

Testing Fifth Forces from the Galactic Dark Matter [†]

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[†] Presented at the meeting Recent Progress in Relativistic Astrophysics, Shanghai, China, 6–8 May 2019.

Published: 5 July 2019

Abstract: Is there an unknown long-range force between dark matter (DM) and ordinary matters? When such a fifth force exists and in the case that it is ignored, the equivalence principle (EP) is violated apparently. The violation of EP was severely constrained by, for examples, the Eöt–Wash laboratory experiments, the lunar laser ranging, the MICROSCOPE satellite, and the long-term observation of binary pulsars. We discuss a recent bound that comes from PSR J1713+0747. When it is combined with the other bounds, a compelling limit on the hypothetical fifth force is derived. For the neutral hydrogen, the strength of such a fifth force should not exceed 1% of the gravity.

Keywords: dark matter; equivalence principle; binary pulsars

1. Introduction

Over the past hundreds of years, while physicists have established a sophisticated picture to delineate the ordinary world around us, we are still lacking a coherent description of the dark world. Two notable substances, the dark matter (DM) and the dark energy, were conjectured, though we do not know much detail of them [1]. In this proceeding we focus on the DM. Up to now, the DM was solely discovered via its gravitational interaction with the ordinary matters. By using the word “discovery”, we mean to look for interactions with our experimental instruments, either directly or indirectly. The primary example for direct searches is to look for the interaction of DM with nucleons in underground laboratories. As an example of indirect searches, by looking for γ -ray excess in the direction of the Galactic Center, we aim to detect DM particles that, via some portal, decay or annihilate into some standard-model particles which eventually couple to photons. Although various means were performed for the past decades, and we have learnt a lot from these direct and indirect experiments, no unanimously accepted clues on the non-gravitational interaction were found yet, and the nature of DM remains largely unknown [2].

Most of past searches looked for possible short-range interactions (say, via a massive force mediator) between the DM and the ordinary matters. We here look for an alternative possibility. We investigate the possibility that, besides the gravitational interaction, there is an extra long-range force between the DM and the ordinary matters [3]. By saying “long-range”, the force mediator should be massless or ultralight, with its Compton wavelength λ larger than the typical length scale of the systems under discussion [4,5]. Here we make use of the Galactic distribution of DM, hence $\lambda \gg \mathcal{O}(10 \text{ kpc})$ and the mass of the force mediator $m \ll 10^{-27} \text{ eV}/c^2$ [6]. We assume $m \rightarrow 0$ in the following study.

The spin-independent potential between body A and the DM, from scalar (“−” sign) or vector (“+” sign) exchange, is [4,5],

$$V(r) = \mp g_5^2 \frac{q_5^{(A)} q_5^{(\text{DM})}}{4\pi r}, \quad (1)$$

where g_5 is the coupling constant and q_5 is the dimensionless fifth-force charge [4,5]. If such a fifth force was ignored by the experimenter, she/he will “discover” an apparent violation of the equivalence

principle (EP) between body *A* and body *B* when performing her/his gravity experiments in the gravitational field of the DM, with an Eötvös parameter $\eta_{\text{DM}}^{(A,B)}$ [3],

$$\eta_{\text{DM}}^{(A,B)} = \pm \frac{g_5^2}{4\pi G u^2} \frac{q_5^{(\text{DM})}}{\mu_{\text{DM}}} \left[\frac{q_5^{(A)}}{\mu_A} - \frac{q_5^{(B)}}{\mu_B} \right], \quad (2)$$

where *G* is the Newtonian gravitational constant, and (q_5/μ) is an object’s charge per atomic mass unit *u*. This is true even that the gravity is still described by the general relativity (GR), and EP is valid if the experimenter is aware of the fifth force. From observations, if EP is observed to hold, one can put a limit on the fifth force. Such tests were performed with the Eöt–Wash laboratory experiment [3,5] and the lunar laser ranging [7,8]. The Eötvös parameter was constrained to be $|\eta_{\text{DM}}| \lesssim 10^{-5}$. Here we discuss an independent test from the binary pulsar PSR J1713+0747 [6], which has some specific distinctions from solar system experiments.

The proceeding is organized as follows. In the next section the relevant observational characteristics of PSR J1713+0747 are introduced [9]. In Section 3, we review the EP-violating signal in the orbital dynamics of a binary pulsar [10,11]. The method is applied to put a limit on the fifth force in Section 4 and the advantages of using neutron stars (NSs) are outlined. The last section discusses the possibility of finding a suitable binary pulsar close to the Galactic Center, that will boost the test significantly.

2. PSR J1713+0747

PSR J1713+0747 is a 4.5 ms pulsar in a binary system with an orbital period $P_b = 68$ day. Its companion is a white dwarf (WD) with mass $m_c = 0.29 M_\odot$. Due to its narrow pulse profile and stable rotation, PSR J1713+0747 is monitored by the North American Nanohertz Gravitational Observatory (NANOGrav), the European Pulsar Timing Array (EPTA), and the Parkes Pulsar Timing Array (PPTA). Splaver et al. [12], Zhu et al. [13], and Desvignes et al. [14] have published timing solutions for this pulsar, and the latest timing parameters from combined datasets are given in Zhu et al. [9]. Some relevant parameters for this proceeding are collected in Table 1.

Table 1. Parameters for PSR J1713+0747 [9]. The numbers in parentheses indicate the uncertainties on the last digit(s).

Parameter	Value
Spin frequency, ν (s^{-1})	218.8118438547250(3)
Orbital period, P_b (day)	67.8251299228(5)
Time derivative of P_b , \dot{P}_b ($10^{-12} \text{ s s}^{-1}$)	0.34(15)
Corrected \dot{P}_b ($10^{-12} \text{ s s}^{-1}$)	0.03(15)
Orbital inclination, i (deg)	71.69(19)
\hat{x} component of the eccentricity vector, e_x	−0.0000747752(7)
\hat{y} component of the eccentricity vector, e_y	0.0000049721(19)
Time derivative of e_x , \dot{e}_x (s^{-1})	$0.4(4) \times 10^{-17}$
Time derivative of e_y , \dot{e}_y (s^{-1})	$-1.7(4) \times 10^{-17}$
Companion mass, m_c (M_\odot)	0.290(11)
Pulsar mass, m_p (M_\odot)	1.33(10)

Because of mass transfer activities in the past, this binary has a nearly circular orbit. Nevertheless, its eccentricity, $e \lesssim 10^{-4}$, can still be measured. For the purposes of this study, we define $e_x \equiv e \cos \omega$ and $e_y \equiv e \sin \omega$ where ω is the longitude of periastron. Using data from 1993 to 2014, the timing sub- μs precision of PSR J1713+0747 was achieved. It renders a previous timing model for small-eccentricity binary pulsars, ELL1 [15], not accurate enough. Zhu et al. [9] developed an extended model, ELL1+, by including higher-order contributions from the eccentricity. The ELL1+ model includes terms up to $\mathcal{O}(e^2)$ in the Römer delay [9]. The measured values for e_x and e_y are listed in Table 1. In addition, the

first time derivatives of e_x and e_y are also given in the table, assuming that the changes are linear in time [9].

The measurements of the orbital decay and the eccentricity evolution were used to put constraints on different aspects of gravitational symmetries [9,16], including

1. the gravitational constant G 's constancy, $|\dot{G}/G| \lesssim 10^{-12} \text{ year}^{-1}$;
2. the universality of free fall for strongly self-gravitating bodies in the gravitational potential of the Milky Way, $|\eta_{\text{Gal}}| < 0.002$; and
3. the parameterized post-Newtonian (PPN) parameter $\hat{\alpha}_3$, $|\hat{\alpha}_3| \lesssim 10^{-20}$.

The first one is based on the orbital decay measurement, and the rest are based on the eccentricity evolution measurements [9]. The second test is our focus here and it is to be discussed below.

3. EP-Violating Signals in a Binary Pulsar

Damour and Schaefer [10] proposed to use small-eccentricity binary pulsars in testing the strong EP. When the EP is violated, a “gravitational Stark effect” takes place and polarizes the binary orbit in a characteristic way. A related phenomenon, the so-called “Nordtvedt effect”, takes place in the Earth–Moon–Sun system when the EP is violated [17]. It also happens for binary pulsars when the PPN preferred-frame parameters α_1 [18,19] and α_3 [9,20] are nonzero. In the current case, the relative acceleration between the pulsar and its companion star reads [10,11],

$$\ddot{\mathbf{R}} = -\frac{GM}{R^2}\hat{\mathbf{R}} + A_{\text{PN}} + A_\eta, \tag{3}$$

where \mathbf{R} is the binary separation, G is the gravitational constant, M is the total mass, and $\hat{\mathbf{R}} \equiv \mathbf{R}/R$. The post-Newtonian (PN) corrections are collected in A_{PN} , and the EP-violating abnormal acceleration is denoted as A_η . At leading order, $A_\eta \simeq \eta_{\text{DM}}\mathbf{a}_{\text{DM}}$ where \mathbf{a}_{DM} is the gravitational acceleration generated by the DM. To be explicit, here we take GR as the gravity, and the A_η term comes from an unknown “fifth force” instead of some modified gravity. If A_η comes from some modified gravity, then there will be extra considerations; for example, in that case the gravitational constant G should be replaced with an effective gravitational constant, \mathcal{G} [11].

We define the eccentricity vector $\mathbf{e}(t) \equiv e\hat{\mathbf{a}}$ to have a length of e , and a direction from the center of mass of the binary towards the periastron, $\hat{\mathbf{a}}$. In the Newtonian gravity, $\mathbf{e}(t)$ is a constant vector due to the fact that the Newtonian interaction has a larger symmetry group than $\text{SO}(3)$. In GR, there is the famous periastron advance where, at leading order, $\mathbf{e}(t)$ rotates uniformly at a rate,

$$\dot{\omega}_{\text{PN}} = \frac{3}{1-e^2} \left(\frac{2\pi}{P_b}\right)^{5/3} \left(\frac{GM}{c^3}\right)^{2/3}. \tag{4}$$

Under the relative acceleration (3), equations of motion get modified. After averaging over an orbit, the most important ones read [11],

$$\left\langle \frac{dP_b}{dt} \right\rangle = 0, \tag{5}$$

$$\left\langle \frac{d\mathbf{e}}{dt} \right\rangle = \mathbf{f} \times \mathbf{l} + \dot{\omega}_{\text{PN}}\hat{\mathbf{k}} \times \mathbf{e}, \tag{6}$$

$$\left\langle \frac{d\mathbf{l}}{dt} \right\rangle = \mathbf{f} \times \mathbf{e}, \tag{7}$$

where $\hat{\mathbf{k}}$ is the direction of orbital norm, $\mathbf{l} \equiv \sqrt{1-e^2}\hat{\mathbf{k}}$, and $\mathbf{f} \equiv 3A_\eta / (16\pi GM/P_b)^{1/3}$. These differential equations can be integrated to give [10,11],

$$\mathbf{e}(t) = \mathbf{e}_\eta + \mathbf{e}_{\text{GR}}(t), \tag{8}$$

where $e_{GR}(t)$ is a uniformly rotating vector with a rate according to GR's prediction (4), and $e_\eta \equiv f_\perp / \dot{\omega}_{PN}$ is a constant vector with f_\perp representing the projection of f on the orbital plane.

4. Constraints on the Fifth Force

From the theoretical side, we have a characteristic evolution of the eccentricity vector, dictated in Equation (8), while from the observational side, we have measured the linear changes in the eccentricity vector, decomposed to \dot{e}_x and \dot{e}_y in Table 1. Therefore, by comparing them, we can perform a test of the existence of the A_η term. Notice that the DM acceleration a_{DM} comes from the Galactic DM distribution. It is different from that of Zhu et al. [9] where the authors, considering a different scenario, used the total acceleration from the whole Milky Way to obtain η_{Gal} . We used an updated Galactic model [21] to calculate the acceleration from the DM. This choice does not change the relative strength in constraining the fifth force from different experiments.

The values of \dot{e}_x and \dot{e}_y from PSR J1713+0747 are consistent with $e_\eta = 0$ in Equation (8), which means that $A_\eta = 0$ and $\eta_{DM} = 0$. A careful analysis gives $|\eta_{DM}| < 4 \times 10^{-3}$ at the 95% confidence level [6]. This limit is weaker than those from the Eöt-Wash laboratory experiments and the lunar laser ranging. However, due to the very nature of the celestial binary system, PSR J1713+0747 has multiple advantageous aspects [6].

- **Driving force.** Because gravity is a manifest of the curved spacetime, free-fall states are ideal in performing gravity tests. Though the measurement precision is not as good as that of the Eöt-Wash group, the MICROSCOPE satellite gains a factor of 500 in the driving force by putting the experiment in space in a free-fall state, thus achieving a better bound on η [22]. On the contrast, binary pulsars are usually worse in testing η due to the smaller driving force from the Milky Way. Nevertheless, if considering the DM as the attracting source, binary pulsars do not have such a disadvantage, for all the experiments performed in the solar system have the same attraction from the Galactic DM distribution. Even better, if a suitable binary pulsar is found in a region that has a larger DM attraction, it may outperform the other tests (see the next section). It is interesting to note that the triple pulsar, PSR J0337+1715, though being excellent in testing the strong EP [23,24], does not probe the fifth force from the Galactic DM.
- **Measurement precision.** The precision in measuring \dot{e} is proportional to $\sigma / \sqrt{NT^3}$ where σ is the rms of time-of-arrival (TOA) residuals, N is the average number of TOAs per unit time, and T is the observational baseline [11]. Therefore, the test from binary pulsars will improve as a function of time, especially with the new instruments, like the five-hundred-meter Aperture Spherical radio Telescope (FAST) in China [25], and the Square Kilometre Array (SKA) in Australia and South Africa [26–28].
- **Material sensitivity.** Unlike the majority of solid materials on the Earth that have similar portions of protons and neutrons, NSs are almost 100% made of neutrons which are different from its WD companion. This gains a factor of $\mathcal{O}(10^2)$ when interpreting the result $|\eta_{DM}| < 4 \times 10^{-3}$ to more fundamental theory quantities. Thus, though the measurement of η_{DM} from PSR J1713+0747 is worse than the other measurements, it has a comparable power when being translated into fundamental theory parameters (see Figure 1 in Ref. [6] for details).
- **Binding energy.** Ordinary materials that were used in the EP test have a mass deficit about $\mathcal{O}(0.1\%)$ due to the nuclear binding energy. NSs, being strongly self-gravitating, have a mass deficit about $\mathcal{O}(10\%)$ due to the gravitational binding. This benefits a lot in probing some specific parameter space that is very hard to investigate with solely terrestrial experiments (see Table 1 and Figure 1 in Ref. [6]).

By combining all existing EP experiments, we reach the following conclusion: if there is a long-range fifth force between the DM and the ordinary matters, its strength should not exceed 1% of the gravitational force for neutral hydrogens [6].

5. Galactic Center Binary Pulsars

As is discussed in the previous section, the driving force from the DM is important in this test. The experiments in the solar system, by definition, cannot be done elsewhere but in the solar system. Due to the static large-scale DM distribution in the Galaxy, the acceleration a_{DM} in the solar system is a fixed quantity, almost zero variation from place to place inside the solar system. Therefore, for these experiments, one cannot enlarge its driving force.

However, binary pulsars in principle can be distributed anywhere in the Galaxy, and in the future that the SKA is to discover all pulsars in the Milky Way that point towards the Earth [26]. Among them, it is likely that there are suitable binary pulsars for this test in the region where the driving force is much larger. In particular, we consider the Galactic Center region where the DM density is much denser. Gondolo and Silk [29] argued that in the inner region of our Galaxy, there might be a DM spike. Such a spike will indeed enhance the driving force significantly when a binary pulsar has a distance smaller than ~ 10 parsec to the Galactic Center. Studies on the pulsar population suggested that the inner parsec could harbor as many as thousands of active radio pulsars that beam at the Earth [30]. Current and future searching plans are on their way (see e.g., Ref. [31]).

Funding: This work was supported by the Young Elite Scientists Sponsorship Program by the China Association for Science and Technology (2018QNRC001), and partially supported by the National Natural Science Foundation of China (11721303), and the Strategic Priority Research Program of the Chinese Academy of Sciences through the Grant No. XDB23010200.

Acknowledgments: We thank Zhoujian Cao, Michael Kramer, and Norbert Wex for helpful discussions.

Conflicts of Interest: The author declares no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DM	Dark matter
EP	Equivalence principle
GR	General relativity
NS	Neutron star
WD	White dwarf

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