k = \mu_j - c_j - \phi \tau_j
\]

Define

\[
\Sigma = \begin{bmatrix}
\sigma_{11} & \cdots & \sigma_{1N} \\
\sigma_{21} & \cdots & \sigma_{2N} \\
\vdots & \ddots & \vdots \\
\sigma_{N1} & \cdots & \sigma_{NN}
\end{bmatrix},
\]

\[
F = \begin{bmatrix}
\mu_1 - c_1 - \phi \tau_1 \\
\mu_2 - c_2 - \phi \tau_2 \\
\vdots \\
\mu_N - c_N - \phi \tau_N
\end{bmatrix},
\]

\[
Q = \begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_N
\end{bmatrix}
\]

where \(\Sigma_{ii} = \begin{cases} \sigma_{ii}, & i \neq j \\ 2\sigma_{ii}, & i = j \end{cases}\). We can obtain that \(\Sigma \cdot Q = F\), so \(Q^* = \Sigma^{-1} \cdot F\).

Assume that there are two sectors in the ETS and we use the subscript 1 to represent values in the IS sector and subscript 2 to represent those in the CM sector, the output are

\[
\begin{align*}
q_1^* &= \frac{2\sigma_{11}(\mu_1 - c_1 - \phi \tau_1) - \sigma_{12}(\mu_2 - c_2 - \phi \tau_2)}{4\sigma_{11}^2 - \sigma_{12}^2} \\
q_2^* &= \frac{2\sigma_{12}(\mu_2 - c_2 - \phi \tau_2) - \sigma_{11}(\mu_1 - c_1 - \phi \tau_1)}{4\sigma_{12}^2 - \sigma_{11}^2}
\end{align*}
\]

The welfare could be represented as:

\[
W(\phi) = -\frac{1}{2} \sigma_{11} (q_1^*)^2 - \sigma_{12} q_1 q_2^* + (\mu_1 - c_1 - \epsilon \tau_1) q_1^* + (\epsilon - a_1) a_1^* - \beta_1 (a_1^*)^2
\]

\[
-\frac{1}{2} \sigma_{22} (q_2^*)^2 - \sigma_{21} q_1 q_2^* + (\mu_2 - c_2 - \epsilon \tau_2) q_2^* + (\epsilon - a_2) a_2^* - \beta_2 (a_2^*)^2
\]

\[
+ (\mu_1 - c_1 - \epsilon \tau_1) (\mu_2 - c_2 - \epsilon \tau_2) q_1 q_2^* + (\epsilon - a_1) a_1 (a_2^*)^2
\]

\[
+ (\mu_1 - c_1 - \epsilon \tau_1) (\mu_2 - c_2 - \epsilon \tau_2) q_1^* q_2^* + (\epsilon - a_1) a_1 q_2^* (a_2^*)^2
\]

\[
+ (\mu_1 - c_1 - \epsilon \tau_1) (\mu_2 - c_2 - \epsilon \tau_2) q_2^* q_2^* + (\epsilon - a_2) a_2 q_1^* (a_2^*)^2
\]

\[
- (\sigma_{12} + \sigma_{21}) \frac{(\mu_1 - c_1 - \epsilon \tau_1) - (\mu_2 - c_2 - \epsilon \tau_2)}{4\sigma_{12}^2 - \sigma_{11}^2} \cdot \frac{(\mu_1 - c_1) - (\mu_2 - c_2)}{4\sigma_{21}^2 - \sigma_{12}^2}.
\]
We can obtain $\phi^*$ by solving the first-order condition of $W(\phi)$:

$$
\phi^* = \frac{2\beta_1 \beta_2}{(4\sigma_1 c_2 - c_{12} c_{21})(c_{12} T_2 - 2c_2 T_1)(\mu_1 - c_1 - \epsilon T_1) - c_1 (c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\} + (4\sigma_1 c_2 - c_{12} c_{21})(c_{21} T_1 - 2c_1 T_2)(\mu_2 - c_2 - \epsilon T_2) - c_2 (c_{21} T_1 - 2c_1 T_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\} - (c_{12} + c_{21})\{(c_{12} T_2 - 2\tau_1 c_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\} + (c_{21} T_1 - 2\tau_1 c_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\}\}}^\frac{\frac{(4\sigma_1 c_2 - c_{12} c_{21})}{(c_{12} T_2 - 2c_2 T_1)(\mu_1 - c_1 - \epsilon T_1)} - c_1 (c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\} + (4\sigma_1 c_2 - c_{12} c_{21})(c_{21} T_1 - 2c_1 T_2)(\mu_2 - c_2 - \epsilon T_2) - c_2 (c_{21} T_1 - 2c_1 T_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\} - (c_{12} + c_{21})\{(c_{12} T_2 - 2\tau_1 c_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\} + (c_{21} T_1 - 2\tau_1 c_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\}\}}{2\beta_1 \beta_2}.$$

(2) Analytical Solution to the Multi_ES Case

Taken Equation (19) to replace the demand function, the first-order conditions of Equation (10) are:

$$
\begin{align*}
\frac{\partial \Pi}{\partial q_j} &= \mu_j - \sum_{k \neq j} \sigma_{jk} q_k - 2c_j q_j - c_j - \phi T_j = 0 \\
\frac{\partial \Pi}{\partial q_j} &= -a_j - 2\beta_j a_j + \phi + \psi_j = 0
\end{align*}
$$

So the emissions of sector $j$ is

$$a_j^* = \frac{\phi - a_j + \psi_j}{2\beta_j},$$

and the output of sector $j$ is the same as that in the base case.

Assume that there are two sectors in the ETS, and we use the subscript 1 to represent values in the IS sector and subscript 2 to represent those in the CM sector, the output are:

$$
\begin{align*}
q_1^* &= \frac{2c_1 \mu_1 - c_1 \phi T_1 - c_{12} (\mu_2 - c_2 - \phi T_2)}{4c_1 c_2 - c_{12} c_{21}} \\
q_2^* &= \frac{2c_2 (\mu_2 - c_2) - \phi T_2 - c_{21} (\mu_1 - c_1)}{4c_1 c_2 - c_{12} c_{21}}
\end{align*}
$$

The welfare could be represented as:

$$
W(\phi) = -\frac{1}{2}\sigma_1 (q_1^*)^2 - c_{12} q_1^* q_2^* + (\mu_1 - c_1 - \epsilon T_1)q_1^* + \epsilon - a_1) a_1^* - \beta_1 (a_1^*)^2 - \frac{1}{2}\sigma_2 (q_2^*)^2 - c_{21} q_1^* q_2^* + (\mu_2 - c_2 - \epsilon T_2)q_2^* + \epsilon - a_2) a_2^* - \beta_2 (a_2^*)^2
$$

$$
W(\phi) = -\frac{1}{2}\sigma_1 \left(\frac{(c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\}}{4c_1 c_2 - c_{12} c_{21}}\right) + (\mu_1 - c_1 - \epsilon T_1)\left(\frac{(c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_1 - c_1) - c_{12} (\mu_2 - c_2)\}}{4c_1 c_2 - c_{12} c_{21}}\right) + (\mu_2 - c_2 - \epsilon T_2)\left(\frac{(c_{21} T_1 - 2c_1 T_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\}}{4c_1 c_2 - c_{12} c_{21}}\right) + (\mu_2 - c_2 - \epsilon T_2)\left(\frac{(c_{21} T_1 - 2c_1 T_2)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\}}{4c_1 c_2 - c_{12} c_{21}}\right)
$$

$$
\frac{(c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\}}{4c_1 c_2 - c_{12} c_{21}} \cdot \frac{(c_{12} T_2 - 2c_2 T_1)\{2\sigma_2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)\}}{4c_1 c_2 - c_{12} c_{21}}.
$$
We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \):

\[
\phi^* = \frac{2\beta_1 \beta_2}{4(\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21})} \begin{cases}
(4\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21})(\sigma_{12} \tau_2 - 2\sigma_2 \tau_1)(\mu_1 - c_1 - \epsilon \tau_1) \\
-\sigma_1(\sigma_{12} \tau_2 - 2\sigma_2 \tau_1)2\sigma_2(\mu_1 - c_1 - \sigma_{12} \mu_2 - c_2) \\
+4\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21}(\sigma_{21} \tau_1 - 2\sigma_1 \tau_2)(\mu_2 - c_2 - \epsilon \tau_2) \\
-\sigma_2(\sigma_{21} \tau_1 - 2\sigma_1 \tau_2)(2\sigma_1(\mu_2 - c_2) - \sigma_{21} \mu_1 - c_1) \\
-(\sigma_{12} + \sigma_{21})((\sigma_{12} \tau_2 - 2\tau_1 \sigma_2)(\sigma_{12} \mu_2 - c_2) - \sigma_{21}(\mu_1 - c_1)) \\
+2(\sigma_{21} \tau_1 - 2\sigma_1 \tau_2)(2\sigma_2(\mu_1 - c_1) - \sigma_{12} \mu_2 - c_2) \\
(4\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21})^2[(\epsilon - \psi_1)\beta_2 + (\epsilon - \psi_2)\beta_1] \\
\end{cases}
\]

(3) Analytical Solution to the Multi_OB Case

Taken Equation (19) to replace the demand function, the first-order conditions of Equation (4) are:

\[
\begin{align*}
\frac{\partial \bar{q}_j}{\partial \bar{a}_j} &= \mu_j - \sum_{k \neq j} \sigma_{jk} q_k - 2\sigma_j a_j - c_j - \phi \tau_j + \phi \tau_j \cdot \nu_j = 0 \\
\frac{\partial \bar{q}_j}{\partial \bar{a}_j} &= -\alpha_j - 2\beta_j a_j + \phi = 0
\end{align*}
\]

For \( j, k \in j \)

So the emissions of sector \( j \) is \( q_j^* = \frac{\phi - \alpha_j}{2\beta_j} \).

The first-order condition on output can be rewritten as:

\[
2\sigma_j \bar{q}_j + \sum_{k \neq j} \sigma_{jk} q_k = \mu_j - c_j - \phi \tau_j (1 - \bar{a}_j)
\]

Define

\[
F_{OB} = \left[ \begin{array}{c}
\mu_1 - c_1 - \phi \tau_1 (1 - \bar{a}_1) \\
\mu_2 - c_2 - \phi \tau_2 (1 - \bar{a}_2) \\
\vdots \\
\mu_N - c_N - \phi \tau_N (1 - \bar{a}_N)
\end{array} \right]
\]

there are \( \Sigma \cdot Q = F_{OB} \), so \( Q^* = (\Sigma)^{-1} \cdot F_{OB} \).

Assume that there are two sectors in the ETS, and we use the subscript 1 to represent values in the IS sector and subscript 2 to represent those in the CM sector, the output are:

\[
\begin{align*}
q_1^* &= \frac{4\sigma_1(\mu_1 - c_1 - \phi \tau_1 (1 - \bar{a}_1)) - \sigma_{12}(\mu_2 - c_2 - \phi \tau_2 (1 - \bar{a}_2))}{4\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21}} \\
q_2^* &= \frac{4\sigma_1(\mu_2 - c_2 - \phi \tau_2 (1 - \bar{a}_2)) - \sigma_{12}(\mu_1 - c_1 - \phi \tau_1 (1 - \bar{a}_1))}{4\sigma_1 \sigma_2 - \sigma_{12} \sigma_{21}}
\end{align*}
\]

The welfare could be represented as:
\[ W(\phi) = -\frac{1}{2}c_1 (q_1^*)^2 - c_{12} q_1^* q_2^* + (\mu_1 - c_1 - \varepsilon \tau_1) q_1^* + (\varepsilon - a_1) a_1^* - \beta_1 (a_1^*)^2 \]
\[ -\frac{1}{2}c_2 (q_2^*)^2 - c_{21} q_1^* q_2^* + (\mu_2 - c_2 - \varepsilon \tau_2) q_2^* + (\varepsilon - a_2) a_2^* - \beta_2 (a_2^*)^2 \]
\[ = -\frac{1}{2}c_1 \left( \frac{c_{12} q_1^* (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} q_1^*} - \frac{c_{12} \tau_2 (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} \tau_2} + \frac{\phi - a_1}{2 \sigma_1^2} \right)^2 \]
\[ + (\mu_1 - c_1 - \varepsilon \tau_1) \left( \frac{c_{12} q_1^* (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} q_1^*} - \frac{c_{12} \tau_2 (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} \tau_2} + \frac{\phi - a_1}{2 \sigma_1^2} \right)^2 \]
\[ + (\varepsilon - a_1) \left( \frac{c_{12} q_1^* (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} q_1^*} - \frac{c_{12} \tau_2 (1 - \beta_1 a_1^*)}{4 \sigma_1^2 - c_{12} \tau_2} + \frac{\phi - a_1}{2 \sigma_1^2} \right)^2 \]
\[ + (\mu_2 - c_2 - \varepsilon \tau_2) \left( \frac{c_{21} q_1^* (1 - \beta_2 a_2^*)}{4 \sigma_2^2 - c_{21} q_1^*} - \frac{c_{21} q_2^* (1 - \beta_2 a_2^*)}{4 \sigma_2^2 - c_{21} \tau_2} + \frac{\phi - a_2}{2 \sigma_2^2} \right)^2 \]
\[ + (\varepsilon - a_2) \left( \frac{c_{21} q_1^* (1 - \beta_2 a_2^*)}{4 \sigma_2^2 - c_{21} q_1^*} - \frac{c_{21} q_2^* (1 - \beta_2 a_2^*)}{4 \sigma_2^2 - c_{21} \tau_2} + \frac{\phi - a_2}{2 \sigma_2^2} \right)^2 \]
\[ - (c_{12} + c_{21}) \frac{2 \sigma_2^2 (\mu_2 - c_2) - c_{21} (\mu_1 - c_1)}{4 \sigma_1^2 - c_{12} q_1^* 4 \sigma_2^2 - c_{21} \tau_2} . \]

We can obtain \( \phi^* \) by solving the first-order condition of \( W(\phi) \):

\[
\phi^* = \frac{2 \beta_1 \beta_2}{4 \sigma_1^2 - c_{12} q_1^* c_{21} q_2^*}
\]

**References**


31. Li, Y.; Zhu, L. Cost of energy saving and CO\textsubscript{2} emissions reduction in China’s iron and steel sector. *Appl. Energy* 2014, 130, 603–616. [CrossRef]


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