DYNAMIC RESPONSE OF COMPOSITE CYLINDRICAL SHELLS TO SHOCK LOADING

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ABSTRACT

In this paper, dynamic response of composite cylindrical shells subjected to shock loading is studied analytically. The equations of motion are based on Sanders theory of thin shell. The effects of transverse shear deformation and rotatory inertia are taken into account. The circular cylindrical thin shell with simply supported ends is subjected to a shock load, represented by an axially symmetric moving load at a constant speed along the longitudinal direction. The governing equations are solved analytically using the Assumed-Modes Method. The influences of geometrical parameters and composite material properties of circular cylindrical shells on the dynamic response are studied.

1. INTRODUCTION

Pipelines, gun tubes, shock tubes and missile systems are common structural elements subjected to moving shock loads. For aerospace applications of these elements, there has been an increasing demand for composite materials whose high strength and structural properties can be combined with highly sophisticated physical and/or chemical features. Dynamic behavior of shell-type structures made of composite materials has been of considerable research interest.

A number of theories for composite cylindrical shells exist in the literature. Many of these theories were developed originally for thin shells and based on the Kirchhoff-Love kinematic hypothesis [1]. Dynamic analysis of composite cylindrical shells has been performed by many researchers by using Donnell's shallow shell theory, Love's first approximation theory, Flügge's shell theory and etc. [2] in which the transverse shear deformation is neglected. These theories are expected to yield sufficiently accurate results when: (a) the radius-to-thickness ratio is large; (b) the dynamic excitations are within the low-frequency range; (c) the material anisotropy is not severe [3]. However, Reddy and Asce [3] show that application of such theories to layered anisotropic composite shells could lead to as much as 30 per cent errors in deflections, stresses and frequencies. They presented an extension of the Sanders shell theory for doubly curved shells to a shear deformation theory of laminated shell taking into account transverse shear strains and rotation about the normal to the shell midsurface.

The transverse shear moduli of composite materials are typically an order of magnitude lower than their in-plane elastic moduli. This is in contrast to isotropic materials, which have shear to elastic moduli ratios of approximately 40 per cent. Thus, composite material shells would be expected to be affected by transverse shear flexibility to a much greater extent than isotropic shells having the same geometric parameters.
The shear deformable analog of Donnell’s shallow shell theory [4] and the shear deformable version of Love’s first approximation [5,6] were used on the vibration of laminated shells. Bert and Kumar [7] formulated a theory for the small amplitude free vibration of circular cylindrical shells laminated of bimodulus composite materials. The theory used in the study was the dynamic, shear deformable (moderately thick shell) analog of the Sanders best first approximation thin shell theory. Noor and Burton [8] reviewed the different approaches used for modeling multilayered composite shells.

Structures subjected to moving loads have been analyzed since 19th century. A number of studies of the moving load problem were referred to Fryba [9]. Dynamic response of a cylinder subjected to moving pressure was studied by Tang [10]. Schiffner and Steele [11] investigated the transient response of cylindrical shell with axisymmetric moving load, step pressure, by using modified saddle-point methods. Jones and Bhuta [12] investigated the moving ring-load problem within a cylinder. Simkins [13] solved the problem of a gun tube subjected to moving ballistic pressure. Recently, Kurtaran et al. [14] studied the dynamic response of circular cylindrical shells to moving shock loading and obtained the equations of motion by use of Nagdi’s shell theory. Türkmen et al. [15] made an experimental study of the dynamic behavior of a thick cylindrical shell subjected to moving detonation load.

This paper presents the dynamic response of composite cylindrical shells subjected to shock loading. The shock load is represented by an axially symmetric moving load at a constant speed along the longitudinal direction of the circular cylindrical shell. Such a load may be generated by a detonation developed in a tube. Equations of motion have been obtained by shear deformable analog of Sanders theory in which transverse shear deformation and rotatory inertia are included. The Assumed-Modes Method is used to solve the governing equations. The influence of geometrical parameters and composite material properties of circular cylindrical shells on the dynamic response are investigated.

2. GOVERNING EQUATIONS

Let us consider an orthotropic cylindrical shell formed from a number of layers. The co-ordinate system and loading condition are shown in Figure 1, where h, R, and L denote wall thickness, mid-surface radius and cylinder length, respectively.

![Figure 1. Geometry](image)

The shear deformable version of Sanders theory of thin shell is used on derivation of the equations of motion. Simplifying assumptions on displacement field that are used to derive the equilibrium equations are as follows:

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(1) The thickness of shell is small compared to the radius of shell (h/R << 1).
(2) The transverse normal stress is negligible.
(3) Normals to the reference surface of the shell before deformation remain straight, but not necessarily normal, after deformation (a relaxed Kirchhoff-Love's hypothesis).

**Displacement Field**

The Weierstrass theorem states that any function that is continuous in an interval may be approximated uniformly by polynomials in this interval. Thus, the displacement field in the shell can be represented by the following relationships

\[
U(x, \theta, z) = u(x, \theta) + z\psi_x(x, \theta) + z^2\gamma_x(x, \theta) + \cdots
\]

\[
V(x, \theta, z) = v(x, \theta) + z\psi_\theta(x, \theta) + z^2\gamma_\theta(x, \theta) + \cdots
\]

\[
W(x, \theta, z) = w(x, \theta) + z\psi_z(x, \theta) + z^2\gamma_z(x, \theta) + \cdots
\]

where \( U, V, \) and \( W \) are the displacement components in the directions of axes \( x, \theta, \) and \( z, \) respectively. The relaxed Kirchhoff-Love hypothesis stated in the third assumption results in the linearly distributed tangential displacements and a constant normal displacement through the thickness of the shell, and hence Eqs. (1) are simplified as follow

\[
U(x, \theta, z) = u(x, \theta) + z\psi_x(x, \theta)
\]

\[
V(x, \theta, z) = v(x, \theta) + z\psi_\theta(x, \theta)
\]

\[
W(x, \theta, z) = w(x, \theta)
\]

where \( u, v, \) and \( w \) are the components of displacement at the middle surface in the \( x, \theta, \) and normal directions, respectively. \( \psi_x \) and \( \psi_\theta \) are the rotations of the normal to the middle surface during deformation about the \( x \) and \( \theta \) axes, respectively.

**Kinematical Relations**

Sanders [16] developed an eight order shell theory from the principle of virtual work. The strain-displacement relations of the theory for a circular cylindrical shell can be expressed as

\[
\varepsilon_x = \frac{\partial u}{\partial x} + \psi_x
\]

\[
\kappa_x = \frac{\partial \psi_x}{\partial x}
\]

\[
\varepsilon_\theta = \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + \frac{w}{R} \right)
\]

\[
\kappa_\theta = \frac{1}{R} \frac{\partial \psi_\theta}{\partial \theta}
\]

\[
\varepsilon_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \theta}
\]

\[
\kappa_{x\theta} = \frac{1}{R} \frac{\partial \psi_x}{\partial \theta} + \frac{1}{R} \frac{\partial \psi_\theta}{\partial x} + \frac{1}{2R} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial \theta} \right)
\]

\[
\varepsilon_{x\theta} = \frac{1}{R} \frac{\partial w}{\partial x} - \frac{v}{R} + \psi_\theta
\]

\[
\varepsilon_{x\theta} = \frac{1}{R} \frac{\partial w}{\partial x} - \frac{v}{R} + \psi_\theta
\]
where, $\varepsilon_x$, $\varepsilon_\theta$, and $\varepsilon_{\theta\theta}$ are the membrane strains of the middle surface; $\kappa_x$, $\kappa_\theta$, and $\kappa_{\theta\theta}$ the bending strains; $\varepsilon_{xz}$ and $\varepsilon_{\theta z}$ the transverse shear strains. Total tangential strains at any point in a shell can be obtained as

$$
\varepsilon_x = \varepsilon_x + z\kappa_x \quad \varepsilon_\theta = \varepsilon_\theta + z\kappa_\theta \quad \text{and} \quad \varepsilon_{\theta\theta} = \varepsilon_{\theta\theta} + z\kappa_{\theta\theta} \tag{4}
$$

Assuming that the geometry and loading have axial symmetry, deformations in the circumferential direction are small and negligible, and the kinematical equations (3) can be reduced as follows

$$
\varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_\theta = \frac{w}{R} \quad \varepsilon_{xz} = \frac{\partial w}{\partial x} + \psi_x \quad \kappa_x = \frac{\partial \psi_x}{\partial x} \tag{5}
$$

### Constitutive Equations

In the plate and shell theory, it is convenient to introduce the force and moment resultants by integrating the stresses over the shell thickness. The constitutive equations of an anisotropic material relate the force and moment resultants to the membrane and bending strains [17]. Here, the bending-stretching coupling is considered in the constitutive equations. With the assumption of axial symmetry stated above, the constitutive equations can be written as

$$
N_x = A_{11}\varepsilon_x + A_{12}\varepsilon_\theta + B_{11}\kappa_x \\
N_\theta = A_{12}\varepsilon_x + A_{22}\varepsilon_\theta + B_{12}\kappa_x \\
M_x = B_{11}\varepsilon_x + B_{12}\varepsilon_\theta + D_{11}\kappa_x \\
M_\theta = B_{12}\varepsilon_x + B_{22}\varepsilon_\theta + D_{12}\kappa_x \\
Q_x = A_{55}\varepsilon_{xz} \tag{6}
$$

where the extensional, the flexural-extensional coupling, flexural, and transverse shear stiffnesses are

$$
\{A_{ij}, B_{ij}, D_{ij}\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \tilde{Q}_{ik} \{1, z, z^2\} \, dz \quad (i,j = 1,2) \\
A_{55} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \tilde{Q}_{55} K^2 \, dz \tag{7}
$$

where $K$, shear correction factor, and the terms $\tilde{Q}_{ij}$ are the stiffnesses of a lamina transformed to the shell coordinates. The stiffnesses of a lamina are defined as

$$
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \quad Q_{55} = \frac{E_2}{2(1 + \nu_{13})} \tag{8}
$$

and $E_1$, $E_2$ are Young's moduli in 1 and 2 material-principal directions, respectively. $\nu_{ij}$ are Poisson's ratios for transverse strain in the $j$th direction when stressed in the $i$th direction, and related to Young's moduli by the reciprocal relation as $\nu_{ij}E_j = \nu_{ji}E_i \,(i=1,2)$. 

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Equations of motion for the dynamic behavior of the circular cylindrical shell considering the assumption of axial symmetry can be reduced from the general equations of the Sanders shell theory [7]:

\[
\begin{align*}
\frac{\partial N_x}{\partial x} &= I_1 \frac{\partial^2 u}{\partial t^2} + \frac{1}{R} \frac{\partial^2 \psi_x}{\partial t^2} \\
\frac{\partial Q_x}{\partial x} &= \frac{N_0}{R} + \delta(x-Vt) = I_1 \frac{\partial^2 w}{\partial t^2} \\
\frac{\partial M_x}{\partial x} &= \frac{I_2}{R} \frac{\partial^2 \psi_x}{\partial t^2} + \frac{I_2}{\partial t^2} \frac{\partial^2 \psi_x}{\partial t^2}
\end{align*}
\]  

(9)

where the term \( \delta(x-Vt) \) defines a ring load, moving with a constant velocity; \( \delta \) the Dirac Delta function; and the normal and rotatory inertias are

\[
(I_1, I_2) = \sum_{k=1}^{N} \int \left[ (1, z^2) \right] \rho \, dz
\]

(10)

Substituting equation (5) and (6) into equation (9) one obtains the governing equations in the following form:

\[
\begin{align*}
A_{11} \frac{\partial^2 u}{\partial x^2} + A_{12} \frac{\partial w}{\partial x} + B_{11} \frac{\partial^2 \psi_x}{\partial x^2} &= ph \left( \frac{\partial^2 u}{\partial t^2} + \frac{h^2}{12R} \frac{\partial^2 \psi_x}{\partial t^2} \right) \\
A_{55} \frac{\partial^2 w}{\partial x^2} - A_{12} \frac{\partial u}{\partial x} - A_{22} \frac{\partial \psi_x}{\partial x} + \delta(x-Vt) &= ph \left( \frac{\partial^2 w}{\partial t^2} \right) \\
B_{11} \frac{\partial^2 u}{\partial x^2} + D_{11} \frac{\partial^2 \psi_x}{\partial x^2} - A_{55} \psi_x - A_{55} \frac{\partial w}{\partial x} &= \frac{ph^3}{12} \left( \frac{1}{R} \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 \psi_x}{\partial t^2} \right)
\end{align*}
\]

(11)

It is assumed that the conditions of simple support are fulfilled at the boundaries of the shell as follows

\[
\begin{align*}
N_x(0,t) &= N_x(L,t) = M_x(0,t) = M_x(L,t) = 0 \\
w(0,t) &= w(L,t) = 0
\end{align*}
\]

(12)

Initial conditions are

\[
\begin{align*}
u(x,0) &= w(x,0) = \psi_x(x,0) = 0 \\
\frac{\partial u(x,0)}{\partial t} &= \frac{\partial w(x,0)}{\partial t} = \frac{\partial \psi_x(x,0)}{\partial t} = 0
\end{align*}
\]

(13)

3. SOLUTION

The governing equations above are solved analytically by using Assumed-Modes method [18]. For the simply supported boundary conditions, the displacement and curvature change function are taken to be

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\[ u(x, t) = \sum_{m=1}^{M} U_m(t) \cos(\alpha_m t) \]
\[ \psi(x, t) = \sum_{m=1}^{M} W_m(t) \sin(\alpha_m t) \]
\[ \psi_s(x, t) = \sum_{m=1}^{M} \Phi_m(t) \cos(\alpha_m t) \]

where \( \alpha_m = \frac{m \pi}{L} \).

The above expressions for the displacement field are substituted into Eq. (II). Then, the first and the third equation are multiplied by \( \cos(\alpha_n x) \) and the second equation is multiplied by \( \sin(\alpha_n x) \) and integrated from zero to \( L \) considering the orthogonality relations of geometric functions. Finally, we obtain the following system of ordinary, coupled three differential equations:

\[
\begin{bmatrix}
(D^2 + k_{11}) & k_{12} & (k_{13} D^2 + k_{14}) \\
k_{12} & (D^2 + k_{22}) & k_{23} \\
(D^2 + k_{31}) & k_{32} & (R D^2 + k_{33})
\end{bmatrix}
\begin{bmatrix}
U_m \\
W_m \\
\Phi_m
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
k_{24} \sin(k_{24} t)
\end{bmatrix}
\]

where

\[
k_{11} = \frac{\alpha_m^2 A_{11}}{\rho h} \quad k_{12} = -\frac{\alpha_m A_{11}}{\rho h R} \quad k_{13} = \frac{h^2}{12 R} \quad k_{14} = \frac{\alpha_m^2 B_{11}}{\rho h} \\
k_{22} = \frac{1}{\rho h R^2} \left( \frac{A_{22}}{A_{55}} \alpha_m^2 \right) \quad k_{23} = \frac{\alpha_m A_{55}}{\rho h} \quad k_{24} = \frac{2 P}{\rho h L} \quad k_{25} = \frac{L}{\rho h R} \quad k_{26} = \frac{12 R}{\rho h^3} \alpha_m B_{11} \\
k_{31} = \frac{12 R}{\rho h^3} \alpha_m B_{11} \quad k_{32} = \frac{A_{55} \alpha_m}{\rho h} \quad k_{33} = \frac{12 R}{\rho h^3} \left( \alpha_m^2 D_{11} + A_{55} \right) D^2 = \frac{d^2}{dt^2} \)

Formal application of Cramer's rule can be used for solving equations (15). Thus, we find

\[
U_m = U_{mp} + \sum_{i=1}^{6} A_i \Phi_i \]
\[
W_m = W_{mp} + \sum_{i=1}^{6} B_i \Phi_i \quad (m=1, 2, \ldots, M) \]
\[
\Phi_m = \Phi_{mp} + \sum_{i=1}^{6} C_i \Phi_i
\]

where \( A_i, B_i, C_i \) are constants that are determined by the use of the initial conditions. \( \Phi_i \)'s are functions depended on the polynomial roots of the determinant of homogenous part. The determinant of the homogenous part gives a cubic equation in \( D^2 \), and there will be three pairs of roots. \( \Phi_i \)'s according to root characteristics are listed in Table 1.
in which

\[ k_{34} = k_{24} \left( (k_{11} - k_{25}^2)(k_{33} - Rk_{25}) - k_{13}k_{25}^2 \right) \]

\[ k_{35} = \left[ -(R - k_{13})k_{25}^4 + k_{36}k_{25}^2 - k_{37}k_{25}^2 + k_{38} \right] \]

\[ k_{44} = -k_{24} \left[ k_{12}(k_{33} - Rk_{25}) + k_{13}k_{32}k_{25} \right] \]

\[ k_{55} = -k_{24} \left[ (k_{11} - k_{25}^2)k_{32} + k_{12}k_{25}^2 \right] \]

\[ k_{56} = [R(k_{11} + k_{22}) + k_{33} - k_{14} - k_{13}(k_{22} + k_{31})] \]

\[ k_{57} = \left[ k_{12}k_{23} + k_{32}(k_{13}k_{25} - k_{23}) + k_{33}(k_{11} + k_{22}) + R(k_{11}k_{22} - k_{12}) \right. \]

\left. -k_{13}k_{31}k_{22} + k_{14}(k_{22} + k_{31}) \right] \]

\[ k_{58} = \left[ k_{23}(k_{12}k_{31} - k_{11}k_{32}) + k_{33}(k_{11}k_{22} - k_{12}) + k_{14}(k_{12}k_{32} - k_{31}k_{22}) \right] \]

<table>
<thead>
<tr>
<th>Root characteristic</th>
<th>Pair of root values*</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>( r_i = a )</td>
<td>( \Phi_i = e^{at} )</td>
</tr>
<tr>
<td></td>
<td>( r_{i+1} = -a )</td>
<td>( \Phi_{i+1} = e^{-at} )</td>
</tr>
<tr>
<td>Imaginary</td>
<td>( r_i = bj )</td>
<td>( \Phi_i = \cos(bt) )</td>
</tr>
<tr>
<td></td>
<td>( r_{i+1} = -bj )</td>
<td>( \Phi_{i+1} = \sin(bt) )</td>
</tr>
<tr>
<td>Complex</td>
<td>( r_i = a + bj )</td>
<td>( \Phi_i = e^{at} \cos(bt) )</td>
</tr>
<tr>
<td></td>
<td>( r_{i+1} = a - bj )</td>
<td>( \Phi_{i+1} = e^{at} \sin(bt) )</td>
</tr>
<tr>
<td>Two-equal pair</td>
<td>( r_i = r_k )</td>
<td>( \Phi_i = t\Phi_k )</td>
</tr>
<tr>
<td></td>
<td>( r_{i+1} = r_{k+1} )</td>
<td>( \Phi_{i+1} = t\Phi_{k+1} )</td>
</tr>
<tr>
<td>Three-equal pair</td>
<td>( r_i = r_k = r_n )</td>
<td>( \Phi_i = t\Phi_k )</td>
</tr>
<tr>
<td></td>
<td>( r_{i+1} = r_{k+1} = r_{n+1} )</td>
<td>( \Phi_{i+1} = t^2\Phi_k )</td>
</tr>
</tbody>
</table>

* \( i = 1, 3, 5 \) \( k = 3, 5 \) \( n = 5 \) \( i \neq k \neq n \) and \( a, b \) are real numbers

Table 1. Complementary functions

Since the degree of characteristic equation for the homogenous part of Eq. (15) is 6, there must be only six constants. The relationship among the eighteen constants \( A_i, B_i, C_i \) \( (i = 1, 6) \), in Eq.(16) can be found by substituting the values of \( U_m, W_m, \Psi_m \) obtained above in the first and second one of Eq.(15). This relationship reduces the number of the unknown constants to six. Remaining constants may be determined by applying the initial conditions (Eq. 13).
4. NUMERICAL RESULTS

Before examining the dynamic response of circular cylindrical shells of composite material, we present some results for isotropic material. The following geometric parameters and material properties of steel are used: \( R=15 \) mm, \( h=1 \) mm, \( L=1800 \) mm, \( E=207 \) GPa, \( \nu=0.3 \), \( \rho=7770 \) kg/m\(^3\). The strain results shown in the figures were obtained for the load \( P=5 \) kN/m, velocity \( V=2500 \) m/s, and shear correction factor \( K=5/6 \). The calculations are carried out by the 500-term series, which the convergence is achieved with 500 term in series. The time history of the circumferential strain \( \varepsilon_\theta \) at the mid-length of the shell is shown in Figure 2 in terms of dimensionless time as \( \tau=t \frac{V}{L} \).

As seen in Figure 2, the strain at the points forward from the shock load is almost negligible, but it takes wave-like form at the points behind the load. In comparison to static values obtained from Timoshenko [19] and present method, Timosenko gives \( \varepsilon_2 = 60.13 \) \( \mu \) and present method gives \( \varepsilon_2 = 58.45 \) \( \mu \) (500 term) and \( \varepsilon_2 = 60.89 \) \( \mu \) (750 term).

![Figure 2. Time history of circumferential strain (Freq. = 54 kHz)](image)

Next, the results of dynamic response of laminated cylindrical shells are presented. The same geometric parameters are taken as used in the isotropic shell and time histories are obtained at the mid-length of the shell. Material properties in a single layer of Aramid-epoxy are as follows: \( E_1=83 \) GPa, \( E_2=7.03 \) GPa, \( G_{12}=G_{13}=1.966 \) GPa, \( \nu_{12}=0.41 \), \( \rho=1387 \) kg/m\(^3\). Results for a single layer are presented in Figures 3-9. Figure 3 shows the convergence by means of the term number in series; 250 term is taken on the calculations for the accuracy of the results. The response to shock load having different velocities is presented in Figure 4.

At the higher velocities inertial forces become important, so the lower amplitudes but higher frequencies are obtained. The effects of the fiber orientation are shown in the Figures 5-9. As orientation angle increases up to certain degree (about 42 deg.), frequencies decrease but at higher angle vice versa. The effect of the fiber orientation angle on the frequency is shown in Figure 10.
The effect $R/h$ and $L/R$ ratios on the response of cross-ply laminate $(90^0/0^0/90^0)$ is shown in Figure 11 and 12, respectively. For the higher $R/h$ ratios, lower frequencies and amplitudes are obtained. The effect of length apart from short shell is negligible.

Figure 3. Convergence (at $0.75\tau$ and $V=1500$ m/s)

Figure 4. The response at different velocities

Figure 5. Time history of circumferential strain ($\theta = 0$ deg.)

Figure 6. Time history of circumferential strain ($\theta = 30$ deg.)
Figure 7. Time history of circumferential strain ($\theta = 45$ deg.)

Figure 8. Time history of circumferential strain ($\theta = 60$ deg.)

Figure 9. Time history of circumferential strain ($\theta = 90$ deg.)

Figure 10. Frequency versus fiber orientation angle

Figure 11. The time history for different R/h ratios

Figure 12. The time history for different L/h ratios
5. CONCLUSIONS

Analytical solution for the dynamic response of simply supported, laminated cylindrical shells to shock load is derived using the shear deformation analog of the Sanders thin shell theory. The shock load is represented by an axially symmetric moving load at a constant speed along the longitudinal direction of the circular cylindrical shell. Transverse shear deformation and rotatory inertia are included in the governing equations. The Assumed-Modes Method is used to solve the governing equations. The influence of geometrical parameters and composite material properties of circular cylindrical shells on the dynamic response are investigated.

The present method can also be used to obtain the response of static loading. The static solution calculated by using present method is good agreement with classical one. At the higher velocities inertial forces become important, so the lower amplitudes but higher frequencies are obtained. Fiber orientation affects considerably the dynamic response. Therefore optimum solutions in terms of fiber orientation angle may be found for the other types of loadings.

Finally, it is concluded that the present method can be employed to examine the effect of discrete stiffeners on the dynamic behavior of laminated composite circular cylindrical shell. The influence of temperature dependent material properties on structural dynamic has a great importance for the circular cylindrical shell made of composite materials and this will be topic of future study.

REFERENCES