Performance Analysis of a Hybrid System Consisting of a Molten Carbonate Direct Carbon Fuel Cell and an Absorption Refrigerator

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Abstract: By integrating an Absorption Refrigerator (AR), a new hybrid system model is established to reuse the waste heat from a Molten Carbonate Direct Carbon Fuel Cell (MCDCFC) for additional cooling production. Various irreversible losses in each element of the system are numerically described. The operating current density span of the MCDCFC that allows the AR to work is derived. Under different operating conditions, the mathematical expressions for equivalently evaluating the hybrid system performance are derived. In comparison with the stand-alone MCDCFC, the maximum attainable power density of the proposed system and its corresponding efficiency are increased by 5.8% and 6.8%, respectively. The generic performance features and optimum operating regions of the proposed system are demonstrated. A number of sensitivity analyses are performed to study the dependences of the proposed system performance on some physical parameters and operating conditions such as operating temperature, operating current density, and pressure of the MCDCFC, cyclic working fluid internal irreversibility inside the AR, thermodynamic losses related parameters and the anode thickness of the MCDCFC. The obtained results may offer some new insights into the performance improvement of an MCDCFC through a reasonable heat management methodology.

Keywords: molten carbonate direct carbon fuel cell; absorption refrigerator; hybrid system; irreversible loss; performance analysis

1. Introduction

The global energy demand dramatically increases with the increases of urbanization, modernization and the human population; however, the world highly depends on fossil fuels such as oil, natural gas and coal [1]. Coal is the most affluent fossil fuel in the world and is widely used in coal-fired plants to generate electricity [2]. A Direct Carbon Fuel Cell (DCFC) enables us to directly transform the chemical energy stored in solid carbon into electrical energy without any gasification or complicated intermediate processes, which offers comparatively higher energy conversion efficiency and lower greenhouse gas emission levels [3,4]. Based on the kinds of electrolytes, DCFCs may be primary assorted into three types: solid oxide [5–7], molten hydroxide [8,9], and molten carbonate DCFCs [10–12]. Due to its high stability, low toxicity, high electrical conductivity and low melting point, the Molten Carbonate Direct Carbon Fuel Cell (MCDCFC) is regarded as a promising technology for efficient, environment-friendly coal utilization [13].

Although the research and development of MCDCFCs have made great progress in aspects such as electrolyte development [14,15], prototype design [16,17] and lifetime extension [18,19], the energy...
conversion efficiency of MCDCFCs is still low [20]. Alternatively, the MCDCFC performance can be also equivalently improved by building cogeneration systems [21–25], since a large proportion of the chemical energy stored in fuels is released as waste heat. Steinberg [21] developed an innovative hybrid system integrating an MCDCFC with a hydrogen plasma black reactor, which converted the biomass and fossil fuels into transportation fuels and electricity. The thermal efficiency was found to be 70–90%. Zhang et al. [22] proposed and simulated a hybrid system mainly composed of an MCDCFC and a thermoelectric generator, which was used to recover the waste heat generated in the MCDCFC for electricity production. Liu et al. [23] presented a hybrid system composed of an MCDCFC, a methane catalytic decomposition reactor, two gas turbines and an internal reforming solid oxide fuel cell. Their studies showed that the MCDCFC performed better at relatively higher loads. The exergy efficiency for such system was 68.24%, while if the waste heat contained in the exhaust gases was further reused, the overall exergy efficiency could be over 80% [24]. Chen et al. [25] used the application of a Carnot heat engine to harvest the waste heat from an MCDCFC, and theoretically studied the effects of some important parameters on the proposed system performance.

The conventional electrically driven vapor refrigeration systems use chlorofluorocarbon refrigerants that may deplete the ozone layer [26]. Absorption refrigerators (ARs), capable of cooling and driven by heat instead of electricity with environmental-friendly working fluid, have attracted attention in fields such as waste heat recovery and low-grade heat utilization [27–29]. Obviously, it is convenient to use the AR to harvest the waste heat from fuel cells [30–33]. Representatively, Silveira et al. [30] used the waste heat from a molten carbonate fuel cell to drive an AR to produce electricity and cold water simultaneously. They demonstrated that this path was feasible from multi-perspectives.

In the present work, a new hybrid system that couples an AR to an MCDCFC to cogenerate electricity and cooling is put forward, so that the overall performance of the MCDCFC can be improved. The irreversible loss in each element within the system is numerically described. The mathematical formulas of power output and efficiency to evaluate the cogeneration system performance are formulated, and the operating current density region of the MCDCFC that enables the bottoming AR to work will be determined. The optimum operating regions for the performance parameters are given. The impacts of some design parameters and operating conditions on the proposed system performance will be revealed by comprehensive sensitivity analyses.

The concrete contents of this paper are arranged as follows. In Section 2, each component within the presented system will be introduced and described, and the equivalent output power density and efficiency of the hybrid system will be deduced by considering various irreversible losses. In Section 3, the generic performance characteristics and the optimum operating ranges for the presented system will be revealed and determined. In Section 4, the effects of some irreversible losses and operating conditions on the hybrid system performance will be analyzed through comprehensive sensitivity analyses.

2. System Description

The hybrid system consists of an MCDCFC, an AR and a regenerator, as illustrated in Figure 1, where the AR is constituted by an evaporator, a generator, an absorber and a condenser. MCDCFC transforms the chemical energy stored in solid carbon into electrical power with heat energy as the by-product. The whole system is operated under atmospheric condition, which may significantly reduce the system complexity. The MCDCFC and AR are configured in the indirect thermal coupling form, in which the produced heat is indirectly transferred to the bottoming AR through a heat exchanger. A part of the waste heat is used to make up the regenerative losses, another part is directly dissipated into the surrounding environment, and the rest is transferred to the generator of the bottoming AR for cooling production. The cycle of the AR working substance comprises of three irreversible isothermal and three irreversible adiabatic processes. In Figure 1, \( P_{\text{MCDCFC}} \) is the electric power output of the MCDCFC, \( q_h \) is the heat flow from the MCDCFC at \( T \) to the working substance in the generator at \( T_1 \), \( q_0 \) is the overall rate of heat transfer from the working substance in the absorber...
and condenser at \( T_3 \) to the environment at \( T_0 \). \( q_c \) is the heat flow between the cooled space at \( T_c \) and the working substance in the evaporator at \( T_2 \). \( q_{re} \) is the rate of regenerative heat loss, \( q_L \) is the heat-leakage rate from the MCDCFC to the ambience. The regenerator functions as a counter-flow heat exchanger that preheats the incoming reactants by means of the heat contained in the exhaust products. The hybrid system is formulated based on the following assumptions:

- Both the MCDCFC and the AR are operated under steady-state conditions;
- Operating temperature and pressure are uniform and constants in the MCDCFC;
- Chemical reactions involved are complete;
- All gases involved are ideal gases;
- Carbon fuel is regarded as a rigid sphere and packed with a simple hexagonal pattern;
- Electrical power required to compress the reactants is excluded in the calculations;
- Working fluid in the AR constantly flows and continuously exchanges heat with the three heat reservoirs;
- Heat transfers within the system obey Newton’s law.

![Figure 1. Schematic diagram of an MCDCFC/AR hybrid system.](image)

2.1. MCDCFC

As described in Refs. [10,22], the output voltage of an MCDCFC is often smaller than the equilibrium potential due to the irreversible losses including activation overpotential, ohmic overpotential, and concentration overpotential. The power output and efficiency of an MCDCFC are, respectively, given by Refs. [10,22].

\[
P_{\text{DCFC}} = I V = j A (E - \Delta V_{\text{act,a}} - \Delta V_{\text{act,c}} - \Delta V_{\text{con}} - \Delta V_{\text{ohm}})
\]

and

\[
\eta_{\text{DCFC}} = \frac{P}{-\Delta H} = -\frac{n_e F V}{\Delta h}
\]
where

\[ E = E_0 + \frac{RT}{nF} \ln \left[ p_{O_2,\text{cat}} \left( p_{CO_2,\text{cat}} \right)^2 \right] \]  

(3)

\[ V_{\text{act},\text{an}} = \frac{RT}{2F} \ln \left\{ \frac{j}{(2j_{0,\text{an}})} + \sqrt{\left[ \frac{j}{(2j_{0,\text{an}})} \right]^2 + 1} \right\} \]  

(4)

\[ V_{\text{act},\text{cat}} = \frac{RT}{2F} \ln \left\{ \frac{j}{(2j_{0,\text{cat}})} + \sqrt{\left[ \frac{j}{(2j_{0,\text{cat}})} \right]^2 + 1} \right\} \]  

(5)

\[ V_{\text{con}} = \frac{RT}{nF} \ln \left[ \frac{j_{\text{lim}}}{(j_{\text{lim}} - j)} \right] \]  

(6)

\[ V_{\text{ohm}} \sum I_{e,j} R_{e,j} + \sum I_{c,j} R_{c,j} + V_{\text{ec}} \]  

(7)

\[ -\Delta H = -\frac{\Delta h}{nF} \frac{j}{A} \]  

(8)

where \( I \) and \( j \) are, respectively, the electric current and current density flowing through the MCDCFC; \( A \) is the polar plate area of MCDCFC; \( V \) and \( E \) are, respectively, the output voltage and equilibrium potential; \( V_{\text{act},\text{an}} \) and \( V_{\text{act},\text{cat}} \) are, respectively, the anode and cathode activation overpotentials \([34,35]\); \( V_{\text{con}} \) is the concentration overpotential \([36,37]\); \( V_{\text{ohm}} \) is the ohmic overpotential; \( V_{\text{ec}} \) is the total ohmic overpotential losses in the cathode and the electrolyte \([10]\); \( -\Delta H \) is the total energy (i.e., both electrical and thermal energies) released per unit time; \( -\Delta h \) is the molar enthalpy change of the electrochemical reactions in the MCDCFC \([10]\). The electrochemical model of MCDCFC has been compared with the one developed by Liu et al. \([36]\) in a previous study \([10]\). It was shown that the adopted MCDCFC model was superior in accuracy to the one from Ref. \([36]\).

2.2. Absorption Refrigerator

When \( q_h \) flows from the MCDCFC to the generator, the AR begins to extract heat from the cooled space. The AR within the hybrid system functions as a three-heat-source absorption refrigerator \([38–41]\). For a total heat-transfer area \( A_R \) and a given heat-transfer rate \( q_h \), the maximum cooling rate \( R \) and its homologous coefficient of performance (COP) \( \varepsilon \) for the AR are, respectively, given by \([39,40]\):

\[ R = q_e = \frac{q_h}{2} \left\{ \left[ a + \frac{I_r T_0 - T_c}{C_q h} \right]^2 - 4T_c \left( \frac{1}{(1 + B)^2 T} - \frac{1 - I_r T_0 / T}{C_q h} \right) \right\}^{0.5} - \left[ a + \frac{I_r T_0 - T_c}{C_q h} \right] \]  

(9)

and

\[ \varepsilon = \frac{1}{2} \left\{ \left[ a + \frac{I_r T_0 - T_c}{C_q h} \right]^2 - 4T_c \left( \frac{1}{(1 + B)^2 T} - \frac{1 - I_r T_0 / T}{C_q h} \right) \right\}^{0.5} - \left[ a + \frac{I_r T_0 - T_c}{C_q h} \right] \]  

(10)

where

\[ a = 1 + \left( T_c - I_r B^2 T_0 \right) / \left[ (1 + B)^2 T \right] \]  

(11)

\[ B = \left( \sqrt{b_2 - 1} \right) / \left( 1 + \sqrt{b_1 b_2} \right) \]  

(12)

\[ A_R = A_h + A_c + A_v \]  

(13)

\[ C = \left( 1 + B \right)^2 / \left( A_R K \right) \]  

(14)

\[ K = K_h / \left[ 1 + \sqrt{b_1 b_2} \right] \]  

(15)

where \( I_r \) is the internal irreversibility factor of the cyclic working fluid, \( b_0 = K_h / K_0 \), \( b_2 = K_h / K_c \), \( K_c \) and \( K_h \) are the heat-transfer coefficients (HTCs) of the evaporator and the generator, \( K_0 \) is the HTC of the absorber or condenser, \( A_R \) is the total heat-transfer area (HTA) of the AR, \( A_c \) and \( A_h \)
are, respectively, the HTAs of the evaporator and generator, \( A_0 \) is the overall HTA of the absorber and condenser.

Considering the exergy content differences between electric power and cooling load, the equivalent power output \( P_{AR} \) and the efficiency \( \eta_{AR} \) for the AR can be, respectively, given by [42]:

\[
P_{AR} = q_c \left| 1 - \frac{T_0}{T_e} \right| = \frac{q_h}{2} \left| 1 - \frac{T_0}{T_c} \right| \left\{ \left( a + \frac{L_{T_0 - T_c}}{C_{dh}} \right)^2 - 4T_c \left( \frac{1}{(1+B)^2 T} - \frac{1-L_{T_0}/T}{C_{dh}} \right) \right\}^{0.5} - \left[ a + \frac{L_{T_0 - T_c}}{C_{dh}} \right] \quad (16)
\]

and

\[
\eta_{AR} = \frac{P_{AR}}{q_h} = \frac{1}{2} \left| 1 - \frac{T_0}{T_e} \right| \left\{ \left( a + \frac{L_{T_0 - T_c}}{C_{dh}} \right)^2 - 4T_c \left( \frac{1}{(1+B)^2 T} - \frac{1-L_{T_0}/T}{C_{dh}} \right) \right\}^{0.5} - \left[ a + \frac{L_{T_0 - T_c}}{C_{dh}} \right] \quad (17)
\]

2.3. Regenerator

With the help of the regenerator, the inlet reactants are preheated from the ambient temperature \( T_0 \) to the operating temperature of MCDCFC with the assistance of the outlet exhaust products. The rate of regenerative heat loss is often given by [33]:

\[
q_{re} = K_{re} A_{re} (1 - \varepsilon) (T - T_0) \quad (18)
\]

2.4. Performance Parameter of the Hybrid System

The rate of heat-leak loss \( q_L \) is supposed to be in proportion to the temperature gap between the MCDCFC and the ambience, and consequently, \( q_L \) and \( q_1 \) can be, respectively, given by [43]:

\[
q_L = K_L A_L (T - T_0) \quad (19)
\]

and

\[
q_h = -\Delta H - P_{DCFC} - q_{re} - q_L = -\frac{A\Delta h}{2F} \left[ (1 - \eta_{DCFC})j - \frac{2F(c_1 + c_2)(T - T_0)}{-\Delta h} \right] \quad (20)
\]

where \( K_L \) and \( A_L \) are, respectively, the heat-leak coefficient and the heat-leak area, \( c_1 = [K_{re}A_{re}(1 - \varepsilon)]/A \) and \( c_2 = K_L A_L / A \) are two temperature-independent constants related to the thermodynamic losses.

It is seen from Equation (20) that the AR starts to extract heat from the cooled space only when in Equation (21) is valid:

\[
-\Delta H - P_{DCFC} > q_{re} + q_L. \quad (21)
\]

Considering Equation (20), Equation (21) can be explicitly revised as:

\[
j > j_c = \left[ \frac{2F}{-\Delta h(1 - \eta_{DCFC})} \right] [(c_1 + c_2)(T - T_0)] \quad (22)
\]

where \( j_c \) is the lower bound of the MCDCFC operating current density, from which the AR starts to work. Based on Equations (16) and (20) and the condition of \( P_{AR} > 0 \), the allowable maximum current density \( j_M \) can also be numerically calculated. Consequently, the effective operating current density interval is given by \( \Delta j = j_M - j_c \).

When \( j \) is in the region of \( j_c < j < j_M \), the equivalent power output \( P \) and efficiency \( \eta \) of the proposed system can be, respectively, given by:

\[
P = P_{DCFC} + q_c \left| 1 - \frac{T_0}{T_c} \right| \left\{ \left( a + \frac{L_{T_0 - T_c}}{C_{dh}} \right)^2 - 4T_c \left( \frac{1}{(1+B)^2 T} - \frac{1-L_{T_0}/T}{C_{dh}} \right) \right\}^{0.5} - \left[ a + \frac{L_{T_0 - T_c}}{C_{dh}} \right] \quad (23)
\]
and

\[
\eta = \frac{P_{\text{DCFC}} + P_{\text{AR}}}{\Delta H} \left\{ \left[ a + \frac{k_2 - \eta}{c_{\text{CH}}^2} \right]^2 - 4T_{c}\left( \frac{1}{(1+\beta)^2} - \frac{1 - \frac{k_1}{c_{\text{CH}}^2}}{c_{\text{CH}}^2} \right) \right\}^{0.5} - \left[ a + \frac{k_2 - \eta}{c_{\text{CH}}^2} \right] \tag{24}\]

When \( j \leq j_C \) or \( j \geq j_M \), \( P \) and \( \eta \) of the proposed system are the same as that of the stand-alone MCDCFC, i.e.,

\[
P = P_{\text{DCFC}} \tag{25}\]

and

\[
\eta = \eta_{\text{DCFC}}. \tag{26}\]

3. Performance Characteristic and Optimum Operating Region

According to the typical parameters listed in Table 1 [10,34,36,44] and the equations in Section 2, the power densities and efficiencies of the stand-alone MCDCFC, AR and hybrid system versus the operating current density of MCDCFC are shown in Figure 2, where \( P_{\text{DCFC}}^\ast \) and \( P_{\text{AR}}^\ast \) are, respectively, the power densities of the MCDCFC and AR; \( \eta_{\text{DCFC}}^\ast \) and \( \eta_{\text{AR}} \) are, respectively, the efficiencies of the MCDCFC and AR; and \( j_C \) and \( j_M \) are, respectively, the lower bound current density and upper bound current density between which the AR is enabled to work, \( P_C^\ast \) and \( \eta_C \) are, respectively, the power density and efficiency at \( j_C \), \( P_M^\ast \) and \( \eta_M \) are, respectively, the power density and efficiency at \( j_M \), \( j_S \) is the stagnation current density from which the MCDCFC is always different from \( P^\ast \). Combining \( j_C \) and \( j_M \), the curves of \( P^\ast \) with \( \eta \) are overlapped with that of \( P_{\text{DCFC}}^\ast \) and \( \eta_{\text{DCFC}} \). It is clearly demonstrated that abstracting the waste heat for cooling production is an effective way to further improve the MCDCFC performance. When \( 0 < j \leq j_C \) or \( j \geq j_M \), the curves of \( P^\ast \sim j \) and \( \eta \sim j \) are overlapped with that of \( P_{\text{DCFC}}^\ast \sim j \) and \( \eta_{\text{DCFC}} \sim j \). This is because the bottoming AR does not engage in the cooling production under these operating conditions.

It is seen from Figure 2 that an increase in \( j \) not only decreases \( P^\ast \) but also lowers \( \eta \) in the range of \( j > j_p \). Combining \( P^\ast \) and \( \eta \), one may easily derive the optimum operating region for \( j \)

\[
j_C < j \leq j_p. \tag{27}\]

Accordingly, the optimum operating regions for \( P^\ast \) and \( \eta \) can be, respectively, given by

\[
P_C^\ast < P^\ast \leq P_{\text{max}}^\ast \tag{28}\]

and

\[
\eta_C > \eta \geq \eta_p. \tag{29}\]
Table 1. Parameters used in the modeling [10,34,36,44].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal gas constant, (R) (J mol(^{-1}) K(^{-1}))</td>
<td>8.314</td>
</tr>
<tr>
<td>Operating pressure, (p) (atm)</td>
<td>1.0</td>
</tr>
<tr>
<td>Faraday constant, (F) (C mol(^{-1}))</td>
<td>96,485</td>
</tr>
<tr>
<td>Number of electrons involved per reaction, (n_e)</td>
<td>4</td>
</tr>
<tr>
<td>Height of packed bed anode, (H) (m)</td>
<td>5.0 \times 10(^{-4})</td>
</tr>
<tr>
<td>Diameter of spherical graphite particle, (D_c) (m)</td>
<td>1.0 \times 10(^{-5}) [10]</td>
</tr>
<tr>
<td>Polar plate area of the MCDCFC, (A) (m(^2))</td>
<td>0.04</td>
</tr>
<tr>
<td>Cathode exchange current density, (j_{0e,cat}) (A m(^{-2}))</td>
<td>5.0 \times 10(^{2}) [10]</td>
</tr>
<tr>
<td>Pre-exponential factor of the backward reaction, (K_B) (A m(^{-2}))</td>
<td>5.8 \times 10(^{8}) [34]</td>
</tr>
<tr>
<td>Mass transport coefficient of CO(<em>2), (K</em>{CO_2}) (m s(^{-1}))</td>
<td>3.5 \times 10(^{-2}) [10]</td>
</tr>
<tr>
<td>Temperature activation of the backward reaction, (E_B) (K(^{-1}))</td>
<td>22,175 [34]</td>
</tr>
<tr>
<td>Constant, (r_1)</td>
<td>-1.250 [10]</td>
</tr>
<tr>
<td>Constant, (r_2)</td>
<td>0.375 [10]</td>
</tr>
<tr>
<td>Temperature of environment, (T_0) (K)</td>
<td>303</td>
</tr>
<tr>
<td>Operating temperature, (T) (K)</td>
<td>923</td>
</tr>
<tr>
<td>Internal irreversibility of AR, (I_p)</td>
<td>1.1</td>
</tr>
<tr>
<td>Temperature of cooled space, (T_c) (K)</td>
<td>290</td>
</tr>
<tr>
<td>HTC of the generator, (K_g) (W K(^{-1}) m(^{-2}))</td>
<td>1163 [44]</td>
</tr>
<tr>
<td>HTA of AR, (A_R) (m(^2))</td>
<td>5.0 \times 10(^{-4})</td>
</tr>
<tr>
<td>Constant, (c_1) (W K(^{-1}) m(^{-2}))</td>
<td>5.0 \times 10(^{-2})</td>
</tr>
<tr>
<td>Constant, (c_2) (W K(^{-1}) m(^{-2}))</td>
<td>5.0 \times 10(^{-2})</td>
</tr>
<tr>
<td>Constant, (b_1)</td>
<td>1.0</td>
</tr>
<tr>
<td>Constant, (b_2)</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Figure 2.** Curves of (a) power densities and (b) efficiencies of the MCDCFC, absorption refrigerator, and hybrid system varying with the operating current density of the MCDCFC.
4. Results and Discussion

As shown in Section 2, the performance of the proposed hybrid system depends on a set of design parameters and operating conditions. In this section, comprehensive sensitivity analyses are undertaken to study the impacts of them on the hybrid system performance using the commercial software MATLAB®. The variables used in the following analyses are given in Table 1 unless they are specified otherwise.

4.1. Effects of \( I_r \)

The internal irreversibility \( I_r = \Delta S_o / (\Delta S_b + \Delta S_c) \) is an important parameter that describes the irreversible effects of the mass transfer, friction, eddy and other irreversible effects inside the cyclic working substance of the AR, where \( \Delta S_o \) is the rate of entropy that escapes out of the cyclic working substance, \( \Delta S_b \) and \( \Delta S_c \) are rates of entropy that enter the working substance. \( I_r \) dramatically affects the AR performance and thus it influences the whole hybrid system performance. In the region of \( j_c < j < j_M \), it is observed that both \( P \) and \( \eta \) are increased as \( I_r \) drops, and \( j_p \) moves to a larger value as \( I_r \) is decreased. \( j_c \) keeps invariant while both \( j_M \) and \( \Delta j \) increase as \( I_r \) is decreased.

![Figure 3. Effects of the internal irreversibility \( I_r \) on the hybrid system performance.](image)

The solid line of Figure 3 indicates an unusual case that the internal irreversible effects within the working substance are negligible. In this context, Equations (23) and (24) may be, respectively, simplified into

\[
P = P_{DCFC} + \frac{q_b}{2} \left[ 1 - \frac{a}{\eta_{DCFC}} \right] \left\{ \left[ \left( a + \frac{T_0 - T_f}{C_{q_0}} \right)^2 - 4T_c \left( \frac{1}{(1+B)^2} - \frac{1-T_0/T_c}{C_{q_0}} \right) \right]^{0.5} - \left[ a + \frac{T_0 - T_f}{C_{q_0}} \right] \right\} \tag{30}
\]

and

\[
\eta = \frac{1}{\eta_{DCFC} + \frac{1}{2} \left[ 1 - \eta_{DCFC} + \frac{2F(c_1+c_2)(T_f-T_0)}{A_M} \right] \left\{ \left[ \left( a + \frac{T_0 - T_f}{C_{q_0}} \right)^2 - 4T_c \left( \frac{1}{(1+B)^2} - \frac{1-T_0/T_c}{C_{q_0}} \right) \right]^{0.5} - \left[ a + \frac{T_0 - T_f}{C_{q_0}} \right] \right\} \tag{31}
\]

where \( a_1 = 1 + (T_c - B_1^2 T_0) / (1 + B_1)^2 \), \( B_1 = (\sqrt{B_2} - 1) / (1 + \sqrt{B_1}) \), \( C_1 = (1 + B_1)^2 / (A_k K_1) \) and \( K_1 = K_b / [1 + \sqrt{B_1}]^2 \).

4.2. Effects of Anode Thickness

The anode ohmic overpotential of the MCDCFC takes a large part in the overall overpotentials. The anode thickness dramatically affects the output voltage of the MCDCFC and thus influences the
overall hybrid system performance, as shown in Figure 4. When \( j_c < j < j_M \), both \( P^* \) and \( \eta \) are monotonically decreasing functions of the anode thickness, and \( j_c, j_P, j_M, \Delta j \) and \( j_S \) move to larger values as the anode thickness decreases. In practice, the anode thickness should be designed to be as small as possible.

![Figure 4](image-url)

**Figure 4.** Effects of anode thickness on the hybrid system performance.

### 4.3. Effects of T

The operating temperature \( T \) not only affects the thermodynamic losses within the overall system but also impacts the performances of both MCDCFC and AR. Similar to the effects of the anode thickness, the effects of \( T \) are in the whole range of \( j \), as illustrated in Figure 5. Both \( P^* \) and \( \eta \) are improved as \( T \) is increased. Furthermore, \( j_C, j_P, j_P \) as well as \( \Delta j \) increase as \( T \) is increased. A greater \( T \) not only improves the equilibrium potential but also lessens the activation, concentration and ohmic overpotentials, which is beneficial to improve the MCDCFC performance. On the other hand, a larger \( T \) creates a bigger temperature difference \( (T - T_0) \), which not only improves the AR performance but also results in larger thermodynamic losses. Since the performance improvements in the MCDCFC and AR are larger than the performance reduction caused by the increased thermodynamic losses, and therefore, a larger \( T \) is always preferred.

![Figure 5](image-url)

**Figure 5.** Effects of operating temperature \( T \) on the hybrid system performance.
4.4. Effects of $c_1$ and/or $c_2$

As shown by Equation (20), $c_1$ and $c_2$ are two temperature-independent composite constants. The thermodynamic losses not only relate to the temperature difference ($T - T_0$) but also associate with the temperature-independent parameters $c_1$ and $c_2$. As indicated by Figure 6, both $j_C$ and $j_M$ are increased as $c_1$ and/or $c_2$ increase, while $\Delta j$ is decreased as $c_1$ and/or $c_2$ are increased. As both $P^*_{DCFC}$ and $\eta_{DCFC}$ are not affected by $c_1$ or $c_2$, $P^*$ and $\eta$ are decreased at small operating current densities while increased at large operating current densities as $c_1$ and/or $c_2$ increase.

![Figure 6](image)

**Figure 6.** Effects of the thermodynamic losses related parameters $c_1$ and $c_2$ on the hybrid system performance.

The solid lines in Figure 6 indicate an unusual case that both $q_F$ and $q_L$ are neglected (i.e., $c_1 = 0$ and $c_2 = 0$). In such a case, the AR begins to extract heat from the cooled space when the MCDCFC works, and Equations (20) and (22) can be, respectively, simplified into

$$q_h = -jA(1 - \eta_{MCFC})\Delta h$$

and

$$j > j_C = 0.$$  

4.5. Effects of $p$

The operating pressure $p$ not only impacts the MCDCFC performance but also impacts the waste heat flowing from the MCDCFC to the AR. Similar to the effects of the operating temperature and anode thickness, the effects of $p$ on the hybrid system performance occur in the whole region of $j$, as shown in Figure 7. Both $P^*$ and $\eta$ are improved as $p$ is increased. In addition, $j_C$, $j_M$, $j_P$, $j_S$ and $\Delta j$ are increased with a greater $p$. Though a greater $p$ is beneficial for performance enhancement, it also needs more electric power to compress the incoming air. 1 atm is the usual option, as illustrated by the solid lines in Figure 7.

From the above sensitivity analyses, the most sensitive parameter is the operating temperature, followed by the anode thickness, operating pressure, internal irreversibility of AR and lastly by thermodynamic loss related constants $c_1$ and $c_2$.
5. Conclusions

For performance improvement, a new hybrid system primally consisting of an MCDCFC and an AR is proposed to reuse the exhaust heat from the MCDCFC for cooling production. The irreversible losses within the proposed system are mathematically described. The effective operating current density interval of the MCDCFC that enables the bottoming AR to work is derived. The mathematical expressions to assess the proposed system performance are given under various operating conditions. It is clarified that the MCDCFC performance could be effectively enhanced by integrating with an AR. The maximum attainable power density of the proposed system and its corresponding efficiency are 5.8% and 6.8% larger than that of the single MCDCFC, respectively. The generic performance characteristics are revealed and the optimum operating regions for performance parameters are given. A number of sensitivity analyses are carried out to discuss the effects of the operating current density, temperature and pressure of MCDCFC, the internal irreversibility of AR, and some thermodynamic losses related parameters on the proposed system performance. The obtained results may offer some theoretical guidance for the performance enhancement of an actual MCDCFC through reasonable heat management.

It should be noted that the performance improvement of the proposed hybrid system is not adequately obvious in comparison with the stand-alone MCDCFC. This is because a large amount of exergy destruction occurs in the cooling processes. Compared with the energetic perspective, the exergetic viewpoint provides more useful information which can directly impact the process design and performance improvement. Therefore, the exergy method will be adopted in our further works. In addition, the theoretical model in this paper is comparatively simplified., An actual experimental system needs to be built to check whether or not the modeling results are in good agreement with experimental results in future works. In addition, experimental tests are needed to check the reasonability of the model assumptions.

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