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Article

Divorce Costs and Marital Dissolution in a One-to-One Matching Framework With Nontransferable Utilities

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Abstract: In this paper, we use a two-period one-to-one matching model with incomplete information to examine the effect of changes in divorce costs on marital dissolution. Each individual who has a nontransferable expected utility about the quality of each potential marriage decides whether to marry or to remain single at the beginning of the first period. Individuals married in the first period learn the qualities of their marriages at the beginning of the second period and then decide whether to stay married or to unilaterally divorce. We show that, for any society, there exist matching environments where the probability of the marital dissolution does not reduce divorce costs under gender-optimal matching rules. In such environments, an allocation effect of divorce costs with an ambiguous sign outweighs an incentive effect that is always negative. We also show that these results may also arise under stable matching rules that are not gender optimal.

Keywords: One-to-one matching; stability; marriage dissolution; divorce; incomplete information

JEL Codes: C78, J12

1. Introduction

An unsettled debate in the economics literature is about the role of divorce laws on the probability of marital dissolution, focusing in particular on the big shift in divorce behavior in the United States (US) in the last five decades. Between 1965 and 1980, the divorce rate in the US more than doubled, reaching a rate of around 8 divorces per 1000 adults, before starting to steadily decline alongside the marriage rate. As a reason for this profound change in the divorce rate, a number of empirical works cite that the majority of American states adopted, in the 1970s, looser divorce laws which allowed spouses to unilaterally file for divorce. While Peters found no impact of unilateral divorce laws on divorce rates between 1975 and 1978 [1], Allen showed, using the same data set, significant and permanent effects when he monitored for geographical differences in divorce propensities [2]. Using US state-level panel data from 1968 to 1988 to control state and year-fixed effects, as well as state-specific time trends, Friedberg demonstrated that unilateral divorce laws had led to a 6% higher divorce rate, thereby explaining 17% of the increase in divorces [3]. Later, Gruber found, using the US census data from 1960, 1970, 1980, and 1990, a similar positive effect of the unilateral divorce laws on the stock of divorced women and men [4]. More recently, using longitudinal data on divorce rates in large samples of European countries, González and Viitanen [5] and Kneip and Bauer [6] showed that the results in the literature with significant positive effects of unilateral divorce laws are not pertinent to the US. However, using an extended data set from 1956 to 1998, and adding some variables into the econometric analysis of Friedberg in order to model the dynamic response of divorce explicitly [3], Wolfers showed that the adoption of unilateral divorce laws increased US divorce rates sharply in the first two years following the adoption, but the effects of this legal change decayed within a decade [7,8]. A theoretical explanation (which we will further address in this section) as to why short-term and long-term effects of a policy switch from a mutual consent regime to a unilateral divorce regime may differ was proposed by Rasul [9].

Given the contradictory results in the empirical literature on the effects of unilateral divorce laws on actual divorce rates, we aim to show in this paper whether the stable matching theory, pioneered by Gale and Shapley [10], can be used to offer, on the described empirical irregularity, some insights that the other models could be missing. In our model, individuals are heterogeneous due to the nature of the one-to-one matching framework and they have incomplete information about the nontransferable utility of marriage with each of their potential mates.[1] At the beginning of the first period, each individual decides whether to marry or to remain single. Those who married in the first period learn the utility of their marriage at the beginning of the second period and then decide whether to stay married or to divorce for the rest of their lives. We show that divorce costs affect not only individuals' decision to divorce in the second period but also their first-period decisions to marry, as well as whom to marry. Interestingly, the average probability of marital dissolution in the society is affected by all these three decision channels, which we simply refer to as "the divorce channel," "the marital status channel," and "the marital composition channel," respectively.

We can separate the effect of divorce costs on marital dissolution through the divorce channel, which we call "the incentive effect," since the lower the divorce costs, the higher the incentive for married couples to divorce in the second period. In addition, we can also identify in our model a composite effect of divorce costs operating through the other two channels, which we call "the allocation effect.". While it is certain that with lower divorce costs more individuals may decide to change their marital status from

[1] As argued by Becker *et al.*, in situations where couples can efficiently transfer utilities, the change in divorce costs (or generally divorce laws), can only affect the distribution of welfare within marriages, not the marriage or divorce rates, thanks to the Coase theorem [11].

single to married in the first period, the extent to which any cost effects will be transmitted through the described marital status channel to the average probability of marital dissolution entirely depends on the divorce likelihoods of the new couples relative to those of the existing couples. The same uncertainty is true about the operation of the marital composition channel, which is novel to our study. To isolate the effect of this new channel, we will restrict ourselves to environments where the controlled change in divorce costs induces no impact on the marital status of individuals.

Our results show that the allocative effect of the marital composition channel is indeterminate: even small variations in divorce costs can yield unpredictable, yet significant, changes in the ordinal preferences of individuals over potential mates, consequently changing the identities of the spouses in equilibrium. Not surprisingly, the ambiguity of the allocation effect leads to the ambiguity of the aggregate effect of divorce costs. Thus, it is not true that the average probability of marital dissolution can always be reduced by tighter divorce laws that propose higher divorce costs. Conversely, it is also not true that divorce rates will be higher under looser divorce laws. Indeed, as the main result of our paper shows, we can find, for any society, matching environments and stable matching rules at which the allocation effect of divorce costs outweighs the incentive effect and thus the average probability of marital dissolution is nondecreasing in divorce costs.

The works that most resemble our study by examining the effects of divorce costs on marital dissolution are Bougheas and Georgellis [12]and Rasul [9], which both consider a multi-period search theoretic model. While Bougheas and Georgellis allow the individuals to divorce unilaterally [12], Rasul examines the mutual consent regime, as well, in order to identify the effect of a policy switch to a unilateral divorce regime [9]. In both studies, the divorce channel and the induced incentive effect (which is called the "pipeline effect" in [9]) work in the same unambiguous way as in our model: the lower the divorce costs, the higher the divorce rate of existing couples in the short term.

The marital status channel operates in [9] by the effect of divorce costs on the reservation signal of individuals, for any signal below which they choose to remain single and draw a new potential match in the next period from the pool of available mates. When divorce costs decrease, the reservation signal of individuals increases, implying better matches among those who choose to marry and, consequently, a fall in the divorce rate. Rasul illustrates how much influence this '‘selection effect" has on the divorce rate of the newly formed marriages and how it may dominate, in the long run, the pipeline effect of lower divorce costs, thus leading to higher divorce rates among existing married couples [9].[2] As remarked by Rasul [9], the tension between the pipeline and the selection effect explains the earlier empirical result of Wolfers [8]. On the other hand, the marital composition channel is missing in the search theoretic matching models of Bougheas and Georgellis [12]and Rasul (2006) [9], since all individuals of the same gender are *ex ante* identical and the potential partner of each individual is randomly matched in the pre-marriage stage; thus all possible couples are *ex ante* subject to the same likelihood of divorce.

We should finally note that the use of the stable matching theory on the problem of marriage formation and marital dissolution was earlier considered by Mumcu and Saglam [13]. Their work showed that even

[2] A similar effect potentially exists, though not yet explicitly developed, also in [12], where divorce costs negatively affect the number of marriages formed in the society, with the effect being more pronounced when the signals received in the pre-marriage stage are less informative. In such situations, many individuals marry when divorce costs are low; however, the high noise in the signal implies that the probability of divorce is also high.

small changes in divorce costs that yield insignificant effects on the post-divorce distribution of welfare may lead to wide fluctuations in the marital status of individuals with heterogeneous endowments. Our paper differs from theirs in that utilities in our case are not transferable among individuals. Our work is, to the best of our knowledge, the first attempt to deal with the problem of marital dissolution in a one-to-one matching framework with "nontransferable" utilities. This framework allows us to study the effect of divorce costs in a society with heterogeneous individuals; in fact, it is solely this heterogeneity that activates the part of the allocation effect of divorce costs operating through the marital composition channel, which is currently missing from the existing search theoretic matching literature.

The organization of the paper is as follows: Section 2 introduces the model, Section 3 presents our results, and Section 4 summarizes the findings.

2. The Model

There are two nonempty, finite and disjoint sets of individuals: a set of men, M and a set of women, W. We assume $\min\{|M|, |W|\} \geq 2$. The society is denoted by $N = M \cup W$.

Each individual has two periods to live. The beginning of the first period involves a ''matching'' stage, where individuals enter a marriage market where all matchings between men and women take place. In this stage, the individuals derive their preferences over the potential mates by calculating, the expected utilities of the potential marriages, and according to these preferences, they decide whether to marry or to remain single under a given matching rule. The outcome of this matching rule is a stable matching at which there exists no matched individual who prefers being single to his/her mate and no pair of individuals who are not matched to each other but prefer that they are.

At the beginning of the second period, each married individual enters a ''perfect learning'' stage where the information about the quality of the marriage is acquired and the individual has to decide, as an optimal response to this new information, whether to stay married or to divorce. We assume that the society has a unilateral divorce regime in which either spouse in a couple has the right to unilaterally end the marriage. For simplicity, we assume that a marriage market does not open in the second period; therefore, individuals that choose to become single in the matching stage will remain single in both periods, whereas married individuals that decide on divorce in the perfect learning stage will become single in the second period. Here, we let $\beta > 0$ to denote the common discount factor of all individuals for the second period.[3] Before deriving the equilibrium in this two-period matching model, we will describe a matching environment.

2.1. Matching Environment

A matching is a one-to-one function, $\mu : N \to N$ such that for each $(m, w) \in M \times W$, $\mu(m) = w$ if and only if $\mu(w) = m$; $\mu(m) \notin W$ implies $\mu(m) = m$ and similarly $\mu(w) \notin M$ implies $\mu(w) = w$. If $\mu(m) = w$, then m and w are matched to one another. For any $i \in M \cup W$, i is single if $\mu(i) = i$. Let $\mathcal{M}_N$ denote the set of all matchings for the society N.

[3] Although we do not drop the conventional assumption that individuals discount future periods in their intertemporal choices, we allow β to exceed unity to model situations in which the first period is shorter than the second period, whereby the remaining singlehood period of a divorced individual would be longer than the period in which he or she was married.

For any $i \in N$, we denote by $A_N(i)$ the set of admissible mates in N; *i.e.*, the set of individuals of the opposite gender. Then, $A_N(m) = W$ for all $m \in M$ and $A_N(w) = M$ for all $w \in W$.

We assume that no individual completely knows in the first period the quality (hence the utility) of any potential marriage. Let θ^i_j denote a random variable representing the perception of individual $i \in N$ of the quality of a marriage with individual $j \in A_N(i)$. For all $i \in N$ and $j \in A_N(i)$, we restrict the support of θ^i_j to the interval $[0, 1]$ and we assume that all individuals know these supports. We let $f^i_j(\theta^i_j)$ denote the probability density function representing the beliefs of $i \in N$ about the quality parameter θ^i_j corresponding to a marriage with $j \in A_N(i)$. We assume that the densities $(f^i_j)_{j \in A_N(i), i \in N}$ are all independent. For each f^i_j, we denote by F^i_j the corresponding distribution function. We assume that for each $i \in N$ and $j \in A_N(i)$, the beliefs $\{f^i_j, f^j_i\}$ are mutually known to both i and j.

For each individual, a function U maps the type space $[0, 1]$ to reals, with $U(\theta)$ representing the instantaneous utility the individual derives from a marriage with the perceived quality θ. We set the lowest utility individuals may get from marriage to zero, *i.e.*, $U(0) = 0$. Moreover, we assume that $U(.)$ is strictly increasing (hence the inverse function $U^{-1}(.)$ is defined and strictly increasing, too).

For each individual, we denote by U^s the instantaneous utility of being single and by c the instantaneous divorce costs, which are the same for both men and women.[4] For convenience, we assume $0 < U^s - c < U(1)$; *i.e.*, the instantaneous net utility any individual obtains after getting divorced is positive and below his/her instantaneous utility from a marriage with the highest quality. Here, we also assume that each individual knows the list $(\beta, U(.), U^s, c)$ and that this list is common for all individuals.

For a given society N, a utility-belief structure is described by the list $\Gamma_N = (\beta, U, U^s, (f^i_j)_{j \in A_N(i)})_{i \in N}$, and a matching environment by the list $\Phi = (\Gamma_N, c)$. Define also the notations $\Phi_- = \Gamma_N$ and $\Phi = (\Phi_-, c)$.

2.2. Equilibrium

Below, we will describe the equilibrium in our two-period matching problem, going backwards from the perfect learning stage in the second period to the matching stage in the first period.

Perfect Learning Stage: Individual $i \in N$ who was matched to $j \in A_N(i)$ in the first period learns the private quality parameter θ^i_j associated with his or her marriage and decides whether to stay married or to divorce. The net utility in the second period becomes $U(\theta^i_j)$ if individual i stays married or $U^s - c$ if individual i divorces. So, individual i decides to stay married to individual j if and only if θ^i_j is not below a calculated threshold $\bar{\theta}(c)$, where

$$\bar{\theta}(c) = U^{-1}(U^s - c). \tag{1}$$

We can immediately note that the quality threshold, $\bar{\theta}(c)$, is nonincreasing in divorce costs, c, and nondecreasing in the instantaneous utility of being single, U^s.

While we have considered above the decision of individual i only, a similar decision is also made by individual j. Therefore, individuals i and j remain married in the second period if and only if $\bar{\theta}(c) \leq$

[4] Thanks to the assumed heterogeneity of beliefs of individuals about the quality parameters of potential marriages, we will be able to obtain in Section 3 the allocation effect of a change in divorce costs through the marital composition channel under our simplistic assumptions that the utility functions, as well as divorce costs, do not differ across individuals or genders.

$\min\{\theta_j^i, \theta_i^j\}$. Let $V^i(\theta_j^i, \theta_i^j)$ denote the actual utility of individual i in the second period conditional upon the individuals i and j being married in the first period. Clearly,

$$V^i(\theta_j^i, \theta_i^j) = \begin{cases} U(\theta_j^i) & \text{if } \bar{\theta}(c) \leq \min\{\theta_j^i, \theta_i^j\}, \\ U^s - c & \text{otherwise.} \end{cases}$$

Matching Stage: Each individual $i \in N$ calculates the two-period expected utility, $E[U_j^i(c)]$, derived from a potential match with individual $j \in A_N(i)$. (Note that $E[.]$ is the expectation operator.) In the first period , individual i does not know the actual value of θ_j^i, the quality of a marriage with individual j. Thus, the first period expected utility of individual i is $\int_0^1 d\theta_j^i\, f_j^i(\theta_j^i)U(\theta_j^i)$. On the other hand, the (discounted) expected value of the second-period utility of individual i, as calculated in the first period, is

$$\beta \int_0^1 \int_0^1 d\theta_i^j d\theta_j^i\, f_i^j(\theta_i^j) f_j^i(\theta_j^i) V^i(\theta_j^i, \theta_i^j) =$$

$$\beta \int_0^{\bar{\theta}(c)} \int_0^{\bar{\theta}(c)} d\theta_i^j d\theta_j^i\, f_i^j(\theta_i^j) f_j^i(\theta_j^i)(U^s - c) + \beta \int_0^{\bar{\theta}(c)} \int_{\bar{\theta}(c)}^1 d\theta_i^j d\theta_j^i\, f_i^j(\theta_i^j) f_j^i(\theta_j^i)(U^s - c) +$$

$$\beta \int_{\bar{\theta}(c)}^1 \int_0^{\bar{\theta}(c)} d\theta_i^j d\theta_j^i\, f_i^j(\theta_i^j) f_j^i(\theta_j^i)(U^s - c) + \beta \int_{\bar{\theta}(c)}^1 \int_{\bar{\theta}(c)}^1 d\theta_i^j d\theta_j^i\, f_i^j(\theta_i^j) f_j^i(\theta_j^i) U(\theta_j^i)$$

$$= \beta F_i^j(\bar{\theta}(c)) F_j^i(\bar{\theta}(c))(U^s - c) + \beta F_i^j(\bar{\theta}(c))[1 - F_j^i(\bar{\theta}(c))](U^s - c) +$$

$$\beta[1 - F_i^j(\bar{\theta}(c))] F_j^i(\bar{\theta}(c))(U^s - c) + \beta[1 - F_i^j(\bar{\theta}(c))] \int_{\bar{\theta}(c)}^1 d\theta_j^i\, f_j^i(\theta_j^i) U(\theta_j^i).$$

After rearranging the terms in the last equation, we can add the (discounted) second-period expected utility of individual i (from a marriage with individual j) to his/her first-period expected utility to obtain

$$\begin{aligned} E[U_j^i(c)] &= \int_0^1 d\theta_j^i\, f_j^i(\theta_j^i) U(\theta_j^i) + \\ &\quad \beta \left[(U^s - c) D_{\Phi_-}^{i,j}(c) + [1 - F_i^j(\bar{\theta}(c))] \int_{\bar{\theta}(c)}^1 d\theta_j^i\, f_j^i(\theta_j^i) U(\theta_j^i) \right], \end{aligned} \tag{2}$$

where

$$D_{\Phi_-}^{i,j}(c) = F_j^i(\bar{\theta}(c)) + F_i^j(\bar{\theta}(c)) - F_j^i(\bar{\theta}(c)) F_i^j(\bar{\theta}(c)) \tag{3}$$

denotes the probability that individuals i and j will divorce in the second period.[5]

For notational convenience, we also define $E[U_i^i(c)] = (1 + \beta)U^s$ for all $i \in N$. Thus, we have defined the expected utility $E[U_j^i(c)]$ of each individual i derived from a marriage with $j \in A_N(i) \cup \{i\}$. Apparently, the preferences represented by these expected utilities are complete and transitive. For any matching environment (Φ_-, c), we define the list $E[U(c)] = (E[U_j^i(c)])_{j \in A_N(i) \cup \{i\},\, i \in N}$ as the

[5] The divorce probability in equation (3) can also be obtained from the fact that under a unilateral divorce law, any pair of individuals i and j married in the first period will not divorce in the second period with probability $[1 - F_j^i(\bar{\theta}(c)][1 - F_i^j(\bar{\theta}(c))]$, since f_j^i and f_i^j are independent.

preference (or expected utility) profile of N and we denote the associated marriage market by the triple $(M, W, E[U(c)])$.

We say that a mate $j \in A_N(i) \cup \{i\}$ is acceptable for $i \in N$ at the preference profile $E[U(c)]$ if $E[U^i_j(c)] \geq E[U^i_i(c)]$. (Obviously, individual i is acceptable for i.) Given a marriage market $(M, W, E[U(c)])$, a matching μ is *individually rational* if for all $i \in N$, the mate $\mu(i)$ is acceptable for individual i. For a given matching μ, (m, w) is a *blocking pair* if they are not matched to one another but prefer one another to their matches at μ; *i.e.*, $\mu(m) \neq w$ and $E[U^m_w(c)] > E[U^m_{\mu(m)}(c)]$ and $E[U^w_m(c)] > E[U^w_{\mu(w)}(c)]$. A matching is *stable* if it is individually rational and if there are no blocking pairs.

A well-known theorem by Gale and Shapley shows the existence of a stable matching for every marriage market [10]. We will require that all matchings realized in the first stage of the first period are stable. This completes the description of the matching stage.

2.3. Gender-Optimal Equilibrium Matchings

As stable matchings in a matching environment are not necessarily unique, we will first restrict ourselves for most of our results to a special selection of equilibrium, namely gender-optimal stable matchings (and the rules that select them), which were first introduced and studied by Gale and Shapley [10]. However, we will relax this restriction at the end of Section 3 to show that our main results may hold true under other stable matching rules, as well.

We say that men strictly prefer μ to μ' if all men like μ at least as well as μ', with at least one man strictly preferring μ to μ'; *i.e.*, $E[U^m_{\mu(m)}(c)] \geq E[U^m_{\mu'(m)}(c)]$ for all $m \in M$, with the inequality being strict for some $m \in M$. We also say that men weakly prefer μ to μ' if either all men strictly prefer μ to μ' or all men are indifferent between μ and μ', *i.e.* $E[U^m_{\mu(m)}(c)] = E[U^m_{\mu'(m)}(c)]$ for all $m \in M$. Similarly, we define strict and weak preference relations for women.

For a given marriage market $(M, W, E[U(c)])$, a stable matching μ is said to be M*-optimal* if men weakly prefer μ to any other stable matching. Similarly, a stable matching ν is said to be W*-optimal* if women weakly prefer ν to any other stable matching. We denote the M-optimal and W-optimal stable matchings for the matching environment Φ by μ^M_Φ and μ^W_Φ, respectively. We also say that a stable matching is gender-optimal if it is M-optimal or W-optimal.

We know that in any matching environment Φ where men and women have strict preferences, the ''Deferred Acceptance Algorithm'' by Gale and Shapley produces μ^M_Φ if men propose to women and μ^W_Φ if women propose to men [10].[6]

Now, for any society we define a matching rule as a function from the set of all matching environments to the set of all matchings. For any matching rule $\varphi(.)$ and any matching environment Φ, we denote by

[6] The Deferred Acceptance Algorithm with men proposing is simply as follows: In the initial step, each man proposes to the most preferred woman among all acceptable women. In the responding stage of this step, each woman rejects all proposals but the one from the most preferred acceptable man who proposed and gets engaged to him. At any step $k \geq 2$, any man who was not engaged in the previous step deletes the woman who rejected him in step $k-1$ from his list of acceptable women and proposes to his favorite woman, if any, in the updated list. Each woman receiving proposals gets engaged to the most preferred acceptable man among the group consisting of the man to whom she may have engaged in step $k-1$ and men who have just proposed in step k, and rejects all other members of this group. The algorithm stops after any step in which no man is rejected. By changing the role of men and women in the above procedure, we can similarly define the Deferred Acceptance Algorithm with women proposing.

$\varphi(\Phi)$ the matching selected at the matching environment Φ. We say that a matching rule $\varphi(.)$ is the M-optimal matching rule if it selects the M-optimal stable matching at all matching environments; *i.e.*, $\varphi(\Phi) = \mu_\Phi^M$ for all Φ. We also define the W-optimal matching rule, analogously. We denote by $\varphi^M(.)$ and $\varphi^W(.)$ the M-optimal and W-optimal matching rule, respectively.

It must be pointed out that the gender-optimal equilibrium matchings need not be perfectly (positive) assortative in the sense that the married partners have similar characteristics (or derive similar qualities from a marriage to each other). The reason is that under incomplete information, no equilibrium matching – whether it be gender-optimal or not – can be based on any observable characteristics of individuals. In fact, equilibrium matchings in our setup are only determined by the preference orderings of individuals over potential mates, while these orderings depend on the beliefs of individuals over the unobservable qualities of the potential marriages. It is well known from Knuth [14] that for preference orderings under which gender-optimal equilibrium matchings differ, the M-optimal stable matching is the least preferred stable matching for women, and oppositely, the W-optimal stable matching is the least preferred stable matching for men. Thus, the mutual satisfaction of the two sides of the matching problem is not always possible without sacrificing the stability of matchings.[7]

Before giving our results in the next section, we finally define, using (3), the average probability of marital dissolution in the society N for the matching environment (Φ_-, c) under the matching rule $\varphi(.)$ as

$$D^N_{\Phi_-}(c, \varphi) = \frac{1}{|W^\varphi_{\Phi_-}(c)|} \sum_{w \in W^\varphi_{\Phi_-}(c)} D^{\varphi(\Phi)(w),w}_{\Phi_-}(c), \tag{4}$$

where $W^\varphi_{\Phi_-}(c) = \{w \in W \mid (w, w) \notin \varphi(\Phi)\}$ is the set of women who were not single in the first period. Likewise, we define (abusing the above notation) the average probability of marital dissolution in the society N for the matching environment (Φ_-, c) under the matching μ as

$$D^N_{\Phi_-}(c, \mu) = \frac{1}{|W^\mu_{\Phi_-}(c)|} \sum_{w \in W^\mu_{\Phi_-}(c)} D^{\mu(w),w}_{\Phi_-}(c), \tag{5}$$

where $W^\mu_{\Phi_-}(c) = \{w \in W \mid (w, w) \notin \mu\}$.

3. Results

Below, we first observe that the probability of marital dissolution under any given matching is nonincreasing in divorce costs.

Proposition 1. *For any society N, any matching environment (Φ_-, c), and any matching $\mu \in \mathcal{M}_N$, we have $D^N_{\Phi_-}(\hat{c}, \mu) \leq D^N_{\Phi_-}(c, \mu)$ if $\hat{c} > c$.*

[7] Nonetheless, one may argue that gender-optimal equilibrium matchings are *approximately* (positive) assortative since these matchings respect the preferences of men and women as much as possible given the sequential order of moves in the Gale and Shapley algorithm. If, for instance, women move first in this algorithm, each unengaged woman at any step will propose to the *best* man in her preference ordering and he will engage to the *best* proposed woman in his preference ordering and reject all others.

The above proposition follows from (1), (3) and (5) together with the facts that $\partial\bar{\theta}(c)/\partial c \leq 0$ and that $dF_j^i(\theta)/d\theta \geq 0$ for all $i \in N$ and $j \in A_N(i)$. We call this effect, which also works in search theoretic models, the ''incentive effect'' of divorce costs; since the higher the divorce costs, the higher the incentive for the marriages that were formed in the first period to continue into the second period as well. Indeed, divorcing becomes totally unattractive for the whole society when divorce costs are sufficiently high; *i.e.*, for all N and $\mu \in \mathcal{M}_N$ we have $\lim_{c \uparrow U^s} D^N_{\Phi_-}(c, \mu) = 0$.

The incentive effect is only a partial specification of the relation between divorce costs and marital dissolution, since given any non-constant matching rule $\varphi(.)$ and any matching environment (Φ_-, c), a change in divorce costs c can also change equilibrium matchings $\varphi(\Phi_-, c)$, which we have fixed in the above proposition.[8] This dependence is evident from equations (1) and (2), since a sufficiently large change in divorce costs can change the ''ordinal'' preferences of the individuals over their potential mates because of the heterogeneity of the individuals' beliefs and utilities. Naturally, impacts of divorce costs on the equilibrium matchings are also transmitted to the average probability of divorce. We refer to this second effect of divorce costs on the marital dissolution as the "allocation effect." Below, we will formally define these two effects.

Given any society N and any matching environment (Φ_-, c), the change in the average probability of marital dissolution under a matching rule $\varphi(.)$ when the cost value is changed from c to $\hat{c}$ is given by

$$D^N_{\Phi_-}(\hat{c}, \varphi(\Phi_-, \hat{c})) - D^N_{\Phi_-}(c, \varphi(\Phi_-, c)) = \Delta^A_\varphi(\hat{c}, c, \Phi_-) + \Delta^I_\varphi(\hat{c}, c, \Phi_-), \tag{6}$$

where

$$\Delta^A_\varphi(\hat{c}, c, \Phi_-) = D^N_{\Phi_-}(\hat{c}, \varphi(\Phi_-, \hat{c})) - D^N_{\Phi_-}(\hat{c}, \varphi(\Phi_-, c)) \tag{7}$$

is the change in the average probability of marital dissolution for the matching environment $(\Phi_-, \hat{c})$ due to a change in the matching from $\varphi(\Phi_-, c)$ to $\varphi(\Phi_-, \hat{c})$, and

$$\Delta^I_\varphi(\hat{c}, c, \Phi_-) = D^N_{\Phi_-}(\hat{c}, \varphi(\Phi_-, c)) - D^N_{\Phi_-}(c, \varphi(\Phi_-, c)) \tag{8}$$

is the change in the average probability of marital dissolution under the matching $\varphi(\Phi_-, c)$ due to a change in the matching environment from (Φ_-, c) to $(\Phi_-, \hat{c})$. We call the terms $\Delta^A_\varphi(\hat{c}, c, \Phi_-)$ and $\Delta^I_\varphi(\hat{c}, c, \Phi_-)$, respectively the allocation and the incentive effect, due to the change in divorce costs from c to $\hat{c}$.

By Proposition 1, we know that the incentive effect is always negative, *i.e.*, $\Delta^I_\varphi(\hat{c}, c, \Phi_-) < 0$ if $\hat{c} > c$. In fact, this is an already well-known result. (See, for example, Bougheas and Georgellis [12] and Rasul [9]). What we aim to accomplish in the rest of this paper is to find the effect of the marital composition channel on the probability of marital dissolution. To isolate the effect of this new channel, we will consider environments where the controlled change in divorce costs will not affect marital status of individuals in the society. Thus, the allocation effect of divorce costs will arise only through the marital composition channel. We will establish that, unlike the incentive effect, this new effect has an indeterminate sign, as will be evident from the comparison of the results in Examples 1 and 2. Below, we will first show that for any gender-optimal matching rule there exists a matching environment in which the allocation and incentive effect may work in opposite directions.

[8] A matching rule is constant if it selects the same matching at all matching environments.

Example 1. Consider first the M-optimal matching rule $\varphi^M(.)$. Consider a society N involving $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$ with Φ_- given by $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, $U^s = 0.15036$, and

$$f_{w_1}^{m_1} = f_{m_1}^{w_1} = f_{m_2}^{w_1} = f_{m_1}^{w_2} = f_{w_2}^{m_2} = f_{m_2}^{w_2} = f^a,$$

$$f_{w_2}^{m_1} = f_{w_1}^{m_2} = f^b,$$

where

$$f^a(\theta) = \begin{cases} 0.1380 & \text{if } \theta \in [0.200, 0.269], \\ 1.0639 & \text{otherwise,} \end{cases}$$

and

$$f^b(\theta) = \begin{cases} 0.1429 & \text{if } \theta \in [0.200, 0.270], \\ 1.0645 & \text{otherwise.} \end{cases}$$

Let $c = 0.01286$. We compute $\bar{\theta}(c) = 0.2701$. Then, the expected utility profile $E[U(c)]$ is calculated as follows:

$E[U_j^i(c)]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.299313	0.299314
m_2	—	0.299216	0.299314	0.299313
w_1	0.299313	0.299306	0.299216	—
w_2	0.299306	0.299313	—	0.299216

From the above table, it follows that

$$\varphi^M(\Phi_-, c) = \begin{pmatrix} m_1 & m_2 \\ w_2 & w_1 \end{pmatrix},$$

i.e., m_1 and m_2 are matched to w_2 and w_1, respectively. We then compute $D_{\Phi_-}^N(c, \varphi^M(\Phi_-, c)) = 0.396624$. It is interesting to note that the equilibrium matching $\varphi^M(\Phi_-, c)$ does not exhibit positive assortative mating in terms of the individuals' preferences over stable matchings, since the W-optimal stable matching swaps the mates of the two men under $\varphi^M(\Phi_-, c)$; *i.e.*,

$$\varphi^W(\Phi_-, c) = \begin{pmatrix} m_1 & m_2 \\ w_1 & w_2 \end{pmatrix}.$$

Now, consider the matching environment $(\Phi_-, \hat{c})$ with $\hat{c} = 0.01291$. We compute $\bar{\theta}(\hat{c}) = 0.2699$. Then, the expected utility profile $E[U(\hat{c})]$ is calculated as follows:

$E[U_j^i(\hat{c})]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.299296	0.299293
m_2	—	0.299216	0.299293	0.299296
w_1	0.299296	0.299290	0.299216	—
w_2	0.299290	0.299296	—	0.299216

It follows that

$$\varphi^M(\Phi_-, \hat{c}) = \begin{pmatrix} m_1 & m_2 \\ w_1 & w_2 \end{pmatrix}.$$

Here, we should immediately note that $\varphi^W(\Phi_-, \hat{c}) = \varphi^M(\Phi_-, \hat{c})$, that is the increase in divorce costs from c to $\hat{c}$ leads to a new stable matching that exhibits positive assortative mating in terms of preferences over stable matchings. Now, we calculate $D^N_{\Phi_-}(\hat{c}, \varphi^M(\Phi_-, \hat{c})) = 0.396656$, which is higher than $D^N_{\Phi_-}(c, \varphi^M(\Phi_-, c))$. We also compute $\Delta^A_\varphi(\hat{c}, c, \Phi_-) = 0.000280$ and $\Delta^I_\varphi(\hat{c}, c, \Phi_-) = -0.000248$.

Finally, by interchanging the names of men and women in the above society, we can simply get the same result for the W-optimal matching rule $\varphi^W(.)$, as well. □

The allocation effect (obtained exclusively through the marital composition channel) for the above example is positive and it dominates the incentive effect. We thus get the increase in the average probability of divorce. It may seem surprising that even though the rise in divorce costs in the above example changes an initial M-optimal stable matching – which is the least preferred by women among all stable matchings – to a new M-optimal stable matching – which is the most preferred stable matching by women – it leads to an increase in the probability of marital dissolution. The underlying reason for this result is that equilibrium matchings in our setup are formed according to the preferences of individuals over prospective mates, represented by the expected utilities of individuals derived from the corresponding marriages. These expected utilities, which are calculated using the beliefs of individuals (represented by some probability density functions) about the unobservable qualities of the potential marriages, depend on all possible moments of the associated probability distributions, while the probability of divorce between couples only depends on a single aspect of these beliefs, namely the size of the lower tail of the couples' probability distributions at the common quality threshold. Since this particular aspect of the individuals' beliefs does not summarize all the relevant information in a probability distribution, we inevitably observe an ambiguous relation between individuals' preferences over equilibrium matchings and the probabilities of divorce at these matchings. These points should be more apparent after examining Table 1, which reports divorce probabilities in Example 1.

Table 1. Divorce Probabilities in Example 1 (Probabilities for Matched Pairs in Boldface).

	$D^{i,j}_{\Phi_-}(c)$		$D^{i,j}_{\Phi_-}(\hat{c})$	
	w_1	w_2	w_1	w_2
m_1	0.396980	**0.396624**	**0.396656**	0.396376
m_2	**0.396624**	0.397013	0.396376	**0.396656**

The above table shows that although the divorce probability decreased for all potential couples in the society in response to an increase in divorce costs from $c = 0.01286$ to $\hat{c} = 0.01291$, the change of matchings from $\varphi^M(\Phi_-, c)$ to $\varphi^M(\Phi_-, \hat{c})$ caused the divorce probabilities of the matched couples to rise. Please note that if the couples (m_1, w_2) and (m_2, w_1), which are married when the cost of divorce is c, remained married when the cost of divorce is $\hat{c}$, the divorce probability of each couple would fall from 0.396624 to 0.396376 (the incentive effect). However, the matchings did not remain the same when divorce costs were increased. The reason is that the preference orderings of both men changed completely, despite the rise in the divorce cost being extremely small. At the higher cost level $\hat{c}$, the married couples turn out to be (m_1, w_1) and (m_2, w_2), each of which faces a divorce probability of 0.396656 instead of 0.396376 (the allocation effect). As the allocation effect dominates the incentive effect, the average probability of divorce becomes higher under the increased divorce costs. Examining the setup in Example 1, we may further note that the increase in divorce costs slightly changes the quality threshold (the sole determinant of divorce probabilities), thereby reducing it from the level $\bar{\theta}(c) = 0.2701$ to $\bar{\theta}(\hat{c}) = 0.2699$ only. At both matching environments (Φ_-, c) and $(\Phi_-, \hat{c})$, the beliefs of the individuals about the quality of potential marriages are such that the cumulative probabilities between the two quality thresholds are almost zero, *i.e.*, $F^a(0.2701) - F^a(0.2699) = 0.0002$ and $F^b(0.2701) - F^b(0.2699) = 0.0002$; therefore, for each potential couple, the probability of divorce, as defined in equation (3), falls only insignificantly in response to a small decrease in the quality thresholds (the incentive effect). Consequently, the incentive effect that is always negative becomes inadequate in size to outweigh the allocation effect that turns out to be positive for this example.

Our next proposition below generalizes Example 1 to state that for any gender-optimal matching rule and for any society there exists a matching environment where increases in divorce costs may raise the average probability of marital dissolution.

Proposition 2. *For any society N and a given gender-optimal matching rule $\varphi^X(.)$ where $X \in \{M, W\}$, there exists a matching environment (Φ_-, c) and divorce costs $\hat{c} > c$ such that we have $D^N_{\Phi_-}(\hat{c}, \varphi^X(\Phi_-, \hat{c})) > D^N_{\Phi_-}(c, \varphi^X(\Phi_-, c))$.*

Proof. Consider first the M-optimal matching rule $\varphi^M(.)$. Consider the society N with the list Φ_- in Example 1. Let $N' = M' \cup W'$ be any larger society such that $M' \supseteq M$ and $W' \supseteq W$ with at least one of the inclusions being strict. Consider the two cost values $c = 0.01286$ and $\hat{c} = 0.01291$. Consider the list $\Phi'_- = (\beta, U, U^s, (f_j'^i)_{j \in A_{N'}(i)})_{i \in N'}$ such that $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, and $U^s = 0.15036$. (Thus, we have extended all utility specifications for N in Example 1 to N'.) By this construction, we have $\bar{\theta}(c) = 0.2701$ and $\bar{\theta}(\hat{c}) = 0.2699$. Now, let $f_j'^i = f_j^i$ if $i \in N$ and $j \in A_N(i)$. Moreover, for all $(i, j) \in M' \times W'$ such that $(i, j) \notin M \times W$, let $f_j'^i = f^c$, where

$$f^c(\theta) = \begin{cases} 0 & \text{if } \theta \in [0.200, 0.260], \\ 1.0638 & \text{otherwise.} \end{cases}$$

Then, we have $E[U_j^i(c)] = 0.299209$ and $E[U_j^i(\hat{c})] = 0.299191$ for all $(i, j) \in M' \times W'$ such that $(i, j) \notin M \times W$. Since we also have $E[U_i^i(c)] = E[U_i^i(\hat{c})] = 0.299216$ for all $i \in N' \backslash N$, it follows that under both c and $\hat{c}$, all individuals in $N' \backslash N$ will decide to remain single in both periods. Thus, we have $\varphi^{M'}(\Phi'_-, c)(i) = \varphi^M(\Phi_-, c)(i)$ and $\varphi^{M'}(\Phi'_-, \hat{c})(i) = \varphi^M(\Phi_-, \hat{c})(i)$ for all $i \in N$ (with $\varphi^M(\Phi_-, c)$ and $\varphi^M(\Phi_-, \hat{c})$ defined in Example 1), and $\varphi^{M'}(\Phi'_-, c)(i) = \varphi^{M'}(\Phi'_-, \hat{c})(i) = \{i\}$ for all $i \in N' \backslash N$. It then follows that $D^{N'}_{\Phi'_-}(c, \varphi^{M'}(\Phi'_-, c)) = D^N_{\Phi_-}(c, \varphi^M(\Phi_-, c))$ and $D^{N'}_{\Phi'_-}(\hat{c}, \varphi^{M'}(\Phi'_-, \hat{c})) = D^N_{\Phi_-}(\hat{c}, \varphi^M(\Phi_-, \hat{c}))$. Finally, by interchanging the names of all men and women above, we can simply get the desired result for the W-optimal matching rule $\varphi^W(.)$, as well. $\square$

The above proposition shows that there are environments in which a policy change that decreases divorce costs, which are clearly a pure deadweight loss, is Pareto improving. However, the following example shows that there are also matching environments in which the opposite conclusion is true; *i.e.*, an increase in divorce costs reduces the average probability of marital dissolution. We show that in such environments the allocation effect (through the marital composition channel) turns out to be negative under gender-optimal matching rules.

Example 2. Consider first the M-optimal matching rule $\varphi^M(.)$. Consider a society N involving $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$ with Φ_- given by $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, $U^s = 0.15036$, and

$$f_{w_1}^{m_1} = f_{m_1}^{w_1} = f_{w_2}^{m_2} = f_{m_1}^{w_2} = f^a,$$

$$f_{w_2}^{m_1} = f_{w_1}^{m_2} = f_{m_2}^{w_1} = f_{m_2}^{w_2} = f^b,$$

where

$$f^a(\theta) = \begin{cases} 10 & \text{if } \theta \in [0.20, 0.22], \\ 0.8163 & \text{otherwise,} \end{cases}$$

and

$$f^b(\theta) = \begin{cases} 6.25 & \text{if } \theta \in [0.20, 0.24], \\ 0.7813 & \text{otherwise.} \end{cases}$$

Let $c = 0.01286$. We compute $\bar{\theta}(c) = 0.2701$. Then, the expected utility profile $E[U(c)]$ is calculated as follows:

$E[U_j^i(c)]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.307935	0.307887
m_2	—	0.299216	0.308197	0.308259
w_1	0.307935	0.308197	0.299216	—
w_2	0.308259	0.307887	—	0.299216

From the above table, it follows that

$$\varphi^M(\Phi_-, c) = \begin{pmatrix} m_1 & m_2 \\ w_1 & w_2 \end{pmatrix}.$$

We then compute $D_{\Phi_-}^N(c, \varphi^M(\Phi_-, c)) = 0.652596$.

Note that the equilibrium matching $\varphi^M(\Phi_-, c)$ does not exhibit positive assortative mating, since the optimal stable matching from the viewpoint of women is

$$\varphi^W(\Phi_-, c) = \begin{pmatrix} m_1 & m_2 \\ w_2 & w_1 \end{pmatrix}.$$

Now, consider the matching environment $(\Phi_-, \hat{c})$ such that $\hat{c} = 0.02286$. We compute $\bar{\theta}(\hat{c}) = 0.2322$. Then, the expected utility profile $E[U(\hat{c})]$ is calculated as follows:

$E[U^i_j(\hat{c})]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.302274	0.303467
m_2	—	0.299216	0.303424	0.302192
w_1	0.302274	0.303424	0.299216	—
w_2	0.302192	0.303467	—	0.299216

It follows that

$$\varphi^M(\Phi_-, \hat{c}) = \begin{pmatrix} m_1 & m_2 \\ w_2 & w_1 \end{pmatrix}.$$

Here, we have $\varphi^W(\Phi_-, \hat{c}) = \varphi^M(\Phi_-, \hat{c})$, that is the increase in divorce costs from c to $\hat{c}$ leads the society to an equilibrium that exhibits positive assortative mating. We can now calculate $D^N_{\Phi_-}(\hat{c}, \varphi^M(\Phi_-, c)) = 0.592446$, $\Delta^A_\mu(\hat{c}, c, \Phi_-) = -0.009866$ and $\Delta^I_\mu(\hat{c}, c, \Phi_-) = -0.050284$.

Finally, by interchanging the names of men and women in the above society, we can simply get the same result for the W-optimal matching rule $\varphi^W(.)$, as well. □

Interestingly, the rise in divorce costs changes in Example 2, like in the previous example, a non-positive assortative mating to a positive assortative mating in terms of preferences over stable matchings. Despite this striking similarity, the sign of the allocation effect is different in the two examples, because of the nontrivial differences in the heterogeneous beliefs of the individuals.

Table 2. Divorce Probabilities in Example 2 (Probabilities for Matched Pairs in Boldface).

	$D^{i,j}_{\Phi_-}(c)$		$D^{i,j}_{\Phi_-}(\hat{c})$	
	w_1	w_2	w_1	w_2
m_1	**0.644968**	0.660223	0.607185	**0.597440**
m_2	0,674823	**0.660223**	**0.587452**	0.597440

Table 2 above shows that the probability of divorce decreased for all potential couples in the society in response to a rise of the cost of divorce from $c = 0.01286$ to $\hat{c} = 0.02286$. If the couples (m_1, w_1) and (m_2, w_2), which are married when the cost of divorce is c, remained to be married when the cost of divorce is $\hat{c}$, the divorce probability would fall from 0.644968 to 0.607185 for the first couple, from 0.660223 to 0.597440 for the second couple, and from 0.652596 to 0.602312 for the society as a whole (the incentive effect). However, at the cost level $\hat{c}$, the married couples change to (m_1, w_2) and (m_2, w_1), which face the divorce probabilities 0.597440 and 0.587452, respectively. Thus, at higher divorce costs, the society is facing an average divorce probability of 0.592446 instead of 0.602312 (the allocation effect). Since the allocation effect for this example is negative like the incentive effect, the average probability of divorce becomes lower under the increased divorce costs. We should also note that, unlike in Example 1, the increase in divorce costs changes the quality threshold significantly, thereby reducing it from the level $\bar{\theta}(c) = 0.2701$ to $\bar{\theta}(\hat{c}) = 0.2322$. Moreover, at both matching environments (Φ_-, c) and $(\Phi_-, \hat{c})$, the beliefs of the individuals about the quality of potential marriages are such that the change in the cumulative probabilities between the two quality thresholds is not negligible (*i.e.*, $F^a(0.2701) -$

$F^a(0.2322) = 0.0309$ and $F^b(0.2701) - F^b(0.2322) = 0.0296$), therefore for each potential couple, the probability of divorce, as defined in equation (3), falls significantly (the incentive effect). Apparently, the observed negative sign of the allocation effect in Example 2 has sufficed for the decrease in the average probability of divorce. However, such a result does not necessarily hinge upon the sign of the allocation effect. In fact, what is failing in the case of Example 1 as to the surprising effect of divorce costs on the average probability of divorce is not that the sign of the allocation effect is positive, but rather that the reduction in divorce costs is not sufficiently high to ensure that the induced incentive effect will be adequately large to outweigh a positive allocation effect.

We will finally discuss whether our main results in Example 1 and Proposition 2 may still hold when the matchings in the society are determined by a stable matching rule that is not gender optimal. Since such rules have not been explicitly studied in the one-to-one matching literature, we need to construct one that will suffice for our purpose. Nevertheless, we will first give some helpful structures, adapted from Roth and Sotomayor [15].

For each matching environment (Φ_{-}, c) and each pair of matchings μ and μ' in $\mathcal{M}_N$, we define the join, $\mu \vee_M \mu'$, as the function that assigns each man to his mate at the most preferred of these matchings, and the meet, $\mu \wedge_M \mu'$ as the function that assigns each man to his mate at the least preferred of the two matchings; *i.e.*,

$$\mu \vee_M \mu'(m) = \begin{cases} \mu(m) & \text{if } E[U^m_{\mu(m)}(c)] > E[U^m_{\mu'(m)}(c)], \\ \mu'(m) & \text{otherwise,} \end{cases}$$

and

$$\mu \wedge_M \mu'(m) = \begin{cases} \mu'(m) & \text{if } E[U^m_{\mu(m)}(c)] > E[U^m_{\mu'(m)}(c)], \\ \mu(m) & \text{otherwise.} \end{cases}$$

Now, consider any society involving $M = \{m_1, m_2, \ldots, m_{n_m}\}$ and $W = \{w_1, w_2, \ldots, w_{n_w}\}$ with $n_m, n_w \geq 2$, and any matching environment Φ. For any pair of matchings μ' and μ'', define a number $i(\mu', \mu'')$ such that $i(\mu', \mu'') = 0$ if there exists no integer $i^* \in \{1, 2, \ldots, n_m\}$ such that $E[U^{m_i}_{\mu'(m_i)}(c)] \geq E[U^{m_i}_{\mu''(m_i)}(c)]$ for all $i \in \{1, 2, \ldots, i^*\}$, and $i(\mu', \mu'') = \max\{i^* \in \{1, 2, \ldots, n_m\} \mid E[U^{m_i}_{\mu''(m_i)}(c)] \geq E[U^{m_i}_{\mu'(m_i)}(c)]$ for all $i \leq i^*\}$, otherwise. Clearly, for any pair of matchings μ' and μ'', we have $i(\mu', \mu'') \neq i(\mu'', \mu')$ if $\mu' \neq \mu''$. Now, consider the matching rule, denoted by $\hat{\varphi}(.)$: If there exists a unique pair of stable matchings μ' and μ'' such that $\mu \neq \mu'$, $\varphi^M(\Phi) \notin \{\mu', \mu''\}$ and $\mu' \vee_M \mu'' = \varphi^M(\Phi)$, then let

$$\hat{\varphi}(\Phi) = \begin{cases} \mu' & \text{if } i(\mu', \mu'') > i(\mu'', \mu'), \\ \mu'' & \text{otherwise,} \end{cases}$$

and otherwise let $\hat{\varphi}(\Phi) = \varphi^M(\Phi)$. Below, we will show that the result in Example 1 is valid under the matching rule $\hat{\varphi}(.)$, as well.

Example 3. Consider a society N involving $M = \{m_1, m_2, m_3, m_4\}$ and $W = \{w_1, w_2, w_3, w_4\}$ with Φ_- given by $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, $U^s = 0.15036$, and

$$f_{w_1}^{m_1} = f_{m_1}^{w_1} = f_{m_2}^{w_1} = f_{m_1}^{w_2} = f_{w_2}^{m_2} = f_{m_2}^{w_2} = f_{w_3}^{m_3} = f_{m_3}^{w_3} = f_{m_4}^{w_3} = f_{m_3}^{w_4} = f_{w_4}^{m_4} = f_{m_4}^{w_4} = f^a,$$

$$f_{w_2}^{m_1} = f_{w_1}^{m_2} = f_{w_4}^{m_3} = f_{w_3}^{m_4} = f^b,$$

$$f_{w_3}^{m_1} = f_{m_1}^{w_3} = f_{w_4}^{m_1} = f_{m_1}^{w_4} = f_{w_3}^{m_2} = f_{m_2}^{w_3} = f_{w_4}^{m_2} = f_{m_2}^{w_4} = f^c,$$

where

$$f^a(\theta) = \begin{cases} 0.1380 & \text{if } \theta \in [0.200, 0.269], \\ 1.0639 & \text{otherwise}, \end{cases}$$

$$f^b(\theta) = \begin{cases} 0.1429 & \text{if } \theta \in [0.200, 0.270], \\ 1.0645 & \text{otherwise}, \end{cases}$$

and

$$f^c(\theta) = \begin{cases} 0.0870 & \text{if } \theta \in [0.200, 0.269], \\ 1.0677 & \text{otherwise}. \end{cases}$$

Let $c = 0.01286$. We compute $\bar{\theta}(c) = 0.2701$. Then, the expected utility profile $E[U(c)]$ is calculated as follows:

$E[U_j^i(c)]$ (i: rows; j: columns)

	m_1	m_2	m_3	m_4	w_1	w_2	w_3	w_4
m_1	0.299216	—	—	—	0.299313	0.299314	0.299199	0.299199
m_2	—	0.299216	—	—	0.299314	0.299313	0.299199	0.299199
m_3	—	—	0.299216	—	0.299199	0.299199	0.299313	0.299314
m_4	—	—	—	0.299216	0.299199	0.299199	0.299314	0.299313
w_1	0.299313	0.299306	0.299199	0.299199	0.299216	—	—	—
w_2	0.299306	0.299313	0.299199	0.299199	—	0.299216	—	—
w_3	0.299199	0.299199	0.299313	0.299306	—	—	0.299216	—
w_4	0.299199	0.299199	0.299306	0.299313	—	—	—	0.299216

For the above environment, there are only four stable matchings given by

$$\mu_1 = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_2 & w_3 & w_4 \end{pmatrix}, \; \mu_2 = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ w_2 & w_1 & w_3 & w_4 \end{pmatrix},$$

$$\mu_3 = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_2 & w_4 & w_3 \end{pmatrix}, \; \mu_4 = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ w_2 & w_1 & w_4 & w_3 \end{pmatrix}.$$

It is easy to check that $\varphi^M(\Phi_-, c) = \mu_4$ and $\mu_2 \vee_M \mu_3 = \varphi^M(\Phi_-, c)$. Moreover, $i(\mu_2, \mu_3) = 2 > 0 = i(\mu_3, \mu_2)$. Therefore, $\hat{\varphi}(\Phi_-, c) = \mu_2$. We then compute $D_{\Phi_-}^N(c, \hat{\varphi}(\Phi_-, c)) = 0.396624$.

Now, consider the matching environment $(\Phi_-, \hat{c})$ with $\hat{c} = 0.01291$. We compute $\bar{\theta}(\hat{c}) = 0.2699$. Then, the expected utility profile $E[U(\hat{c})]$ is calculated as follows:

$E[U_j^i(c)]$ (i: rows; j: columns)

	m_1	m_2	m_3	m_4	w_1	w_2	w_3	w_4
m_1	0.299216	—	—	—	0.299296	0.299293	0.299199	0.299199
m_2	—	0.299216	—	—	0.299293	0.299296	0.299199	0.299199
m_3	—	—	0.299216	—	0.299199	0.299199	0.299296	0.299293
m_4	—	—	—	0.299216	0.299199	0.299199	0.299293	0.299296
w_1	0.299296	0.299290	0.299199	0.299199	0.299216	—	—	—
w_2	0.299290	0.299296	0.299199	0.299199	—	0.299216	—	—
w_3	0.299199	0.299199	0.299296	0.299290	—	—	0.299216	—
w_4	0.299199	0.299199	0.299290	0.299296	—	—	—	0.299216

It is easy to check that the only stable matching at this preference profile is

$$\varphi^M(\Phi_-, \hat{c}) = \varphi^W(\Phi_-, \hat{c}) = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_2 & w_3 & w_4 \end{pmatrix}.$$

By the definition of $\hat{\varphi}(.)$, we have $\hat{\varphi}(\Phi_-, \hat{c}) = \varphi^M(\Phi_-, \hat{c})$. Now, we can calculate $D^N_{\Phi_-}(\hat{c}, \hat{\varphi}(\Phi_-, \hat{c})) = 0.396656$, which is higher than $D^N_{\Phi_-}(c, \varphi^M(\Phi_-, c))$. Finally, we compute $\Delta^A_\mu(\hat{c}, c, \Phi_-) = 0.000279$ and $\Delta^I_\mu(\hat{c}, c, \Phi_-) = -0.000248$. □

As should be obvious from the proof of Proposition 2, one can generalize Example 3 such that given the described matching rule $\hat{\varphi}(.)$ and an arbitrary society involving at least four men and four women, there always exists a matching environment where increases in divorce costs would raise the average probability of marital dissolution.

4. Conclusions

In this paper, we have studied the effect of changes in divorce costs on marital dissolution in a two-period one-to-one matching model with nontransferable utilities under incomplete information. We show that divorce costs affect not only individuals' decisions to divorce and to marry, but also their decision whom to marry. Consequently, the average probability of marital dissolution in the society is determined by these three decision channels, which we call "the divorce channel," "the marital status channel" and "the marital composition channel," respectively. Divorce costs always operate through the divorce channel in an unambiguous way, yielding a negative incentive effect on the average probability of marital dissolution (Proposition 1), which was earlier found in search theoretic models of matching. In this study, we also identify the allocation effect of divorce costs. The allocation effect through the marital status channel has already been studied in the search theoretic models: Rasul calls it the selection effect, and it is always working in the negative direction of the incentive effect, *i.e.*, the pipeline effect [9]. However, the allocation effect through the marital composition channel is novel to this study, thanks to the existence of heterogeneous individuals in our one-to-one matching framework. To isolate the effect of the marital composition channel in our study, we consider environments where the controlled change in divorce costs does not cause individuals to alter their marital status. Thus, the allocation effect in our study only arises through the marital composition channel. Our results show that the effect obtained through this new channel is ambiguous. For any gender-optimal matching rule that always selects a stable matching which is optimal for men or women, we can find matching environments in which the allocation effect, through the marital composition channel, has a positive sign and dominates the incentive effect (Example 1), as well as environments in which the allocation effect, through the marital composition channel, has a negative sign and reinforces the incentive effect (Example 2). Moreover, we show that the main results obtained for gender-optimal stable matching rules may also hold for other stable matching rules (Example 3). We should remark that, in our findings, the ambiguous sign of the allocation effect (through the marital composition channel) should not be surprising, since in our one-to-one matching framework the equilibrium notion (stability) that determines the final matching allocations involves individual comparisons of expected utilities under alternative matchings, but not comparisons of the corresponding expected divorce rates.

Following a similar classification of Rasul [9], we call the incentive effect and the allocation effect of a change in divorce costs to be the short-term effect and the long-run effect, respectively. While a decrease in divorce costs always increases divorce rates through the short-term effect, there are environments (as implied by Proposition 2) where the long-term effect may outweigh the short-term effect and eventually lead to a fall in the divorce rate. This theoretical result may help to explain the empirical observation in Wolfers that a sharp immediate rise in the US divorce rate after the divorce reform in the 1970s is reversed in the next decade to such an extent that the divorce rate is lower 15 years after the reform [7,8].

We believe that another contribution of this study is to show that the one-to-one matching theory with nontransferable utilities which has been heavily used in studying stable matchings for given economic environments can also be successfully employed to study the issue of separation once an algorithm or rule that produces stable matchings is chosen. While a search theoretic framework has a clear advantage in terms of formulating the equilibrium in a simple way, in large matching environments where the information about the potential partners is more limited, the one-to-one matching framework may be powerful in small environments that allow for assortative mating.

Finally, we believe that future research may benefit from the search theoretic models of Bougheas and Georgellis [12] and Rasul [9] to analyze, in a one-to-one matching framework, the effects of imperfect learning about potential partners on the marital dissolution, along with several potential strategic issues in acquiring knowledge. Further research questions that can be addressed in our new setup are the sensitivity of our results to alternative models of divorce which may allow remarriage or allow a correlation between an individual's qualities or traits with divorce costs.

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