VIBRATION CONTROL METHODS BY MODE LOCALIZATION:
A COMPARATIVE STUDY

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Abstract

This study is an attempt to generalize different active mode localization strategies. It is demonstrated that similar results can be obtained by either a passive design which exhibits normal mode localization or by an active controller which is obtained by employing some of the recently proposed design methods of active control methods. Different active control methods proposed for achieving mode localization are compared. It is shown that all active control methods proposed for mode localization can be considered as special cases of a general control method called "Space Domain Control".

1 Introduction

Almost all structures are subjected to noise and vibration due to the presence of various sources. These sources often excite unwanted structural resonances which can cause damage or the transmission of vibrational energy to distant parts or regions where they can not be tolerated. Therefore, it may be of interest to contain the vibrational energy near to the points of excitation, and prevent the propagation of large vibrational amplitudes to other parts of a structure. Such systems include aerospace and ship structures, large communication antennas and flexible robotic manipulators carrying very sensitive payloads. In these cases, it is also desirable to suppress the vibrations at a faster rate at those sensitive regions or stations.

Recent research has shown that irregularities or disorder in nearly periodic structures may lead to free vibration modes which are localized to small geometric regions, and may confine the vibrational energy to the vicinity of the source of excitation [1]. This phenomenon, known as mode localization may either be catastrophic as it leads to larger amplitudes or beneficial, as a passive damping mechanism.

The possibility of occurrence of mode localization due to random disorder has some implications on control design for such structures. If a control strategy requires modal estimation, (e.g., Independent Modal Structure Control), it should be used with care in structures which are susceptible to mode localization. Since the assumed mode shapes as well as the natural frequencies of an ordered structure may be quite different than those of the actual structure which may be disordered, it leads large errors in the modal estimation. Another difficulty will be on the placement of actuators. Since the actual locations of nodal points are not precisely known, the degree of controllability of a certain mode can not be assessed apriori. Another important issue to be addressed is the robustness of the closed loop performance to parameter uncertainties. Clearly, if the mode localization is produced by small disorder (in weakly-coupled systems), the system will be very sensitive to any parameter changes. However, for the case of large disorder, this is not the case, since small changes in disorder will not make the coupling/disorder ratio significantly different.
Recently, some researchers proposed active mode localization as an active control strategy for both lumped and distributed parameter systems [2-5]. Feedback control is used to alter the system closed loop eigenvectors or eigenfunctions such that the vibrations is confined in a prescribed region of the system. The objective of this study is to survey the existing active mode localization techniques and to find a general problem statement to represent all cases.

2 Passive Mode Localization: A Simple Example

It is well known that a small amount of disorder in weakly coupled systems may lead to strong mode localization. If the structure is a collection of substructures which are weakly coupled, then, by introducing some small amount of disorder, one can produce localized modes. In this section, a brief overview of normal mode localization as it occurs passively in structures is given, and some of the concepts will be illustrated through a simple two degrees-of-freedom system shown in Fig. 1.

When \( k_1 = k_2 = k \) and \( m_1 = m_2 = m \) the system is ordered, or tuned; otherwise it is mistuned, or disordered. The stiffness \( k_c \) represents the coupling between the two subsystems. The equations of motion are given by:

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2
\end{bmatrix}
+
\begin{bmatrix}
  k_1 + k_c & -k_c \\
  -k_c & k_2 + k_c
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
=
\begin{bmatrix}
  f_1 \\
  f_2
\end{bmatrix}
\]  

(1)

The mode shapes of the tuned system are given by the modal matrix as:

\[
U_0 = \begin{bmatrix}
  1.0 & 1.0 \\
  1.0 & -1.0
\end{bmatrix}
\]

(2)

When the system is disordered, for example by an additional stiffness of \( k_e \), such that \( k_1 = k + k_e \), the new mode shapes are given by

\[
U = \begin{bmatrix}
  1.0 \\
  (k_c/k_e)
\end{bmatrix}
\begin{bmatrix}
  \sqrt{(k/k_e)^2 + 0.25 - 0.5} \\
  \sqrt{(k/k_e)^2 + 0.25 + 0.5}
\end{bmatrix}
\begin{bmatrix}
  1.0 \\
  (k_c/k_e)
\end{bmatrix}
\]

(3)

Clearly, for small values of the ratio \( k_c/k_e \), which represents the ratio of coupling to disorder, the modes will be strongly localized. For example, when \( k_c/k_e = 1 \), the mode shapes are \([1 \ 10]^T\) and \([10 \ -1]^T\). This is a simple example of passive mode localization, or eigenvalue loci veering. The degree of localization depends on the ratio of coupling to disorder strength. Most studies on passive mode localization are concerned
with the effect of small disorder. In order to see localized modes for small disorder, the coupling $k_c$ should be very small. Hence, studies of passive mode localization mostly deal with weakly-coupled systems. However, if one is interested in producing localized modes, this need not be the case, and mode localization can be achieved by large disorder for strongly-coupled systems, as long as the coupling to disorder ratio is kept small. In what follows different active control methods proposed for achieving a somewhat similar mode localization are briefly summarized.

3 Mode Localization By Distributed Feedback

The dynamics of structures considered here is described by the following linear partial differential equation:

$$L_1[v(P,t)] + M_1 \frac{\partial^2 v(P,t)}{\partial t^2} = -F_L(P,t) \quad P \in D$$

$$B_i[v(P,t)] = 0 \quad i = 1, 2, \ldots, p \quad P \in S$$

where $L_1$ and $M_1$ are linear differential operators of orders $2p$, and $2q$, respectively. $t$ is time, $P$ is a point of the domain $D$, and for every point on the boundary $S, B_i, i = 1, 2, \ldots, p$, is a set of linear differential operators of order $2p - 1$ characterizing the boundary conditions. The spatial operators $L_1$ and $M_1$ are such that $p > q$. $F_L$ is a distributed feedback force that consists of a linear combination of the displacement, velocity and acceleration fields, and their spatial partial derivatives and can be written as:

$$F_L(P,t) = L_2[v(P,t)] + C_2 \frac{\partial v}{\partial t}(P,t) + M_2 \frac{\partial^2 v}{\partial t^2}(P,t)$$

where $L_2, C_2$ and $M_2$ are linear spatial operators. The substitution of Eq. (6) in Eq. (4) yields the following eigenvalue problem associated with the undamped system:

$$L[v(P,t)] - \omega^2 M[v(P,t)] = 0$$

where $L = L_1 + L_2$ and $M = M_1 + M_2$.

The operators $L_2$ and $M_2$ are selected such that they provide the missing spatial partial derivatives in $L_1$ and $M_1$ up to the orders of $2p - 1$ and $2q - 1$, respectively. The operator $C_2$ represents the energy dissipating terms which are responsible for vibration suppression.

The approach, for confining vibrations in flexible structures consists of converting the original mode shapes into exponentially decaying functions of the spatial coordinates by an appropriate selection of the feedback force $F_L(P,t)$ in Eq.(6). If the size of the structure is large, then, spatial confinement of vibration becomes dual to the case of time decaying functions for $0 \leq t < \infty$.

As an example, consider the axial vibrations of a long bar with axial stiffness $EA$ and linear mass density $m$. A possible feedback force that leads to confinement is given by [5]

$$F_L(x,t) = 2EA\alpha \frac{\partial v}{\partial x} + EA\left(\alpha^2 - \frac{m}{EA}\gamma^2\right)v - 2m\gamma \frac{\partial v}{\partial t}$$

The characteristic equation associated with the eigenvalue problem is given as
By applying the boundary conditions of the bar, the resulting set of natural frequencies and eigenfunctions are found as

\[ \omega_n = \frac{n\pi}{L} \sqrt{\frac{EA}{m}} \quad n = 1, 2, 3, \ldots (10) \]

\[ V_n(x) = A_n e^{-\alpha x} \sin \left( \frac{n\pi x}{L} \right)\]  

where \( L \) is the length of the bar. It can be seen that by selecting the free parameter \( \alpha \), a desired spatial decay rate can be achieved. For this example, note that the eigenvalues of the original structure are preserved, but the eigenmodes are altered for the purpose of vibration confinement. The decay rate \( \alpha \) determines the size of the region where the modes are localized.

Let the bar have the following initial conditions

\[ v(x,0) = 0, \quad \frac{\partial v}{\partial t}(x,0) = K\delta(x - x_0) \]  

where \( \delta(x - x_0) \) is a Dirac delta function characterizing an impulse of magnitude \( K \) at \( x_0 \). It can be shown that the time response of the bar to the distributed force given in Eq. (8) is

\[ v(x,t) = \sum_{n=1}^{\infty} \frac{2K}{L\omega_n} e^{-\alpha(x-x_0)} \sin \frac{n\pi x_0}{L} \sin \frac{n\pi x}{L} e^{-\alpha t} \sin \omega_n t \]  

(13)

It is clear from Eq. (13) that the vibrations will be confined close to the origin. This strategy can be generalized as follows: Given the boundary value problem described by Eqs. (4)-(5), for any arbitrary complex number \( \alpha \), whose real part is positive, find a control force \( F_L(P,t) \) such that the following form is satisfied

\[ v(P,t) = e^{-\alpha P} w(P,t) \quad P \in D \]  

(14)

where \( w(P,t) \) is a time periodic function. In general, a control law leading to Eq. (14) is called “Space Domain Control” [6]. Note that in a conventional control law the resulting exponential decay is for time only as opposed to spatial variables. It is also straightforward to show that for a persistent excitation close to origin the response will also be confined, i.e., the regions far away from the excitation will have much smaller amplitudes.

4 Mode Localization by Discrete Feedback

The distributed feedback requires distributed sensors and actuators. In Reference [3] a method is presented to allow the use of discrete actuators. For complex structures, however, closed form solutions are not available for the boundary value problem. In
most cases only a finite element model may be available. It is therefore desirable to achieve a similar mode localization using a discrete model, and by employing discrete sensors and actuators. In this section, the essential features of active mode localization by discrete feedback is given. For brevity, only the equations used for calculating the control gains are presented.

Consider the set of controlled $n$-dimensional discrete mass systems whose dynamics is described by

$$M\ddot{x} + Kx = Bf$$

where, $M$ and $K$ are the mass and stiffness matrices, and $x$ and $f$ are the displacement and feedback force vectors and $B$ is the input matrix. A proper alteration of the eigenvectors and eigenvalues aims at achieving the prescribed eigenstructure and suppressing simultaneously the structural vibration. We assume that the mass matrix is symmetric and positive definite and the stiffness matrix is symmetric and positive semi-definite (or positive definite). Note that if a consistent approach is used to obtain the mass matrix, typically a finite element model of a lightly damped structure will have these characteristics as well.

Assume that the desired eigenstructure is represented by the modal matrix $U$ and the diagonal matrix of eigenvalues $\Lambda$. The columns of the modal matrix are the desired eigenvectors, and the diagonal elements of $\Lambda$ are the desired eigenvalues (square of the undamped natural frequencies). It is assumed that the desired eigenvectors are selected such that the modal matrix $U$ can be inverted. Assume that the desired modal damping ratios are given by $\zeta_i$. A diagonal damping matrix $\Gamma$ can be constructed as $\gamma_{ii} = 2\zeta_i\omega_i, \ i = 1,...,n$, and the feedback force is given by

$$f(t) = -B_L^{-1}(\bar{M}\ddot{x} + \bar{C}x + \bar{K}x)$$

where $B_L^{-1}$ is the left pseudo-inverse defined as

$$B_L^{-1} = (B^TB)^{-1}B^T$$

and $\bar{M}, \bar{C}$ and $\bar{K}$ are constant square matrices of appropriate dimensions given by

$$\bar{M} = \frac{1}{1-\alpha}\left\{K\Lambda^{-1}\Lambda^{-1} - M\right\}$$

$$\bar{C} = (M + \bar{M})U\Lambda U^{-1} = \frac{1}{1-\alpha}\left(-\alpha M + K\Lambda^{-1}\Lambda^{-1}\right)U\Lambda U^{-1}$$

$$\bar{K} = \alpha\bar{M}\Lambda U^{-1} = \frac{\alpha}{1-\alpha}\left(K - M\Lambda U^{-1}\right) \text{ with } \alpha \neq 1$$
Fig. 2: System response, (a) with mode localization; (b) with classical velocity feedback, key, solid: Node 1, dashed: Node 2, dashed-dot: Node 3.

where $\alpha$ is a scaling parameter to be selected. The case with $\alpha = \infty$ corresponds to the full state feedback without acceleration feedback [5].

This strategy can be generalized as follows: Given the initial value problem described by Eq. (15), for a series of arbitrary complex numbers $\rho_i \in \mathbb{C}$, and $|\rho_i| < 1$ $(i = 1, 2, ..., 2n - 1)$, find a control force vector $f(t)$ such that the following form is satisfied for the states of the system, $y = [x \quad \dot{x}]$

$$y_i = \rho_i y_{i-1}$$

(21)

Similar to the distributed feedback case, a control law leading to Eq. (21) is called “Space Domain Control” for lumped parameter systems [6].

If $\rho_i$ is selected such that $\rho_i = e^{-\alpha \tau \omega(P_i, t)}$, then it is expected that the result will be similar to that of the distributed feedback. As an example, the method is applied to control of the axial vibrations in a long bar ($E A = 100$ N, $L = 10$ m, and $m = 1$ kg/m). Instead of using a distributed feedback described in Section 3, discrete feedback proposed in Section 4 is now applied to the discretized model of the system obtained through a standard FEM procedure with linear shape functions. For simplicity, three actuators are used to control the displacements at three nodes, the design parameters are $\alpha = 0.5$, and $\gamma_n = 0.5$. Figure 2a shows the performance of the control for a similar initial condition given for the example in Section 3 (an initial velocity of 1 m/s at Node 1). As expected the displacements at the Nodes 2 and 3, which are located far away from the origin are much smaller than that of Node 1. In other words, vibrations are confined at Node 1. The utility of this control would be clearer if it is compared to a more conventional controller. Figure 2b shows the result of a classical velocity feedback controller with the same damping ratio. It is seen that while the vibrations are damped out at about the same time, all of the three nodes have the same decay rate. With mode localization, however, the Nodes 2 and 3 are brought to rest much faster than Node 1. Furthermore, the amplitudes at these nodes are much smaller. This may be of significance if these nodes are to be protected from severe vibrations.

The example given above clearly demonstrates that with the proposed control, if the structure is disturbed by a transient excitation such as an impulse (or by initial
conditions), the resulting vibrations will be localized around Node 1. In case of a persistent excitation such as a steady disturbance acting in the localization region (Node 1 in this example), the results will be similar in the sense that the displacements away from the excitation will be smaller. This can easily be illustrated by the frequency response plots (not shown here due to space restrictions). In these plots, the attenuation ratios at Nodes 2 and 3 are much smaller than the attenuation ratio at Node 1 for all frequencies.

6 Concluding Remarks

It is demonstrated that similar results can be obtained by either a passive design which exhibits normal mode localization or by an active controller which is obtained by employing some of the recently proposed design methods of active mode localization. Different active control methods proposed for achieving mode localization are presented. It is shown that all active control methods proposed for mode localization can be considered as special cases of a general control method called “Space Domain Control”. It is anticipated that the ideas proposed in this study will be useful in developing systematic methods for integrated structure/control designs.

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References