Carbon Taxes and Carbon Right Costs Analysis for the Tire Industry

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Abstract: As enterprises are the major perpetrators of global climate change, concerns about global warming, climate change, and global greenhouse gas emissions continue to attract attention, and have become international concerns. The tire industry, which is a high-pollution, high-carbon emission industry, is facing pressure to reduce its carbon emissions. Thus, carbon prices and carbon trading have become issues of global importance. In order to solve this environmental problem, the purpose of this paper is to combine mathematical programming, Theory of Constraints (TOC), and Activity-Based Costing (ABC) to formulate the green production decision model with carbon taxes and carbon right costs, in order to achieve the optimal product mix decision under various constraints. This study proposes three different scenario models with carbon taxes and carbon right used to evaluate the effect on profit of changes in carbon tax rates.

Keywords: Activity-Based Costing (ABC); carbon emissions; tire industry; carbon trading; mathematical programming

1. Introduction

The international community has paid considerable attention to environmental protection in recent years. Carbon taxes are considered as effective tools to curb greenhouse gas emissions. Analysis of the empirical evidence of carbon tax benefits, including indirect effects on technological developments and other pollutants (i.e., co-benefits) also added suppliers to improve energy price expectations in the future, and empirically propose the first green research contribution [1] and the United Nations has successively enacted relevant laws and regulations, such as the international Greenhouse Gas Emission Act and the 2015 Paris Accord [2], in the hope that the environment we live in will not continue to deteriorate. Due to high technology and industrial development, anthropocentric greenhouse gases are continually generated and accumulate, leading to increased global warming, and deteriorating climate and environmental ecology. In the past decade, it has been well recognized that environmental and economic performance are mutually reinforcing, with improved environmental performance leading to lower costs, increased sales, and greater economic efficiency. That joint effects will help companies to obtain temporary financing and minimize the damage to the environment stemming from carbon emissions [3]. Different perspectives and empirical studies [4] suggest that more attention should be paid to the causal relationship between eco-efficiency and different environmental management approaches, as well as their economic consequences; for example, the implementation of a carbon tax will help to improve the environment, promoting carbon taxes as a policy to improve the cost-effectiveness of carbon emissions. However, in the current practice of environmental policy, only a few countries implement taxation based on carbon content. According to empirical results, the carbon tax may be an interesting policy choice, and the negative impact can be compensated by designing taxes and the fiscal revenue generated by the use [5]. Various environmental policy tools are used in critical assessment criteria, including cost-effectiveness, equitable distribution, risk minimization
of uncertainty and political feasibility. Additional considerations include carbon taxes, tradable emission allowances, emission reduction subsidies, and research and development subsidies [6]. In the actual situation of the EU, the implementation of the carbon price policy can achieve the goal of emission reduction, and the carbon trading results also show that carbon emissions can be controlled through carbon trading [7]. But when the regulatory company gets compensation, efficiency needs to be paid among the companies to balance the damage caused by the marginal probability. Applying this basic economic logic to the analysis of the compensation rules proposed under the EU Emissions Trading Scheme, emissions permits can be allocated free of charge to carbon-intensive and trade-exposed industries [8]. However, various environmental policy instruments, including carbon emission taxes, tradable emission allowances, emission reduction subsidies, performance standards, technical tasks, and research and development subsidies, can meet major evaluation criteria, including cost-effectiveness, fair distribution, and risk minimization of uncertainty, and political feasibility [9]. Investigating carbon prices in China, the results show that it has important policy implications for regional markets to be included in national carbon trading [10]. The impact of the Emissions Trading Scheme (ETS) is one of the most effective mitigation measures [11]. The tire industry’s production process greatly increases atmospheric greenhouse gas concentrations. At present, many countries have adopted a “carbon tax” levy to solve corporate carbon emission problems with the related costs borne by enterprises. Sathre and Gustavsson believed that taxation on environmental issues can reduce pollution [12]. Therefore, the impact of carbon pricing policies on the economy is very significant [13]. In Australia, the carbon premium has become higher and higher after the implementation of the carbon tax, and indicates that a stable long-term policy is needed in the future to enable the carbon pricing mechanism to play its full role [14]. Therefore, the carbon price and carbon trading policy results discussed by the aforementioned research institutes have clearly become the most important issues for governments around the world. In this paper, the carbon taxes and carbon right costs analysis holds only for direct carbon taxes or for cap-and-trade schemes as well through a market determined price for carbon emission allowances.

According to the above research, the effective way to reduce emissions is to implement carbon tax collection and formulate carbon rights trading mechanism. This paper aims to analyze the relevant analysis of carbon emissions in the tire industry. Then the tire industry can employ the Activity-Based Costing (ABC) method and perform related cost calculations. The ABC system provides a means for analyzing non-value-added activity workbooks, such as procedures and production that neglect resource constraints in activity [15,16]. A better solution to the reduction of environmentally harmful products is provided by the Theory of Constraint (TOC) [17,18].

The purpose of this paper is to combine mathematical programming, TOC, and ABC to formulate a green production decision model with carbon taxes and carbon right costs, in order to achieve the optimal product mix decision under various constraints and to analyze the effect on profit of carbon taxes. The tire industry is one of the industries with high pollution and high energy consumption, where its carbon pollution will seriously affect environmental issues [19]. Therefore, further assumes that three different scenario models with carbon taxes and carbon right for tire companies are used for scenario analysis. Hoinka and Ziebik [20] applied mathematical methods to analyze complex energy management systems. In summary this paper will discuss the green ABC production decision-making model. The contribution of this study is that it incorporates the cost of carbon emissions into the mathematical programming, enabling the tire industry to evaluate emissions projects. In recent years, studies on the implementation of carbon trading have provided useful insights and helped to reduce carbon emissions [21–23].
2. Research Background

2.1. Carbon Emissions Cost and Carbon Tax

The increasing attention to global warming and climate change has become a common concern among governments around the world; therefore, it is imperative to adopt effective measures to promote energy conservation and carbon reduction, and implement corporate carbon emissions policies. Carbon pricing is a hot topic in climate policy debates. There are still many problems due to numerous reasons for implementing global carbon prices. Therefore, the main arguments for carbon pricing are proposed for analysis and discussion. The relatively low environmental cost of carbon pricing helps to improve the social acceptability of climate policy. This includes correcting the price of property that stimulates rapid environmental innovation. These arguments are underestimated in open debates, where pricing strategies are often overlooked and innovation policy misconceptions are common. However, carbon pricing and technology policies are largely complementary and therefore necessary for effective climate policy. In addition, other tools are used to complement the carbon pricing, and suggestions for solving these problems are proposed. Whether through carbon taxes or carbon emissions trading, reflections on the implementation of global carbon prices, including energy research and innovation research, go beyond the traditional arguments of environmental economics [24].

Due to the abnormal climate change in the world, the impact of air pollution is becoming more and more serious, and people’s health is also seriously threatened. Therefore, effective action must be taken to develop policies to reduce greenhouse gas emissions. People are increasingly concerned about the market instruments in the form of carbon pricing mechanisms. The study found that carbon pricing policies include carbon taxes, limits and transactions, emission reduction credits, clean energy standards and reductions in fossil fuel subsidies. Organize the experience and implementation methods of developing countries. And put forward the carbon pricing applicability for decision-making reference [25]. In South America, a carbon pricing strategy is also proposed for climate issues to cope with environmental pollution problems [26]. Environmental pollution has become a major social and political challenge around the world [27,28], and some studies have yielded positive results. That the application of carbon price is a carbon reduction tool [29]. Because companies with high carbon emissions have higher carbon risks. Recent ex-post evaluations of the European Union (EU) Emissions Trading Scheme (ETS) on regulated industries in the industrial and power industries have found that issues such as carbon dioxide emissions, economic performance and competitiveness, and innovation can all evolve with international developments. Propose more relevant future research directions, continue to expand on these innovative and internationally important issues [30].

In addition, there is an empirical research on the impact of the EU Emissions Trading Scheme on German stock returns. It was found that in the first few years of the program, companies that received national free carbon credits performed significantly better than companies that did not receive tax-free carbon credits. This result is very interesting, if there is a large “carbon premium”, the cash flow from the free allocation of carbon allowances increases. Companies with high carbon emissions, higher carbon exposures, show higher expected returns [31]. For example, The Netherlands [32], Denmark, and Sweden have begun implementing carbon tax policies. Carbon and energy taxes will increase fiscal measures to reduce carbon emissions [12,33,34]. The results of these studies show that the implementation of carbon emission cap-and-trade mechanisms can effectively reduce aviation industry carbon emissions [35], and the means used to reduce CO₂ emissions by this policy have been theoretically assessed [36]. And considered the use of two different manufacturing technologies for their carbon emission quota model [37], and the analysis results can reduce the total amount of carbon emissions and pollution of the environment. In considering a mathematical model for carbon emission quota, the results show that carbon emission restrictions will always encourage restricted capital goods manufacturers to produce more remanufactured products to maximize profits and reduce carbon emissions [38]. In addition, the New Zealand Emissions Trading Scheme (NZ ETS) is unique in that it
allowed unlimited use of Kyoto grants until 2013. As a result, New Zealand ETS provides an impact on the pricing and limits of carbon pricing for small systems. Using the quota import and export data, empirical analysis, the study found that carbon pricing offset is mainly the price determinant [39]. That the inclusion of flexible pricing mechanisms and the differentiation of tax rates can effectively reduce carbon emissions [29].

Ecological protection has increasingly attracted international attention, and the dramatic increases in various environmentally harmful substances and energy consumption have led to severe air pollution. In view of protecting human health and environmental ecology by preventing the deterioration of air quality, emission reduction policies have become the important issue in the tire industry; in the complex process of manufacturing tires, the carbon emitted cannot be ignored. Higher carbon emissions will lead to higher carbon taxes, which will increase the costs of the tire industry.

The present research, as related to the tire industry, mainly focuses on the analysis of waste tire recycling [40], low carbon or zero carbon production system innovation to reduce carbon dioxide emissions [41]. There is no relevant research analyzing carbon emission costs and carbon trading in combination with mathematical programming models; therefore, the focus of this article is on the costs and expenses of the tire manufacturing process, while adding carbon emission costs to carry out three different scenario models analysis with carbon taxes and carbon right for tire companies. Analysis of the results can be used for decision reference for the tire industry.

2.2. ABC Applied to Green Production and Environmental Protection in the Tire Industry

The ABC method is mainly used for calculating the activity cost of each product, where resources are allocated to activities or activity centers according to resource drivers. Activity costs are allocated to products based on activity drivers, and the resources allocated to activities constitute the cost elements of the activity [42]. This article uses tire companies as the research object. After process differentiation, the cost of each green process project is estimated in order to analyze the total cost; therefore, we consider the green tire manufacturing process as a cost target. In the complex tire industrial process, this article divides cost into direct material cost, direct labor cost, handling cost, carbon tax, and other costs, and also regards the maintenance costs and depreciations of the machines, plants, etc. as the total fixed cost. The ABC method is used to configure the tire industry’s manufacturing process and allocate resource costs to corresponding process projects, in order to determine the most suitable green product mix to maximize total profits.

In recent years, due to environmental concerns, new green manufacturing technologies have been more widely applied and explored to improve operation technology [43]; the issue of global warming has drawn ever-greater attention. Economists and international organizations demand the introduction of carbon taxes as a cost-effective way to decrease Greenhouse Gas (GHG) emissions [44]. Moreover, the carbon duty guidelines can promote the growth and application of renewable power sources [45], and assist in the growth of the global economy [46]. With the rapid technological progress, the tire industry has been pushed to recognize the need for environmental protection for the sustainable development of its enterprises regarding indispensable factors, such as raw materials. Processes and products must support the care of the environment and cherish the required resources by embracing the concept of environmental protection, as well as improved product design, equipment, and activities. The tire production process is complicated, and there is little research that analyzes the process costs of this industry. In addition, the tire industry must consider the cost of carbon emissions, in order to accurately predict tire production costs and reduce the impact of environmental pollution. Therefore, this paper proposes three green decision models, including a model without carbon rights trading and two models with two kinds of carbon rights trading costs, in order to analyze the carbon emission costs in the tire industry for managerial decisions.
2.3. Application of TOC Theory of Constraints

This article applies mathematical programming models and analyzes the most profitable tire product mix using the ABC approach. In addition, through fixed operating costs and systematic analysis of overhead costs [41], product mix decisions are evaluated via mathematical programming and TOC. The ABC method can be customized to analyze different types of board decision making, including price and product-mix [47–49], design and development [50], environmental management [51,52], green building project, strategy, and construction method selection [53]. Plenert’s economic planning included TOC [54], which is an integer linear programming method, to produce a portfolio with diverse restricted capital and employed TOC-based arithmetic to obtain the best combined product mix resolution [49]. This essay uses the TOC approach to determine production priorities. This approach can be applied to a variety of study themes [55,56] to derive the best product mix.

3. Green Production Decision Model for the Tire Industry

3.1. Tire Manufacturing Process

The manufacturing processes of the tire industry are complex and have six main processes: kneading, pressing out, cutting off, forming, vulcanizing, and inspecting (as shown in Figure 1):

1. **Kneading** (j = 1): The first process mixes all the rubber materials and soot, and transports them to the kneader for kneading to change the strength of the rubber.
2. **Pressing out** (j = 2): The second process uses glue and friction to generate heat, allowing the rubber to mature.
3. **Cutting off** (j = 3): The third process mainly uses a cutting machine to cut off various tire sizes.
4. **Forming** (j = 4): The fourth process is to perform the molding assembly work to assembly all the materials, strips, and steel rings to complete the tire prototype.
5. **Vulcanizing** (j = 5): The fifth process is to perform the vulcanization process. The main purpose of tire vulcanization is to rearrange the molecules of the rubber, meaning to heat and mold the tires through steam. The tires that complete this process will become the finished tires.
6. **Inspecting** (j = 6): Finally, the finished tires are inspected and the tires are moved to the warehouse for shipment.

![Figure 1. Tire manufacturing flow chart.](image-url)
3.2. The Model Formulation

From the aforementioned ABC model and the main tire manufacturing process, the corresponding costs are included in the mathematical model. First, this article assumes that the unit product prices are constant within a certain relevant range. Second, the green activity for each production process, as well as its activity driver and related resources, are determined and selected. Third, five materials (natural rubber, soot, synthetic rubber, cord, and bead) used in the production process are the direct material inputs, while other insignificant materials are not included in this study. Fourth, direct labor resources can be increased by additional overtime work with higher wage rates. Fifth, assume that carbon emission costs are considered as variable costs, which depend on carbon emissions quantity and different tax rates. Organic compounds are chemicals that contain carbon and are found in all living things. In addition to carbon, carbon contains other elements, such as hydrogen, oxygen, fluorine, chlorine, bromine, sulfur or nitrogen. Assume that the relationship between carbon tax and carbon emission quantity has been set up by using the concept of carbon equivalency. Sixth, this research is limited by the difficulty of obtaining relevant information, thus, we only consider the direct materials, direct labor, disposal costs, and carbon emissions costs, and assume that all other costs are fixed costs. The data used in this study are in metric tons and U.S. dollars, as shown in Table 1.

<table>
<thead>
<tr>
<th>Product</th>
<th>PCR (i = 1)</th>
<th>TBR (i = 2)</th>
<th>MC (i = 3)</th>
<th>Available Capacity (thousand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum demand: (thousand)</td>
<td>f</td>
<td>j</td>
<td>B_i</td>
<td>1000</td>
</tr>
<tr>
<td>Selling price: (USD)</td>
<td>C_i</td>
<td>310</td>
<td>1010</td>
<td>160</td>
</tr>
<tr>
<td>Direct material: (USD/ton)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k = 1</td>
<td>21</td>
<td>E_{ij}</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>k = 2</td>
<td>11</td>
<td>E_{ij}</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>k = 3</td>
<td>6</td>
<td>E_{ij}</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>k = 4</td>
<td>38</td>
<td>E_{ij}</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>k = 5</td>
<td>22</td>
<td>E_{ij}</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Machine hour constraint:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kneading</td>
<td>Machine hours</td>
<td>1</td>
<td>b_{ij}</td>
<td>5</td>
</tr>
<tr>
<td>Pressing out</td>
<td>Machine hours</td>
<td>2</td>
<td>b_{ij}</td>
<td>2</td>
</tr>
<tr>
<td>Cutting off</td>
<td>Machine hours</td>
<td>3</td>
<td>b_{ij}</td>
<td>2</td>
</tr>
<tr>
<td>Forming</td>
<td>Machine hours</td>
<td>4</td>
<td>b_{ij}</td>
<td>3</td>
</tr>
<tr>
<td>Vulcanizing</td>
<td>Machine hours</td>
<td>5</td>
<td>b_{ij}</td>
<td>2</td>
</tr>
<tr>
<td>Inspecting</td>
<td>Machine hours</td>
<td>6</td>
<td>b_{ij}</td>
<td>3</td>
</tr>
<tr>
<td>Material handling constraint:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine hours (h)</td>
<td>G_i = 50</td>
<td>1</td>
<td>b_{ij}</td>
<td>2</td>
</tr>
<tr>
<td>Cost (USD)</td>
<td>G_i = 150</td>
<td>a_{ij}</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Direct labor constraint:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>T_{lj} = 7040</td>
<td>T_{lj} = 11,000</td>
<td>T_{lj} = 2640</td>
<td>d_i</td>
</tr>
<tr>
<td>Labor hours (h)</td>
<td>L_{ij} = 1760</td>
<td>L_{ij} = 2200</td>
<td>L_{ij} = 2640</td>
<td>r_i = 9</td>
</tr>
<tr>
<td>Wage rate (USD/h)</td>
<td>t_0 = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon emission constraint:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost (USD/ton)</td>
<td>C_{ij} = 20,400</td>
<td>C_{ij} = 46,800</td>
<td>UCE = 4000</td>
<td>l_i</td>
</tr>
<tr>
<td>Emission quantities</td>
<td>C_{ij} = 2040</td>
<td>C_{ij} = 2340</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax rate (USD/ton)</td>
<td>TR_1 = 10</td>
<td>TR_2 = 20</td>
<td>TR_3 = 30</td>
<td></td>
</tr>
<tr>
<td>Total fixed cost: (USD)</td>
<td>22,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Data for illustrative example (Company A).

Since the traditional cost system analyzes the cost, most of it is based on direct raw materials, direct labor, and indirect manufacturing costs. The ABC cost system allocates resources to the activity or activity center according to the resource driver, and the activity cost is distributed to the product according to the activity drivers. Through the implementation of the analysis method, the cost of each activity can be correctly calculated. In addition, the TOC theory proposes some standardization methods for defining and eliminating constraints in manufacturing activities. The enterprise is a system, the goal is to obtain more profits, and all the factors that hinder the achievement of the overall goal are constraints, in order to measure the realization. The performance and effectiveness of the target, TOC breaks the traditional concept of accounting costs. In summary, this paper inspires the
innovative thinking, applies ABC cost system, TOC theory combined with mathematical planning to establish the model, the objective function and its constraints are described in the sections that follow.

3.2.1. Objective Function

This study establishes the mathematical planning model of the tire industry through the ABC cost system and the TOC theory. Since it is difficult to obtain real cost and expense data, it can be assumed that the factors hindering the target are constraints. Therefore, the following is a discussion of the objective function of the ABC green production model. The objective of the model is to determine the optimal product mix to achieve maximal profit under the constraints of various resources and carbon emissions. The following is the objective of the model:

$$\text{Maximize } Z = (\text{Total revenue}) - (\text{Direct material cost}) - (\text{Direct labor cost})$$

$$(\text{Material handing cost}) - (\text{carbon emission cost}) (\text{Other fixed cost})$$

$$= \sum_{i=1}^{n} C_i B_i - \sum_{k=1}^{s} \left\{ D_k \sum_{i=1}^{n} (E_{ik} B_i) \right\} - (\text{TR}_0 + e_1(\text{TLC}_1 - \text{TR}_0) + e_2(\text{TLC}_2 - \text{TR}_0))$$

$$(1)$$

where $B_i$ is the production quantity of product $i$; $C_i$ is the unit sales price of product $i$; $D_k$ is the unit cost of material $k$; $E_{ik}$ is the requirements of material $k$ for producing a unit of product $i$; $\text{TLC}_0$ is total direct labor costs at $LS_0$; $\text{TLC}_1$ is total direct labor costs at $LS_1$; $\text{TLC}_2$ is total direct labor costs at $LS_2$; $(e_0, e_1, e_2)$ is a set of non-negative variables of SOS2 (special ordered set of type 2), where at most two adjacent variables may be non-zero in the order of a given set [57,58]; $G_i$ is the other material handling cost, with the exception of machine depreciation per batch of material handling for product $i$; $\beta_i$ is the number of batches for material handling of product $i$; $CT_i$ is the total carbon emission cost in $CE_i$; $F$ is the fixed cost.

The profit function of $Z$ in the model is shown in Equation (1). Total revenue is $\sum_{i=1}^{n} C_i B_i$ of Equation (1) in the first term. Direct material cost is $\sum_{k=1}^{s} \left\{ D_k \sum_{i=1}^{n} (E_{ik} B_i) \right\}$ of Equation (1) in the second term, direct labor cost is $(\text{TLC}_0 + e_1(\text{TLC}_1 - \text{TR}_0) + e_2(\text{TLC}_2 - \text{TR}_0))$ of Equation (1) in the third term, material handing cost is $(G_1 \beta_1 + G_2 \beta_2 + G_3 \beta_3)$ of Equation (1) in the fourth term, carbon emission cost is $(\text{TR}_1 \text{W}_1 + \text{TR}_2 \text{W}_2 + \text{TR}_3 \text{W}_3)$ of Equation (1) in the fifth term, and the other fixed cost is $F$ of Equation (1) in the sixth term. Some other costs are fixed; for example, the depreciations of property, plants, and equipment are fixed, thus, they will have the associated constraints of capacity. While fire insurance charges and fundamental utility fees are also fixed, they will not have the associated constraints of capacity. These fixed costs in total can be expressed as a constant $F$ (USD $22,000 in this example data).

3.2.2. The Associated Constraints

(1) Direct material quantity constraints:

$$\sum_{i=1}^{n} (E_{ik} B_i) \leq F_k \ \ k = 1, 2, \ldots, s$$

(2) Direct labor hour constraints:

$$\text{TDL} = d_1 B_1 + d_2 B_2 + d_3 B_3 \leq LS_0 + e_1(\text{LS}_1 - \text{LS}_0) + e_2(\text{LS}_2 - \text{LS}_0)$$

$$e_0 - m_1 \leq 0$$

$$e_1 - m_1 - m_2 \leq 0$$
where \( d_i \) is the requirement of direct labor hours for one unit of product \( i \); \( B_i \) is the production quantity of product \( i \); \( LS_0 \) is the normal direct labor hours available; \( LS_1 \) is the maximal working hours at the first overtime rate plus the normal direct labor hours available; \( LS_2 \) is the maximal working hours at the first and second overtime rate plus the normal direct labor hours available. Equations (3)–(11) are the constraints associated with direct labor. TDL in Equation (3) is the total direct labor hours. The direct labor cost is \( (TLC_0 + e_1(TLC_1 - TLC_0) + e_2(TLC_2 - TLC_0)) \) of Equation (1) in the third term, and the direct labor cost function is shown in Figure 2. \( (e_0, e_1, e_2) \) is a set of non-negative variables of \( SOS2 \) (special ordered set of type 2), where at most two adjacent variables may be non-zero in the order of a given set \([39,40]\); \( (m_1, m_2) \) is a set of 0–1 variables of \( SOS1 \) (special ordered set of type 1), where only one must have one variable that will be non-zero \([39,40]\). If \( m_1 = 1 \), then \( m_2 = 0 \) from Equation (8), \( e_2 = 0 \) from Equation (6), \( e_0, e_1 \leq 1 \) from Equations (4) and (5), and \( e_0 + e_1 = 1 \) from Equation (7). Therefore, total direct labor hours and total labor costs are \( LS_0 + e_1(LS_1 - LS_0) \) and \( TLC_0 + e_1(TLC_1 - TLC_0) \), respectively, which means that the company works overtime at the first overtime rate. On the other hand, if \( m_2 = 1 \), then \( m_1 = 0 \) from Equation (8), \( e_0 = 0 \) from Equation (4), \( e_1, e_2 \leq 1 \) from Equations (5) and (6), and \( e_1 + e_2 = 1 \) from Equation (7). Thus, the total direct labor hours required is \( LS_0 + e_1(LS_1 - LS_0) + e_2(LS_2 - LS_0) \) from Equations (3) and the total labor cost is \( TLC_0 + e_1(TLC_1 - TLC_0) + e_2(TLC_2 - TLC_0) \), respectively, which means there will be overtime work at the second overtime rate. When \( m_1 = 1 \) and \( e_0 = 1 \), then \( m_2 = e_1 = e_2 = 0 \) from Equations (4)–(9), total direct labor hours \( = d_1B_1 + d_2B_2 + d_3B_3 \leq LS_0 \), and the total labor cost is \( TLC_0 \).

![Figure 2. Direct labor cost.](image)

(12) Machine hour constraints:

\[
\sum_{i=1}^{n} b_j B_i \leq H_j, \quad j = 1, 2, 3, 4, 5, 6
\]

where \( b_j \) is the number of machine hours required to produce one unit of product \( i \) in activity \( j \); \( B_i \) is the production quantity of product \( i \); and \( H_j \) is the number of machine hours \( i \) available for activity \( j \). Each process requires a different machine for related activities and Equation (12) is the machine hour constraints.
(4) Material handing constraints:

\[(B_i - a_{ij} \beta_i) \leq 0, \ i = 1, 2, \ldots, n, \ j = 7\]  
\[\sum_{i=1}^{n} b_{ij} \beta_i \leq H_j, \ j = 7\]  

where \(B_i\) is the production quantity of product \(i\); \(\beta_i\) is the number of batches for product \(i\); \(a_{ij}\) is the quantity of product \(i\) in a batch of material handling activity; \(b_{ij}\) is the requirement of machine hours for product \(i\) in a batch of material handling activity; \(H_j\) is the machine hours available for material handling. The material handling activity is a batch-level activity. Equation (13) is the constraints regarding the relationship between the quantity of product \(i\) (\(B_i\)) and the number of batches for product \(i\); Equation (14) is the resource constraint for the machine hours of material handling activity. In the tire manufacturing process, the relevant materials must be moved from the warehouse to the first process, and then, move the finished tires from the last process to the warehouse. Total material handling cost, with the exception of the depreciation of the material handling machine (it is included in \(F\)) is, as follows:

Total material handling cost = \(G_1 \beta_1 + G_2 \beta_2 + G_3 \beta_3\)

This is the fourth term of Equation (1), where \(G_i\) is the other material handling cost, with the exception of machine depreciation per batch of material handling for product \(i\), and \(\beta_i\) is the number of batches for material handling of product \(i\).

(5) Carbon emissions constraints:

\[\text{GCE} = I_1 B_1 + I_2 B_2 + I_3 B_3 = W_1 + W_2 + W_3\]  
\[W_1 + W_2 + W_3 \leq \text{UCE}\]  
\[0 \leq W_1\]  
\[W_1 \leq CE_1 * n_1\]  
\[CE_1 * n_2 < W_2\]  
\[W_2 \leq CE_2 * n_2\]  
\[W_3 > CE_2 * n_3\]  
\[n_1 + n_2 + n_3 = 1\]

where \(\text{GCE}\) is the total carbon emission quantity in the tire company; \(I_i\) is the carbon emission quantity for one unit of product \(i\); \(B_i\) is the production quantity of product \(i\); \(W_i\) represents the total carbon emission quantity in the tire company if it falls within the \(i\)th range of the total carbon emissions in Figure 3; \(\text{UCE}\) is the upper limit of the carbon emissions, as distributed to the tire company by the government; \(n_i\) is the indicator variable and \(n_i = 1\) means that the total carbon emission quantity of the tire company falls within the \(i\)th range of the total carbon emissions in Figure 3; \(CE_1\) and \(CE_2\) are the highest carbon emission quantities for the first and second carbon emission ranges of the carbon emission cost function in Figure 3, respectively.

Levying taxes could increase GDP. Energy taxes will affect the costs of energy-consumption industries due to rising energy costs [59]. On the other hand, energy taxes can help to reduce the concentration of air pollutants, thereby, improving air quality. In addition, calculating capital and operating expenses is based on a simulation of carbon emissions and economic performance under efficiency taxes, as based on the obtained quality and energy balances; the research results of levying carbon taxes show that all of the assessed environmental impact indicators are performing well [60], the new green manufacturing technology (GMT) can reduce carbon emissions [61], which uses a Life Cycle Assessment (LCA) method to measure emissions costs. LCA is a method for assessing
the environment over the entire life cycle of a product or service. Based on previous research, we consider all the environmental impacts of carbon emissions in the tire manufacturing process, and reduce carbon footprint through the development of innovative materials and manufacturing sustainability. Other studies have reduced the carbon footprint resistance of trucking by developing innovative low-rolling, reducing truck tires by at least 20% rolling resistance, roughly equivalent to a 5% reduction in fuel consumption and carbon dioxide emissions. Truck tire wear (10% improvement) and safety performance are further improved [43]. According to the method of Ward and Chapman [62], we quantify carbon emissions in Equation (15). The tax rates for carbon emissions differ, and the total emission cost function is a piecewise discontinuity function (as shown in Figure 3). As carbon emissions increase, taxes will increase [63]. That is: TR_1W_1, TR_2W_2, and TR_3W_3 represent the carbon emission cost when total carbon emission falls within the first, the second, and the third range of Figure 3. Total carbon emission costs are, as follows:

\[
\text{The total cost of carbon emissions} = TR_1W_1 + TR_2W_2 + TR_3W_3
\]

This is the fifth term of Equation (1), i.e., total carbon emissions cost. The related constraints are Equations (15)–(22). In Equations (17)–(22), (n_1, n_2, n_3) is a SOS2 set of 0–1 variables, and only one and just one will be 1. If n_1 = 1, then n_2, n_3 = 0 from Equation (22), 0 ≤ W_1 ≤ CE_1 from Equations (17) and (18), W_2 = W_3 = 0 from Equations (19)–(21); that is, the total carbon emission quantity falls within the first range [0, CE_1] of Figure 3, and the total carbon emission cost is TR_1W_1. Similarly, if n_2 = 1, then the total carbon emission quantity falls within the first range (CE_1, CE_2] of Figure 3, and the total carbon emission cost is TR_2W_2; if n_3 = 1, then the total carbon emission quantity falls within the third range (CE_2, ∞] of Figure 3, and the total carbon emission cost is TR_3W_3. The cost function of Figure 3 is a carbon emission cost function with full progressive tax rates. In summary, the more the tire company generates carbon emissions, the more it should pay for carbon emission taxes.

![Figure 3. Carbon emission costs (taxes).](image)

3.3. Summary

In summary, the above discussion includes the manufacturing process of the tire industry, the application of ABC cost system and TOC theory in mathematical programming, revenue, various costs, and carbon emissions in the objective function and the associated constraints. In the following sections, scenario analysis will be conducted using the assumed three carbon emission cost models. The results of the analysis will be provided for management decision-making.
4. Illustration

The tire company burns exhaust during the production process, and the emission of carbon seriously affects air pollution. This paper proposes a hypothetical discussion of a green decision model for tire Company A. As an illustration, company A in the tire industry has been established in Taiwan for more than 50 years. There are 11 tire manufacturing companies in the world, with more than 24,000 employees worldwide and paid-in capital exceeding 30 billion yuan. The main products are all kinds of tires and rubber products. Company A is used to illustrate how to apply the green decision-making model proposed in this paper. We assume that Company A’s costs for producing their products include direct material cost, direct labor cost, kneading activity cost, pressing out activity cost, cutting off activity cost, forming activity cost, vulcanizing activity cost, inspecting activity cost, material handing activity cost, and carbon emission cost (tax).

4.1. Model 1: Green Decision Model without Carbon Rights Trading, but Has a Carbon Cap Allocated for Use by the Government

When using Equations (1)–(22), let Company A consider the production of three main products: Passenger Car Radial (PCR) product (i = 1), Truck & Bus Radial (TBR) product (i = 2), and Motorcycle (MC) product (i = 3), and that these products consume the same direct materials. Then, each product’s quantity is shown in thousands, and the costs and input costs are in U.S. dollars. The data for the case are shown in Table 1. Total fixed cost is a constant (USD $22,000 in this example). This study uses the illustrative example data in Table 1 for the three green decision models explored in this paper, including a model without carbon rights trading and two models with two kinds of carbon rights trading costs, in order to analyze the carbon emission costs in the tire industry for managerial decisions.

The green product mix decision model without carbon rights trading for the case data in Table 1 is shown in Table 2. We solved this 0-1 mixed integer programming (0–1 MIP) model by using LINGO 16.0 software, and the optimal solution is shown at the bottom area of Table 2.

The total profit Z is USD $53,225,000. It is assumed that the government allocates a carbon emission quantity to Company A, which has been converted to the carbon emission quantity UCE = 4,000,000 metric tons, and the carbon emission cost (tax) is shown in Figure 3. The carbon tax cost function \( f_z \) are, as follows:

\[
\begin{align*}
   f_z (GCE) &= \begin{cases} 
   TR_1 \cdot GCE & \text{if } 0 \leq GCE \leq CE_1 \\
   TR_2 \cdot GCE & \text{if } CE_1 < E \leq CE_2 \\
   TR_3 \cdot GCE & \text{if } GCE > CE_2 
   \end{cases}
\end{align*}
\]

The optimal solution in Table 2 indicates that the total carbon emission quantity is 335,000 metric tons and the carbon emission cost is USD $3,350,000, which falls in the first range of carbon emission quantity in Figure 3. If the remaining 3,665,000 metric tons of carbon emission quantity can be sold to other companies that need to purchase additional carbon credits, according to the EU (2008–2012) under the emissions trading scheme, and is sold at USD $30 per metric ton of carbon dioxide, the tire industry could add an additional profit of USD $109,950,000 to increase its existing maximum gross profit to USD $163,175,000 (=USD $53,225,000 + USD $109,950,000).

According to the results of this scenario analysis as shown in Table 2, if the tire industry controls the carbon emission cost according to this mathematical programming model and sells the remaining carbon emissions allocated from the government to the companies that need to purchase carbon emissions. In addition to the original income of the products, the company can also get a considerable income to sell the remaining carbon emissions. It can be seen that carbon emissions are indeed a very important indicator variable affecting the tire company’s profit, and the tire industry should carefully control carbon emissions to avoid cost explosion on the premise of maximizing profits.
The model and its optimal solutions with the illustrative data for Model 1.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max ( Z = (310B_1 + 1010B_2 + 160B_3) - (21<em>4 + 11</em>2 + 6<em>5 + 38</em>2 + 22*2)B_1 )</td>
<td>Subject to direct material:</td>
</tr>
<tr>
<td>( - (21<em>6 + 11</em>10 + 6<em>15 + 38</em>6 + 22<em>15)B_2 - (21</em>2 + 11<em>1 + 6</em>2 + 38<em>1 + 22</em>1)B_3 )</td>
<td>( 4B_1 + 6B_2 + 2B_3 \leq 10,600; )</td>
</tr>
<tr>
<td>( - 7040 - (11,000 - 7040)e_1 - (15,840 - 7040)e_2 - 50B_1 - 150B_2 - 10B_3 )</td>
<td>( 2B_1 + 10B_2 + 1B_3 \leq 8100; )</td>
</tr>
<tr>
<td>( - 10W_1 - 20W_2 - 30W_3 - F )</td>
<td>( 5B_1 + 10B_2 + 1B_3 \leq 13,300; )</td>
</tr>
</tbody>
</table>

\( 3B_1 + 5B_2 + 1B_3 \leq 22,100; \)

\( 2B_1 + 3B_2 + 1B_3 \leq 22,100; \)

\( 3B_1 + 6B_2 + 1B_3 \leq 26,500; \)

\( 2B_1 + 3B_2 + 1B_3 \leq 13,300; \)

\( 3B_1 + 6B_2 + 1B_3 \leq 6700; \)

\( 1B_1 + 1.5B_2 + 0.5B_3 - 1760-440e_1 - 880e_2 \leq 0; \)

\( e_0 - m_1 \leq 0; \)

\( e_1 - m_1 - m_2 \leq 0; \)

\( e_2 - m_2 \leq 0; \)

\( e_0 + e_1 + e_2 = 1; \)

\( m_1 + m_2 = 1; \)

\( 0 \leq e_0 \leq 1; 0 \leq e_1 \leq 1; 0 \leq e_2 \leq 1; m_1, m_2 = 0,1; \)

Subject to carbon emission:

\( GCE = 0.2B_1 + 0.1B_2 + 0.1B_3, \)

\( GCE = W_1 + W_2 + W_3; \)

\( W_1 + W_2 + W_3 \leq 4000; \)

\( 0 \leq W_1; W_1 \leq 2040n_1; \)

\( 2040n_2 < W_2; W_2 \leq 2340n_2; \)

\( W_3 > 2340n_3; \)

\( n_1 + n_2 + n_3 = 1. \)

The optimal solutions for the example data

\( B_1 = 895,000 \quad B_2 = 85,000 \quad B_3 = 1,475,000 \quad F = 22,000,000 \)

\( e_0 = 1 \quad e_1 = 0 \quad e_2 = 0 \quad Z = 53,225,000 \)

\( \beta_1 = 181,000 \quad \beta_2 = 8000 \quad \beta_3 = 1,475,000 \quad GCE = 335,000 \)

\( n_1 = 1 \quad n_2 = 0 \quad n_3 = 0 \)

\( W_1 = 335,000 \quad W_2 = 0 \quad W_3 = 0 \)

\( m_1 = 1 \quad m_2 = 0 \)

4.2. Model 2: Green Decision Model with Constant Carbon Right Cost

Model 2 assumes that the company can purchase carbon rights with a constant unit cost (R). The carbon right cost function is equal to the \( f_4(GCE - UCE) \), as shown in Figure 4, if the company has the need to buy carbon right for additional sales.

![Carbon Right Cost](Figure 4. Carbon right cost.)
The company’s carbon emission cost function is equal to the carbon tax cost plus the carbon right purchase cost. The carbon emission cost function $f_2(GCE)$ is as follows:

$$f_2(GCE) = \begin{cases} f_z(GCE) & \text{if } 0 \leq GCE \leq UCE \\ f_z(GCE) + f_4(GCE - UCE) & \text{if } GCE > E \end{cases}$$

where $f_z(GCE)$ is the carbon tax cost function and $f_4(GCE - UCE)$ is carbon right cost function. When the company’s carbon emission quantity does not exceed the carbon emission quota (UCE), the carbon emission cost only includes the carbon tax cost $f_z(GCE)$, otherwise, the company needs to buy carbon right, and the company’s carbon emission cost is $f_z(GCE) + f_4(GCE - UCE)$.

The objective for Model 2 is shown as follows:

Maximize $Z = (\text{Total revenue}) - (\text{Direct material cost}) - (\text{Direct labor cost}) - (\text{Material handling cost}) - (\text{carbon emission cost}) - (\text{Other fixed cost})$

$$= \sum_{i=1}^{n} C_i B_i - \sum_{k=1}^{3} \left\{ D_k \sum_{i=1}^{n} (E_{ik} B_i) \right\} - (TLC_0 + e_1 (TLC_1 - TLC_0) + e_2 (TLC_2 - TLC_0))$$

$$- (G_1 \beta_1 + G_2 \beta_2 + G_3 \beta_3) - \{(TR_1 W_1 + TR_2 W_2 + TR_3 W_3) + \theta_1$$

$$+ (TR_1 W_1 + TR_2 W_2 + TR_3 W_3) + R \ast (GCE - UCE)) \ast \theta_2) - F$$

The additional associated constraints are, as follows:

$$GCE = l_1 B_1 + l_2 B_2 + l_3 B_3 = CQ_1 + CQ_2$$

$$0 \leq CQ_1$$

$$CQ_1 \leq UCE \ast \theta_1$$

$$UCE \ast \theta_2 < CQ_2$$

$$CQ_2 \leq (UCE + UCPQ) \ast \theta_2$$

$$\theta_1 + \theta_2 = 1, \theta_1, \theta_2 = 0, 1$$

where $R$ is the unit cost of carbon right; $UCPQ$ is the maximum quantity of carbon right that the company can purchase. $\theta_1$ and $\theta_2$ are indicator variables of 0–1 variables. If $\theta_1 = 1$, then $\theta_2 = 0$ from Equation (29), $CQ_2 = 0$ from Equations (27) and (28), and $0 \leq GCE = CQ_1 \leq UCE$ from Equations (25) and (26). That is, the company need not purchase the carbon rights, and the total carbon emission cost is $(TR_1 W_1 + TR_2 W_2 + TR_3 W_3)$ from Equation (23). On the other hand, if $\theta_2 = 1$, then $\theta_1 = 0$ from Equation (29), $CQ_1 = 0$ from Equations (25) and (26), and $UCE \leq GCE = CQ_2 \leq (UCE + UCPQ)$ from Equations (27) and (28). That is, the company needs to purchase carbon rights at the quantity, $(GCE - UCE)$, and the total carbon emission cost is $[(TR_1 W_1 + TR_2 W_2 + TR_3 W_3) + R \ast (GCE - UCE)]$. Other variables and parameters are as previously defined.

Equations (2)–(29) form Model 2, as explored in this paper. In Model 2, it is assumed that the carbon rights can be purchased regarding the required carbon emissions, and according to the EU (2008–2012) under the emissions trading scheme, the constant cost per unit tonne of carbon dioxide is USD $30 (R). Assume also that the maximal quantity of carbon rights that the company can purchase ($UCPQ$) is 160 tonnes of carbon dioxide. Model 2 with $R = USD 30$, $UCPQ = 160$, and the illustrative data in Table 1, is as shown in Table 3. We solve this problem and the optimal solution is shown at the bottom area of Table 3. The optimal solution indicates that the required carbon footprint of Company A exceeds the maximum amount of carbon credits, as allocated by the government.
Table 3. The model and its optimal solutions with the illustrative data for Model 2.

<table>
<thead>
<tr>
<th>Subject to:</th>
<th>Subject to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $Z = (310^*B_1 + 1010^*B_2 + 160^*B_3) - (21^*4 + 11^*2 + 6^*5 + 38^*2 + 22^*2)^*B_1$</td>
<td>Subject to: direct material:</td>
</tr>
<tr>
<td>$- (21^*6 + 11^*10 + 6^*15 + 38^*6 + 22^*15)^*B_2 - (21^*2 + 11^*1 + 6^*2 + 38^*1 + 22^*1)^*B_3$</td>
<td>$4^*B_1 + 6^*B_2 + 2^*B_3 \leq 10,600;$</td>
</tr>
<tr>
<td>$- 7040 - (11,000 - 7040)^<em>e_1 - (15,840 - 7040)^<em>e_2 - 50^</em>\beta_1 - 150^</em>\beta_2 - 10^*\beta_3$</td>
<td>$2^*B_1 + 10^*B_2 + 1^*B_3 \leq 8100;$</td>
</tr>
<tr>
<td>$- [(10^*W_1 - 20^*W_2 - 30^*W_3)^*B_1 + [(10^*W_1 - 20^<em>W_2 - 30^<em>W_3) + 30^</em>(GCE - 4000)]^</em>\theta_1 - F$</td>
<td>$5^*B_1 + 10^*B_2 + 1^*B_3 \leq 13,300;$</td>
</tr>
</tbody>
</table>

Subject to direct labor:

- $e_0 - m_1 \leq 0;
- e_1 - m_1 - m_2 \leq 0;
- e_2 - m_2 \leq 0;
- e_3 + e_1 + e_2 = 1; m_1 + m_2 = 1;
- 0 \leq e_0 \leq 1; 0 \leq e_1 \leq 1; 0 \leq e_2 \leq 1; m_1, m_2 = 0,1$;

Subject to carbon emission:

- $GCE = 0.2 + B_1 + 0.1 * B_2 + 0.1 * B_3$;
- $GCE = W_1 + W_2 + W_3$;
- $W_1 + W_2 + W_3 \leq 4000$;

Subject to carbon emission right purchasing:

- $GCE = CQ_1 + CQ_2$;
- $0 \leq CQ_1$;
- $CQ_1 \leq 4000^*\theta_1$;

The optimal solutions for the example data:

<table>
<thead>
<tr>
<th>$B_1 = 905,000$</th>
<th>$B_2 = 80,000$</th>
<th>$B_3 = 1,470,000$</th>
<th>$F = 22,000,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0 = 1$</td>
<td>$e_1 = 0$</td>
<td>$e_2 = 0$</td>
<td>$Z = 53,050,000$</td>
</tr>
<tr>
<td>$\beta_1 = 181,000$</td>
<td>$\beta_2 = 8000$</td>
<td>$\beta_3 = 1,470,000$</td>
<td></td>
</tr>
<tr>
<td>$n_1 = 1$</td>
<td>$n_2 = 0$</td>
<td>$n_3 = 0$</td>
<td>$W_1 = 336,000$</td>
</tr>
<tr>
<td>$W_2 = 336,000$</td>
<td>$W_3 = 0$</td>
<td>$W_3 = 0$</td>
<td></td>
</tr>
<tr>
<td>$CQ_1 = 336,000$</td>
<td>$CQ_2 = 0$</td>
<td>$GCE = 336,000$</td>
<td></td>
</tr>
<tr>
<td>$m_1 = 1$</td>
<td>$m_2 = 0$</td>
<td>$m_2 = 0$</td>
<td>$\theta_1 = 1$</td>
</tr>
<tr>
<td>$\theta_2 = 0$</td>
<td></td>
<td></td>
<td>$\theta_2 = 0$</td>
</tr>
</tbody>
</table>

The results of this scenario analysis are compared with scenario 1 and it is found that scenario 2 assumes that carbon rights are purchased, the maximum profit will be reduced by USD $175,000 (from USD $53,225,000 to USD $53,050,000) and carbon emissions will increase by 1000 metric tons. (by 335,000 metric tons increased to 336,000 metric tons). The results of this scenario analysis also prove that carbon emissions are indeed variables that affect the total profitability. It can also provide the tire industry in the future when purchasing carbon rights, we should carefully consider the company’s overall revenue, and avoid the purchase and use of carbon rights, so that the cost can be optimally controlled, so that the company will continue to operate.

4.3. Model 3: Green Decision Model with a Minimum Purchase Quantity of Carbon Right

Model 3 assumes that the company should purchase the carbon right with a minimum purchase quantity (LCPQ) that has a constant charge (CF) if the company has the need for additional sales, as shown in Figure 5 and the carbon emission cost function $f_3$ (GCE) is as follows:

$$f_3 (GCE) = \begin{cases} f_3 (GCE) \text{ if } \theta \leq \text{UCE} \\ f_3 (GCE) + \text{CF if } \text{UCE} < \text{GCE} \leq (\text{UCE} + \text{LCPQ}) \\ f_3 (GCE) + \text{CF} + f_3 (\text{GCE} - \text{UCE} - \text{LCPQ}) \times R \text{ if } (\text{UCE} + \text{LCPQ}) < \text{GCE} \leq (\text{UCE} + \text{LCPQ}) \end{cases}$$
where, there are three scenarios for the carbon emission cost function: (1) the company’s carbon emission quantity does not exceed the carbon emission quota (UCE), (2) the company needs to buy the carbon right whose quantity does not exceed the LCPQ, (3) the company needs to buy the carbon right whose quantity between LCPQ and UCPQ.

![Graph showing carbon right cost](image)

**Figure 5.** Carbon right cost.

The objective for Model 3 is shown as follows:

\[
\text{Maximize} \ Z = (\text{Total revenue}) - (\text{Direct material cost}) - (\text{Direct labor cost}) - (\text{Material handling cost}) - (\text{carbon emission cost}) (\text{Other fixed cost})
\]

\[
= \sum_{i=1}^{n} c_i b_i - \sum_{k=1}^{\delta} \left\{ d_k \sum_{i=1}^{n} (e_k b_i) \right\} - (\text{TLC}_0 + e_1 (\text{TLC}_1 - \text{TLC}_0) + e_2 (\text{TLC}_2 - \text{TLC}_0))
\]

\[
- (\beta_1 + \beta_2 + \gamma_1 (\text{TR}_1 W_1 + \text{TR}_2 W_2 + \text{TR}_3 W_3) + \theta_1)
\]

\[
+ [\text{TR}_1 W_1 + \text{TR}_2 W_2 + \text{TR}_3 W_3 + \text{CF}] + \theta_2
\]

\[
+ [\text{TR}_1 W_1 + \text{TR}_2 W_2 + \text{TR}_3 W_3 + \text{CF} + R (\text{GCE} - \text{UCE} - \text{LCPQ}) + \theta_3] - F
\]

\[ (30) \]

The additional associated constraints are, as follows:

\[
\text{GCE} = l_1 b_1 + l_2 b_2 + l_3 b_3 = \text{CQ}_1 + \text{CQ}_2 + \text{CQ}_3
\]

\[ (31) \]

\[
0 \leq \text{CQ}_1
\]

\[ (32) \]

\[
\text{CQ}_1 \leq \text{UCE} \times \theta_1
\]

\[ (33) \]

\[
\text{UCE} + \theta_2 < \text{CQ}_2
\]

\[ (34) \]

\[
\text{CQ}_2 \leq (\text{UCE} + \text{LCPQ}) \times \theta_2
\]

\[ (35) \]

\[
(\text{UCE} + \text{LCPQ}) \times \theta_3 < \text{CQ}_3
\]

\[ (36) \]

\[
\text{CQ}_3 \leq (\text{UCE} + \text{UCPQ}) \times \theta_3
\]

\[ (37) \]

\[
\theta_1 + \theta_2 + \theta_3 = 1, \ \theta_1, \ \theta_2, \ \theta_3 = 0, 1
\]

\[ (38) \]

where R is the unit cost of carbon right; LCPQ is the minimum purchase quantity; and UCPQ is the maximum quantity of carbon right that the company can purchase. \( \theta_1, \ \theta_2, \ \text{and} \ \theta_3 \) are indicator variables of 0–1 variables. If \( \theta_1 = 1 \), then \( \theta_2 = \theta_3 = 0 \) from Equation (38), \( \text{CQ}_2 = \text{CQ}_3 = 0 \) from Equations (34)–(37), and \( 0 \leq \text{GCE} = \text{CQ}_1 \leq \text{UCE} \) from Equations (32)–(33). That is, the company need not purchase carbon rights, and the total carbon emission cost is \( (\text{TR}_1 W_1 + \text{TR}_2 W_2 + \text{TR}_3 W_3) \) from Equation (30). If \( \theta_2 = 1 \), then \( \theta_1 = \theta_3 = 0 \) from Equation (38), \( \text{CQ}_1 = \text{CQ}_3 = 0 \) from Equations (32)–(33) and (36)–(37), and \( \text{UCE} \leq \text{GCE} = \text{CQ}_2 \leq (\text{UCE} + \text{LCPQ}) \) from Equations (34)–(35). That is, the company needs to purchase carbon rights at the minimum purchase quantity, LCPQ, and the total carbon emission cost is \( [\text{TR}_1 W_1 + \text{TR}_2 W_2 + \text{TR}_3 W_3 + \text{CF}] \). If \( \theta_3 = 1 \), then \( \theta_1 = \theta_2 = 0 \). from
Equation (38), \( CQ_1 = CQ_2 = 0 \) from Equations (32)–(35), and \((UCE + LCPQ) \leq GCE = CQ_3 \leq (UCE + UCPQ) \) from Equations (36)–(37). That is, the company needs to purchase carbon rights exceeding the minimum purchase quantity, LCPQ, and the total carbon emission cost is \([TR_1*W_1 + TR_2*W_2 + TR_3*W_3] + CF + R^*(GCE – UCE – LCPQ)\). Other variables and parameters are as previously defined.

Equations (21)–(22) and (30)–(38) form Model 3, as studied in this paper. As in Model 2, it is assumed that R = USD $30 and UCPQ = 160. Assume also that the minimum purchase of carbon rights is 60,000 metric tonnes, i.e., LCPQ = 60; then, CF = R^*UCPQ = $1800. Model 3, with these data and the illustrative data in Table 1, is shown in Table 4. We solve this problem and the optimal solution is shown at the bottom area of Table 4. The optimal solution indicates that the required carbon footprint of 4,060,000 tonnes for Company A is equal to \((UCE + LCPQ)\). It means that Company A needs to obtain just the minimum purchase quantity of 60,000 metric tonnes of carbon rights in addition to the permissible no need to buy carbon rights emission quantity (UCE), as allocated by the government. As a result, the optimal green portfolio decision and the maximum total profit obtained is USD $53,050,000.

### Table 4. The model and its optimal solutions with the illustrative data for Model 3.

| Max Z = (310*B_1 + 1010*B_2 + 160*B_3) – (21*4 + 11*2 + 6*5 + 38*2 + 22*2)*B_1 – (21*6 + 11*10 + 6*15 + 38*6 + 22*15)*B_2 – (21*2 + 11*1 + 6*2 + 38*1 + 22*1)*B_3 – 7040 – (11,000 – 7040)*e_1 – (15,840 – 7040)*e_2 – 50^*\beta_1 – 150^*\beta_2 – 10^*\beta_3 – [(10^*W_1 – 20^*W_2 – 30^*W_3)^*\theta_1 + (10^*W_1 – 20^*W_2 – 30^*W_3 + 1800)^*\theta_2 + [(10^*W_1 – 20^*W_2 – 30^*W_3 + 1800)^*\theta_1] – F | Subject to direct material; 4^*B_1 + 6^*B_2 + 2^*B_3 \leq 10,600; 2^*B_1 + 10^*B_2 + 1^*B_3 \leq 8100; 5^*B_1 + 15^*B_2 + 2^*B_3 \leq 8700; 2^*B_1 + 6^*B_2 + 1^*B_3 \leq 7100; 2^*B_1 + 15^*B_2 + 1^*B_3 \leq 7900; Subject to machine hours; 5^*B_1 + 10^*B_2 + 1^*B_3 \leq 13,300; 2^*B_1 + 4^*B_2 + 1^*B_3 \leq 22,100; 2^*B_1 + 3^*B_2 + 1^*B_3 \leq 22,100; 3^*B_1 + 5^*B_2 + 1^*B_3 \leq 26,500; 2^*B_1 + 3^*B_2 + 1^*B_3 \leq 13,300; 3^*B_1 + 6^*B_2 + 1^*B_3 \leq 6700; Subject to direct labor; 1^*B_1 + 1.5^*B_2 + 0.5^*B_3 – 1760 – 2200*e_1 – 2640*e_2 = 0; e_0 – m_1 \leq 0; e_1 – m_1 – m_2 \leq 0; e_2 + e_1 + e_2 = 1; m_1 + m_2 = 1; 0 \leq e_0 \leq 1, e_0 \leq 0, e_0 \leq 1, e_0 = 0, 0 = m_1, m_2 = 0; Subject to batch handling; B_1 – 5^*\beta_1 = 0; B_2 – 10^*\beta_2 = 0; B_3 – 1^*\beta_3 = 0; 2^*\theta_1 + 3^*\beta_2 + 1^*\beta_3 \leq 1860; Subject to carbon emission; GCE = 0.2B_1 + 0.1B_2 + 0.1B_3, GCE = W_1 + W_2 + W_3; W_1 + W_2 + W_3 \leq 4000; Subject to carbon emission right purchasing; GCE = CQ_1 + CQ_2 + CQ_3; 0 \leq CQ_1; CQ_1 \leq 4000^*\theta_1; \frac{4000}{2250}^*\theta_2 < CQ_2; CQ_2 \leq (4000 + 60)^*\theta_2. The optimal solutions for the example data B_1 = 905,000 B_2 = 80,000 B_3 = 1,470,000 e_0 = 1 e_1 = 0 e_2 = 0 \beta_1 = 181,000 \beta_2 = 8000 \beta_3 = 1,470,000 n_1 = 1 n_2 = 0 n_3 = 0 W_1 = 336,000 W_2 = 0 W_3 = 0 GCE = 336,000 CQ_1 = 336,000 CQ_2 = 0 CQ_3 = 0 \theta_1 = 1 \theta_2 = 0 \theta_3 = 0 m_1 = 1 m_2 = 0 m_3 = 0 F = 22,000,000 Z = 53,050,000
tax is levied, in the cost control, special attention should be paid to the quantity of carbon emissions for which there is no need to buy carbon rights to avoid cost explosions.

4.4. Sensitivity Analysis

One of important things for enterprises is the maximization of total profits, thus, this article focuses on high-carbon emission and high-polluting tire industries. With the rise of environmental awareness, corporate carbon emissions policies have been implemented in the branches of other countries. Insufficient carbon emissions due to the allocation of the country will require the additional acquisition of carbon rights, which will result in an additional burden on costly carbon emissions. To further understand the impact of the three key variables of the carbon emissions trading model on total profits in this study, three types of model carbon tax rates were increased or decreased by 5% to 30% for a total of 12 cases for sensitivity analysis (Tables 5-7). The test results are, as follows:

1. **Model 1**: The original total profit of the company is USD $53,225, and the carbon tax rate is increased by 5%, 10%, 15%, 20%, 25%, and 30%, respectively. The total profit found was reduced by 0.31% (USD $53,057,500), 0.62% (USD $52,890,000), 0.94% (USD $52,772,500), 1.25% (USD $52,555,000), 1.57% (USD $52,387,500), and 1.88% (USD $52,200,000), respectively; at the relative decreases of 5%, 10%, 15%, 20%, and 25%, then the total profit was found to increase by 0.31% (USD $53,392,500), 0.62% (USD $53,560,000), and 0.94% (USD $53,727,500), 1.25% (USD $53,895,000), 1.57% (USD $54,062,500), and 1.88% (USD $54,230,000), respectively. Therefore, as the carbon tax rate decreases, the total profit of the enterprise will increase. Conversely, increasing the carbon tax rate will lead to a decrease in total corporate profits.

2. **Model 2**: The original total profit of the company was USD $53,225, the carbon tax rate was increased by 5%, 10%, 15%, 20%, and 25%, respectively, and the total profit was found to decrease by 0.31% (USD $52,882), 0.63% (USD $52,714), 0.95% (USD $52,546), 1.26% (USD $52,378), 1.58% (USD $52,210), and 1.9% (USD $52,042), respectively; at the relative decrease of 5%, 10%, 15%, 20%, 25%, and 30%, it is found that the total profit increased by 0.31% (USD $53,218), 0.63% (USD $53,386), 0.95% (USD $53,554), 1.26% (USD $53,722), 1.58% (USD $53,890), and 1.9% (USD $54,058), respectively. Therefore, as the carbon tax rate decreases, the total profit of the company will increase. Conversely, increasing the carbon tax rate will lead to a decrease in total corporate profits.

3. **Model 3**: The original total profit of the company is USD $53,050, and the carbon tax rate is increased by 5%, 10%, 15%, 20%, 25%, and 30% respectively. The total profit found was reduced by 0.31% (USD $52,882), 0.63% (USD $52,714), 0.95% (USD $52,546), 1.26% (USD $52,378), 1.58% (USD $52,210), and 1.9% (USD $52,042), respectively. At the relative decrease of 5%, 10%, 15%, 20%, 25%, and 30%, it is found that the total profit increased by 0.31% (USD $53,218), 0.63% (USD $53,386), 0.95% (USD $53,554), 1.26% (USD $53,722), 1.58% (USD $53,890), and 1.9% (USD $54,058), respectively. Therefore, as the carbon tax rate decreases, the total profit of the company will increase. Conversely, increasing the carbon tax rate will lead to a decrease in total corporate profits.
In summary, three different scenario models with carbon taxes and carbon right are analyzed in this study by sensitivity analysis, and the results show that the increase or decrease of the carbon tax rate is one of the major factors affecting the total profit of the tire industry. However, under the current international focus on greenhouse gas emission reduction and environmental protection, how to reduce carbon emissions has become an important issue. The implementation of the national carbon emission system and carbon rights trading system is the most important issue for international environmental protection today. Therefore, the tire industry should understand the response to the implementation of the carbon tax, as well as the additional costs of carbon emission exchanges, in order to avoid affecting the company’s biggest profit decline.
5. Conclusions

The tire industry is a high-carbon and high-pollution industry. As global warming, climate change, and global emissions of greenhouse gases have become international environmental issues of concern, in recent years, some countries have encountered air pollution and environmental problems, and have begun to adopt the carbon price mechanism to implement carbon taxes. Therefore, the carbon tax and the carbon trading mechanism are the main strategies used by various governments around the world to respond to global warming and promote a green economy. Based on this concept, this paper applied a mathematical programming model and put forward three green optimal carbon price models for analysis. In Model 1, the results show that the tire company can sell the remaining 3,664,000 metric tonnes to other companies in need, and thus, earn more money. In addition, based on the assumptions of the two carbon rights purchase cost models of Model 2 and Model 3, analysis shows that the tire companies do not need to purchase additional carbon rights; however, compared with Model 1, if there is a carbon rights transaction, the total profit will decrease. Therefore, in order to reduce the impact of carbon tax and carbon trading on the profits of tire companies, tire companies could reduce carbon emissions as much as possible, and they can significantly reduce costs to create higher profits.

Based on the mathematical programming model, this article developed three different scenario models with carbon tax and carbon right for tire companies and conducted sensitivity analysis. The analysis results can provide references for tire industry management’s decision-making. In addition, according to the analysis results, we know that the operating costs will increase if a company wants to purchase additional carbon rights, which may lead to a substantial reduction in profits. Therefore, management should pay more attention to the quantity of carbon rights purchased. In addition, the mathematical programming model with carbon tax and carbon rights trading, as developed in this paper, can be extended to other relevant industries, such as construction, aviation, paper, steel, and other high pollution industries with high carbon emissions. In addition, this paper can be extended to the development of a green tax system through the design of a long-term and short-term mixed tax policy [64] to reflect the correct price of carbon tax and to reduce carbon emissions and air pollution. It is also possible to consider the policy of carbon emission and carbon pricing from the perspective of economics. Besides, this research can also be extended to consider the carbon emission permit allocation factors or the game-theoretic problem involving more than one company in a cap-and-trade system [65].

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References


47. Kee, R. The sufficiency of product and variable costs for production-related decisions when economies of scope are present. *Int. J. Prod. Econ.* 2008, 114, 682–696. [CrossRef]


