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Generalized (σ, ξ)-Contractions and Related Fixed Point Results in a P.M.S

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Abstract: In this paper, we present the concept of $\Theta - (\sigma, \xi)_{\Omega}$-contraction mappings and we nominate some related fixed point results in ordered p-metric spaces. Our results extend several famous ones in the literature. Some examples and an application are given in order to validate our results.

Keywords: fixed point; generalized contraction; complete ordered p-metric space

1. Introduction

The Banach contraction principle (BCP) [1] is an applicable instrumentation to solve problems in nonlinear analysis. The BCP has been modified in variant procedures (see e.g., [2–11]).

Definition 1. *[12] The function $\xi : [0, +\infty) \to [0, +\infty)$ verifying:*

1. *ξ is non-decreasing and continuous;*
2. *$\xi(t) = 0$ iff $t = 0$,*

is said to be an altering distance function.

Heretofore, many authors have concentrated on fixed point theorems depended on altering distance functions (see, e.g., [2,12–19]).

The concept of a b-metric space was nominated by Czerwik in [20]. Later, many interesting results about the existence of fixed points in b-metric spaces have been acquired (see, [2,21–33]).

Definition 2. *([20]) Let X be a (nonempty) set and $\varsigma \geq 1$ be a real number. A function $d : X \times X \to R^+$ is a b-metric if for all $\zeta, \nu, \mu \in X$,*

(b_1) $d(\zeta, \nu) = 0$ *iff* $\zeta = \nu$;
(b_2) $d(\zeta, \nu) = d(\nu, \zeta)$;
(b_3) $d(\zeta, \mu) \leq \varsigma[d(\zeta, \nu) + d(\nu, \mu)]$.

If $\varsigma = 1$, the b-metric is a metric.

Let $\to$ be the set of strictly increasing continuous functions $\Omega : [0, \infty) \to [0, \infty)$ such that $\Omega(0) = 0$ and $t \leq \Omega(t)$ for $t \geq 0$. Motivated by [20], we state the following.

Definition 3. *[34] Let X be a (nonempty) set. A function $\rho : X \times X \to R^+$ is a p-metric iff there is $\Omega \in \to$ so that*

(p_1) $\rho(\zeta, \nu) = 0$ *iff* $\zeta = \nu$,
(p_2) $\rho(\zeta, \nu) = \rho(\nu, \zeta)$,
(p_3) $\rho(\zeta, \mu) \leq \Omega(\rho(\zeta, \nu) + \rho(\nu, \mu))$,

for all $\zeta, \nu, \mu \in X$. (X, d) is said to be a p.m.s. (or an extended b-metric space).

It should be mentioned that, the class of p-metric spaces is considerably comprehensive than the class of b-metric spaces. Note that a b-metric (with a coefficient $\varsigma \geq 1$) is a p-metric, when $\Omega(t) = \varsigma t$. If $\Omega(t) = t$, a p-metric is a metric.

Example 1. *[34] Let (X, d) be a metric space. Take $\rho(\zeta, \nu) = e^{d(\zeta,\nu)} - 1$. Then ρ is a p-metric with $\Omega(t) = e^t - 1$.*

The following example shows that a p-metric need not be a b-metric.

Example 2. *[34] Let (X, d) be a b-metric space (with a coefficient $\varsigma \geq 1$). Consider $\rho(\zeta, \nu) = \sinh[d(\zeta, \nu)]$. Then ρ is a p-metric with $\Omega(t) = \sinh(\varsigma t)$, $t \geq 0$.*

For $\varsigma = 1$, $\zeta = 2$, $\nu = -3$, $\mu = 0$ and $d(\zeta, \nu) = |\zeta - \nu|$, we have

$$\rho(\zeta, \nu) = \sinh(5) > \sinh(2) + \sinh(3) = \rho(\zeta, \mu) + \rho(\mu, \nu).$$

Definition 4. *[34] Let (X, ρ) be a p.m.s. A sequence $\{\mu_n\}$ in X*

(a) p-converges iff there is $\mu \in X$ so that $\rho(\mu_n, \mu) \to 0$, as $n \to +\infty$. In this case, we write $\lim_{n\to\infty} \mu_n = \mu$;

(b) is p-Cauchy iff $\rho(\mu_n, \mu_m) \to 0$ as $n, m \to +\infty$.

Note that a p.m.s (X, ρ) is p-complete if every p-Cauchy sequence in X is p-convergent.

Lemma 1. *Let (X, ρ) be a p.m.s. Suppose that $\{\mu_n\}$ and $\{\nu_n\}$ p-converge to μ, ν, respectively. Then*

$$(\Omega^2)^{-1}(\rho(\mu, \nu)) \leq \liminf_{n \longrightarrow \infty} \rho(\mu_n, \nu_n) \leq \limsup_{n \longrightarrow \infty} \rho(\mu_n, \nu_n) \leq \Omega^2(\rho(\mu, \nu)).$$

Additionally, if $\mu = \nu$, then $\lim_{n \longrightarrow \infty} \rho(\mu_n, \nu_n) = 0$. Also, for any $z \in X$,

$$\Omega^{-1}(\rho(\mu, z)) \leq \liminf_{n \longrightarrow \infty} \rho(\mu_n, z) \leq \limsup_{n \longrightarrow \infty} \rho(\mu_n, z) \leq \Omega(\rho(\mu, z)).$$

The idea of Θ-contraction has been introduced by Jleli and Samet in [35] which provides an interesting generalization of BCP. Zhang and Song generalized the BCP using two altering distance functions [36]. Our approach provides a generalization of Zhang-Song result using the idea of Θ-contraction. In fact, we present the notion of generalized $\Theta - (\sigma, \xi)_\Omega$-contractive mappings (where σ and ξ are altering distance functions) and we inaugurate some related fixed point results in complete ordered p-metric spaces. We also give some examples and an application.

2. Main Results

We first provide the notion of $\Theta - (\sigma, \xi)_\Omega$-contractions.

Let Υ be a self-map on the ordered p.m.s $(X, \preceq, \rho)$. Consider

$$P(x, y) = \max\left\{\rho(x, y), \rho(x, \Upsilon x), \rho(y, \Upsilon y), \frac{\Omega^{-1}[\rho(x, \Upsilon y) + \rho(y, \Upsilon x)]}{2}\right\}.$$

Motivated by [35], denote by Δ the set of functions $\Theta : [0, \infty) \to [1, \infty)$ so that

(Θ_1) Θ is continuous and non-decreasing;

(Θ_2) for any $\{t_n\} \subseteq (0, \infty)$, $\lim_{n\to\infty} \Theta(t_n) = 1$ iff $\lim_{n\to\infty} t_n = 0$.

Definition 5. *Let $(X, \preceq, \rho)$ be an ordered p.m.s. The mapping $\Upsilon : X \to X$ is an ordered $\Theta - (\sigma, \xi)_\Omega$-contraction if there are $\Theta \in \Delta$, $\Omega \in \omega$ and two altering distance functions σ and ξ, so that*

$$\Theta(\sigma(\Omega^2(\rho(\Upsilon x, \Upsilon y)))) \leq \frac{\Theta(\sigma(P(x,y)))}{\Theta(\xi(P(x,y)))} \tag{1}$$

for all comparable elements $x, y \in X$.

Our first result is

Theorem 1. *Let $(X, \preceq, \rho)$ be an ordered p-complete p.m.s. Suppose that $\Upsilon : X \to X$ is an ordered non-decreasing continuous $\Theta - (\sigma, \xi)_\Omega$-contractive mapping. If there is $r_0 \in X$ such that $r_0 \preceq \Upsilon r_0$, then Υ admits a fixed point.*

Proof. Let $r_0 \in X$ satisfy $r_0 \preceq \Upsilon r_0$. Consider a sequence (r_n) in X so that $r_{n+1} = \Upsilon r_n$ for each $n \geq 0$. Since $r_0 \preceq \Upsilon r_0 = r_1$ and Υ is non-decreasing, we have $r_1 = \Upsilon r_0 \preceq r_2 = \Upsilon r_1$. Inductively, we have

$$r_0 \preceq r_1 \preceq \cdots \preceq r_n \preceq r_{n+1} \preceq \cdots .$$

If $r_k = r_{k+1}$ for some $k \in \mathbb{N}$, so r_k is a fixed point of Υ. Suppose that $r_n \neq r_{n+1}$ for each $n \geq 0$. According to (1) and the fact $\Omega \in \to$, we have

$$\begin{aligned} \Theta(\sigma(\rho(r_n, r_{n+1}))) &\leq \Theta(\sigma(\Omega^2(\rho(r_n, r_{n+1})))) \\ &= \Theta(\sigma(\Omega^2(\rho(\Upsilon r_{n-1}, \Upsilon r_n)))) \\ &\leq \frac{\Theta(\sigma(P(r_{n-1}, r_n)))}{\Theta(\xi(P(r_{n-1}, r_n)))}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} P(r_{n-1}, r_n) &= \max\left\{\rho(r_{n-1}, r_n), \rho(r_{n-1}, \Upsilon r_{n-1}), \rho(r_n, \Upsilon r_n), \frac{\Omega^{-1}[\rho(r_{n-1}, \Upsilon r_n) + \rho(r_n, \Upsilon r_{n-1})]}{2}\right\} \\ &\leq \max\left\{\rho(r_{n-1}, r_n), \rho(r_n, r_{n+1})\right\}. \end{aligned} \tag{3}$$

From (2) to (3) and the assumptions on σ and ξ, we deduce that

$$\begin{aligned} \Theta(\sigma(\rho(r_n, r_{n+1}))) &\leq \frac{\Theta(\sigma\left(\max\left\{\rho(r_{n-1}, r_n), \rho(r_n, r_{n+1})\right\}\right))}{\Theta(\xi\left(P(r_{n-1}, r_n)\right))} \\ &< \Theta(\sigma\left(\max\left\{\rho(r_{n-1}, r_n), \rho(r_n, r_{n+1})\right\}\right)). \end{aligned} \tag{4}$$

If for some n,

$$\max\left\{\rho(r_{n-1}, r_n), \rho(r_n, r_{n+1})\right\} = \rho(r_n, r_{n+1}),$$

then by (4) we have

$$\begin{aligned} \Theta(\sigma(\rho(r_n, r_{n+1}))) &\leq \frac{\Theta(\sigma(\rho(r_n, r_{n+1})))}{\Theta(\xi(P(r_{n-1}, r_n)))} \\ &< \Theta(\sigma(\rho(r_n, r_{n+1}))), \end{aligned}$$

which gives a contradiction. Thus,

$$\max\left\{\rho(r_{n-1},r_n),\rho(r_n,r_{n+1})\right\}=\rho(r_{n-1},r_n),\quad \text{for each } n\geq 0.$$

Therefore, (4) yields that

$$\Theta(\sigma(\rho(r_n,r_{n+1})))\leq\frac{\Theta(\sigma(\rho(r_n,r_{n-1})))}{\Theta(\xi(P(r_{n-1},r_n)))}<\Theta(\sigma(\rho(r_n,r_{n-1}))),\quad \text{for each } n\geq 0. \tag{5}$$

Since $\Theta\in\Delta$ and σ is non-decreasing, the positive sequence $\{\rho(r_n,r_{n+1})\}$ is non-increasing. Thus, there is $r\geq 0$ so that

$$\lim_{n\to\infty}\rho(r_n,r_{n+1})=r.$$

Taking $n\to\infty$ in (5), we get

$$\Theta(\sigma(r))\leq\frac{\Theta(\sigma(r))}{\Theta(\xi(\lim_{n\to\infty}P(r_{n-1},r_n)))}\leq\Theta(\sigma(r)).$$

Therefore, $\Theta(\xi(\lim_{n\to\infty}P(r_{n-1},r_n)))=1$ which supplies that $\xi(\lim_{n\to\infty}P(r_{n-1},r_n))=0$, and so $r=0$, that is,

$$\lim_{n\to\infty}\rho(r_n,r_{n+1})=0. \tag{6}$$

Next, we demonstrate that $\{r_n\}$ is a p-Cauchy sequence in X. By contradiction, there is $\varepsilon>0$ for which we can gain $\{r_{m_i}\}$ and $\{r_{n_i}\}$ of $\{r_n\}$ so that

$$n_i>m_i>i,\;\;\rho(r_{m_i},r_{n_i})\geq\varepsilon \tag{7}$$

and

$$\rho(r_{m_i},r_{n_i-1})<\varepsilon. \tag{8}$$

The p-triangular inequality leads to

$$\begin{aligned}\varepsilon&\leq\rho(r_{m_i},r_{n_i})\\&\leq\Omega(\rho(r_{m_i},r_{m_i-1})+\rho(r_{m_i-1},r_{n_i}))\\&\leq\Omega(\rho(r_{m_i},r_{m_i-1})+\Omega(\rho(r_{m_i-1},r_{n_i-1})+\rho(r_{n_i-1},r_{n_i}))).\end{aligned}$$

Exploiting (6), (7) and (8), we have

$$(\Omega^2)^{-1}(\varepsilon)\leq\liminf_{i\longrightarrow\infty}\rho(r_{m_i-1},r_{n_i-1}).$$

Likewise,

$$\rho(r_{m_i-1},r_{n_i-1})\leq\Omega(\rho(r_{m_i-1},r_{m_i})+\rho(r_{m_i},r_{n_i-1})).$$

Handling (6) and (8), we have

$$\limsup_{i\longrightarrow\infty}\rho(r_{m_i-1},r_{n_i-1})\leq\Omega(\varepsilon), \tag{9}$$

Moreover,

$$\rho(r_{m_i},r_{n_i})\leq\Omega(\rho(r_{m_i},r_{n_i-1})+\rho(r_{n_i-1},r_{n_i})).$$

Appling (5) and (8), we have

$$\limsup_{i\longrightarrow\infty}\rho(r_{m_i},r_{n_i-1})\geq\Omega^{-1}(\varepsilon),$$

In addition,

$$\rho(r_{m_i-1}, r_{n_i}) \leq \Omega(\rho(r_{m_i-1}, r_{m_i}) + \Omega[\rho(r_{m_i}, r_{n_i-1}) + \rho(r_{n_i-1}, r_{n_i})]).$$

Using (6) and (8), we have

$$\limsup_{i\longrightarrow\infty} \rho(r_{m_i}, r_{n_i-1}) \leq \Omega^2(\varepsilon).$$

Moreover,

$$\rho(r_{m_i}, r_{n_i}) \leq \Omega(\rho(r_{m_i}, r_{m_i-1}) + \rho(r_{m_i-1}, r_{n_i})).$$

Appling (6) and (8), we get

$$\limsup_{i\longrightarrow\infty} \rho(r_{m_i-1}, r_{n_i}) \geq \Omega^{-1}(\varepsilon).$$

From (1),

$$\begin{aligned} \Theta(\sigma(\Omega^2(\rho(r_{m_i}, r_{n_i})))) &= \Theta(\sigma(\Omega^2(\rho(\Upsilon r_{m_i-1}, \Upsilon r_{n_i-1})))) \\ &\leq \frac{\Theta(\sigma(P(r_{m_i-1}, r_{n_i-1})))}{\Theta(\xi(P(r_{m_i-1}, r_{n_i-1})))}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} &P(r_{m_i-1}, r_{n_i-1})] \\ &= \max\left\{\rho(r_{m_i-1}, r_{n_i-1}), \rho(r_{m_i-1}, \Upsilon r_{m_i-1}), \rho(r_{n_i-1}, \Upsilon r_{n_i-1}), \frac{\Omega^{-1}[\rho(r_{m_i-1}, \Upsilon r_{n_i-1}) + \rho(r_{n_i-1}, \Upsilon r_{m_i-1})]}{2}\right\} \\ &= \max\left\{\rho(r_{m_i-1}, r_{n_i-1}), \rho(r_{m_i-1}, r_{m_i}), \rho(r_{n_i-1}, r_{n_i}), \frac{\Omega^{-1}[\rho(r_{m_i-1}, r_{n_i}) + \rho(r_{n_i-1}, r_{m_i})]}{2}\right\}. \end{aligned} \tag{11}$$

Taking $i \to \infty$ in (11) and using (6), we achieve that,

$$\begin{aligned} &\limsup_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1}) \\ &= \max\{\limsup_{i\longrightarrow\infty} \rho(r_{m_i-1}, r_{n_i-1}), 0, 0, \limsup_{i\longrightarrow\infty} \rho(r_{m_i}, r_{n_i-1})\} \leq \Omega^2(\varepsilon). \end{aligned} \tag{12}$$

Similarly,

$$(\Omega^2)^{-1}(\varepsilon) \leq \liminf_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1}). \tag{13}$$

Now, taking $i \to \infty$ in (10) and using (7) and (12),

$$\begin{aligned} \Theta(\sigma(\Omega^2(\varepsilon))) &\leq \Theta(\sigma(\Omega^2(\limsup_{i\longrightarrow\infty} \rho(r_{m_i}, r_{n_i})))) \\ &\leq \frac{\Theta(\sigma(\limsup_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1})))}{\Theta(\liminf_{i\longrightarrow\infty} \xi(P(r_{m_i-1}, r_{n_i-1})))} \\ &\leq \frac{\Theta(\sigma(\Omega^2(\varepsilon)))}{\Theta(\xi(\liminf_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1})))}. \end{aligned}$$

It yields that

$$\xi(\liminf_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1})) = 0,$$

so, $\liminf_{i\longrightarrow\infty} P(r_{m_i-1}, r_{n_i-1}) = 0$, a contradiction to (13). Thus, $\{r_{n+1} = \Upsilon r_n\}$ is a p-Cauchy sequence in the p-complete space X, so there is $u \in X$ so that $r_n \to u$. According to the continuity of Υ,

$$\lim_{n\to\infty} r_{n+1} = \lim_{n\to\infty} \Upsilon r_n = \Upsilon u. \tag{14}$$

The p-triangular inequality leads to

$$\begin{aligned}\rho(u,\Upsilon u) \leq & \Omega(\rho(u,\Upsilon r_n)+\rho(\Upsilon r_n,\Upsilon u)) \\ = & \Omega(\rho(u,r_{n+1})+\rho(\Upsilon r_n,\Upsilon u)) \\ \leq & \Omega[\Omega(\rho(u,r_n)+\rho(r_n,r_{n+1}))+\rho(\Upsilon r_n,\Upsilon u)].\end{aligned}$$

The continuity of Ω together with and (14) imply that

$$\rho(u,\Upsilon u) \leq \Omega[\Omega(\lim_{n\to\infty}\rho(u,r_n)+\lim_{n\to\infty}\rho(r_n,r_{n+1}))+\lim_{n\to\infty}\rho(\Upsilon r_n,\Upsilon u)] = 0.$$

We find that $\Upsilon u = u$. □

The continuity of Υ in Theorem 1 can be substituted by the following reservation:

An ordered p.m.s $(X,\preceq,p)$ possesses the sequential limit comparison property (s.l.c.p) if for each nondecreasing sequence $\{r_n\}$ in X, converging to some $x\in X$, we have $r_n \preceq x$ for each $n\in\mathbb{N}$.

Theorem 2. *Having the same assumptions of Theorem 1, by replacing the continuity of Υ with the s.l.c.p. property of $(X,\preceq,\rho)$, Υ encompasses a fixed point.*

Proof. Reviewing the lines of the proof of Theorem 1, we have that $\{r_n\}$ is an increasing sequence in X so that $r_n \to u$, for $u\in X$. Using the *s.l.c.p.* obligation on X, we have $r_n \preceq u$, for any $n\in\mathbb{N}$. We claim that $\Upsilon u = u$. By (1),

$$\begin{aligned}\Theta(\sigma(\Omega^2(\rho(r_{n+1},\Upsilon u)))) & = \Theta(\sigma(\Omega^2(\rho(\Upsilon r_n,\Upsilon u)))) \\ & \leq \frac{\Theta(\sigma(P(r_n,u)))}{\Theta(\xi(P(r_n,u)))},\end{aligned} \tag{15}$$

where

$$\begin{aligned}& P(r_n,u) \\ & = \max\left\{\rho(r_n,u),\rho(r_n,\Upsilon r_n),\rho(u,\Upsilon u),\frac{\Omega^{-1}[\rho(r_n,\Upsilon u)+\rho(u,\Upsilon r_n)]}{2}\right\} \\ & = \max\left\{\rho(r_n,u),\rho(r_n,r_{n+1}),\rho(u,\Upsilon u),\frac{\Omega^{-1}[\rho(r_n,\Upsilon u)+\rho(u,r_{n+1})]}{2}\right\}.\end{aligned} \tag{16}$$

Making $n\to\infty$ in (16) and using Lemma 1, we get

$$\limsup_{n\longrightarrow\infty} P(r_n,u) = \rho(u,\Upsilon u). \tag{17}$$

Likely, we can obtain

$$\liminf_{n\longrightarrow\infty} P(r_n,u) = \rho(u,\Upsilon u). \tag{18}$$

The the upper limit as $n\to\infty$ in (15) together with Lemma 1 and (17) imply that

$$\begin{aligned}\Theta(\sigma(\rho(u,\Upsilon u))) & = \Theta(\sigma(\Omega(\Omega^{-1}(\rho(u,\Upsilon u))))) \\ & \leq \Theta(\sigma(\Omega^2(\limsup_{n\longrightarrow\infty}\rho(r_{n+1},\Upsilon u)))) \\ & \leq \frac{\Theta(\sigma(\limsup_{n\longrightarrow\infty} P(r_n,u)))}{\Theta(\liminf_{n\longrightarrow\infty}\xi(P(r_n,u)))} \\ & \leq \frac{\Theta(\sigma(\rho(u,\Upsilon u)))}{\Theta(\xi(\liminf_{n\longrightarrow\infty} P(r_n,u)))}.\end{aligned}$$

Therefore, $\xi(\liminf_{n\longrightarrow\infty} P(r_n,u)) \to 0$, equivalently, $\liminf_{n\longrightarrow\infty} P(r_n,u) = 0$. Thus, from (18) we get $u = \Upsilon u$ and hereupon u is a fixed point of Υ. □

Remark 1. *Substituting $\Theta(t) = e^t$ in (1), we obtain the following contractive condition:*

$$\sigma(\Omega^2(\rho(\Upsilon x, \Upsilon y))) \leq \sigma(P(x,y)) - \xi(P(x,y))$$

which is the Zhang-Song contractive condition in a p-metric space.

Corollary 1. *Let $(X, \preceq, \rho)$ be an ordered p-complete p.m.s. Let $\Upsilon : X \to X$ be an ordered non-decreasing mapping. Assume there is $k \in [0,1)$ so that*

$$\Omega^2(\rho(\Upsilon x, \Upsilon y)) \quad \leq k \max\left\{\rho(x,y), \rho(x,\Upsilon x), \rho(y,\Upsilon y), \frac{\Omega^{-1}[\rho(x,\Upsilon y)+\rho(y,\Upsilon x)]}{2}\right\},$$

for all comparable elements $x, y \in X$. If there is $r_0 \in X$ so that $r_0 \preceq \Upsilon r_0$, then Υ admits a fixed point provided that either Υ is continuous, or $(X, \preceq, p)$ enjoys the s.l.c.p.

Proof. It follows using Theorems 1 and 2 by taking $\Theta(t) = e^t$, $\sigma(t) = t$ and $\xi(t) = (1-k)t$. □

Corollary 2. *Let $(X, \preceq, \rho)$ be an ordered p-complete p.m.s. Let $\Upsilon : X \to X$ be an ordered non-decreasing mapping. Assume that there are $\alpha, \beta, \gamma, \delta \in [0,1)$ with $\alpha + \beta + \gamma + \delta \in [0,1)$ so that*

$$\Omega^2(\rho(\Upsilon x, \Upsilon y)) \leq \alpha\rho(x,y) + \beta\rho(x,\Upsilon x) + \gamma\rho(y,\Upsilon y) + \delta\frac{\Omega^{-1}[\rho(x,\Upsilon y)+\rho(y,\Upsilon x)]}{2},$$

for all comparable elements $x, y \in X$. If there is $r_0 \in X$ such that $r_0 \preceq \Upsilon r_0$, then Υ has a fixed point provided that either Υ is continuous, or $(X, \preceq, p)$ possesses the s.l.c.p.

The following corollary is an enlargement of BCP in a p.m.s., where $\rho(x,y) = e^{d(x,y)} - 1$.

Corollary 3. *Let Υ be a non-decreasing self-mapping on an ordered p-complete p.m.s $(X, \preceq, \rho)$. Assume that there is $\alpha \in [0,1)$ such that*

$$e^{\rho(\Upsilon x, \Upsilon y)} - 1 \leq \alpha\rho(x,y),$$

for all comparable elements $x, y \in X$. If there is $r_0 \in X$ such that $r_0 \preceq \Upsilon r_0$, then Υ has a fixed point provided that either Υ is continuous, or $(X, \preceq, p)$ enjoys the s.l.c.p.

Remark 2. *A subset W in an ordered set X is well ordered if each two elements of W are comparable. In Theorems 1 and 2, Υ admits a unique fixed point whenever the fixed points of Υ are comparable.*

Remark 3. *For any p-metric space (X, ρ), the conclusion of Theorems 1 and 2 remains true if σ, ξ are only non-decreasing on $diam(X) = \sup_{x,y \in X} \rho(x,y)$.*

Corollary 4. *Let $(X, \preceq, \rho)$ be a partially ordered p-complete p-metric space. Let $\Upsilon : X \to X$ be an ordered non-decreasing mapping. Suppose that there exists $k \in [0,1)$ such that*

$$\Omega^2(\rho(\Upsilon x, \Upsilon y)) \quad \leq k \max\left\{\rho(x,y), \rho(x,\Upsilon x), \rho(y,\Upsilon y), \frac{\Omega^{-1}[\rho(x,\Upsilon y)+\rho(y,\Upsilon x)]}{2}\right\},$$

for all comparable elements $x, y \in X$. If there is $r_0 \in X$ such that $r_0 \preceq \Upsilon r_0$, then Υ has a fixed point provided that either Υ is continuous, or $(X, \preceq, p)$ enjoys the s.l.c.p.

Proof. It follows from Theorems 1 and 2, by taking $\Theta(t) = e^t$, $\sigma(t) = t$ and $\xi(t) = (1-k)t$ for each $t \in [0, +\infty)$. □

Corollary 5. *Let $(X, \preceq, \rho)$ be a partially ordered p-complete p-metric space. Let $\Upsilon : X \to X$ be an ordered non-decreasing mapping. Suppose that there are $\alpha, \beta, \gamma, \delta \in [0, 1)$ with $\alpha + \beta + \gamma + \delta \in [0, 1)$ such that*

$$\Omega^2(\rho(\Upsilon x, \Upsilon y)) \quad \leq \alpha\rho(x, y) + \beta\rho(x, \Upsilon x) + \gamma\rho(y, \Upsilon y) + \delta\frac{\Omega^{-1}[\rho(x,\Upsilon y)+\rho(y,\Upsilon x)]}{2},$$

for all comparable elements $x, y \in X$. If there is $r_0 \in X$ such that $r_0 \preceq \Upsilon r_0$, then Υ has a fixed point provided that either Υ is continuous, or $(X, \preceq, p)$ enjoys the s.l.c.p.

Example 3. *Take $X = \{0, 1, 2, 3\}$. Define on X the partial order $\preceq$:*

$$\preceq := \{(0, 0),(1, 1),(2, 2),(3, 3),(1, 2),(0, 1),(0, 2)\}.$$

Define the metric

$$d(x, y) = \begin{cases} 0, & \text{if } x = y, \\ x + y, & \text{if } x \neq y \end{cases}$$

and let $\rho(x, y) = \sinh[d(x, y)]$. Note that (X, ρ) is a p-complete p-metric space [Here, $\Omega(t) = \sinh(t)$ for $t \geq 0$].

Define the self-map Υ by

$$\Upsilon = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix}.$$

We see that Υ is an ordered increasing mapping and $(X, \preceq, \rho)$ enjoys the s.l.c.p. Define $\sigma(t) = \sqrt{t}$ and $\xi(t) = \dfrac{t^2}{15 + t^2}$ and $\Theta(t) = 1 + t^2$. We show that Υ is an ordered non-decreasing $\Theta - (\sigma, \xi)_\Omega$-contractive mapping. Indeed, let $x, y \in X$ with $x \preceq y$. If $(x, y) \in \{(0, 0),(1, 1),(2, 2),(3, 3),(0, 1)\}$, then we have nothing to prove. Thus, we need to only check the following cases:

Case 1. $(x, y) = (1, 2)$. Here,

$$\begin{aligned} \sigma(\Omega^2(\rho(\Upsilon x, \Upsilon y))) &= \sqrt{\sinh^3(\Upsilon 1 + \Upsilon 2)} \\ &= \sqrt{\sinh^3(0 + 1)} \\ &= 1.623, \end{aligned}$$

$$\sigma(P(x, y)) = \sqrt{P(x, y)} = \sqrt{\sinh 3} = 3.16,$$

$$\xi(P(x, y)) = \frac{P(x, y)^2}{15 + P(x, y)^2} = \frac{(\sinh 3)^2}{15 + (\sinh 3)^2} = 0.86,$$

$$\begin{aligned} \Theta(\sigma(\Omega^2(\rho(\Upsilon x, \Upsilon y)))) &= \Theta(1.623) \\ &= 3.63 \leq 6.31 = \frac{3.16^2 + 1}{0.86^2 + 1} \\ &= \frac{\Theta(\sigma(P(x, y)))}{\Theta(\xi(P(x, y)))} \end{aligned}$$

Case 2. $(x, y) = (0, 2)$. We have

$$\begin{aligned} \sigma(\Omega^2(\rho(\Upsilon x, \Upsilon y))) &= \sqrt{\sinh^3(\Upsilon 0 + \Upsilon 2)} \\ &= \sqrt{\sinh^3(0 + 1)} \\ &= 1.623, \end{aligned}$$

$$\sigma(P(x,y)) = \sqrt{P(x,y)} = \sqrt{\sinh 2} = 1.904,$$

$$\xi(P(x,y)) = \frac{P(x,y)^2}{15+P(x,y)^2} = \frac{(\sinh 2)^2}{15+(\sinh 2)^2} = 0.467,$$

$$\Theta(\sigma(\Omega^2(\rho(\Upsilon x,\Upsilon y)))) = \Theta(1.623) = 3.634 \leq 3.797 = \frac{1.904^2+1}{0.467^2+1} = \frac{\Theta(\sigma(P(x,y)))}{\Theta(\xi(P(x,y)))}.$$

Also, any two fixed points of Υ *are comparable. Thus, all of the conditions of Theorem 2 are satisfied, and so* Υ *has a unique fixed point, which is,* 0.

Remark 4. *Taking* $(x,y) = (0,2)$ *in Example 3, we have*

$$\sigma(\Omega^2(\rho(\Upsilon x,\Upsilon y))) = 1.623 > 1.437 = 1.904 - 0.467 = \sigma(P(x,y)) - \xi(P(x,y)).$$

Thus, we can not apply the main result of Roshan et al. [30]. Also, we have $|\Upsilon 1 - \Upsilon 2| = |0-1| = |1-2|$ *and* $1 \preceq 2$*. Thus,* Υ *is neither a Banach contraction, nor an ordered Banach contraction, with the usual metric. This example shows that our result is a real generalization of the similar results in literature in the setting of b-metric spaces and metric spaces.*

Corollary 6. *Let* $(X,\preceq,\rho)$ *be an ordered p-complete p-metric space. Let* $\Upsilon : X \to X$ *be an ordered non-decreasing continuous ordered mapping and suppose that there exist altering distance functions* σ,ξ *satisfying*

$$1+\ln(1+(\sigma(\Omega^2(\rho(\Upsilon x,\Upsilon y)))) \leq \frac{1+\ln(1+(\sigma(P(x,y)))}{1+\ln(1+(\xi(P(x,y)))}. \quad (19)$$

If there is $r_0 \in X$ *such that* $r_0 \preceq \Upsilon r_0$*, then* Υ *has a fixed point. Moreover if any two fixed points of* Υ *are comparable, then the fixed point of* Υ *is unique and for any* $r_0 \in X$*, the iterated sequence* $\{\Upsilon^n(r_0)\}_{n\in\mathbb{N}}$ *converges to the fixed point.*

In much the same way as in Theorem 2 we can prove:

Theorem 3. *Let* $(X,\preceq,\rho)$ *be an ordered p-complete p-metric space. Let* $\Upsilon : X \to X$ *be an ordered continuous non-decreasing mapping satisfying*

$$\Theta(\sigma(\Omega(\rho(\Upsilon x,\Upsilon y)))) \leq \frac{\Theta(\sigma(\rho(x,y)))}{\Theta(\xi(\rho(x,y)))} \quad (20)$$

for all $x,y \in X$ *with* $x \preceq y$*. If there is* $r_0 \in X$ *such that* $r_0 \preceq \Upsilon r_0$*, then* Υ *has a fixed point. Moreover, if any two fixed points of* Υ *are comparable, then the fixed point of* Υ *is unique and for any* $r_0 \in X$*, the iterated sequence* $\{\Upsilon^n(r_0)\}_{n\in\mathbb{N}}$ *converges to the fixed point.*

Theorem 4. *Let* $(X,\preceq,\rho)$ *be an ordered p-complete p-metric space. Let* $\Upsilon : X \to X$ *be a non-decreasing mapping satisfying*

$$\Theta(\sigma(\Omega(\rho(\Upsilon x,\Upsilon y)))) \leq \frac{\Theta(\sigma(\rho(x,y)))}{\Theta(\xi(\rho(x,y)))} \quad (21)$$

for all $x,y \in X$ *with* $x \preceq y$*. Assume that* $(X,\preceq,p)$ *enjoys the s.l.c.p. If there is* $r_0 \in X$ *so that* $r_0 \preceq \Upsilon r_0$*, then* Υ *has a fixed point. Moreover, if any two fixed points of* Υ *are comparable, then the fixed point of* Υ *is unique and for any* $r_0 \in X$*,* $\{\Upsilon^n(r_0)\}_{n\in\mathbb{N}}$ *converges to the fixed point.*

Example 4. *Let* $X = [5,6]$*. Given the p-metric* $\rho(\zeta,\nu) = e^{|\zeta-\nu|} - 1$ *(Here,* $\Omega(t) = e^t - 1$*).*

Consider on X: $\zeta \preceq \nu$ iff $\nu \leq \zeta$. Given $\Upsilon : X \to X$ as

$$\Upsilon\zeta = 3\ln(1+\zeta)$$

Take $\sigma(t) = 2\ln(1+t)$ and $\xi(t) = 2\ln(1+t) - 0.9t$ for each $t \geq 0$. Now, we show that Υ is an ordered $\Theta - (\sigma, \xi)_{\Omega}$-contractive mapping with $\Theta(t) = 1 + [\ln(t+1)]^2$.

Let $\zeta \preceq \nu$, that is $\nu \leq \zeta$. The mean value theorem for $s \longmapsto 3\ln(1+s)$ yields that

$$\begin{aligned}
\sigma(\Omega(\rho(\Upsilon\zeta, \Upsilon\nu))) &= 2\ln(\Omega(\rho(\Upsilon\zeta, \Upsilon\nu)) + 1)) \\
&= 2\rho(\Upsilon\zeta, \Upsilon\nu) \\
&= 2(e^{\frac{|\Upsilon\zeta - \Upsilon\nu|}{2}} - 1) \\
&= 2(e^{\frac{3\ln(1+\zeta) - 3\ln(1+\nu)}{2}} - 1) \\
&= 2(e^{\frac{3}{(1+c(\zeta,\nu))}(\zeta-\nu)} - 1) \\
&\leq 2(e^{\frac{1}{2}(\zeta-\nu)} - 1) \\
&\leq (e^{(\zeta-\nu)} - 1).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Theta(\sigma(\Omega(\rho(F\zeta, F\nu)))) &\leq \Theta(e^{|\zeta-\nu|} - 1) \\
&= 1 + |\zeta - \nu|^2 \leq \frac{1 + [\ln(2|\zeta-\nu| + 1)]^2}{1 + [\ln(2|\zeta-\nu| - 0.9(e^{|\zeta-\nu|} - 1) + 1)]^2} \\
&= \frac{\Theta(\sigma(\rho(\zeta, \nu))}{\Theta(\xi(\rho(\zeta, \nu)))}
\end{aligned}$$

where $c(\zeta, \nu)$ is a constant dependent on ζ, ν, obtained from mean value theorem such that $3\ln(1+\zeta) - 3\ln(1+\nu) = \frac{3}{1+c(\zeta,\nu)}(\zeta - \nu)$. So, we conclude that Υ is a $\Theta - (\sigma, \xi)_{\Omega}$-contractive mapping. Thus, all of the hypotheses of Theorem 3 are verified and hence Υ has a fixed point in $[5, 7]$. Moreover, since any two elements of $[5, 7]$ are comparable, the fixed point of Υ is unique and for any $r_0 \in X$, the iterated sequence $\{\Upsilon^n(r_0)\}_{n\in\mathbb{N}}$ is convergent to the fixed point.

Note that we can not apply the main result of Roshan et al. [30]. Indeed, for $\zeta = 5$ and $\nu = 6$, we get

$$\begin{aligned}
\sigma(\Omega(\rho(\Upsilon\zeta, \Upsilon\nu))) &= 2\ln(\Omega(\rho(\Upsilon\zeta, \Upsilon\nu)) + 1)) \\
&= 2\rho(\Upsilon\zeta, \Upsilon\nu) \\
&= 2(e^{|\Upsilon\zeta - \Upsilon\nu|} - 1) \\
&= 2(e^{|3\ln(6) - 3\ln(7)|} - 1) \\
&= 2(\frac{7^3}{6^3} - 1) \\
&= 1.175 \\
&> 1.546 \\
&= 0.9(e - 1) \\
&= 2\ln(e - 1 + 1) - (2\ln(e - 1 + 1) - 0.9(e - 1)) \\
&= 2\ln(e^{|\zeta-\nu|} - 1 + 1) - (2\ln(e^{|\zeta-\nu|} - 1 + 1) - 0.9(e^{|\zeta-\nu|} - 1)) \\
&= \sigma(\rho(\zeta, \nu)) - \xi(\rho(\zeta, \nu)).
\end{aligned}$$

3. Application

For $T > 0$, consider

$$\zeta(s) = p(s) + \int_0^T \lambda(s,r)f(r,\zeta(r))dr, \quad s \in I = [0,T] \tag{22}$$

Here, we give an existence theorem for a solution of (22) in $X = C(I, [0, \ln(\frac{20}{9})])$ using Theorem 2. Take

$$\rho(\zeta,\nu) = e^{||\zeta-\nu||_\infty} - 1$$

for all $\zeta, \nu \in X$. Note that X is a p-complete p-metric space with $\Omega(s) = e^s - 1$, where $||\zeta||_\infty = \sup_{q\in I} |\zeta(q)|$.

X is endowed with the partial order $\preceq$:

$$\zeta \preceq \nu \Longleftrightarrow \zeta(s) \leq \nu(s),$$

for each $s \in I$. Note that $(X, \preceq, \rho)$ is regular. Assume that

(*i*) $f : I \times [0, \ln(\frac{20}{9})] \to [0, \ln(\frac{20}{9})]$ and $p : I \to [0, \ln(\frac{20}{9})]$ are continuous;
(*ii*) $\lambda : I \times I \to [0, \infty)$ is continuous;
(*iii*) For all ζ, ν with $\zeta \preceq \nu$
$$0 \leq f(r,\nu) - f(r,\zeta) \leq \nu - \zeta.$$
(*iv*) $\max_{s\in I} \int_0^T | \lambda(s,r) | dr \leq \frac{1}{2}$;
(*v*) There exists a continuous function $\alpha : [0,T] \to [0, \ln(\frac{20}{9})]$ so that
$$\alpha(s) \leq p(s) + \int_0^T \lambda(s,r)f(r,\alpha(r))dr.$$

Theorem 5. *Under the conditions (i)-(v), (22) has a solution in* $X = C([0,T], \ln(\frac{20}{9})]$.

Proof. Take $F : X \to X$ as
$$F(\zeta(s)) = p(s) + \int_0^T \lambda(s,r)f(r,\eta(r))dr.$$
For $\zeta \preceq \nu$,
$$f(s,\zeta) \leq f(s,\nu),$$
the operator F is ordered increasing. Having that $\lambda(s,r) > 0$, so

$$F(\zeta(s)) = p(s) + \int_0^T \lambda(s,r)f(r,\zeta(r))dr \leq p(s) + \int_0^T \lambda(s,r)f(r,\nu(r))dr = F(\nu(s)).$$

Now, take $\Theta(s) = 1 + [\ln(s+1)]^2$, $\sigma(s) = 2\ln(1+s)$ and $\xi(s) = 2\ln(1+s) - 0.9s$. Note that ξ is increasing iff $0 \leq s \leq \frac{11}{9}$. For $\zeta, \nu \in X$, we have $0 \leq ||\zeta - \nu||_\infty \leq \ln(20/9)$, hence $0 \leq \rho(\zeta,\nu) = e^{||\zeta-\nu||_\infty} - 1 \leq 11/9$. Thus, $diam(X) = \sup_{\zeta,\nu\in X} \rho(\zeta,\nu) = \frac{11}{9}$.

Now,

$$\begin{aligned}\sigma(\Omega(\rho(F\zeta,F\nu))) &= 2\ln(\Omega(e^{||F\zeta-F\nu||_\infty}-1)+1)\\ &= 2\ln(e^{e^{||F\zeta-F\nu||_\infty}-1}-1+1)\\ &= 2(e^{||F\zeta-F\nu||_\infty}-1)\\ &\leq 2(e^{\max_{s\in I}|\int_0^T\lambda(s,r)[f(r,\zeta(r))-f(r,\nu(r))]dr|}-1)\\ &\leq 2(e^{(\max_{s\in I}\int_0^T|\lambda(s,r)|dr)||\zeta-\nu||_\infty}-1)\\ &\leq 2(e^{\frac{||\zeta-\nu||_\infty}{2}}-1)\\ &\leq e^{||\zeta-\nu||_\infty}-1.\end{aligned}$$

Therefore,

$$\begin{aligned}\Theta(\sigma(\Omega(\rho(F\zeta,F\nu)))) &\leq \Theta(e^{||\zeta-\nu||_\infty}-1)\\ &= 1+||\zeta-\nu||^2 \leq \frac{1+[\ln(2||\zeta-\nu||+1)]^2}{1+[\ln(2||\zeta-\nu||-.9(e^{||\zeta-\nu||_\infty}-1)+1)]^2}\\ &= \frac{\Theta(\sigma(\rho(\zeta,\nu))}{\Theta(\xi(\rho(\zeta,\nu)))}.\end{aligned}$$

Due to assumption (v),

$$\alpha \preceq F(\alpha).$$

By Theorem 4, there is $\zeta \in X$ such that $\zeta = F(\zeta)$, which is a solution of (22). □

Note that we can not apply the theorem of Roshan et al. [30] to have a solution of (22). Indeed,

$$\begin{aligned}e^{||\zeta-\nu||_\infty}-1 &> 2||\zeta-\nu|| - (2||\zeta-\nu|| - 0.9(e^{||\zeta-\nu||_\infty}-1))\\ &= 2\ln(e^{||\zeta-\nu||_\infty}-1+1) - (2\ln(e^{||\zeta-\nu||_\infty}-1+1) - 0.9(e^{||\zeta-\nu||_\infty}-1))\\ &= \sigma(\rho(\zeta-\nu)) - \xi(\rho(\zeta-\nu)).\end{aligned}$$

4. Conclusions

We introduced contraction type mappings by intervening Θ-contractions of Jleli and Samet [35] and some control functions including altering distance functions. We gave some fixed point theorems related to above mappings in the class of p-metric spaces. The obtained results have been illustrated by some concrete examples and an application on integral equations.

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