The Regeneration of Information: A Model-Theoretic Approach †

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Regeneration is an operation whereby an organism restores a segment that has been severed from one of its limbs. Analogs to this process can be found in non-biological contexts, such as restoration. When an area in a painting has faded or been damaged, experts apply a variety of techniques in their efforts to restore the original work. Image processing provides yet another context; faced with a photograph from which a portion is missing, we can use a number of methods for its restoration. Another example, very relevant to vision, can be found in the eye’s “filling in”, or completion, of missing visual information. In order to describe what takes place in regeneration, we use words such as “extrapolation” or “interpolation” to indicate what we mean. This does not yet approach formal analysis, however, nor any methodical investigation of the operation.

1.

In my paper I wish to propose a definition for a certain type of regeneration, and will do so in general terms of the concept of model theory. More specifically, I wish to apply here an analysis of expansion of functions in mathematics and definitions of the process of natural expansions to the study of regeneration in general [1]. I will propose several requisite concepts and point out a number of basic characteristics and questions.

The type of regression that I intend to treat can be found in the following example. Consider the natural numbers with addition and multiplication. It is important that they be given as a set on which functions are defined, and that they conform to certain laws. Now, “erase” an In the new situation, let us assume that 1 + 7 do not equal 8 and becomes, in effect, undefined. We have obtained a model in which the function of addition is partially defined. Something in this model is “flawed” and yet it is, nevertheless, given to repair. Should we wish to retain the commutativity of addition in the world without 1 + 7 we will have no choice but to expand upon 1 + 7 to equal 8.

Note that this provides no proof of the fact that 1 + 7 are 8 but rather an expansion of the function's scope of definition. The distinction is clear. While in the case of proof we seek to determine the truth value of the claim defined, here we face a situation in which a function is used to define something that previously was undefined.

We can also erase 7 +1 and it will still be possible to repair the model, only that in this case we shall not approach the law of commutativity but the law of associativity. Erasing the second equation requires that we seek an alternative avenue for regeneration. It is likely that should we continue to erase the model will still be reparable and restorable, but should we erase a sufficiently large part it will be impossible to restore the model to its previous state. This is what we will also find in the case of biological regeneration. We know that if we sever a small piece of a lizard’s tail that the lizard will succeed in restoring it, but sever a large enough piece and the restoration will fail. There are also levels in regeneration: While there are organisms with the wondrous capacity to restore different limbs, mammals have limited abilities of regeneration. So it is with the restoration of images: If we have the beginnings of a drawing of a hand and additional parts of the image we will, to a certain extent, be able to complete it. If we have removed a small spot of color, it seems that there are not too many means of restoring it. Something similar is shared by the image and the mathematical model.
So it is with music; if we omit a note there are many ways to replace it. In any case, so it is in certain musical styles.

2.

These analogies hint at the connection between the mathematical case of the restoration of functions and the restoration of an image. In view of this, let us try to abstract and define the regeneration of a function, and then to complete the list of requisite definitions.

The removal of an object from the domain of a function’s definition requires no special effort. We need but to accept the existence of models with partial functions. The more important question is that of the idea that the model knows how to regenerate itself. The model looks into itself, as it were, and finds a certain constancy which enables it to restore itself. Here I would like to implement a concept of natural expansion, or forced expansions that have been studies in [1]. Let us take the complement of a function where the model imposes a definition. I am thinking here of the expansion of the power function on 0:

3. \( x = 1 \)

This is forced by the rule B:

4. \( x = 2 \)

In this case we will write that the rule B imposes the expansion.

However—and this simple though meaningful—this is exactly what we are doing in the case of restoring \( 1 + 7 = 8 \)

where here in the role of B we use the commutative law C:

5. \( x + y = y + x \)

with this in mind we introduced the following notation “\( F(L, h(a) = b) \)” means the law \( L \) forces the expansion \( h(a) = b \). Thus, in the examples above:

“\( F(B, 2^0 = 1) \)” is true in the appropriate model and “\( F(C, 1 + 7 = 8) \)” is true in another model.

Given a model \( M \) for a first order language \( L \) that includes functions and identity, we can expand \( L \) with expressions of the form “\( F(K, h(t) = s) \)” where “\( h(\ ) \)” is a function symbol and \( t, s \) are terms and define:

\[
M \models F(K, h(a) = b)
\]

A rigorous definition is given in [1] (p.) but informally we explain that there is an expansion of the domain of \( h(\ ) \) that satisify \( K \), and that any manner of expanding the model so that it will retain \( K \) must necessarily be consistent with \( h(a) = b \).

Now we may define the main concept:

**Definition 1.** Partially Regenerative at \( h(a) = b \)

Let model \( M \) for language \( L \) which includes the expression “\( h(\ ) \)” and wherein \( h(a) = b \). We will say that the model is partially regenerative at point \( a \) and function \( h(\ ) \) if, following the removal of \( a \) from the domain of the definition of \( h \), there is a Law \( C \), that imposes \( h(a) = b \) anew.

More specifically:

\[
M \models h(a) = b
\]

\( M' \) is the model after erasing \( a \) from the domain of \( h(\) ).

And there is a sentence \( C \)

\[
M' \models F(C, h(a) = b)
\]

One may limit, in advance, those statements that impose expansion, and say that \( C \) is a universal law. From here onward we will simplify and assume this unless otherwise stated.
On this basis, we can add various definitions. Some of them are given to definition in a very natural way. The model is partially regenerative on a subset of a model A and not only at a point \( h(a) = b \). Another way is to strengthen the way the model restore itself.

**Definition 2.** Universally Regenerative at \( h(a) = b \)

M is partially regenerative at \( h(a) \), and all the possible ways to restore \( h(a) \), by a universal law are compatible with \( h(a) = b \).

In the paper I wish to explore the potential of the definition above. There are two channels we can follow: we can either study the logic of regeneration in the usual way we study constructions and concepts in model theory, or we may look at cases of regeneration and extract from them possible guidelines for this exploration. The interesting part is, of course, where these strategies converge. In the following I will start with few steps in each direction.

**Conflicts of Interest:**

**References**


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