Multi-Objective Emergency Material Vehicle Dispatching and Routing under Dynamic Constraints in an Earthquake Disaster Environment

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Abstract: Emergency material vehicle dispatching and routing (EMVDR) is an important task in emergency relief after large-scale earthquake disasters. However, EMVDR is subject to dynamic disaster environment, with uncertainty surrounding elements such as the transportation network and relief materials. Accurate and dynamic emergency material dispatching and routing is difficult. This paper proposes an effective and efficient multi-objective multi-dynamic-constraint emergency material vehicle dispatching and routing model. Considering travel time, road capacity, and material supply and demand, the proposed EMVDR model is to deliver emergency materials from multiple emergency material depositories to multiple disaster points while satisfying the objectives of maximizing transport efficiency and minimizing the difference of material urgency degrees among multiple disaster points at any one time. Furthermore, a continuous-time dynamic network flow method is developed to solve this complicated model. The collected data from Ludian earthquake were used to conduct our experiments in the post-quake and the results demonstrate that: (1) the EMVDR model adapts to the dynamic disaster environment very well; (2) considering the difference of material urgency degree, the material loss ratio is −10.7%, but the variance of urgency degree decreases from 2.39 to 0.37; (3) the EMVDR model shows good performance in time and space, which allows for decisions to be made nearly in real time. This paper can provide spatial decision-making support for emergency material relief in large-scale earthquake disasters.

Keywords: emergency dispatching and routing; multi-objective; dynamic constraints; network flow; spatial decision support

1. Introduction

In recent years, large-scale earthquakes have caused great damages to people’s lives and property, such as the Wenchuan earthquake in 2008 [1], the L’Aquila earthquake in 2009 [2], the Lushan earthquake in 2013 [3], the Ludian earthquake in 2014 [4], a Nepalese earthquake in 2015 [5], and so on. Undoubtedly, the primary task in the wake of disaster is to meet fundamental human material needs, especially in the first 72 h [6]. This requires that emergency materials be delivered to disaster points as fast as possible. Much effort has gone into disaster management [7,8], disaster assessment [9], and decision-making on rescue plans [10–12] with the support of geographic information science [13–15]. However, complicated dynamic earthquake disaster environments and
specific application requirements (e.g., strong timeliness, weak profit pursuit) make emergency material relief very difficult.

To provide effective spatial decision-making support from a macro perspective, the features of emergency material relief in a large-scale earthquake disaster should be comprehensively considered as follows:

- **Strong timeliness.** The primary task in the initial phases of a disaster response is to meet fundamental human material needs [16] without considering the economic cost, which is the main difference from the goal of commercial logistics.

- **Detailed solutions.** Emergency material relief is not only a dispatching problem, but also a routing problem. The dispatching problem is to answer the question of how many vehicles depart from the origin to the destination. The routing problem is to plan the driving path. Hence, the solving methodology requires calculating a detailed solution with high accuracy in the least possible time.

- **Multiple objectives for multiple reserve centers and disaster points.** Firstly, the primary objective in the initial phase of emergency relief is to deliver materials with the highest transport efficiency, in order to meet fundamental human needs in the affected areas. Secondly, as there are multiple emergency material depositories as well as multiple disaster points in severe disaster situations [17], the quantity of material supply and demand in multiple emergency material depositories and disaster points are different and change as time goes on, which is called “dynamic” in the following [18]. Therefore, multiple objectives should be treated, e.g., minimizing transport cost, minimizing the number of emergency material depositories, minimizing the unsatisfied demand, maximizing the minimum satisfaction, and so on [19,20]. In this paper, both transport efficiency and fairness of distribution among multiple disaster points are considered.

- **Multiple highly dynamic constraints.** Emergency relief in large-scale earthquake disasters is a complicated system involving many components. The dynamic constraints affecting EMVDR lie in the emergency materials and transportation network [21,22]. Firstly, the emergency material supplies come from the government supply repositories, purchase, donation, production, and so on. However, material supplies are uncertain due to damage to infrastructure, the randomness of donations, the unpredictability of raising funds, and so on. Secondly, emergency material demand is affected by the earthquake magnitude, the damage-suffering population, the casualties, damage to houses, the economic level, and so on, some of which change as time goes on within the rescue time horizon [23]. Thirdly, the transportation network can be damaged by mud slides or secondary earthquakes in a large-scale natural disaster, resulting in dynamic traffic conditions in each period of the planning horizon [24]. Thus, flexibility should be considered in this emergency material relief model.

In this paper, a multi-objective multi-dynamic-constraint emergency material vehicle dispatching and routing (EMVDR) model is proposed for decision-making in earthquake disaster environments, and an efficient continuous-time dynamic network flow algorithm is used to solve the EMVDR model.

The main contributions of this paper are shown in three aspects. For the objective functions, the proposed model is not only to maximize transport efficiency for the requirement of strong timeliness, but also to minimize the difference in material demands among multiple disaster points for the fairness of dispatching emergency supplies. In other words, the materials delivered to multiple disaster points should be dependent on the disaster conditions, not only the distance from emergency material depositories. To our knowledge, no related studies have considered this second objective so far. For constraint conditions, this paper considers multiple dynamic constraints, including travel time, road capacity, material supplies at multiple emergency material depositories, and demands for earthquake disaster points. For the solution solving algorithm, a novel continuous-time dynamic network flow algorithm with a low computational complexity and exact solution is proposed to solve the EMVDR model. Both strong timeliness and high accuracy are guaranteed.
The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces emergency material relief in large-scale earthquake disasters and the proposed EMVDR model. Section 4 presents a novel continuous-time dynamic network flow algorithm to solve the EMVDR model. Section 5 discusses a case study and important insights. Section 6 concludes the paper by providing an overview of results and future research.

2. Literature Review

Emergency material relief issue has been extensively investigated in the past decades [25]; many challenges have been solved and new ones are emerging [26]. Related studies vary in terms of the model hypothesis, method parameters, and optimization objectives.

2.1. Optimization Objectives

According to the number of objectives, the literature can be classified into single- and multiple-objective(s).

For the single-objective models, Barbaroso and Arda [27] concentrated on transport planning of emergency materials while minimizing transportation costs to describe the material flows over a network. Sheu et al. [28] solved the large-scale disaster relief distribution problem with the main goal of minimizing the total number of fatalities. Özdamar et al. [29] treated the emergency material relief issue as a hybrid model integrating multi-commodity network flow problem and a vehicle routing problem minimizing the total unmet material quantity. Sheu and Pan [30] sought to minimize the impact of imbalanced relief supply-demand under the constraints of a time-varying multi-source relief supplier. Yan and Shih [31] minimized the length of time required for emergency roadway repair and relief distribution to plan the repairs and relief distribution routes and schedules.

For the multi-objective models, Chang et al. [32] minimized the unsatisfied demand for materials, time to delivery, and transportation costs in a dynamic environment. The objectives of Bozorgi-Amiri et al. [33] were to minimize the sum of expected value and the total cost variance of the relief chain, and to maximize the affected points’ satisfaction levels. Yi and Özdamar [34] minimized the weighted sum of unsatisfied demand over multiple commodities and unserved wounded people. Huang et al. [35] considered three different objectives related to humanitarian relief: lifesaving utility, delay cost, and equality. There are also some studies that aim to address the fairness criteria using a fairness method, an equality-based method, and a utility-based method; related reviews can be found in Huang et al. [35]. However, these fairness criteria do not involve the dynamic demand in the time dimension among multiple disaster points.

2.2. Constraints

Besides the objectives considered in the models, constraints are also a key element of an emergency material relief system. Haghani and Oh [36] considered the constraints from capacities of both vehicles and roads. Yi and Özdamar [34] thought emergency logistics are subject to the capacity of the vehicle and facility, the upper bound on service capacity expansion/contraction, unmet demand, and the dynamic addition of vehicles. The constraints of Huang et al. [35] included the flow balance constraint, flow capacity constraint, the demand-limited received resources, non-negativity, and the zero allocation to any non-demand. Yi and Özdamar [34] considered multiple constraints such as material supply quantity, road capacity, limitations on total vehicles on the road, vehicle travel time, and hospital capacity for wounded people. In conclusion, the constraints in disaster environment come from time, material, and transportation infrastructure.

The main shortcomings of the existing literature are: dynamic materials and transportation network with multiple emergency material depositories and disaster points are seldom comprehensively considered; the objectives do not consider the difference in material demand urgency in the time dimension when dispatching vehicles to multiple disaster points, which is not fair to remote
disaster points; and strong timeliness involves computational performance, which is not solved well by traditional methods.

2.3. Methodology

The methodologies for emergency material relief are diverse, including stochastic optimization approach [37–39], linear programming [30,31,40], metaheuristic algorithms [41], fuzzy multi-objective programming [18], the greedy method [29], immune intelligence [42], dynamic network flow models [36,43], and so on. Among all these methods, dynamic network flow models can capture the features in both time and space dimension, and thus attract considerable attention. The dynamic network flow problem is to find feasible flows (or materials) that satisfy specific objectives under some constraints in a capacitated network [44]. Moreover, the flow supplies, flow demands, road travel times, and road capacities in a dynamic network flow model can represent the material supplies of emergency material depositories, the material demands of disaster points, the travel time passing through road, and the maximum entering rate of road in the real-world emergency material relief system, respectively. As the dynamic network flow model is matched well with an emergency material relief system, the material dispatching and routing are modeled as the dynamic network flow model in this paper.

However, dynamic network flow problems are difficult to solve, especially for the continuous-time models in which time passes in continuous increments. The continuous-time dynamic network flow models overcome two fatal drawbacks of the discrete-time model, i.e., the decision-making time points have to be fixed in advance before the problem is solved, and the network size is enormous to design an effective algorithm. Many endeavors have been made [44]. However, existing continuous-time models center merely on a polynomial-time algorithm (but approximate solution) or an exact solution (but a non-polynomial time algorithm). Moreover, they always require quite strict assumptions. In this paper, a novel algorithm with low computational complexity and an exact solution is proposed to solve the EMVDR model.

Considering the shortcomings of the above studies and the advantages of dynamic network flow models, we focus on making an EMVDR model using the continuous-time dynamic network flow algorithm.

3. Problem Description and Formulation

In this section, both emergency material relief issues in the real world and the notations employed in the relevant mathematical model are described.

3.1. Problem Description

Once a large-scale earthquake happens, the government starts an emergency response. Generally, the primary process of emergency material relief is shown in Figure 1. The command and dispatch center collects near-real-time information about emergency material supply, material demand, and the transportation network, and then quickly makes the most appropriate decisions on vehicle dispatching and routing. The vehicle dispatching is to answer the question of how many vehicles depart from any reserve center to any disaster point. In other words, a dynamic origin-destination (OD) matrix is supposed to be determined because the vehicle departure rates are dynamic in the disaster environment. The routing problem is to plan a path for each vehicle. Thus, the solution contains a dynamic OD matrix and a dynamic vehicle path set.
In order to mathematically describe the problem, the notations employed in the relevant mathematical model are given in Table 1. The mathematical programming form of emergency material relief issue is to achieve the specific objective function(s) under several constraints, as shown in the next section.

Table 1. The notations employed in the mathematical model.

<table>
<thead>
<tr>
<th>Notation</th>
<th>The Meaning in the Real World</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G(N, A)$</td>
<td>The graph that represents transportation network in the real world</td>
</tr>
<tr>
<td>$i \in N$</td>
<td>A node that represents the intersection of transportation network</td>
</tr>
<tr>
<td>$(i, j) \in A$</td>
<td>An link that represents the road in transportation network</td>
</tr>
<tr>
<td>$u_{ij}(t)$</td>
<td>The dynamic capacity that represents the maximum vehicle flow rate of road $(i, j)$ at time moment $t$</td>
</tr>
<tr>
<td>$\tau_{ij}(t)$</td>
<td>The travel time for any vehicle to pass through road $(i, j)$ at entry time moment $t$</td>
</tr>
<tr>
<td>$f_{ij}(t)$</td>
<td>The vehicle flow rate on road $(i, j)$ at entry time moment $t$</td>
</tr>
<tr>
<td>$S_i \in SP$</td>
<td>A supply point that represents a reserve center</td>
</tr>
<tr>
<td>$S_{ui}(t)$</td>
<td>The dynamic material supply quantity of reserve center $S_i$ at time moment $t$</td>
</tr>
<tr>
<td>$D_i \in DM$</td>
<td>A demand point that represents a disaster point</td>
</tr>
<tr>
<td>$D_{ui}(t)$</td>
<td>The dynamic material demand quantity of disaster point $D_i$ at time moment $t$</td>
</tr>
</tbody>
</table>

3.2. Multiple Objectives

The aim of emergency material relief is to meet fundamental human material needs in the fastest possible way. Timeliness is thus the main characteristic of emergency material relief. To this end, emergency materials should be transported from emergency material depositories to disaster points as quickly as possible. In other words, the objective function is to maximize the total quantity $W$ of transported emergency materials during a given time horizon $(0, T)$, or to minimize the time horizon $T$ after the transported emergency material quantity reaches a given value of $W$. For convenience, the objective function for strong timeliness is to maximize the transport efficiency $\frac{W}{T}$.

Emergency material relief for a large-scale earthquake disaster has multiple emergency material depositories and multiple disaster points. If only a single objective (i.e., maximizing the transport efficiency) is considered, then the vehicles would preferentially go to the nearest disaster point. In this case, an imbalance of dispatching materials to multiple disaster points could be caused due to road capacity (Figure 2a). In order to overcome this flaw, the material difference received by multiple disaster points in the time dimension should be minimized. Let $W_{D_i}(t)$ be the received material
quantity of disaster point $D_i \in DM$ at time $t$; let $\delta_{D_i}(t) = 1 - \frac{W_{D_i}(t)}{D_{D_i}(t)}$ be the material demand urgency degree of disaster point $D_i$ at time $t$, where $0 \leq W_{D_i}(t) \leq D_{D_i}(t)$ and $D_{D_i}(t) > 0$. The other objective is to minimize

$$\delta_{D_i}(t) = \max_{D_i \in DM} \delta_{D_i}(t).$$  \quad (1)

To our knowledge, this is the first time the difference of received materials in the time dimension has been considered (Figure 2b).

![Figure 2. Single VS multiple objectives of vehicle dispatching: (a) single-objective; (b) multiple objectives.](image)

### 3.3. Multiple Dynamic Constraints

In a dynamic disaster environment, the constraints of emergency material relief come from time, emergency material, transportation infrastructure, and information, as shown in Figure 3.

1. The time constraint involves two aspects: the time horizon $(0, T)$ and the time consumed making decisions. The emergency materials must be transported to disaster points in the limited time horizon $(0, T)$, so that the value in use of materials can be maximized. This requires the optimal solution for EMVDR issues. As the disaster environment changes fast, making decisions in the least time can improve the timeliness of a solution. As a result, improving both the solution accuracy and computational efficiency is the key to alleviating time constraints.

2. Emergency material constraints involve limited and dynamic material supplies and demands. Due to random donations and unpredictable fundraising, the emergency materials available in emergency material depositories are dynamic. Moreover, the emergency material demands depend on the population density, economic index, seismic intensity, and so on. The material demands even at the same disaster point vary over time.

3. The emergency material relief by vehicles is limited by transportation infrastructure. The road damage due to earthquake destructiveness, the time-varying vehicle number, and traffic accidents cause congestion and accessibility problems. As a result, the road capacity and travel time are dynamic. The decision-making for EMVDR issues should consider the limitations of dynamic transportation network conditions to avoid traffic jams so that the emergency materials can be transported in the most effective way.

4. A lack of information is always a barrier to making optimal decisions. Real-time information is adopted to predict the changing trends of transportation network states, material supplies, and demand. In this paper, time-dependent information combining near-real-time and predicted future information is used. Considering these constraints, the resulting requirements for the EMVDR model are shown in Figure 3.
Formally, multi-objective, multi-dynamic-constraint models for emergency material vehicle dispatching and routing issue can be described as follows:

Maximize

\[ \frac{1}{\delta_{D_i}(t)} + \varepsilon \times \frac{W}{T}, \] (2)

subject to:

\[ f_{ij}(t) \leq u_{ij}(t), \ t \in (0, T) \] (3)

\[ \sum_{(i,j) \in A} f_{ij}(t_0 + \tau_{ij}(t_0)) = \sum_{(u,i) \in A} f_{ui}(t), \ t \in (0, T), \ t_0 + \tau_{ij}(t_0) = t, \ i \in N \setminus (SP \cup DM) \] (4)

\[ \int_{t=0}^{T} \delta_{S_{ij}}(t) = \int_{t=0}^{T} \sum_{D_i \in DM} \sum_{(i,j) \in A} f_{i,D_i}(t) \] (5)

\[ \int_{\theta=0}^{t} \sum_{(S_{ij}) \in A} f_{S_{ij}}(\theta) \leq S_{ui}(t), \ S_i \in SP, \ t \in (0, T) \] (6)

\[ \int_{\theta=0}^{t} \sum_{(j,D_i) \in A} f_{j,D_i}(\theta) \leq D_{ui}(t), \ D_i \in DM, \ t \in (0, T), \] (7)

where

\[ \delta_{D_i}(t) = \max_{D_i \in DM} \delta_{D_i}(t) \] (8)

\[ \delta_{D_i}(t) = 1 - \frac{W_{D_i}(t)}{D_{C_{D_i}}(t)}. \] (9)

In order to maintain consistency, minimizing \( \delta_{D_i}(t) \) is formalized as maximizing \( \frac{1}{\delta_{D_i}(t)} \), where \((0, T)\) is the time horizon or the time span when all materials reach the disaster points; one of \( T \) and \( W \) is a fixed value, and the other is variable. Equation (3) is the network capacity constraint, which denotes that the vehicle flow rate entering the road is always less than the road capacity at any time moment \( t \). Equations (4) and (5) give the vehicle flow rate conservation, that is, the transportation network constraint; Equations (6) and (7) are the emergency material supply and demand constraints, respectively.

4. Solution Methodology

A continuous-time dynamic network flow algorithm is developed to solve the EMVDR model (as shown in Figure 4).
4.1. The Continuous-Time Dynamic Residual Network

Before presenting the solving method, we would like to introduce the continuous-time dynamic residual network (CTDRN). Given an original underlying network \( G(N, A) \), the capacity \( u_{ij}(t) \) and flow \( f_{ij}(t) \) on each \((i, j) \in A\) are known. Different from conventional concepts of a residual network, the residual capacity and flow on arcs are a continuous-time function rather than a fixed value. The corresponding CTDRN \( G_f(N, A_f, U_f, \tau_f) \) is constructed as follows: \( G \) and \( G_f \) are of the same node \( N \), but have different links. If there exists flow \( f_{ij}(t) > 0 \) for link \((i, j) \in A\), then a new reversed link \((j, i) \in A_f\) is added to \( G_f \), satisfying the following constraints:

\[
\begin{align*}
    u_{ij}^f(t + \tau_{ij}(t)) &= f_{ij}(t) \quad (10) \\
    f_{ij}^f(t + \tau_{ij}(t)) &= 0 \quad (11) \\
    \tau_{ij}^f(t + \tau_{ij}(t)) &= -\tau_{ij}(t). \quad (12)
\end{align*}
\]

If there exists \( u_{ij}(t) - f_{ij}(t) > 0 \) for any link \((i, j) \in A\), then the link \((i, j) \in A_f\) with \( u_{ij}^f(t) = u_{ij}(t) - f_{ij}(t) \) and \( \tau_{ij}^f(t) = \tau_{ij}(t) \), \( f_{ij}^f(t) = 0 \) is kept.
4.2. The Continuous-Time Dynamic Network Flow Model

Different from the traditional static augmenting method [45], our augmenting method works on CTDNRN. Four main steps, dynamically choosing the sink \( D \), searching for the earliest dynamic path, calculating the continuous-time dynamic residual flows, and augmenting the continuous-time residual flows, are iteratively executed until the termination conditions.

4.2.1. Choosing the Sink \( D \)

Considering the second objective, i.e., minimizing the difference of received materials in time dimension, the sink \( D \) at each iteration is dynamically selected, so that the materials can be appropriately delivered to multiple disaster points.

As introduced in Section 3.2, the material demand urgency degree \( \delta_D(t) \) of disaster point \( D_i \) at time \( t \) is defined as follows:

\[
\delta_D(t) = 1 - \frac{W_D(t)}{D_E(t)}.
\]

(13)

At each iteration, the disaster point \( D_k \) with \( \delta_D(t_k) = \max_{D_i \in DM} \delta_D(t_i) \) at time \( t_k \) is chosen as the sink \( D \), where \( t_i \) is the earliest time arrived to \( D_i \) in current CTDNRN.

4.2.2. Searching for the Dynamic Earliest Path

In order to maximize the transport efficiency, the emergency vehicles always drive on the earliest (or shortest) paths with a given destination and departure time.

**Function**: searching for the dynamic earliest path

**Input**: departure time set \( R_S \) of any reserve center \( S_i \), the most urgent disaster point \( D \), dynamic residual flow \( r_{ij}(t) \) and dynamic travel time \( \tau_{ij}(t) \) for any link \( (i,j) \in A \)

**Output**: the dynamic earliest path \( P_{OD}(t_0), t_0 \in R_S \)

**Procedure**:

Step 1: Initialize the reachable time interval \( R_i = \emptyset \) for any nodes \( i \notin SP \), except \( R_{S_i} \neq 0 \); put \( R_{S_i} \) into queue \( Q \);

Step 2: If \( Q \) is not empty, select the element \( R_i' \) from \( Q \);

Step 3: For any residual link \( (i,j) \in A_f \), let \( T_{ij} \) be the time set satisfying \( r_{ij}(t) > 0 \) for any \( t \in T_{ij} \), \( R_i' = \{ t + \tau_{ij}(t) \mid t \in T_{ij} \} \), if \( T_{ij} \) is empty, then set \( R_i = R_i \cup R_i' \), put \( R_i' \) into \( Q \);

Step 4: Steps 2 and 3 are executed repeatedly until \( Q \) is empty;

Step 5: Let \( d_D \) be the earliest time moment in \( R_D \), \( d_i = \infty \) for any \( i \neq D \);

Step 6: Depth-First Search method [46] starting from \( D \) is used to obtain the earliest path \( P_{OD} \), where the constraints \( d_i = d_i + \tau_{ij}(d_i), d_i \in R_i, r_{ij}(d_i) > 0 \) must be satisfied when determining whether a residual link \( (i,j) \) belongs to \( P_{OD} \) or not;

Step 7: If no source \( O \in P_{OD} \), set \( d_D = d_D + 1 \), go to Step 6; otherwise, go to Step 8;

Step 8: \( P_{OD}(d_D) \) is the dynamic earliest path;

Step 9: Terminate.

This operation first calculates all reachable time sets for all nodes in Steps 2–4, then the dynamic earliest path is figured out from \( D \) at the earliest time arrived. Any departure time \( t \in R_{S_i} \) of \( S_i \) satisfies \( t \in () \) and \( Su_i(t) > 0 \).

4.2.3. Calculating the Continuous-Time Dynamic Residual Flows

Due to road capacity, the number of vehicles traveling on each road at any time is limited to avoid traffic congestion.
whereby too many materials are intensively transported to a single disaster point, the material quantity \( \delta \) and results, and gives our discussion. conducted in Ludian, Yunnan, China. This section describes the study area, data, and experimental results, and gives our discussion.

### 4.2.4. Augmenting the Continuous-Time Residual Flows

Before executing the operation of augmenting residual flow, we would like to introduce the concept of “the augmenting time interval.” In order to avoid an imbalance in material dispatching whereby too many materials are intensively transported to a single disaster point, the material quantity \( M_k \) to be augmented is limited at each iteration \( k \). Let time interval \((T_1, T_2) \subset (0, T)\) be the time horizon, where

\[
M_k = \int_{T_1}^{T_2} r_{OD}(t) dt
\]

and \( \delta_D(T_2) \) is not the maximum material demand urgency degree from \( T_2 \).

### Example

The time of residual flow \( r_{ij}(t) \) of any link \((i, j) \in P_{OD}(d_0)\) moves forward by \( d_{Oi}(t) \) when calculating the intersection of \( r_{ij}(t) \) and \( r_{OD}(t) \). When augmenting the residual flows \( r_{OD}(t) \) to \((i, j) \in P_{OD}(d_0)\) within \((T_1, T_2)\), the time of \( r_{OD}(t) \) moves backward by \( d_{Oi}(t) \). These operations of moving the time interval forward and backward are inversely related and their aims are to keep the time consistent even if the travel time it takes to pass through \((i, j)\) is dynamic.

The above four steps are executed until all arrived time \( t > T \), or no more materials exist in the reserve center(s), or the demands of all disaster points are met.

### 5. Case Study

To validate the performance of the presented model and solving method, an experiment was conducted in Ludian, Yunnan, China. This section describes the study area, data, and experimental results, and gives our discussion.
5.1. Study Area and Data

An earthquake with a moment magnitude of 6.1 struck Ludian County, Yunnan, China, on 3 August 2014. After this event, at least 617 people were killed, 2400 others were injured, and over 100 people were missing.

The macro epicenter of this earthquake was Longtoushan, 23 km from the center of Ludian County [4]. People in the affected countryside suffered more from a lack of emergency materials. In our experiment, 12 disaster points in the meizoseismal region (Figure 5) are intentionally chosen. Four emergency material depositories (Figure 5) are Zhaotong City (O₁), Huize (O₂), Qiaojia (O₃), and Ludian (O₄) counties.

The transportation network contains multi-grade roads with different attributes. The capacity and free velocity of each grade road are shown in Table 2. The emergency transportation network is extracted from the Chinese road network using the following approach: (1) the all-pairs shortest paths (the red lines in Figure 5) from all reserve centers to all disaster points are figured out; (2) K-order roads of these shortest paths are found, where road \((i, j)\) is a K-order road of \((u, v)\) if the minimum number of roads connecting \((i, j)\) and \((u, v)\) is K. We set \(0 \leq K \leq 16\) in this experiment (blue lines in Figure 5); (3) those peripheral roads that have no chance to transport materials are deleted. Finally, a transportation network with 1517 nodes and 1958 roads is obtained.

![Figure 5](image_url)

**Figure 5.** The emergency transportation network for the Ludian earthquake in 2014.

<table>
<thead>
<tr>
<th>Grade</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>≥5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (vehicles/h)</td>
<td>960</td>
<td>1350</td>
<td>1500</td>
<td>1700</td>
<td>2000</td>
</tr>
<tr>
<td>Free velocity (km/h)</td>
<td>80</td>
<td>75</td>
<td>60</td>
<td>55</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 2. The road attributes in an earthquake disaster environment.

In a disaster environment, the emergency transportation network is damaged by secondary disasters. Moreover, unexpected relief vehicles also drive into disaster points through the emergency transportation network. As a result, transportation network conditions change over time. Fifteen vulnerable roads, shown in Figure 6, are selected.
In our experiments, material supply, demand, road capacity, and travel time are dynamic. (1) The dynamic material supplies are set as shown in Figure 7a according to related real reports and the construction standard of disaster relief material reserves in China. (2) In this experiment, the initial demands are fixed and then piece-wise increase considering consumption periodically, as shown in Figure 7b; (3) Dynamic travel time on vulnerable roads is considered in Figure 7c; (4) Time-dependent road capacities are set in Figure 7d.

5.2. Results

In this section, the proposed EMVDR model is tested considering multiple objective(s) and constraints.
5.2.1. Material Dispatching and Routing with Multiple Objectives

In the early stages of emergency relief, the material demands of disaster points are enormous. With a given time horizon \( T = 5h \), the single-objective emergency material dispatching result is shown in Figure 8a for a static situation, in which the supplies are enough, the transportation network attributes are constant, and only the first objective (i.e., maximizing the transport efficiency) is considered. In Figure 8a, the value on the horizontal axis is the material dispatching time within time horizon \((0, T)\), and the value \( \rho_i(t) \) on the vertical axis is the material dispatching rate of node \( i \): \( \rho_i(t) > 0 \) is the material departure rate from each reserve center, and \( \rho_i(t) < 0 \) is the material arrival rate to each disaster point. The unit of \( \rho_i(t) \) is pcu/h, which indicates how many vehicles depart (or arrive) in unit time. The materials are packed in a vehicle, and material heterogeneity is not considered in this paper. Figure 8a demonstrates that emergency material depositories steadily send materials in the prophase and gradually decrease the departure rate in the anaphase. The received materials of disaster points reveal a symmetrical trend.

When the other objective (i.e., minimizing the difference in material demand urgency for every moment in time dimension) is further considered, different material dispatching results are seen Figure 8b. Compared with Figure 8a, more fluctuations occur in the multi-objective model due to the equilibrium distribution principle.

As for the solution, the emergency relief issue is treated as not only the material dispatching problem, but also the vehicle routing problem. The material dispatching problem is to attain the dynamic OD matrix, and the vehicle routing problem is to determine the specific path for each vehicle. Table 3 gives an example of both vehicle dispatching and routing solutions. From this result, the detailed task (Table 3) for each vehicle is clear and the corresponding paths (Figure 9) are specified. The proposed EMVDR model dispatches the vehicles to different roads in a reasonable way, so that the transportation network can be utilized sufficiently.

Table 3. An example of a vehicle dispatching and routing solution.

<table>
<thead>
<tr>
<th>Departure Time</th>
<th>Origin</th>
<th>Destination</th>
<th>Vehicle Information</th>
<th>Path</th>
<th>Expected Arrived Time</th>
</tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Departure Rate</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>Loaded Materials</td>
<td></td>
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<td>Path</td>
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<td></td>
</tr>
<tr>
<td>18:25</td>
<td>O₄</td>
<td>D₄</td>
<td>12</td>
<td>Tent, quilt</td>
<td>P₁₀₉</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D₅</td>
<td>8</td>
<td>drinking water, food</td>
<td>P₁₁₀</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D₆</td>
<td>5</td>
<td>Medicine</td>
<td>P₇</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D₇</td>
<td>15</td>
<td>Lifesaving equipment</td>
<td>P₁₀₉</td>
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</table>
5.2.2. Material Dispatching and Routing with Multiple Dynamic Constraints

In the real-world scenario, a disaster environment is dynamic due to secondary disasters and traffic jams. In this section, an EMVDR model is executed considering dynamic variables in a disaster environment.

1. Dynamic material supply

There are not enough emergency materials in storage in the early stages of emergency relief but they are recharged periodically in later stages due to material collection. Emergency relief is to deliver limited materials to disaster points within a given time window. In this scenario, the dispatching results are shown in Figure 10a and intermittent dispatching materials are observed due to a shortage of supplies.

![Figure 9. The utilized road in the emergency network.](image)

![Figure 10. The multi-objective emergency material dispatching results in the dynamic situation: (a) dynamic limit emergency material supply; (b) dynamic emergency material demand; (c) dynamic travel time; (d) dynamic road capacity; (e) multiple dynamic constraints.](image)
2. Dynamic material demand

In a disaster environment, the material demand of each disaster point is dynamic due to the dynamic situations. The corresponding material dispatching results for this scenario are given in Figure 10b and intermittent receiving materials are observed due to the limitations on material demand.

3. Dynamic travel time

As transportation conditions change over time, it takes a different amount of travel time for those vehicles to pass through a vulnerable road. There is no doubt that dynamic travel time has a great effect on the efficient transport of emergency materials. The material dispatching results are shown in Figure 10c and fluctuate more than in Figure 8b owing to dynamic vehicle travel time.

4. Dynamic road capacity

With reasons similar to those given for dynamic travel time, dynamic road capacity is a limitation to material transportation. As a result, dynamic road capacity is an indispensable factor of emergency material relief. The emergency material dispatching results are shown in Figure 10d. Not only fluctuations but also undulations are observed due to the equilibrium distribution principle and the dynamic road capacity.

5. Mixed dynamic constraints

In the large-scale earthquake disaster situation, both the material (e.g., emergency material supply and demand) and infrastructure (e.g., the road capacity and travel time) constraints play a role in emergency material relief. Considering all the above constraints and objectives, the emergency material dispatching results are shown in Figure 10e.

5.3. Discussion

The EMVDR model involves multiple objectives and multiple constraints, resulting in different material dispatching results in different scenarios, as shown in Section 5.2. In this section, we discuss the effects of each dynamic variable on the material dispatching based on the above experimental results.

5.3.1. Single VS Multiple Objectives

In the single-objective case, our model for this simplified scenario is actually the dynamic maximum network flow because all constraints are static and only one objective is considered. Thus, the material quantity transported to disaster points is at a maximum. However, the result in Figure 8a demonstrates that only seven disaster points have received materials and the other five disaster points got nothing. The reason is that the finite materials go to nearby disaster points due to limitations on road capacity. The theoretical interpretation can be seen in Section 3.2 and Figure 2a.

In order to overcome the disadvantages of single-objective dispatching, multiple-objective dispatching is done in Figure 8b. Two indexes are presented to evaluate the results. One is the material loss ratio:

\[ LS = \frac{RS'}{RS} - 1 \]  

where \( RS \) and \( RS' \) are the total transported materials to all disaster points under single and multiple objectives, respectively. The material loss ratio is used to evaluate how many fewer materials are delivered due to the equilibrium distribution principle compared with single-objective optimization.

In this scenario, \( LS = -10.7\% \), calculated from accumulated received vehicles, where the vertical axis values are as in Figures 11 and 12:

\[ RS = \sum_{D_i \in DM} \int_0^T \rho_{D_i}(t) dt \]  

\[ LS = \frac{RS'}{RS} - 1 \]  

where \( RS \) and \( RS' \) are the total transported materials to all disaster points under single and multiple objectives, respectively. The material loss ratio is used to evaluate how many fewer materials are delivered due to the equilibrium distribution principle compared with single-objective optimization.
and \( \rho_{D_i}(t) \) is the material received rate of node \( i \) in Figure 8a,b. The other index is the variance \( \Delta_{\delta} \) of material demand urgency degree \( \delta_{D_i}(t) \):

\[
\Delta_{\delta} = \int_0^T \sum_{D_i \in DM} (\delta_{D_i}(t) - \bar{\delta}(t))^2 dt
\]

\[
\bar{\delta}(t) = \frac{\sum_{D_i \in DM} \delta_{D_i}(t)}{|DM|}
\]

where \( \bar{\delta}(t) \) is the average demand urgency degree at time moment \( t \) and \( |DM| \) is the number of disaster points. The material demand of each disaster point is set as the maximum possible accumulated received materials in the case that only one disaster point exists (if \( De_{D_i}(t) = 0 \), the set \( De_{D_i}(t) = \xi \), where \( \xi \) is a very small value). In this scenario, \( \Delta_{\delta} \) is improved from 2.39 to 0.37 according to the experimental results in Figure 8a,b when a second objective is considered.

From the comparison between single- and multi-objective models, it is concluded that: (1) The disadvantage that five disaster points cannot receive materials is overcome by the multi-objective model; (2) different disaster points have different abilities to receive materials; (3) those disaster points that find it difficult to receive materials “rob” some from other disaster points that share the same roads when considering multiple objectives; (4) the second objective (i.e., minimizing the difference in the material demand urgency degree) guarantees the fairness of dispatching materials among multiple disaster points. In the above, emergency material dispatching is underway in a situation with multiple reserve centers, multiple disaster points, and multiple interactive routing choices, resulting in the difficulty of emergency relief. Besides encouraging timeliness, the fairness criterion can guarantee the equilibrium of dispatching emergency materials, which is close to the reality of disaster relief. Hence, multiple objectives are considered in the following discussion due to their practical significance.

**Figure 11.** The accumulated sent materials of each reserve center, considering dynamic material supply.

**Figure 12.** The accumulated received materials of each disaster point considering dynamic material demand.
5.3.2. Static VS Dynamic Constraints

As mentioned above, the dynamic constraints involve material supply, demand, road travel time, and capacity.

1. Static VS Dynamic Material Supply

If the material supply at a reserve center is insufficient and dynamic, the dispatching results in Figure 10a compared with Figure 8b demonstrate that: (1) The emergency material vehicles persistently depart from emergency material depositories in the early stages due to sufficient material reserves; (2) After a period of time, the supplies are disrupted because the reserved materials are used up; (3) If materials are replenished in the later stages, the material dispatching process is restarted; (4) The accumulated materials (Figure 11) sent from emergency material depositories are consistent with the material supply quantity; (5) Each disaster point receives materials in stages because varying material urgency degrees are considered for multiple disaster points to guarantee the fairness of dispatching emergency materials. The other reason is that a limitation of material supplies causes a discontinuity of material dispatching.

2. Static VS Dynamic material demand

When material demands of multiple disaster points are different and dynamic, the material dispatching changes over time to adapt to the dynamic demand and three results can be found in Figure 12, as concluded from Figure 10b: (1) Each disaster point receives materials in several stages because of periodical material demands, and alternately receives materials in each stage; (2) The accumulated received materials periodically increase as the periodical material demands increase as well; (3) All disaster points attain satisfactory emergency materials after the relief is over.

3. Static VS Dynamic travel time

Compared with the static case (Figure 8b), the dispatching results considering dynamic travel time (Figure 7c) are of a similar general tendency, but there are more fluctuations locally. This is because the vehicles always drive in the earliest path with a given origin \(O\) and destination \(D\), and the earliest paths are dynamic due to dynamic travel time. The material loss ratio decreased from \(-10.7\) to \(-12.3\)%. That means that 1.6% fewer materials are transported due to the increase in travel time.

4. Static VS Dynamic road capacity

Road capacity plays an important role in emergency material dispatching. The experimental results in Figure 10d verified this, and demonstrated that: (1) \(O_2\) and \(O_3\) dispatch materials have a similar changing trend as that of road capacity (Figure 7e), i.e., piecewise constant. The reason is that their served disaster points are close in geographical location and their rescue routes share the same vulnerable roads. (2) \(O_1\) and \(O_4\) dispatch materials in a highly fluctuating manner. This is because vulnerable roads are distributed in different routes, and the emergency material depositories alternately dispatch materials to different rescue routes for the fairness of material demands in multiple disaster points. (3) Even though there are fluctuations in the material dispatching rate, the accumulated sent (and received) materials keep increasing at a smooth rate because the dispatching rate is the slope of the accumulated materials. The vehicle flows in a contiguous time interval are complementary, resulting in smoothness from a long-term point of view. (4) The material loss rate decreases from \(-10.7\) to \(-33.3\)%, which means that 22.6% fewer materials are transported due to the decrease in road capacity. Compared with a dynamic case in which travel time is dynamic, dynamic road capacity plays a more significant effect on transport efficiency.

5. Static VS Dynamic mixed constraints

Compared with static or single-dynamic-constraint-based scenarios, the emergency material dispatching results with mixed dynamic constraints in Figure 10e are different. This emergency
material dispatching process is divided into several stages due to limited supplies. Under the constraints of road capacity and travel time, each disaster point receives emergency materials to satisfy its demands, considering the difference in urgency. Different disasters have different features and are simultaneously affected by multiple factors. Multiple factors play different roles and cause different results. The model proposed in this paper is tested in the Ludian earthquake, and can be extended to other scenarios.

5.4. Performance

To evaluate the proposed CTDTF algorithm for solving the EMVDR problem, both the time and space efficiency are tested and compared with the time–space network model [35]. Figure 13 shows the space performance where the CTDTF and time–space network models consume about 0.3 MB and 212.5 MB memory, respectively. From the result, the continuous-time dynamic network flow algorithm has considerable advantages in space efficiency compared with the time–space network model. This is because only the original underlying network and a few extra memories about the change of vehicle flows in the network are recorded by the CTDTF model, while the time–space network model needs information for every time moment. For the same reason, the memories consumed by the CTDTF algorithm do not dramatically increase with the increasing time horizon.

Figure 14 demonstrates the time performance of the CTDTF model. The overall computational time is always less than 10 min when the time horizon $T \leq 10$ h for solving EMVDR issues in Ludian earthquake, whereas the time–space network model runs about 300 times slower than the CTDTF model. Overall, the CTDTF model can satisfy the requirement of attaining a solution for emergency material relief in a large-scale earthquake disaster.

![Figure 13. The space performance comparison of solution methodology.](image1)

![Figure 14. The time performance of the continuous-time dynamic network flow algorithm in solving the EMVDR problem.](image2)

6. Conclusions and Future Research Directions

In this paper, we analyzed the timeliness, fairness, and dynamics of emergency material relief in a large-scale earthquake disaster, which demands accuracy and efficiency in terms of a solution methodology. Then, multiple objectives (e.g., maximizing transport efficiency and minimizing the difference in material demand urgency degrees) and multiple constraints (e.g., dynamic material supply, demand, road capacity, and travel time for emergency material vehicles dispatching and
routing) were discussed for material relief. Furthermore, a mathematical model (i.e., the EMVDR model) and corresponding solution methodology (i.e., the continuous-time dynamic network flow algorithm) were proposed. The advantages of the proposed model and algorithm are: (1) the relatively comprehensive constraints on emergency material relief in an earthquake disaster are dealt with; (2) the dynamics of multiple constraints are considered; (3) to our knowledge, it is the first time the fairness in the time dimension of dispatching materials among multiple disaster points has been considered; (4) the solving algorithm can attain a high-precision solution including detailed dispatching and a routing plan with high computational efficiency.

The experiments on the Ludian earthquake in 2014 demonstrated that: (1) compared with the single objective (i.e., maximizing transport efficiency), multiple objectives in dispatching result in a −10.7% material loss ratio, but a decrease from 2.39 to 0.37 in the variance of material demand urgency degree; (2) the decrease of road capacity and increase of vehicle travel time passing through vulnerable roads cause a −22.6% and −1.6% higher material loss ratio, respectively; (3) the EMVDR model can adapt to dynamic emergency material supply and demand. In conclusion, our proposed model can achieve emergency material relief in a large-scale earthquake disaster.

Even though the relatively comprehensive features of emergency material relief in a large-scale earthquake are considered in this paper, the heterogeneities of vehicles and material items are missing. For example, trucks and motorcycles run at different speeds even though they experience the same road conditions. In the future, the priority of varying emergency materials and multiple vehicle speed-flow relationships will be considered and we intend to choose the multi-commodity dynamic network flow model as the solution methodology.

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Author Contributions: Jincheng Jiang designed the approach and conceived the experiments; Qingquan Li and Lixin Wu contributed analyses and discussions; Jincheng Jiang and Wei Tu wrote most of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

7. Singh, N.; Tuttle, M.; Bhaduri, B. Enhancing Disaster Management: Development of a Spatial Database of Day Care Centers in the USA. ISPRS Int. J. Geo-Inf. 2015, 4, 1290–1300. [CrossRef]


