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Optimal Investment Timing and Scale Choice of Overseas Oil Projects: A Real Option Approach

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Abstract: This article presents a real option model for helping investors to determine the optimal investment timing and scale of overseas oil projects. The model is suitable for the highly uncertain environments in which many oil companies operate, where they have to suspend upstream investment, stop new oilfield construction, and wait for proper oil prices in order to avoid losses. Considering the uncertainty of oil prices and exchange rates, the results of analytical solutions presented in this paper show the critical oil price that can be seen as the investment threshold for triggering oilfield development as well as the optimal recoverable factor for every oil price level to indicate the optimal investment scale. Results of the case project show the critical oil price, which is 82.32 US dollar per barrel, and the selection of optimal investment scale. The article also demonstrates the impact of investment scale on investment timing in overseas oil projects. The policy implication from the case project is that investment decisions are finitely impacted by geological conditions. Besides, the existence of tax holiday directly contributes to a lower investment threshold. In addition, reducing unit operation cost can obviously enlarge optimal investment scale and initiate oil projects in a relatively low level of investment threshold.

Keywords: overseas oil project; real option; investment timing; continuous scaling

1. Introduction

From 2008 to 2017, international oil prices have declined and fluctuated. The upstream oil investment will obviously decrease if the low levels of oil price are sustained [1]. For example, mining and exploration investment, which reflects petroleum exploration and development, declined 35% in 2015, the second largest year-over-year decline since the U.S. Bureau of Economic Analysis (BEA) began reporting the series in 1948 [2]. In the era of low oil prices, many international oil companies, in an effort to avoid greater losses, have suspended upstream investment, stopped new oilfield construction, and waited for the recovery of oil prices.

Compared with domestic oil projects, overseas projects are an ideal way for oil companies as it helps them to continuously expand for survival and seek international profits [3]. As a result, oil companies are willing to seek investment opportunities for overseas projects. However, it should be noted that overseas oil projects are more complex because they contain risks of changes in policy and uncertainties of economic factors [4]. For international oil companies, the choices of optimal investment timing and scale are key aspects in overseas oil projects because of oil price fluctuation and heavy sunk cost. Investment decisions made at an inappropriate time can cause significant losses in the value of oil projects. Furthermore, the investment scale choice of oil projects is equally important under uncertainty. On the one hand, expanding investment scale can enable companies to increase their oil production and make it possible to obtain more benefits in the future. On the other hand, this may lead to large sunk costs and force companies to bear greater losses in the era of low oil prices. Based on the current situation, this article explores two issues from the perspective of overseas oil projects: (1) the critical oil price, which can be seen as the investment threshold for triggering oilfield development and (2) optimal recoverable factor for every oil price level, which indicates the optimal investment scale.

Under highly uncertain environment, oil projects with management flexibility have irreversible capital investment within a long project period. Discounted cash flow method (DCF), the major technology of project valuation, has disadvantages because it cannot capture the value of management flexibility. The real option approach, one of the most important option pricing methods in irreversible capital investment evaluation, can help investors to value oil projects correctly [5]. The real option approach introduces the concept of "option", and its purpose is to consider the value of management flexibility from the uncertainties in oil prices and exchange rates and explain "how firms should make decisions about investments that involve sunk cost" [6].

Therefore, under the current background of oil price and US dollar exchange rate, the purpose of this article is to build a real option model that can determine the optimal investment timing and scale of overseas oil projects. The real option model combines a few significant decision options (timing and scale of investment) and uncertainty factors (oil price and US dollar exchange rate). At the same time, the article demonstrates the impact of investment scale on investment timing in oil projects. In addition, the influence factors analysis of investment strategy can be attained based on this model.

The remaining article proceeds as follows: Section 2 makes a literature review. Section 3 describes the model of this article. Section 4 presents a case project and analyzes the results; it also focuses on demonstrating the impact of investment scale on investment timing. Finally, the conclusion and policy implications for overseas oil projects are provided in the last section.

2. Literature Review

In this article, the main objectives are to reasonably choose the optimal investment timing and scale of overseas oil projects. It is clear that overseas oil projects involve large-scale irreversible investments and are in complex economic environments that cause large fluctuations in oil prices. Although the traditional net present value method (NPV) is simple and easy to implement, it cannot reasonably handle uncertainties because of its irrational assumptions, which make it impossible to accurately evaluate oil projects [7]. In the past decades, real option approach developed by Black and Scholes [8], Merton [9], and Cox and Ross [10], which takes into account uncertainties and management flexibility, have been widely used in investment evaluation of oil projects [11–13].

In order to choose the optimal investment timing of energy projects, many scholars have adopted the real option approach, taking into account the management flexibility and various uncertainties [14–17]. Based on this approach, Zhang [18] and Panteghini [19] studied the effect of taxation on the choice of investment timing with the fixed oilfield investment scale. In an African oilfield project, Fonseca et al. [20] analyzed the impact of oil price uncertainties on investment timing of decision-making through the application of real option approach. Considering uncertainty, irreversibility, and management flexibility, Tang et al. [21] used trinomial tree model to analyze the investment opportunity in oilfield development and production phases. Moreover, the tree model of real option approach was used to evaluate the optimal retrofit timing of CCS investment by Zhang et al. [22]. Using the real option approach, Zhang et al. [23] analyzed the exercising price threshold for a gold mine and estimated the probabilities of exercising the option. The potential value for waiting is taken into account in the model, which is helpful in choosing optimal timing. In addition, the effect of postponing delivery commodity, which reflects the time of production, has also been discussed in mining investment [24]. Considering technology innovation in the long term, Zhang et al. [25] analyzed the correlation between investment timing adjustment and declining production cost. Although the aforementioned literatures considered decision-making flexibility and various uncertainties, these scholars viewed the projects' scale as fixed, which set the output and investment cost as constants. It is significant to consider the impact of investment scale in the choice of optimal investment timing. For international oil companies, the optimal investment timing needs to be revised if the sunk cost and output changes.

In the study of investment scale selection, many scholars have used the method of integrated modeling, which aims to coordinate oilfield and maximize the net present value of oil projects. Due to its ability to reasonably describe all aspects of an oil project, it can help to choose the actual and optimal investment scale. The model, which aims to minimize overall cost for offshore oilfields, was initially created by Devine and Lesso [26], and this article discussed the basic limitations and possible utility of the model. Iyer et al. [27] presented a multiperiod mixed-integer linear programming (MILP) model in order to plan and schedule offshore oilfield facilities. The model could optimize the selected economic indicator (for example, NPV) by a general objective function. In addition, Carvalho and Pinto [28] and Redutskiy [29] used this method to choose oilfield optimal investment planning. However, the nonlinear planning models do not fully consider uncertainties such as oil prices and exchange rates. Therefore, the lack of consideration of risks, as well as the neglect of the value of management flexibility to the uncertainties, leads to bias in calculation of the value of oil projects.

Based on the deficiencies in the nonlinear programming model and fluctuation in oil prices and exchange rates, real option approach can be used to study the choice of investment scale of oil projects, taking full account of uncertainty and management flexibility. Under the consideration of risk from oil prices and exchange rates, international oil companies can use the real option approach to evaluate the value of oil projects and make optimal investment decisions. However, the literature on the choice of investment scale in oil projects have rarely used the real option approach and do not consider the investment timing, which reduces the value of management flexibility. Lund [30], for example, allowed oilfield investment scale to change freely under different tax regimes and used real option approach to study the impact of taxes on investment scale. Blake and Roberts [31] applied Lund's approach to rank tax distortion of several international tax regimes. The timing of investment, however, remained fixed in these researches. Meister et al. [32] studied optimal strategies for oilfield exploitation by real option approach, considering the size of the reserves and the variation in the oil price as uncertainties. Zhu et al. [33] applied real options theory and Monte Carlo simulation to study overseas oil investment evaluation incorporating various uncertainties. The model aimed to value overseas oil investment projects under different resource tax systems and distinct oilfield sizes.

In previous studies, the fixed investment scale or investment timing limited the flexibility of a firm's investment decisions on an uncertain environment. To fully consider the uncertainty and management flexibility, it would be ideal to select both optimal timing and scale of investment in order to obtain the optimal investment decisions in oil projects. Applying the real option approach, Dias et al. [34] evaluated the investment option value and considered discrete production scale. Considering the uncertainties and optimal timing, Liu and Jiang [35] focused on the investment proportion in two stages and presented a two-stage investment model. However, the total scale was fixed in this theoretical model. It is worth noting that a few distinct investment scales have limited its application in practice for international oil companies. Bockman et al. [36], Boomsma et al. [37], and Kitzing et al. [38] presented a method for evaluating renewable energy projects that were subject to uncertainties (electricity price and policy). Considering logistics demand volatility, Zhang et al. [39] evaluated a logistics park investment, which aimed to determine the choice of optimal timing and size. These researchers presented a real options-based method with continuous timing and scaling.

In particular, this continuous scaling selection method fully considered the uncertain environment and value of management flexibility. In addition, renewable energy projects, like overseas oil projects, are also irreversible capital investments. Using such a method can help international oil companies make their own optimal investment decisions on overseas oil projects that are more suitable for current practice.

According to the relevant studies, the new evaluation model should combine a few significant decision options (timing and scale of investment) and uncertainty factors (oil price and US dollar exchange rate). The variable investment cost can enlarge the flexibility of decision-making. Therefore, in this paper, we construct a real option model that can determine the optimal investment timing and scale of overseas oil projects under the condition that the current oil price shows a downward trend and fluctuates greatly. For large-scale irreversible investment, considering both optimal investment timing and scale, this model can take full account of the value of management flexibility, which can give international oil companies more accurate and practical policy implications. At the same time, the model considers the uncertainties of international oil prices and the US dollar exchange rate because these key factors directly affect the profits of overseas oil projects. In addition, unlike renewable energy projects with almost fixed power generation in each period, this model demonstrates and considers variable oil production in the whole production period of oilfields.

3. Model Description

The following model is used to determine the optimal investment timing and scale of overseas oil projects, where their major properties are irreversible capital investment, high uncertainty, and management flexibility. Considering these properties, the first step in this model is to define the relationship between oilfield production, revenue, operation cost, and investment cost. According to this preparation, the analytical solutions of optimal investment scale and its corresponding value of oil projects can be determined. Then, uncertainties of oil prices and US dollar exchange rate, which is presented in historical data, are considered in the function of option value. Postponing investment can be seen as holding an option, and the option value depends on the project itself. The significant decision options (investment timing and scale) are combined in the real option model, taking into account the value of management flexibility. Finally, through the critical investment conditions, the optimal investment timing and scale choice can be determined based on the consideration of delaying investment. The model framework is presented in Figure 1.

3.1. Value of Oil Projects

It is important to define the relationship between oilfield production, revenue, operation cost, and investment cost because of the nature of irreversible capital investment. Then, the analytical solutions of the optimal investment scale and its corresponding project value can be obtained by calculating the project's cash inflow and outflow.

Based on information such as oil prices, after-tax return rates, and production, the oil project value depends on its discounted full-period revenue, operation cost, and investment cost.

$$V = \int_0^T e^{-rt}[R(t) - TC(t)]\mathrm{d}t - I \tag{1}$$

where V is the oil project value, r is the risk-adjusted discount rate, I is the investment cost, and T is the length of contract period of the oil project. $R(t)$ and $TC(t)$ denote the revenue function and operation cost function at time t, respectively.

The investment cost in this model is assumed to be generated in the initial time of the contract period, which indicates the irreversible capital investment of overseas oil projects. The oil project revenue and operation cost at each moment are also calculated and discounted to that of the beginning of the project. It is clear that determining the relationship between irreversible investment and project profit is the foundation for optimal investment decisions.

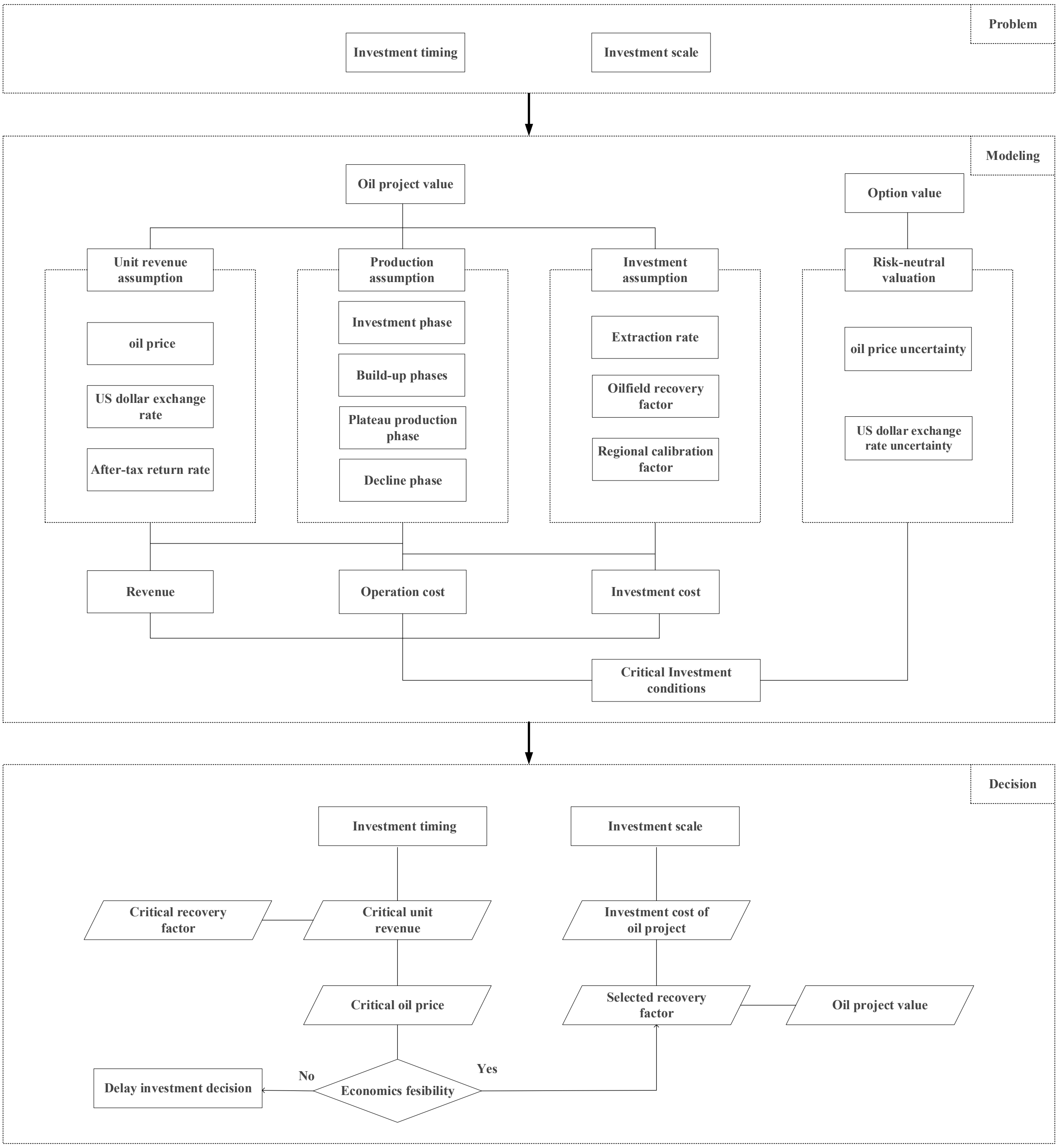

Figure 1. Framework for the choices of optimal investment timing and scale in overseas oil projects.

3.1.1. Oilfield Production

Oilfield production has its variation rule that directly decides investment cost, operation cost, and revenue, which is related to irreversible capital investment. In this model, the oil resource is assumed to be fully explored. Therefore, the production is determined by the volume of oilfield, the selected recovery factor, and the extraction rate. The volume of in situ oil reservoir is fixed and called *OIP*, which means the volume of oil-in-place. The selected recovery factor measures the selected portion of in situ resource that will actually be produced. This implies that the recoverable reserves is determined by the volume of oilfield (*OIP*) and the selected recovery factor. The extraction rate indicates the ratio of plateau oil production to recoverable reserves.

Based on previous research about oilfield production [40–42], the following is known:

Investment, build-up, plateau production, and decline phase are four production phases in an oilfield's lifetime. Firstly, oil companies mainly conduct oilfield infrastructure work, and the oil production is 0 during this investment phase. Secondly, oil production increases in proportion to time during the build-up period. Thirdly, the production of oilfield maintains at a relatively high level and can be approximated as a constant during the plateau production period. Lastly, the production of oilfield can be approximately seen as a constant percentage decline (exponential) during the decline phase.

In summary, we can obtain the production function of oilfield in its whole contract period.

$$m(t) = \begin{cases} 0, t \in [0, T_0) \\ a \cdot s \cdot OIP \cdot \dfrac{t}{T_1}, t \in [T_0, T_1) \\ a \cdot s \cdot OIP, t \in [T_1, T_2) \\ a \cdot s \cdot OIP \cdot e^{-\omega(t-T_2)}, t \in [T_2, T_3] \end{cases} \tag{2}$$

where s and a denote the selected recovery factor and extraction rate of the oilfield, respectively. ω is the oil production decline rate. T_0, T_1, T_2, and T_3 denote the termination of oilfield investment, build-up, plateau production phase, and the length of contract period, respectively.

3.1.2. Revenue of Oil Project

In the uncertain environment, fluctuating oil prices and the US dollar exchange rate directly affect the profit of overseas oil projects. Considering the cash inflow of overseas oil projects, the revenue mainly depends on the production, the fiscal regime of investee country, the oil price, and the US dollar exchange rate.

$$R = \int_0^T e^{-rt} \cdot m(t) \cdot \theta(t)\mathrm{d}t = \int_0^T e^{-rt} \cdot m(t) \cdot \frac{\eta P(t) \cdot E(t)}{E_0}\mathrm{d}t \tag{3}$$

where R denotes the revenue of overseas oil projects, and $\theta(t)$ denotes the oil's unit revenue per barrel at time t. $\theta(t)$ depends on the after-tax return rate η, oil price, and US dollar exchange rate. In order to simplify the analysis in this model, the after-tax return rate η is treated as a constant during the contract period. E_0 denotes the benchmark of US dollar exchange rate. $P(t)$ and $E(t)$ denote the oil price and US dollar exchange rate at time t, respectively.

According to previous research [13,21,33,43,44], the oil price and US dollar exchange rate are assumed to be governed by the geometric Brownian motion, which indicates their variation and implies their influence on revenue.

$$\mathrm{d}P = u_P P\mathrm{d}t + \sigma_p P\mathrm{d}z_P \tag{4}$$

$$\mathrm{d}E = u_E E\mathrm{d}t + \sigma_E E\mathrm{d}z_E \tag{5}$$

where u_E and u_P are the drift of oil price and US dollar exchange rate, σ_P and σ_E are the volatility of oil price and US dollar exchange rate. The $\mathrm{d}z_p$ and $\mathrm{d}z_E$ are independent increments of Wiener process.

Therefore, the unit revenue is governed by the Geometric Brownian motion.

$$\mathrm{d}\theta = u\theta\mathrm{d}t + \sigma\theta\mathrm{d}z \tag{6}$$

$$u = u_P + u_E \tag{7}$$

$$\sigma = \sqrt{\sigma_P^2 + \sigma_E^2 + 2\rho_{E,P}\sigma_P\sigma_E} \tag{8}$$

where u and σ are the drift and volatility of unit revenue, $\rho_{P,E}$ is the correlation coefficient between oil price and exchange rate, and $\mathrm{d}z$ is the independent increment of a Wiener process.

Therefore, the revenue of each phase can be discounted to that of the beginning. Considering the uncertainties, the revenue function of overseas oil projects can be written as Equation (9):

$$R(s;\theta) = E\left[\int_{T_0}^{T_3} m(t)\cdot\theta(t)\cdot e^{-rt}\mathrm{d}t|\theta(0)=\theta\right] = R_1 + R_2 + R_3 = s\cdot\theta\cdot\xi \tag{9}$$

$$\begin{aligned}\xi = &\left\{(\frac{a\cdot OIP}{T_1})\left[\frac{e^{-(r-u)T_0} - e^{-(r-u)T_1}}{(r-u)^2} + \frac{T_0 e^{-(r-u)T_0} - T_1 e^{-(r-u)T_1}}{r-u}\right]\right\} + \left\{\frac{a\cdot OIP}{r-u}\left[e^{-(r-u)T_1} - e^{-(r-u)T_2}\right]\right\} \\ &+\left\{\frac{a\cdot OIP\cdot e^{\omega T_2}}{r+\omega-u}\left[e^{-(r+\omega-u)T_2} - e^{-(r+\omega-u)T_3}\right]\right\}\end{aligned} \tag{10}$$

where R_1, R_2, and R_3 denote the revenue of build-up, plateau production, and decline phase, respectively. ζ denotes the expression of discounted revenue coefficient.

3.1.3. Operation Cost of Oil Project

Cash outflow can be divided into investment cost and operation cost. Based on previous literature [21,43], the unit operation cost of oil production is set as a constant, which can easily determine relationship between revenue and investment cost in the later analysis. Based on the background of global climate change, more and more governments are focusing on the ecological safety and development sustainability [45]. Considering the environmental policy of reducing emission and pollution, we have also set a unit environmental cost for emission policy charges in this part. The unit environment cost, which is reflected in taxation, can also be seen as the pressure of environmental constrains.

$$TC = \int_0^T e^{-rt}\cdot(c_1+c_2)\cdot m(t)dt = \int_0^T e^{-rt}\cdot c\cdot m(t)dt \tag{11}$$

where TC is the operation cost, and c denotes the unit operation cost. c_1 and c_2 denote the unit production cost and unit environment cost in oil projects, respectively.

Therefore, the operation cost of each phase can be discounted to that of the beginning. The operation cost function of overseas oil projects can be written as Equation (12).

$$TC(s) = E\left[\int_{T_0}^{T_3} m(t)\cdot c\cdot e^{-rt}\mathrm{d}t\right] = TC_1 + TC_2 + TC_3 = s\cdot c\cdot\kappa \tag{12}$$

$$\begin{aligned}\kappa = &\left\{(\frac{a\cdot OIP}{T_1})\left[\frac{e^{-rT_0} - e^{-rT_1}}{r^2} + \frac{T_0 e^{-rT_0} - T_1 e^{-rT_1}}{r}\right]\right\} + \left\{\frac{a\cdot OIP}{r}\left[e^{-rT_1} - e^{-rT_2}\right]\right\} + \\ &\left\{\frac{a\cdot OIP\cdot e^{\omega T_2}}{r+\omega}\left[e^{-(r+\omega)T_2} - e^{-(r+\omega)T_3}\right]\right\}\end{aligned} \tag{13}$$

where TC_1, TC_2, and TC_3 denote the operation cost of build-up, plateau production, and decline phase, respectively. κ is the expression of discounted operation cost coefficient.

3.1.4. Investment Cost of Oil Projects

Oil projects have high irreversible capital investments and this part is to define the function of investment cost. Previous researches [46–49] have proposed a function of investment costs, which establishes a relationship between investment cost and oilfield production.

$$I(s) = A\cdot C(s)\cdot s\cdot OIP\cdot a^{\lambda} = A\cdot(3s)^{\zeta}\cdot s\cdot OIP\cdot a^{\lambda} \tag{14-a}$$

The investment cost of oil projects is determined by the volume of oilfield (OIP), the selected recovery factor (s), extraction rate (a) and parameters (ζ, λ and A). A in this function denotes regional calibration factor and reflects local conditions, which can be seen as "the amount of investment required per initial daily barrel of production"[48].

To simplify the investment cost function, this model sets $B = A \cdot 3^{\zeta} \cdot a^{\lambda}$ and $\gamma = \zeta + 1$:

$$I(s) = (A \cdot 3^{\zeta} \cdot a^{\lambda}) \cdot s^{\gamma} \cdot OIP = B \cdot s^{\gamma} \cdot OIP \tag{14-b}$$

where γ and λ denote the elasticity of investment cost with respect to the selected recovery factor and extraction rate. This indicates that if the selected recovery factor or extraction rate increases by 1%, the investment cost will increase γ% or λ%. The figures are all greater than 1, reflecting the characteristics of increasing marginal cost. If the investors choose a relative higher level of recovery factor and want to produce more oil, the investment cost will rise to a notable high level. We can realize a similar relationship between extraction rate and investment cost. Furthermore, the investment function indicates that the investment cost has the constant return to scale for the oilfield volume (*OIP*).

3.1.5. Choice of Optimal Investment Scale

Due to the importance of irreversible capital investment of oil projects, this part uses the constructed functions, which contain oilfield production, revenue, operation cost, and investment cost, to obtain the analytical solutions of optimal investment scale and the corresponding value of oil projects. The investment scale, which determines the oilfield development, is influenced by the volume of oilfield, the selected recovery factor, and the extraction rate. The volume of oilfield, which means the volume of in situ oil reservoir, is exogenous. This model sets both volume of oilfield and extraction rate as constants. Choosing recovery factor, which determines the degree of oilfield development, is the key point for investors to select the investment scale and investment cost of oil projects. The theoretical two-stage economic model established by Zhang and Kleit [50] also proves that choosing the optimal mining rate can significantly increase the mine's profit. To sum up, the choice of investment scale can be transformed to the choice of recovery factor in this model.

When the optimal investment scale, which means the optimal recovery factor, is selected, the project's marginal revenue should be equal to the marginal cost, and the project's net present value could be maximized. Based on Sections 3.1.2 to 3.1.4, the optimal recoverable factor can be calculated from the revenue function, operation cost function, and investment cost function.

$$\max[R(s;\theta) - TC(s) - I(s)] \Rightarrow \frac{\partial}{\partial s} R(s;\theta) = \frac{\partial}{\partial s} TC(s) + \frac{\partial}{\partial s} I(s) \tag{15}$$

Therefore, the analytical solution of the optimal recovery factor ($s_{optimal}$) can be written as follows:

$$s_{optimal} = \left[\frac{\theta\xi - c\kappa}{B \cdot OIP \cdot \gamma} \right]^{\frac{1}{\gamma-1}} \tag{16}$$

The corresponding project value expression can be written as follows:

$$V(\theta; s_{optimal}) = \left[\left(\frac{1}{B \cdot OIP \cdot \gamma} \right)^{\frac{1}{\gamma-1}} - B \cdot OIP \cdot \left(\frac{1}{B \cdot OIP \cdot \gamma} \right)^{\frac{\gamma}{\gamma-1}} \right] \cdot (\theta\xi - c\kappa)^{\frac{\gamma}{\gamma-1}} \tag{17}$$

It is true that the analytical solution of the optimal recovery factor could give guidance to create balance between irreversible capital investment and project revenue.

3.2. Value of the Real Option

Overseas oil projects are located in highly uncertain environments, and the value of real option in this model considers the uncertainties (oil prices and US dollar exchange rate), taking management flexibility into account. The following step is to value the real option, called F, which directly relates to the investment timing. It assumes that the option to invest in an oil project can be exercised by investors at any time [51]. The decision of delaying investment is also the right of oil project investors. Therefore, the value of right should be taken into account in order to capture the value of management flexibility. The stochastic factor affecting the oil project's value is the unit revenue. The unit revenue is governed by geometric Brownian motion (GBM). Over a short period of time, holding the option to invest can get the return from the expected value change. The lifetime of the oil project is limited because our model is aimed at overseas oil investment, where the contract period is fixed. With reference to the research by Conrad and Kotani [44], we do not use a contingent claims approach and avoid estimating the convenience yield of oil.

Therefore we get the following:

$$rF\mathrm{d}t = E(\mathrm{d}F) \tag{18}$$

The quadratic differential equation can be obtained using Ito's Lemma.

$$\frac{1}{2}\sigma^2\theta^2 F''(\theta) + u\theta F'(\theta) - rF = 0 \tag{19}$$

The general solution to this equation can be seen in Equation (20):

$$\begin{gathered} F(\theta) = D_1\theta^{\beta_1} + D_2\theta^{\beta_2} \\ \beta_1 = \frac{1}{2} - \frac{u}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{u}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \beta_2 = \frac{1}{2} - \frac{u}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{u}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \end{gathered} \tag{20}$$

where D_1 and D_2 are unknown constants, β_1 and β_2 are the two roots of the quadratic equation.

According to the boundary condition that $F(0) = 0$, it follows that $D_2 = 0$ because $\beta_2 < 0$. We set D equal to D_1. As above, we can get the value function of real option, which can be seen in Equation (21):

$$\begin{aligned} &F(\theta) = D\theta^{\beta_1} \\ &\beta_1 = \frac{1}{2} - \frac{u}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{u}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \end{aligned} \tag{21}$$

3.3. Optimal Investment Strategy

Considering management flexibility of delaying investment and uncertainties of oil prices and the US dollar exchange rate, the choice of optimal investment timing and scale in overseas oil projects is based on a comparison of net present value and option value. In the highly uncertain environment, the investment decision is a matter of balancing the following [36]: Firstly, the interest from investment cost, I(s), will be saved if the investors do not invest. Secondly, investing in that moment means that the cash flow of the oil project is produced. Thirdly, investing in that moment means that if the oil price drops to a low level in the future, the investors will suffer the losses. The optimal investment strategy follows from two critical investment conditions that are in line with the above tradeoff [38,44,51].

The first one is the "value-matching" condition, which says that the real option value equals the net present value of oil projects when investment occurs.

$$\begin{aligned} F(\theta^*) &= D\theta^{*\beta} = \mathrm{V}\left(\theta^*, s_{optimal}(\theta^*)\right) - TC(s_{optimal}(\theta^*)) - I\left(s_{optimal}(\theta^*)\right) \\ &= \left[\left(\frac{1}{B\cdot OIP\cdot\gamma}\right)^{\frac{1}{\gamma-1}} - B\cdot OIP\cdot\left(\frac{1}{B\cdot OIP\cdot\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right]\cdot(\theta^*\xi - c\kappa)^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{22}$$

where $F(\theta)$ denotes the option value when the unit revenue is θ.

The second one is the "smooth pasting" condition, which entails that both functions have the same slope.

$$\begin{aligned} F'(\theta^*) &= D\beta\theta^{*(\beta-1)} \\ &= \left[\left(\frac{1}{B\cdot OIP\cdot\gamma}\right)^{\frac{1}{\gamma-1}} - B\cdot OIP\cdot\left(\frac{1}{B\cdot OIP\cdot\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right]\cdot\frac{\gamma\xi}{\gamma-1}\cdot(\theta^*\xi - c\kappa)^{\frac{1}{\gamma-1}} \end{aligned} \tag{23}$$

Solving Equations (22) and (23), the following analytical solutions can be obtained:

$$\theta^* = \frac{(\gamma-1)c\kappa\beta}{\xi[(\gamma-1)\beta-\gamma]} \tag{24}$$

$$s^* = \left\{\frac{c\cdot\kappa}{B\cdot OIP\cdot[(\gamma-1)\beta-\gamma]}\right\}^{\frac{1}{\gamma-1}}(s_{optimal}\geq s^*) \tag{25}$$

$$P^* = \frac{(\gamma-1)c\kappa\beta}{\eta\xi[(\gamma-1)\beta-\gamma]} \tag{26}$$

where θ*, P*, and s* denote the critical unit revenue, critical oil price, and critical recovery factor, respectively.

Considering the investment timing, the critical oil price is the investment threshold for triggering oilfield development. If the oil price equals the critical oil price, the optimal recovery factor at that time will equal the critical recovery factor. For international oil companies, if the oil price is below the critical oil price, it is never optimal to invest. If the oil price is higher than the critical oil price, it is optimal to invest immediately, and the optimal recovery factor, which determines the optimal investment scale and cost, can be selected by Equation (16) at that time. Above all, the oil project value can be obtained according to Equation (27):

$$V = \begin{cases} R\Big(\theta, s_{optimal}(\theta)\Big) - TC(s_{optimal}(\theta)) - I\Big(s_{optimal}(\theta)\Big), P > P^* \\ F(\theta), P \leq P^* \end{cases} \tag{27}$$

In summary, it is true that this model is in line with three important properties of overseas oil projects: irreversible capital investment, high uncertainty, and management flexibility.

3.4. Theoretical Analysis of the Impact of Investment Scale on Investment Timing

Changing the drift and volatility of unit revenue revises the optimal investment decisions of overseas oil projects. In highly uncertain environments, the fixed investment scale limits the flexibility of investment timing. The investment scale that can be selected freely allows international oil companies to make comparatively reasonable investment decisions on oil projects and increase the project value of management flexibility from the uncertainties. In this Section, we compare the critical oil price difference with and without the possibility of choosing the investment scale freely. We suppose that investors can choose optimal investment scale when they can choose oilfield recoverable factor freely.

According to Section 3.3, the critical oil price, which determines the investment timing, can be rewritten as follows:

$$P^* = \frac{(\gamma-1)c\kappa\beta}{\eta\xi[(\gamma-1)\beta-\gamma]} = \frac{\beta c\kappa s^* + \beta I(s^*)}{\eta(\beta-1)\xi s^*} \tag{28}$$

where

$$s^* = \left\{\frac{c\cdot\kappa}{B\cdot OIP\cdot[(\gamma-1)\beta-\gamma]}\right\}^{\frac{1}{\gamma-1}}$$

Supposing that the oilfield recovery factor is fixed, the corresponding critical oil price (P_0) can be written as Equation (29):

$$P_0 = \frac{\beta c \kappa s_0 + \beta I(s_0)}{\eta(\beta - 1)\xi s_0} \tag{29}$$

The function G is set to compare the critical oil price difference between the fixed investment scale and the variable one. In this part, the fixed investment scale is optimal under the initial condition.

$$\begin{aligned} G &= P^* - P_0 = \frac{\beta c \kappa s^* + \beta I(s^*)}{\eta(\beta - 1)\xi s^*} - \frac{\beta c \kappa s_0 + \beta I(s_0)}{\eta(\beta - 1)\xi s_0} = \frac{\beta}{\eta(\beta - 1)\xi}(\frac{I(s^*)}{s^*} - \frac{I(s_0)}{s_0}) \\ &= \frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi}(s^{*\gamma-1} - s_0^{\gamma-1}) = (\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi}) \cdot (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1}) \end{aligned} \tag{30}$$

Under the initial condition, $G = 0$ and $s_0 = s^*$ because the parameters of the drift and volatility of unit revenue are the same.

Firstly, we focus on the drift rate and compare the critical oil price difference with and without the possibility of choosing the investment scale freely. The following is known by derivation of Equation (30):

$$\frac{\partial G}{\partial u} = \frac{\partial(P^* - P_0)}{\partial u} = (\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi}) \cdot (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1})'_u + (\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi})'_u (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1}) \tag{31}$$

Under the initial condition, $s_0 = s^*$, we know the following:

$$(\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi})'_u \frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1} = 0, \frac{\partial \kappa}{\partial u} > 0, \frac{\partial \beta}{\partial u} < 0$$

Therefore,

$$\frac{\partial G}{\partial u} > 0 \tag{32}$$

Secondly, we focus on the volatility rate and compare the critical oil price difference with and without the possibility of choosing the investment scale freely. The following is known by derivation of Equation (30):

$$\frac{\partial G}{\partial \sigma} = \frac{\partial(P^* - P_0)}{\partial \sigma} = (\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi}) \cdot (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1})'_\sigma + (\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi})'_\sigma \cdot (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1}) \tag{33}$$

Under the initial condition, $s_0 = s^*$, we know the following:

$$(\frac{\beta \cdot B \cdot OIP}{\eta(\beta - 1)\xi})'_\sigma \cdot (\frac{c \cdot \kappa}{B \cdot OIP \cdot [(\gamma - 1)\beta - \gamma]} - s_0^{\gamma-1}) = 0, \ \frac{d\beta}{d\sigma} < 0$$

So,

$$\frac{\partial G}{\partial \sigma} > 0 \tag{34}$$

Based on Equations (32) and (34), the differences in investment thresholds mean different changes in optimal investment timing. The above theoretical analysis for the drift and volatility of unit revenue indicates that the critical oil price can be greater adjusted when the investment scale is variable. The fixed oilfield investment scale limits the flexibility for the choice of optimal investment timing.

3.5. Theoretical Analysis of the Impact of Tax Holiday on Oil Project Value and Government Tax Revenue

For many oil resource investees, a large proportion of their fiscal revenue comes from oil-related taxes. In the era of low oil prices, the number of new oil projects has been drastically reduced, so the future financial revenues of oil resource investees may be significantly reduced. Therefore, host governments may set a tax holiday of several years in order to encourage international oil companies to carry out exploration and development in their territory [52]. Oil project investors can sometimes also obtain this term through contract negotiation. Next, we will analyze the impact of tax holiday on oil project value and government tax revenue by theoretical analysis. In this part, the length of tax holiday is set as two years, which means that the period of build-up phase is tax-free.

The expressions of oil project value and government tax revenue are presented in the condition of existence of tax holiday. When there is no tax holiday, the oil project value and government tax revenue V_{HG} are shown in Equations (35) and (36), respectively.

$$V = s\theta(\xi_1 + \xi_2 + \xi_3) - sc\kappa - B \cdot OIP \cdot s^{\gamma} \tag{35}$$

$$V_{HG} = \frac{s\theta(\xi_1 + \xi_2 + \xi_3)(1-\eta)}{\eta} + \mathrm{sc}_2\kappa \tag{36}$$

where ξ_1, ξ_2, ξ_3 denote the expression of discounted operation cost coefficient during the build-up, plateau production, and decline phase, respectively.

In the condition of existence of the term, the oil project value V^{TH2} and government tax revenue V_{HG}^{TH2} are shown in Equations (37) and (38), respectively.

$$V^{TH2} = s\theta(\xi_1/\eta + \xi_2 + \xi_3) - sc\kappa - B \cdot OIP \cdot s^{\gamma} \tag{37}$$

$$V_{HG}^{TH2} = \frac{s\theta(\xi_2 + \xi_3)(1-\eta)}{\eta} + sc_2\kappa \tag{38}$$

The tax revenue reduction of the government when there is a two-year tax holiday, called ΔV_{HG}^{TH2}, is also expressed in Equation (39):

$$\Delta V_{HG}^{TH2} = -\Delta V^{TH2} = -\frac{s\theta\xi_1(1-\eta)}{\eta} \tag{39}$$

4. Model Application: Case Project Results and Discussion

Section 4 firstly studies a case project using the aforementioned investment decision model. Secondly, it demonstrates the impact of investment scale on investment timing in overseas oil projects. Finally, the impact of influence factors on investment decision is analyzed.

4.1. Data Preparation

Because of the lack of actual data, most parameters in this case project, which aims to demonstrate the use of investment decision model, are derived from published articles among the web of science core database. This means that international oil companies can apply their own data to get precise results in practical application. We obtained all parameters about oil projects in our case, and these are shown in Table 1.

Investment, build-up, plateau production, and decline phase are four production phases in an oilfield's lifetime. According to Zhao et al. [41] and Fan [53], both the investment phase and build-up phase are one year. The plateau production phase is about four years. The length of contract period is set as 20 years, as referred to by previous articles. The unit production cost and unit environment cost are referenced from Tang et al. [21] and Kobari et al. [54]. Parameters in the investment cost function, including A, a, s, γ, λ, and OIP, is referenced from Smith [48] and Smith and Paddock [55]. Based on the research of Fan and Zhu [43], the oil production decline rate is set as 10%. A simple royalty regime is selected in order to simplify later analyses that the fixed tax rate is 45% of gross sales value [48].

Considering the uncertainties of overseas oil projects, the international oil prices of time series are the average prices of Brent and West Texas Intermediate crude oil collected between January 2008 and December 2017. The parameters of oil price variation process are estimated by weekly spot prices. The corresponding US dollar trade-weighted index (TWDI) is used to estimate the uncertainty of US dollar exchange rate. The risk-adjusted discount rate is set as 10%, which is always applied for oil companies [56,57].

Table 1. Parameters in the case project.

Parameter	Model Symbol	Value	Unit
Investment phase termination	T_0	1	year
Build-up phase termination	T_1	2	year
Plateau production termination	T_2	6	year
Length of contract period	T_3	20	year
Volume of oilfield	OIP	300	million barrels
Regional calibration factor	A	610	
Recovery factor cost elasticity	ζ	3	
Extraction rate cost elasticity	λ	1.68	
Oil production decline rate	ω	10	%/year
Extraction rate	a	7	%/year
After-tax return rate	η	55	%
Unit revenue drift rate	u	2.130	%/year
Unit revenue volatility rate	σ	28.268	%/year
Discount rate	r	10	%/year
Unit production cost	c_1	12.67	US $/bbl
Unit environment cost	c_2	1.5	US $/bbl

4.2. Case Study of Oil Project

Based on the functions in Section 3 and the parameters in Table 1, the results of a case project were obtained (See Table 2). According to the results, the critical oil price is 82.32 US$/bbl, and the corresponding critical recovery factor is 27.47%. When the oil price is 82.32 US$/bbl, the corresponding investment cost and oil project value are about $323 million and $969 million, respectively. In other words, the investment threshold for triggering oilfield development is 82.32 US$/bbl. If the oil price is below 82.32 US dollars per barrel, investors should not invest and delay the investment decision because the option value is higher than the actual value of project; if the oil price is higher than that, it is optimal to invest immediately and select optimal investment scale based on Equation (16). For example, if the oil price rises to 90 US dollars per barrel, it is optimal to start investment. According to Equations (16) and (27), the optimal recovery factor of oilfield, which implies the optimal investment scale, is 28.60%. Based on the decision of optimal investment scale, the corresponding investment cost and oil project value are $379 million and $1138 million, respectively. One point that should be clarified here is that investors should consider relatively stable oil prices for investment decision-making. Using the recent average oil price is more appropriate than the daily price because of the volatility in the oil market.

Table 2. Results for the case project.

Variable	Value	Unit
Profit discount coefficient ξ	0.143	$\times 10^9$
Operation cost discount coefficient κ	0.124	$\times 10^9$
β_1	1.833	
Critical unit revenue θ^*	45.27	US$/bbl
Critical recovery factor s^*	27.47	%
Critical oil price P^*	82.32	US$/bbl
Investment cost (P = 82.32 US$/bbl)	0.323	$\times 10^9$ US$
Oil project value (P = 82.32 US$/bbl)	0.969	$\times 10^9$ US$

4.3. Empirical Analysis of the Impact of Investment Scale on Investment Timing

It is clear that changes in the market environment influences the optimal investment decisions of oil projects. The critical oil price, which is affected by many factors, indicates the investment threshold.

When the drift or volatility of unit revenue is changed, comparing the difference between the critical oil price with and without the possibility of choosing the investment scale freely focuses on demonstrating the impact of investment scale on investment timing. Section 3.4 demonstrates the limitation of fixed investment scale of oil projects by theoretical analysis. In this part, the effect of variable investment scale is explained by empirical analysis.

The drift of unit revenue, which reflects the market environment, affects the expected return and option value of oil projects. The unit revenue is one of the most important profitability factors for oil projects. As the drift rises, the critical oil price and the expected return of the oil project rise. Therefore, it is better to postpone the investment in order to wait for the proper investment threshold. Because the decision of postponing investment is relatively cheaper, investment in the future is more attractive. This also implies that the levels of oil price in the future will be not very low and that investors should choose a relatively larger investment scale. Furthermore, increasing drift leads to a higher value of the option to delay. The variation in the critical oil price with a fixed investment scale is relatively small when the drift of unit revenue is changing. Due to the value of management flexibility, the sensitivity of investment threshold when investment scale can be selected optimally is comparatively high.

The volatility implies the uncertainty of a project's profit. When volatility of unit revenue changes, the investment threshold will be much more flexible with variable investment scale than with fixed investment scale. Increasing volatility of unit revenue leads to a higher critical oil price for investors because the market is more volatile and chances of loss are greater. The fixed investment scale makes investors unable to adjust investment decisions effectively and limits the flexibility of investment timing, as already shown in Section 3.4. In addition, choosing the investment scale freely increases the value of waiting investment and makes investment threshold more and more flexible, which can reflect the obvious interaction between optimal timing and volatility.

In general, the variable investment scale of overseas oil projects can make the optimal investment timing more adjustable and can obtain more value from management flexibility when the market environment changes.

4.4. Influence Factors Analysis of Investment Strategy

4.4.1. The Impact of Geological Condition on Choices of Optimal Investment Timing and Scale in Oil Projects

This Section examines the impact of the geological condition of oil resource investees on the choices of optimal investment timing and scale. In this model, geological condition, which relates to the project revenue and irreversible capital investment, is reflected by oil production decline rate and regional calibration factor.

Different areas have different oil production decline rates. The International Energy Agency (IEA) survey indicates that the oil production decline rate is different from region to region, with the rate of developed oilfields in Middle East having the lowest level and OECD countries having the highest level [58]. The oil production decline rate in the case of Section 4.2 has been used as the business as usual scenario (BAU). In this part, high oil production decline rate scenario (HDR) and low oil production decline rate scenario (LDR) are set as plus 10% and minus 10% of the initial oil production decline rate level, respectively. In Figure 2, we show the critical oil price and its corresponding value of oil project for different oil production decline rates. As oil production decline rate increases, the critical oil price increases and the oil project value decreases. However, the changes in critical oil price are small, so the oil production decline rate affects the investment threshold finitely.

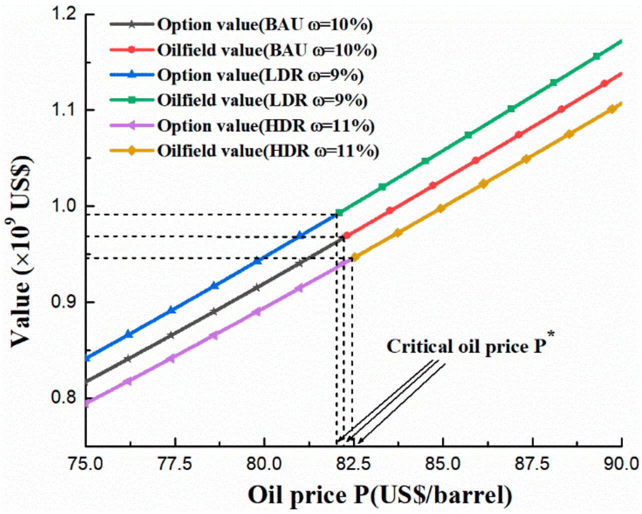

Figure 2. Oil project value and option value for different oil production decline rate scenario.

Regional calibration factor reflects the required investment amount for unit oil production in the region. The higher the regional calibration factor, the worse is the geological condition of the oil resource investee. Equation (26) shows that the regional calibration factor does not influence the critical oil price. However, the regional calibration factor affects the investment costs and influences the choice of investment scale. The regional calibration factor in the case of Section 4.2 has been used as the business as usual scenario (BAU). In this part, high regional calibration factor scenario (HRF) and low regional calibration factor scenario (LRF) are set as plus 10% and minus 10% of the initial regional calibration factor, respectively. When the oil price is equal to the critical oil price, Figure 3 shows that the lower scenario enlarges the investment cost of the oil project and increases its value, although these values do not change much. This means that international oil companies should enlarge their investment scale in the region that has low regional calibration factor. In other words, higher quality of oilfield allows oil companies to increase their investment.

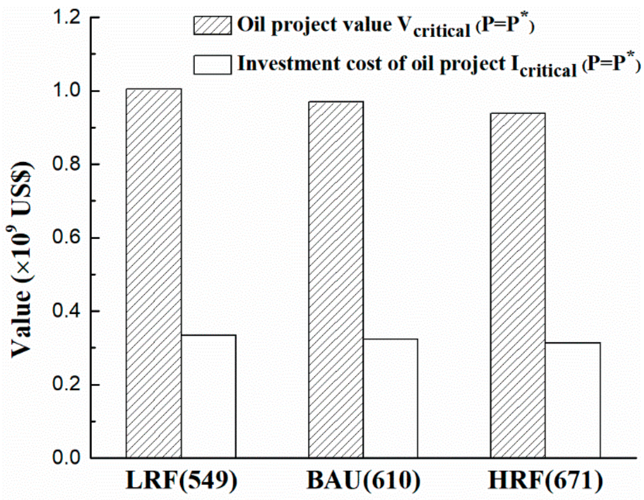

Figure 3. Oil project value for different regional calibration factor scenario ($P = P^*$).

Therefore, when international oil companies make overseas investment decisions, the investment timing in each oil resource investee is roughly the same. Considering the irreversible capital investment and project revenue, investment scale should be selected according to the different geological conditions of each country.

4.4.2. The Impact of Contract Period and Tax Holiday on the Choices of Optimal Investment Timing and Scale in Oil Projects

Changing the length of contract period affects the oil project value and option value. The contract period in the case of Section 4.2 has been used as the business as usual scenario (BAU). In this part, the long length of contract period scenario (LCP) and short length of contract period scenario (SCP) are set as plus 10% and minus 10% of the initial length of contract period level, respectively. Figure 4 shows that extending the length increases the oil project value and option value but decreases the critical oil price. Delaying investment decisions is appropriate when the length of contract period is extended. However, changing the length of contract period to a certain degree does not significantly influence the investment timing of oil projects and has limited effect on investment decisions.

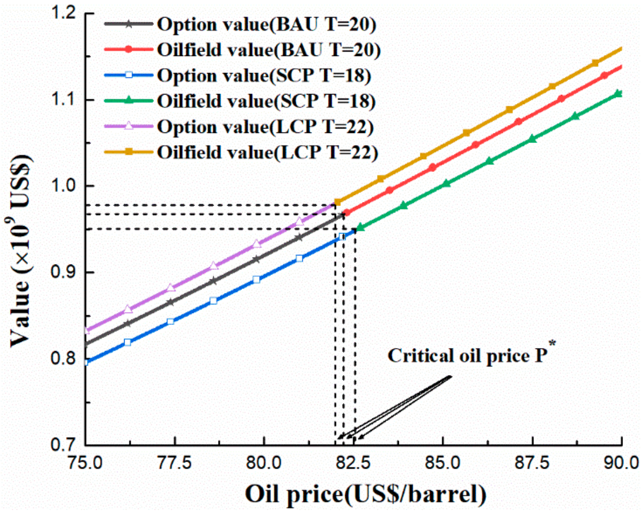

Figure 4. Oil project value and option value for different length of contract period scenario.

Next, the effect of tax holiday will be discussed by empirical analysis. Based on the theoretical analysis in Section 3.5 and parameters in Section 4.1, the calculation results of the case project are presented under the existence of tax holiday (See Table 3). Compared with the case results in Section 4.2, it can be seen that the existence of tax holiday do not affect the choice of the oilfield recovery factor. This means that the existence of the term does not affect the choice of investment scale. However, the existence of tax holiday will allow international oil companies to reduce the investment threshold and start investment at a lower oil price. The existence of this term can offset the burden of taxation for investors to some extent and increase the oil project value. Sunk costs are mainly determined by unit operation cost and investment costs and do not directly relate to the taxation system, so the optimal investment scale does not change.

Table 3. Case results about tax holiday.

Variable	No Existence of Tax Holiday	Existence of Tax Holiday
Oil project value ($\times 10^9$ US$)	0.969	0.969
Tax revenue of government ($\times 10^9$ US$)	1.504	1.267
Critical recovery factor s^* (%)	27.47	27.47
Critical oil price P^* (US$/bbl)	82.32	76.23
Critical unit revenue θ^* (US$/bbl)	45.27	41.93

Therefore, paying attention to negotiation for tax holiday in the era of low oil prices is important because it can enable international oil companies to reduce the investment threshold and start investment at lower levels of oil price.

4.4.3. The Impact of Unit Operation Cost on the Choices of Optimal Investment Timing and Scale in Oil Projects

Unit operation cost directly relate to the emission policy charges and technology of management and production. The unit environment cost and unit production cost in the case of Section 4.2 has been used as the business as usual scenario (BAU). In this part, high environment cost scenario (HEC) and low production cost scenario (LPC) are set as plus 1.5 $/bbl and minus 1.5 $/bbl of the initial unit operation cost level, respectively. Based on the case project in Section 4.2, Figure 5 shows the impact of unit operation cost on the critical oil price and optimal investment scale.

It is clear that reducing unit production cost can reasonably decrease critical oil price and increase oil project value. Compared to previous analysis of geological condition and length of contract period, the change in unit operation cost greatly affects the critical oil price. Considering the optimal investment timing, the reduction of unit operation cost enables international oil companies to significantly reduce their critical oil price, which allows them to reduce the investment threshold and start investment decisions at relatively low levels of oil price. The results also imply that technical progress regarding reducing unit operation cost is important for international oil companies. In addition, they indicate that increasing environment cost obviously affects the optimal investment decision and oil project value. In the background of global climate change, different environmental costs obviously alter the investment threshold. In the future, more and more emission reduction policies will be promoted in every country in order to guarantee development sustainability. Therefore, it is important for investors to consider the pressure of environmental constrain in the investee country, which in turn influences the investment decision.

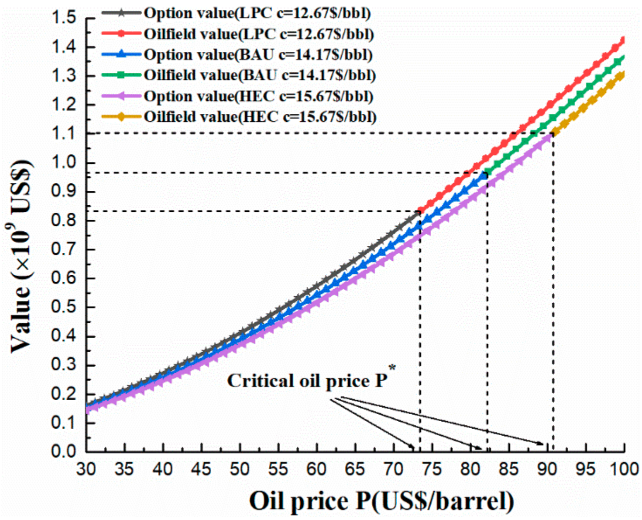

Figure 5. Oil project value and option value for different unit operation cost scenario.

5. Conclusions and Policy Implications

This article builds a real option model that can determine the optimal investment timing and scale of overseas oil projects. The real option model combines a few significant decision options (timing and scale of investment) and uncertainty factors (oil prices and US dollar exchange rate). Considering irreversible capital investment and management flexibility, the analytical solutions of optimal recoverable factor and critical oil price in this model indicate the investment threshold for triggering oilfield development and investment scale of overseas oil projects in highly uncertain environments. For international oil companies, the relationship between the recent average oil price and critical oil price determines whether or not to invest in an oil project at that time. The optimal investment scale can also be selected by analytical solutions and the recent average price. The results show that international oil companies can make rational investment decisions by choosing investment scale freely. In addition, the impact of geological condition, length of contract period, tax holiday, and unit operation cost on the choices of optimal investment timing and scale have also been analyzed. The proposed model offers a few new insights and important findings as follow:

(1) *Choosing oilfield investment scale freely contributes to making more suitable investment decisions*

The uncertainties of oil prices and exchange rates affect the unit revenue, which may potentially revise the optimal investment timing. If international oil companies perceive that the market environment has changed, they should adjust their critical oil price as the previous investment threshold may no longer be suitable for them. Among investment decisions, investors have the right to select investment scale for overseas oil projects. Compared to the fixed investment scale, choosing investment scale freely can greatly transform critical oil price and provide much more value from management flexibility by seizing investment opportunity. To sum up, it is important for international oil companies to abandon fixed investment scale in the highly uncertain environment.

(2) *The geological condition does not have significant impact on investment decisions*

Although we agree that the role of interest rate, institutions, and culture are important [59,60], it should be noted that the geological condition, which relates to the irreversible capital investment, has limited impact on investment timing in practice. For investment timing, the investment thresholds in oil resource investees, which is determined by the critical oil price, are roughly the same for international oil companies. The impact of geological condition on investment scale is also relatively small. Therefore, the geological condition is not a crucial factor for oil project investment compared to other factors, such as political, economic, and fiscal conditions.

(3) *The existence of tax holiday directly contributes to a lower investment threshold*

The clear analysis of tax term is an important step for international oil companies to start their negotiation period. Changing the length of contract period has little effect on investment decisions. When the oil price is relatively low, some investee countries will set a tax holiday in order to encourage investment because governments should ensure taxation sustainability for economic growth [61]. Section 4.4.2 proves that the investment scale will not change even if there is a tax holiday. However, the existence of tax holiday encourages international oil companies to reduce the investment threshold and start project investment in relatively low levels of oil price because it can reduce tax burden to some extent. From this perspective, the negotiation of tax items, particularly in tax holiday, can contribute to easily meeting investment opportunity, especially in the era of low oil prices.

(4) *Unit operation cost effectively affects investment timing and oil project value*

According to the analysis above, changing unit operation cost is of great significance and effective in altering optimal investment timing and oil project value. Compared with other factors, such as geological condition, the length of contract period, and tax holiday, investment threshold is quite sensitive to unit operation cost, which includes environmental cost and production cost. Reducing unit production cost can obviously decrease critical oil price, and international oil companies can take the initiative of oilfield development at a low level of investment threshold. In a competitive environment, the operation cost and sunk cost potentially affect companies in many aspects, such as merger activity [62,63]. In practice, international oil companies should strive to use more efficient management techniques and optimal oil production technology to decrease unit operation costs. Considering policies to restrict pollution and emission, environmental charges also play a key role in the investment decision-making process, which alters the optimal investment timing and project value. Investors should pay more attention to the environmental constrains and sustainability.

In this article, the real option model does not fix the degree of development, which helps investors to select both the optimal investment timing and scale. Previous researches have always set the investment cost as the constant in their real option model; by contrast, the investment scale is variable in this model. Compared with relevant studies, the value of management flexibility and uncertainties of economics factors can be greatly taken into account. In order to reflect the actual practice of oil projects, characteristics of oil production and tax regimes are considered in this model.

However, it is clear that using one model cannot capture the variables that influence optimal investment decision in the uncertain environment [64,65]. This research may have some limitations in application. Firstly, oil prices are affected by various factors [66–68], and it is hard to determine the currently stable oil price for making optimal investment decision. Therefore, choosing the appropriate time extent to calculate the recent average oil price is important for investors. The daily price and weekly price are not suitable for supporting investment decision-making because of instability. Second, the parameters in this model should be adjusted by actual production practice. Overseas oil projects combine many uncertainties and realistic details of investment and operation, which was not discussed in this article. In further research, this model should be extended to consider the uncertainty of investment cost and operation cost, which can reflect the local political or geological conditions. From the perspective of social responsibility, the cost of green practice and contributions for fighting global warming should be considered more carefully [69]. In addition, it will be important to study the impact of enhanced oil recovery (EOR) and specific tax regimes in overseas oil projects [70]. These issues are interesting and attractive for us to address in our future work.

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