

Article

Is Bitcoin a Relevant Predictor of Standard & Poor's 500?

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Abstract: The paper investigates whether Bitcoin is a good predictor of the Standard & Poor's 500 Index. To answer this question we compare alternative models using a point and density forecast relying on Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS). According to our results, Bitcoin does not show any direct impact on the predictability of Standard & Poor's 500 for the considered sample.

Keywords: cryptocurrency; Bitcoin; forecasting; point forecast; density forecast; dynamic model averaging; dynamic model selection; forgetting factors

1. Introduction

The idea of cryptocurrency and the related technology, Blockchain, was suggested in 2009 by an anonymous user known as Satoshi Nakamoto. He posted a paper to a cryptographic mailing list introducing a new electronic cash system with very low transaction costs able to avoid the presence of a central bank: the Bitcoin, see [Nakamoto \(2009\)](#). In the last ten years, cryptocurrencies have become more and more popular among researchers and investors, with around 2000 cryptocurrencies available at the time of writing. In recent months, the Bitcoin has experienced a dramatic price increase and consequently, the global interest in cryptocurrencies has spiked substantially. Despite the price increase, there are other numerous reasons for this intensified interest, just to mention a few: Japan and South Korea have recognised Bitcoin as a legal method of payment ([Bloomberg 2017a](#); [Cointelegraph 2017](#)); some central banks are exploring the use of the cryptocurrencies ([Bloomberg 2017b](#)); a large number of companies and banks created the Enterprise Ethereum Alliance¹ to make use of the cryptocurrencies and the related technology called blockchain ([Forbes 2017](#)). Finally, the Chicago Mercantile Exchange (CME) started the Bitcoin futures on 18 December 2017, see [Group \(2017\)](#), Nasdaq and the Tokyo Financial Exchange will follow, see [Bloomberg \(2017b\)](#).

Although Bitcoin is a relatively new currency, there have already been some studies on this topic: [Hencic and Gourieroux \(2015\)](#) applied a non-causal autoregressive model to detect the presence of bubbles in the Bitcoin/USD exchange rate. The study of [Cheah and Fry \(2015\)](#) focused on the same issue. [Fernández-Villaverde and Sanches \(2016\)](#) analysed the existence of price equilibria among privately issued fiat currencies and [Yermack \(2015\)](#) wondered whether the cryptocurrency can be considered a real currency. [Sapuric and Kokkinaki \(2014\)](#) measured the volatility of the Bitcoin exchange rate against six major currencies. [Chu et al. \(2015\)](#) provided a statistical analysis of the log-returns of the exchange rate of Bitcoin versus the USD. [Catania and Grassi \(2018\)](#) analysed the main characteristics of cryptocurrency volatility.

¹ Source: <https://entethalliance.org/members/>.

Moreover [Bianchi \(2018\)](#) tried to investigate some of the key features of cryptocurrency returns and volatilities, such as their relationship with traditional asset classes, as well as the main driving factors behind the market activity. He found that returns on cryptocurrencies are moderately correlated with commodities and a few more assets.

Other studies have analyzed cryptocurrency manipulation and predictability. For instance, [Hotz-Behofsits et al. \(2018\)](#) applied a time-varying parameter VAR with t-distributed measurement errors and stochastic volatility. [Griffin and Shams \(2018\)](#) investigated whether Tether (another cryptocurrency backed by USD) is directly manipulating the price of Bitcoin, increasing its predictability. [Catania et al. \(2019\)](#) studied cryptocurrencies' predictability using several alternative univariate and multivariate models. They found statistically significant improvements in point forecasting when using combinations of univariate models and in density forecasting when relying on a selection of multivariate models.

Many institutions tried to investigate the relationship between Bitcoin and the stock market. In some articles, it was speculated that the Bitcoin can improve stock market's predictability, in this case, Bitcoin could be used as a leading indicator. In an article by [Bloomberg \(2018\)](#), Morgan Stanley's analysts stated that "big investors may be dragging Bitcoin toward Market correlation": the increasing risk of this cryptocurrency may have had an attraction for investors who were seeking for high gains. [Stavroyiannis and Babalos \(2019\)](#) examine the dynamic properties of Bitcoin and the Standard & Poor's 500 (S&P500) index. They study whether Bitcoin can be classified as a possible hedge, diversifier, or safe-haven with respect to the US markets. They found that it does not hold any of the hedge, diversifier, or safe-haven properties and it exhibits intrinsic attributes not related to US markets.

To the best of our knowledge, there are still no studies to confirm that Bitcoin is a good stock market predictor. This paper tries to fill this gap, analyzing whether Bitcoin could be used as a leading indicator for the S&P500.

To answer this question, we allow for parameter and model uncertainty, avoiding Markov Chain Monte Carlo (MCMC) estimation at the same time. This is accomplished using the forgetting factors methodology (also known as discount factors) which have been recently proposed by [Raftery et al. \(2010\)](#) and found to be useful in economic and financial applications, see [Dangl and Halling \(2012\)](#) and [Koop and Korobilis \(2012\)](#) (KK). Another advantage of this methodology is to provide, in close form, both the marginal and predictive likelihood (PL), which are useful in model selection.

The rest of the paper proceeds as follows: Section 2 presents the general model and the estimation strategy; Section 3 presents the Dataset; Section 4 discusses the empirical results; finally, Section 5 reports some conclusions.

2. Models and Estimation Strategy

Let $\mathbf{y}_t \equiv (y_1, \dots, y_t)'$ denote the time series of interest and $\mathbf{x}_t \equiv (x_1, \dots, x_t)'$ the series of exogenous variables, then the model can be written as:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{z}_t \boldsymbol{\gamma}_t + \boldsymbol{\varepsilon}_t, & \boldsymbol{\varepsilon}_t &\sim N(0, \mathbf{H}_t), \\ \boldsymbol{\gamma}_t &= \boldsymbol{\gamma}_{t-1} + \boldsymbol{\eta}_t, & \boldsymbol{\beta}_t &\sim N(0, \mathbf{Q}_t), \end{aligned} \tag{1}$$

where \mathbf{y}_t is a scalar representing the observed time series at time t , $\mathbf{z}_t = \{y_{t-1}, \dots, y_{t-p}, x_{t-1}, \dots, x_{t-q}\}$ is a $1 \times m$ vector ($m = p + q$) stacking all the lags of the series of interest and of the exogenous variable; $\boldsymbol{\gamma}_t = \{\gamma_{1,t}, \dots, \gamma_{j,t}\}$ is an $m \times 1$ vector containing the time varying states $\boldsymbol{\gamma}_t$, which are assumed to follow a random-walk dynamic. Finally, the errors, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\beta}_t$, are assumed to be mutually independent at all leads and lags. The \mathbf{H}_t contains the time-varying volatilities of the series. The state space model (SSM) of Equation (1) has been used in several recent papers, see among others, [Primiceri \(2005\)](#) and [Koop and Korobilis \(2012\)](#).

In order to estimate the quantities of interest, maximum likelihood or Bayesian estimation based on MCMC can be used. However, these two estimation approaches end up being computationally

complex and, most of the time, infeasible. To reduce the computational burden, KK proposed two main adjustments to the usual MCMC.

The first is to replace the variance-covariance matrix Q_t with an approximation. Latent states— γ_t —can then be obtained with a closed-form expression avoiding maximum likelihood or MCMC, see Supplementary Materials. The second adjustment is to replace the measurement error variance matrix H_t with an Exponential Weighted Moving Average (EWMA) type filter.

As discussed in Supplementary Materials, this methodology requires the specification of the hyperparameters λ , α and κ and the specification of the initial condition of the states γ_0 and Σ_0 . Refer to KK for an extensive discussion of the problem.

3. Dataset Description

Table 1 reports the dataset used for the analysis, with the transformation and the data source. The sample goes from 11 August 2015 to 19 July 2018 and consists of 740 daily observations. The crypto-market is open 24 h a day, seven days a week; hence, for computing returns we use the closing price at midnight (UTC). As discussed in Catania et al. (2019), the data are available from <https://coinmarketcap.com/> with daily frequency; unfortunately, hourly data that could allow for a more precise analysis are not freely available. To investigate non-stationary issues, three Unit-Root tests have been performed: Augmented Dickey-Fuller (ADF) Test, Phillips-Perron (PP) Test and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) Test. All of them confirm the stationarity of each transformed series, results are available from the authors upon request.

Table 1. Data overview and transformation. The table reports the series divided by type, Financial predictors, Commodity predictors and Crypto predictors. The series are available for the period 11 August 2015 to 19 July 2018. For each variable the table reports the abbreviation code, the full name, the data source and the transformation applied.

| Code | Full Name | Transformation | Data Source |
|-----------------------------|---|----------------------|------------------------|
| Analyzed serie | | | |
| S&P500 | Standard & Poor’s 500 | Log-First-Difference | Thomson Reuters Eikon |
| Financial predictors | | | |
| EF300 | FTSEuroFirst300 | Log-First-Difference | Thomson Reuters Eikon |
| NASDAQ | Nasdaq 100 Index | Log-First-Difference | Thomson Reuters Eikon |
| VIX | CBOE Market Volatility Index | Log-First-Difference | Thomson Reuters Eikon |
| 1mUS | 1-month US Treasury Constant Maturity Rate | First-Difference | Federal Reserve System |
| 10yUS | 10-years US Treasury Constant Maturity Rate | First-Difference | Federal Reserve System |
| Commodity predictors | | | |
| OIL | ICE Brent Crude Eletronic Energy Future | Log-First-Difference | Thomson Reuters Eikon |
| GOLD | SPDR Gold Shares | Log-First-Difference | Thomson Reuters Eikon |
| Crypto predictors | | | |
| BTC | Bitcoin | Log-First-Difference | Coinmarketcap |
| BHL | Bitcoin High minus Bitcoin Low | Log | Coinmarketcap |

Figure 1 reports Bitcoin closing price (BTC) which shows a steep rise in 2017 reaching the value of almost 20,000 US dollars in December 2017. This ascending trend was severely interrupted at the beginning of 2018, when price quickly dropped down to \$6000. At the time of writing, BTC’s price is fluctuating between 5000 and 6000 dollars.

The series reported in Table 1 are divided in: financial predictors, such as VIX; commodity predictors, such as GOLD and crypto predictors such as BTC. Among the financial predictors, the VIX, see Figure S1 in Supplementary Materials (Panel (c)), is the most volatile, as expected. It displays a very steep peak between January and February 2018, the same period in which BTC’s price started to fall.

Table S1 in Supplementary Materials reports the correlation matrix of the predictors. As the table shows the BTC appears to be highly positively correlated with all the financial indexes: S&P500, EF300 and NASDAQ.

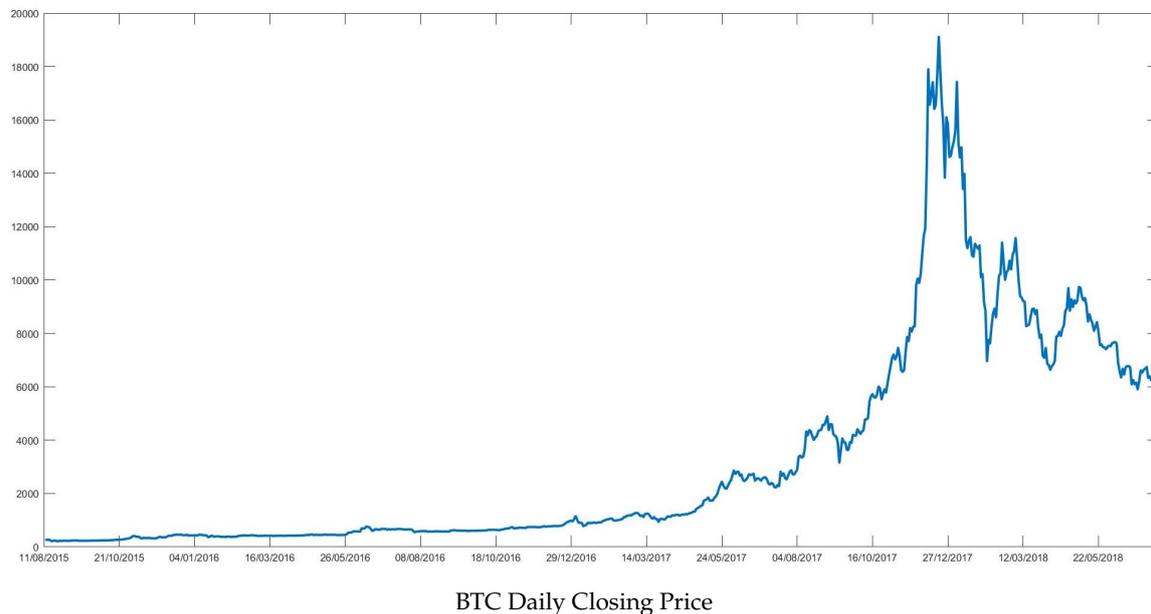


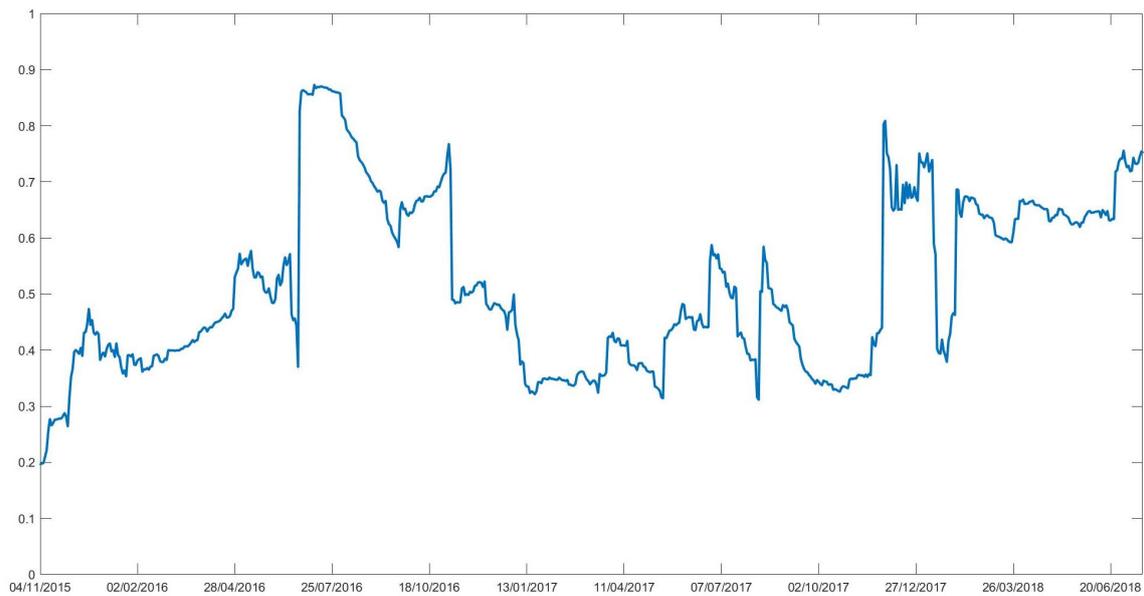
Figure 1. The figure reports the Bitcoin (BTC) closing price from August 2015 to July 2018. The plot clearly shows the steep rise in the price 2017 and the sharp drop in 2018.

4. Analysis

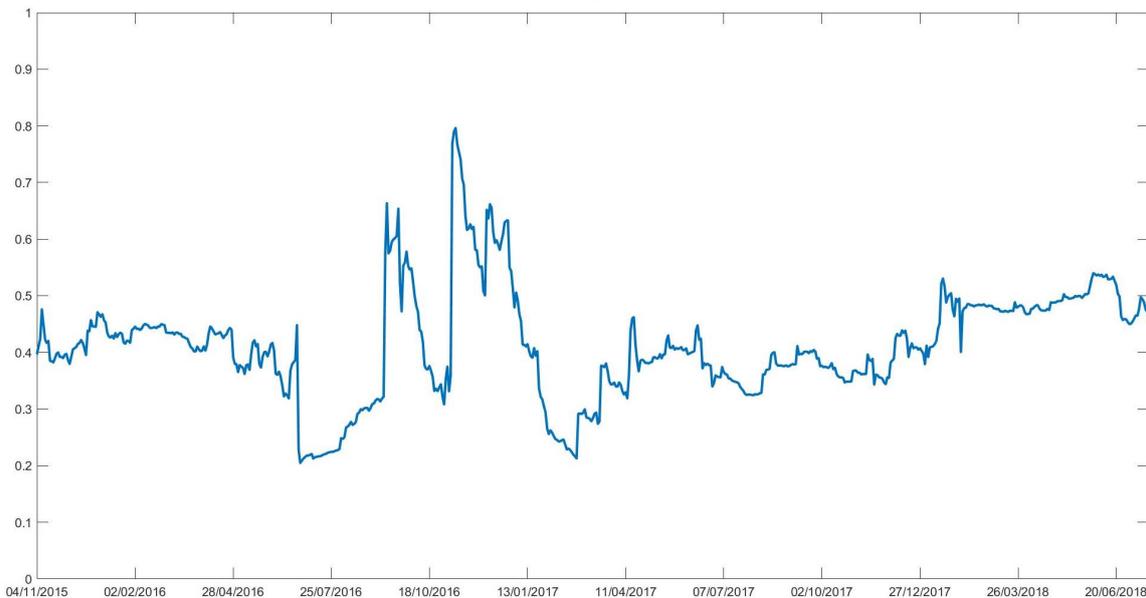
The out-of-sample period begins on 1st September 2016 and the forecast horizon ranges from $h = 1$ to $h = 7$ days ahead. The analysis compares the performances of two models: the first— M_1 —includes all the predictors: financial, commodity and crypto predictors, see Table 1. The second, M_2 , excludes crypto predictors. The benchmark model, denoted with M_0 , is an ARMA(1,1)-GARCH(1,1) model. M_1 and M_2 can suffer from massive model uncertainty due to the number of possible predictor's combination at each time point t . For example, M_1 has $2^9 = 512$ models at each point in time. To mitigate this fact, we use the DMA and DMS as described in Koop and Korobilis (2012) and reported in Supplementary Materials. As already mentioned, the methodology requires fixing three hyperparameters: the forgetting factor λ for the parameter variation; the decay factor κ for the EWMA; and, finally, the discount weight α that weights each model based on forecast performances.

The results reported in this section are based on $\kappa = 0.94$. This value suits daily data, see Riskmetrics (1996) and Prado and West (2010). The other parameters are set to $\alpha = 0.99$ and $\lambda = 0.99$, coherently with Raftery et al. (2010). In Section 4.2 a robustness analysis for the forgetting factors α and λ is carried out. Moreover, we also tried to optimize at each time point λ_t using a standard data-driven approach minimizing the expected prediction error. Unfortunately, the optimized λ_t with crypto time series seems to be very unstable; we leave this issue as a topic of further research.

The analysis begins with the investigation of the posterior inclusion probabilities of each predictor: the higher the probability the higher the predictor's influence over the dependent variable. Figure 2 depicts the posterior probabilities of BTC (Panel (a)), and of BHL (Panel (b)). Time-varying posterior probabilities of inclusion for the other exogenous variables are reported in Supplementary Materials.



(a) Posterior inclusion probability for BTC.



(b) Posterior inclusion probability for BHL.

Figure 2. Posterior inclusion probabilities. Panel (a) shows posterior inclusion probability for BTC. Panel (b) shows posterior inclusion probability for BHL.

The figures show that the importance of each predictor switches rapidly over time, with a high inclusion probability of BTC in some specific periods. One important change is in 2016 when the inclusion probability suddenly jumped from 0.5 to 0.9 increasing the correlation with the S&P500 and potentially its role as a leading indicator.

After a calm period during 2017, the BTC gained importance once again at the end of the same year with a steep rise in price. During this period, a lot of articles pointed out a correlation between BTC and financial markets. [Bloomberg \(2018\)](#) stated that “big investors may be dragging Bitcoin toward market correlation” and see BTC as an asset which guarantees the highest potential risk/return combination in the market. This may have attracted the interest of big investors able to move huge amounts of funds and consequently correlate BTC to the USA stock market. Another article by [Cointelegraph \(2018\)](#) asserted that BTC might be correlated with VIX, but there is no evidence that it may influence the

S&P500 index. An extensive analysis of the latter issue is carried out in the next sections using point and density forecast.

4.1. Forecast Metrics

To assess the leading property of BTC we use point and density forecast. For the point forecasts, we use the mean absolute forecast error (MAFE) for each forecast horizon, $h = 1, \dots, 7$:

$$MAFE_h = \frac{1}{T - R} \sum_{t=R}^{T-h} \left| \hat{y}_{i,t+h|t} - y_{i,t+h} \right|, \tag{2}$$

where T is the number of observations, R is the length of the rolling window, $\hat{y}_{i,t+h|t}$ is the S&P500 forecast made at time t for horizon h and $y_{i,t+h}$ is the realization.

To evaluate the density forecasts, we use predictive log score (LS) that is commonly viewed as the broadest measure of density accuracy, see [Geweke and Amisano \(2010\)](#). As for the MAFE, we compute the LS for each horizon:

$$s_h(y_i) = \sum_{t=R}^{T-h} \ln(f(y_{t+h}|I_t)), \tag{3}$$

where $f(y_{t+h}|I_t)$ is the predictive density for y_{t+h} constructed using information up to time t .

We report the MAFEs and the LSs as a ratio of each model's with respect to the baseline. Entries smaller than 1 indicate that a given model yields forecasts that are more accurate than those from the baseline and differences in score relative to the baseline, such as a negative number, indicates a model that beats the baseline. In order to statistically assess the differences between alternative models, we apply the [Diebold and Mariano \(1995\)](#) test for equality of the average loss (with loss defined as squared error and negative log score) of each model versus the ARMA(1,1)-GARCH(1,1) benchmark and we also employ the Model Confidence Set procedure of [Hansen et al. \(2011\)](#) using the R package MCS detailed in [Bernardi and Catania \(2016\)](#) to jointly compare all predictions. Differences are tested separately for each forecast horizon.

4.2. Point Forecast

Point forecast is evaluated through MAFE for both DMA and DMS as well as for their special case, Bayesian Model Averaging (BMA). For each forecast horizon, the errors are calculated using the following combination of forgetting and discount factors: $\lambda = \alpha = 0.99$, $\lambda = \alpha = 0.95$, $\lambda = 1$ and $\alpha = 0.99$, $\lambda = 0.99$ and $\alpha = 1$, and finally $\lambda = \alpha = 1$. In all the cases, the decay factor is fixed to $\kappa = 0.94$.

Table 2 compares point forecast for M_1 and M_2 as a ratio M_0 (top) and against M_0 (bottom). From the upper table, it emerges that the errors are increasing in accordance with the forecast horizon. Moreover, when h increases, the ratio increases, meaning that the benchmark model displays better results than DMA and DMS. Table S8 in Supplementary Materials B shows that increasing the forecasting horizon to $h = 10$ does not improve the forecasting performance of M_1 and M_2 . However, Section 4.3, which analyses density forecasts results, reveals different outcomes.

Another peculiarity is that forecasts improve when α and λ tend to 1. When $\alpha = \lambda = 0.95$ we get the worst forecast performance for DMA and DMS, while the best results are obtained with BMA. This may be due to the nature of the series: the presence of outliers and high peaks in BTC series may distort the point forecast.

To see if BTC improves predictability over the S&P 500 a DM test is performed with a level of significance equal to $\alpha = 95\%$. Results are reported in Supplementary Materials Table S2. There is no evidence of an improvement in prediction when the BTC is added to the set of predictors. Further results for different forecast horizons are reported in Supplementary Materials.

Using point forecast it seems that BTC does not improve predictability over the S&P 500 index.

Table 2. Point forecast: M_1 vs. M_0 top table and M_2 vs. M_0 bottom table. Results are reported as the ratio between the model considered over the benchmark. From both the tables it emerges that the simplest model, the ARMA(1,1)-GARCH(1,1), forecasts better than both DMA and DMS.

| M_1 vs. M_0 | | DMA | DMS | DMA | DMS | DMA | DMS | DMA | DMS | BMA |
|--|------|------------------|------------------|------------------|------------------|------------------|------------------|-----------------|-----------------|---------------|
| | | $\lambda = 0.99$ | $\lambda = 0.99$ | $\lambda = 0.95$ | $\lambda = 0.95$ | $\lambda = 0.99$ | $\lambda = 0.99$ | $\lambda = 1$ | $\lambda = 1$ | $\lambda = 1$ |
| | | $\alpha = 0.99$ | $\alpha = 0.99$ | $\alpha = 0.95$ | $\alpha = 0.95$ | $\alpha = 1$ | $\alpha = 1$ | $\alpha = 0.99$ | $\alpha = 0.99$ | $\alpha = 1$ |
| $\kappa = 0.94$ | | | | | | | | | | |
| $h = 1$ | MAFE | 1.006 | 1.000 | 1.063 | 1.089 | 1.013 | 1.011 | 1.014 | 1.013 | 1.026 |
| $h = 2$ | MAFE | 1.388 | 1.392 | 1.474 | 1.467 | 1.393 | 1.402 | 1.403 | 1.403 | 1.408 |
| $h = 3$ | MAFE | 1.651 | 1.668 | 1.760 | 1.798 | 1.651 | 1.659 | 1.667 | 1.655 | 1.658 |
| $h = 4$ | MAFE | 1.903 | 1.904 | 2.104 | 2.133 | 1.913 | 1.916 | 1.920 | 1.917 | 1.934 |
| $h = 5$ | MAFE | 2.121 | 2.125 | 2.368 | 2.401 | 2.118 | 2.118 | 2.151 | 2.150 | 2.188 |
| $h = 6$ | MAFE | 2.285 | 2.280 | 2.499 | 2.528 | 2.287 | 2.287 | 2.369 | 2.358 | 2.366 |
| $h = 7$ | MAFE | 2.481 | 2.466 | 2.699 | 2.702 | 2.483 | 2.482 | 2.619 | 2.621 | 2.611 |
| M_2 vs. M_0 | | DMA | DMS | DMA | DMS | DMA | DMS | DMA | DMS | BMA |
| | | $\lambda = 0.99$ | $\lambda = 0.99$ | $\lambda = 0.95$ | $\lambda = 0.95$ | $\lambda = 0.99$ | $\lambda = 0.99$ | $\lambda = 1$ | $\lambda = 1$ | $\lambda = 1$ |
| | | $\alpha = 0.99$ | $\alpha = 0.99$ | $\alpha = 0.95$ | $\alpha = 0.95$ | $\alpha = 1$ | $\alpha = 1$ | $\alpha = 0.99$ | $\alpha = 0.99$ | $\alpha = 1$ |
| $\kappa = 0.94$ | | | | | | | | | | |
| $h = 1$ | MAFE | 1.002 | 1.000 | 1.059 | 1.065 | 1.013 | 1.011 | 1.017 | 1.015 | 1.039 |
| $h = 2$ | MAFE | 1.386 | 1.387 | 1.462 | 1.460 | 1.391 | 1.390 | 1.403 | 1.402 | 1.403 |
| $h = 3$ | MAFE | 1.652 | 1.663 | 1.752 | 1.782 | 1.655 | 1.661 | 1.669 | 1.657 | 1.670 |
| $h = 4$ | MAFE | 1.904 | 1.910 | 2.112 | 2.128 | 1.908 | 1.910 | 1.917 | 1.914 | 1.923 |
| $h = 5$ | MAFE | 2.123 | 2.131 | 2.367 | 2.399 | 2.110 | 2.109 | 2.159 | 2.159 | 2.154 |
| $h = 6$ | MAFE | 2.288 | 2.289 | 2.506 | 2.527 | 2.291 | 2.295 | 2.385 | 2.374 | 2.386 |
| $h = 7$ | MAFE | 2.482 | 2.473 | 2.692 | 2.687 | 2.483 | 2.483 | 2.618 | 2.613 | 2.614 |

4.3. Density Forecast

Density forecast is more informative than point forecast, as it is a measure of the prediction uncertainty. The PL, which is the basis of density forecast, comes as a by-product of the adopted estimation strategy. Table 3 reports the LS: the evidence is striking and the results are almost opposite to those in Section 4.2. Both M_1 and M_2 provide statistically superior forecasts with respect to M_0 .

Table 3. Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (ARMA(1,1)-GARCH(1,1)) for which the absolute score is reported. Values in **bold** indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold-Mariano test at the 5% confidence level. Grey cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level 10%. As the table shows, the difference between M_1 and M_2 is very poor.

| Days Ahead | M_0 | $M_1 - M_0$ | $M_2 - M_0$ |
|------------|-----------|-------------|-------------|
| $h = 1$ | -1659.997 | -1023.949 | -1024.600 |
| $h = 2$ | -1651.352 | -1767.424 | -1763.904 |
| $h = 3$ | -1647.144 | -2188.181 | -2187.612 |
| $h = 4$ | -1643.063 | -2496.529 | -2498.355 |
| $h = 5$ | -1639.526 | -2733.322 | -2740.850 |
| $h = 6$ | -1636.366 | -2930.505 | -2927.831 |
| $h = 7$ | -1631.968 | -3097.841 | -3093.876 |

The first column reports the PL for the benchmark model (M_0), and the other columns report the differences of M_1 and M_2 with respect to the benchmark. Among the three models, M_0 shows the worst results in contrast with the results of Section 4.2. The best forecast result is obtained for M_1 when $h = 1$; however, the difference between M_1 and M_2 is almost irrelevant. Following the same strategy previously adopted the DM test is carried out, with a significance level of 95%.

The DM statistics, equal to -2.236 , suggest that the null hypothesis of equal forecasting ability is rejected. As discussed in Harvey et al. (1997), the DM test could be over-sized when the forecast horizon is greater than one, so in those cases, we used the modified test given by:

$$S_1^* = \left[\frac{P+1-2h+P^{-1}h(h-1)}{P} \right]^{1/2} S_1,$$

where S_1 is the original statistics, h is the forecast horizon, P is the forecast evaluation period. The modified version of DM test maintains the same null hypothesis of equal forecasting ability. Whereas, the alternative is that model M_2 is more accurate than model M_1 .

H_1 is accepted in this case since the test shows a very high p -value (0.987). In other words, model M_2 is performing better than M_1 in terms of forecasting. Finally, the MCS indicates that DMS and DMA has similar performance across horizons and they are both superior to the benchmark.

Therefore, the density forecast shows a different outcome to that of the point forecast. While the benchmark model performs better than DMA and DMS in terms of MAFEs, the opposite is true when the density forecast is considered. DMA and DMS give much better predictions for the S&P 500 when the PL is considered.

The main goal of the paper is to understand whether BTC can be assumed to be a good predictor for the S&P 500. Point forecast does not give any contribution to answering this question. A more precise result is reached when the density forecast is used. Even though the PL are close to each other, it has been found that the model that excludes the BTC related series outperforms the one that includes them at lag one. For the other lags, the results are mixed and almost all the models are included in the MCS without a clear superior model. This indicates that BTC does not seem to improve the predictability of the S&P 500 index.

Table S9 in Supplementary Materials reports the results for $\alpha = \lambda = 0.99$ and $\kappa = 0.94$ when the Dow Jones (DJ) index is substituted to S&P500. It emerges that, using DJ, the BTC improves the

result of both point and density forecast for shorter horizon (one or two days ahead). These results are promising and carrying out an extensive analysis for different markets is a topic for further research.

5. Conclusions

This work investigates whether BTC can be used as a leading indicator for the S&P500 index. We use the methodologies recently introduced by Raftery et al. (2010), which allow the predictor's weight to change dynamically over time. The study is based on two distinct models: the first— M_1 —includes all the predictors; the second— M_2 —excludes the crypto-predictors. The benchmark model— M_0 —is estimated using an ARMA(1,1)-GARCH(1,1).

The forecasting analysis is based on both point and density forecast. Results coming from the point forecast are not very satisfactory: the M_0 outperforms both M_1 and M_2 . Density forecast provides a totally different outcome: M_1 and M_2 strongly outperform M_0 . Unfortunately, in this case, the DM and the MCS do not give clear evidence about which of the two models is the best. Accordingly, from our results we can conclude that BTC does not show any predictive power over the S&P500 index. In the coming years, cryptocurrencies will surely receive more and more consideration and there is the possibility that our result may be disavowed.

Supplementary Materials: The supplementary materials are available online at <http://www.mdpi.com/1911-8074/12/2/93/s1>.

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