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# A Note on Health Insurance under Ex Post Moral Hazard

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**Abstract:** In the linear coinsurance problem, examined first by Mossin (1968), a higher absolute risk aversion with respect to wealth in the sense of Arrow–Pratt implies a higher optimal coinsurance rate. We show that this property does not hold for health insurance under ex post moral hazard; i.e., when illness severity cannot be observed by insurers, and policyholders decide on their health expenditures. The optimal coinsurance rate trades off a risk-sharing effect and an incentive effect, both related to risk aversion.

**Keywords:** health insurance; ex post moral hazard; coinsurance

**JEL:** D1; D8; I1

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## 1. Introduction

The linear coinsurance problem—originally examined by Mossin [1] —plays an important role in the analysis of economic and financial decisions under risk, and this is for at least two reasons. Firstly, this model is suitable for tractable comparative statics analysis, in order to study wealth and income effects on insurance demand in various settings (e.g., with or without background risk, in a static or dynamic setting, etc.). Secondly, its conclusions can be straighforwardly adapted to the analysis of static portfolio choices when agents can invest in one risk-free asset and in one risky asset. An important property of this model states that the individual's degree of absolute risk aversion with respect to wealth in the sense of Arrow–Pratt goes hand in hand with a higher optimal coinsurance rate: more risk-averse individuals choose a higher coinsurance rate.

In this note, we will show that this property does not hold for health insurance under ex post moral hazard. There is ex post moral hazard in medical insurance when insurers do not observe the severity of illness, and policyholders may exaggerate their health care expenses—Arrow [2], Pauly [3] and Zeckhauser [4]. This should be distinguished from ex ante moral hazard that occurs when the insurance contract distorts the policyholder's incentives to make precautionary efforts. Linear coinsurance under ex post moral hazard (i.e., when insurers pay the same fraction of the health care cost regardless of the individuals' expenses) has been considered by many authors, including Zeckhauser [4], Feldstein [5], Arrow [6], and Feldman and Dowd [7] to analyze the trade-off between two conflicting objectives: providing risk coverage on one side, and incentivizing policyholders to moderate their health expenses on the other side.

In order to show that ex post moral hazard breaks the link between the degree of risk aversion and the optimal coinsurance rate, we will proceed through a simple model, in which utility depends on wealth and health in an additive way, and where the utility derived from health is linear. Furthermore, the only private information of individuals is about the severity of their illness. All other preference parameters, including health risk exposure and risk aversion, are either observed by insurers or rather recovered from observable variables such as age, education, occupation, marital status, or from past loss experience. These very crude assumptions are obviously not chosen for the sake of realism, but because they allow us to focus on the ex post moral hazard problem in a simple way, without interfering with adverse or advantageous selection issues.

It will turn out that, in this model, the positive effect of absolute risk aversion on the optimal coinsurance rate may vanish. In particular, an increase in initial wealth does not affect the optimal coinsurance rate, even if the policyholder displays decreasing or increasing absolute risk aversion. We will also consider a computable example with constant absolute risk version with respect to wealth. In that case, the optimal coinsurance rate does not depend on the degree of absolute risk aversion: it is fully determined by the probability distribution of health states, independent of the policyholder's risk aversion.

The intuition for these results goes through two effects of an increase in the coinsurance rate. On one hand, for a given pattern of health care expenses, a larger coinsurance rate offers a better risk protection to risk-averse individuals: thus, the larger the degree of risk aversion, the larger the benefit drawn for this more complete risk coverage. This is the standard channel that links the intensity of risk aversion and the optimal insurance coverage. On the other hand, an increase in coverage exacerbates health care overexpenses, and this ultimately leads to an increase in the cost borne by the policyholder. From this standpoint, coinsurance works as a self-discipline device, and this incentive mechanism will be more beneficial to more risk-averse people. When the index of absolute risk aversion increases, the risk protection effect and the incentive effect push the optimal coinsurance rate upwards and downwards, respectively.

This mechanism will be illustrated in two different ways. In Section 2, we introduce our model of linear coinsurance under ex post moral hazard. We show how risk protection and incentives interact in the determination of the optimal coinsurance rate, and how both effects are affected by the degree of absolute risk aversion. More specifically, we also show that changes in the policyholder's wealth do not affect the optimal coinsurance rate, although they may make the policyholder more or less risk averse when absolute risk aversion is not constant. In Section 3, we consider the case where the individual displays constant risk aversion w.r.t. wealth: the optimal coinsurance rate can then be explicitly calculated. It is shown that this rate is independent of the index of absolute risk aversion: it only depends on the probability distribution of health states. In other words, in that case, the variations in the risk protection effect and in the incentive effect exactly balance each other when risk aversion changes, and ultimately the coinsurance rate remains unchanged.

## 2. Optimal Coinsurance Under Ex Post Moral Hazard

Let us consider an individual whose welfare depends both on monetary wealth $R$ and health level $H$, with a bi-variate von Neumann–Morgenstern utility function $U(R, H)$. Some preliminary comments have to be made at this stage. Many studies on economic decision-making have focused attention on the effect of risk aversion on optimal choices under risk, including insurance coverage, financial choices, prevention behavior, and numerous other topics. In the case of health care, the problem is further complicated by the definition of risk aversion itself, because of the bi-variate nature of utility.

Firstly, we should distinguish the usual risk aversion for gambles on wealth alone from multivariate risk aversion. Multivariate risk aversion has been considered by several authors, following seminal papers by Keeney [8,9]. Roughly speaking, a decision maker who faces multivariate lotteries with "good" and "bad" outcomes for two attributes is considered multivariate risk-averse if she prefers getting some of the "good" and some of the "bad" to taking a chance on all of the "good" or all of the "bad". She is multivariate risk-neutral if she is indifferent between these two prospects, and she is risk-seeking when her preferences are reversed. Multivariate risk preferences do not depend on risk preferences for gambles on any attribute alone. In particular, in our setting, an individual may display risk aversion with respect to her wealth $R$ and be multivariate risk-averse or risk-seeking w.r.t. $R, H$.[1]

Secondly, in an expected utility setting, the optimal choice of a decision maker who may substitute an attribute for another one is simultaneously affected by her risk preference for gambles on each attribute alone, and by her multivariate risk aversion. For example, when the two attributes correspond to time-dating of wealth, multivariate risk aversion is often referred to as correlation aversion. In that case, the intertemporal elasticity of substitution depends at the same time on atemporal risk aversion and on correlation aversion.[2]

We will analyze health insurance choices in a model with a separable utility function

$$U(R, H) = u(R) + v(H),$$

with $u' > 0, u'' < 0$. Thus, the individual is assumed to be risk-averse w.r.t. wealth and bivariate risk-neutral. The assumption of bivariate risk-neutrality is not made for its realism, but (in addition to technical simplicity) because it allows us to separate the effects of risk aversion w.r.t. wealth from those of bivariate risk-averse or risk-seeking preferences.[3] For the sake of technical simplicity, we also assume $v(H) = \beta H, \beta > 0$. Hence, the marginal utility of health is constant and equal to $\beta$, which means that the individual is risk-neutral w.r.t. health. Her Marginal Willingness to Pay ($MWP$) for a health improvement is

$$MWP = \frac{dR}{dH}_{|U=const.} = \frac{\beta}{u'(R)},$$

and, for given $R$, the larger $\beta$, the larger this marginal willingness to pay.[4]

Health may be negatively affected by illness, but it increases with the health care expenses. This is written as

$$H = h_0 - x(1 - m),$$

where $h_0$ is the initial health endowment, $x$ is the severity of illness, and $m$ is the health care expense level. Illness severity is distributed as a random variable $X$ over an interval $[a, b]$, with $a > 0$ and $b < h_0$, and the parameters of the problem are such that $m \in [0, 1]$. Thus, the health level $H$ increases linearly from $h_0 - x$ to $h_0$ when $m$ increases from 0 to 1.

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1 In our health insurance setting, the decision maker is considered multivariate risk-averse if for any $R_0 < R_1$ and any $H_0 < H_1$, she prefers lottery $\mathcal{L}^1$, which gives an even chance for $(R_0, H_1)$ or $(R_1, H_0)$ to lottery $\mathcal{L}^2$, which gives an even chance for $(R_0, H_0)$ or $(R_1, H_1)$, or, equivalently, if

$$\frac{1}{2}U(R_0, H_1) + \frac{1}{2}(R_1, H_0) > \frac{1}{2}U(R_0, H_0) + \frac{1}{2}U(R_1, H_1).$$

A necessary and sufficient condition for multivariate risk aversion (risk seeking, risk neutrality) is $\partial^2 U/\partial R \partial H < 0$ (>0, =0)—see [9]. Since we may simultaneously have $\partial^2 U/\partial R^2 < 0$ and $\partial^2 U/\partial R \partial H > 0$, a decision maker may be risk-averse for gambles on $R$ alone and be multivariate risk-seeking.

2 See Bommier (2007) and [10] on the link between risk aversion w.r.t. wealth, correlation aversion, and the intertemporal elasticity of substitution. Many macroeconomic models postulate an additive intertemporal utility function, which corresponds to correlation neutrality. In such a case, the atemporal risk aversion—often measured by the index of relative risk aversion—simultaneously determines preferences among gambles in a given period, and the propensity of the representative consumer to substitute wealth across time.

3 Moreover, there is no consensus among health economists about the sign of the cross derivative $\partial^2 U/\partial R \partial H$, and thus about whether individuals are bivariate risk-averse or risk-seeking when they face gambles related to wealth and health; see Viscusi and Evans [11], Evans and Viscusi [12], and Finkelstein et al. [13].

4 Hence, any change in the utility function $u(R)$—for instance, a change in a parameter that would make the individual more risk-averse—may affect the marginal willingness to pay. However, parameter $\beta$ provides one degree of freedom in the value of this marginal willingness to pay. A non-expected utility setting—such as prospect theory—would provide more flexibility in order to characterize the attitude toward financial risk, independent of the marginal willingness to pay for a health improvement. See Abdellaoui et al. [14] for an experimental approach, and Bleichrodt et al. [15] for an application to medical decision analysis.

The individual's insurance contract specifies that a fraction $\theta$ of the monetary expenses are reimbursed, and that the insurance premium $P$ is actuarial. In what follows, $\theta$ is called the co-insurance rate.[5] Thus, the individual's wealth is

$$R = w - (1-\theta)m - P.$$

It is assumed that insurers observe all the characteristics of insurance seekers, including their risk exposure and risk aversion. In more concrete terms, insurers are supposed to be able to recover these through observable characteristics, such as age, gender, or level of education.[6]

Let $\widetilde{m}(x,\theta,w,P)$ denote the health expenses in state $x$ when the individual owns initial wealth $w$ and she has an insurance contract with coinsurance rate $\theta$ and premium $P$. She chooses the medical expenses that maximize her utility. Thus, we have

$$\widetilde{m}(x,\theta,w,P) \in \underset{m\in[0,1]}{\arg\max}\,\{u(w-(1-\theta)m-P) + \beta[h_0 - x(1-m)]\}$$

Let us consider an interior solution where $\widetilde{m}(x,\theta,w,P) \in (0,1)$ for all $x$ is characterized by the first-order optimality condition

$$-(1-\theta)u'(w-(1-\theta)\widetilde{m}-P) + \beta x = 0, \tag{1}$$

for all $x$. Differentiating (1) yields the partial derivatives of function $\widetilde{m}$:

$$\widetilde{m}'_x = -\frac{\beta}{(1-\theta)^2 u''(\widehat{R})}, \tag{2}$$

$$\widetilde{m}'_\theta = \frac{\widetilde{m}}{1-\theta} + \frac{1}{\widehat{A}(1-\theta)^2}, \tag{3}$$

$$\widetilde{m}'_w = \frac{1}{1-\theta}, \tag{4}$$

$$\widetilde{m}'_P = -\frac{1}{1-\theta}, \tag{5}$$

where

$$\widehat{R} \equiv \widehat{R}(x,\theta) \equiv u'^{-1}(\beta x/(1-\theta)), \tag{6}$$

and

$$\widehat{A} \equiv A(\widehat{R}) \equiv -u''(\widehat{R})/u'(\widehat{R})$$

is the absolute risk aversion index.

We assume that there is no transaction cost, and thus the insurer charges actuarial premiums. Hence, we have

$$P = \theta\mathbb{E}[\widetilde{m}(x,\theta,w,P)],$$

which gives an equilibrium insurance premium $P = \widetilde{P}(\theta,w)$, with

$$\widetilde{P}'_\theta = \frac{\mathbb{E}(\widetilde{m}) + \theta\mathbb{E}(\widetilde{m}'_\theta)}{1-\theta\mathbb{E}(\widetilde{m}'_P)} = \mathbb{E}(\widetilde{m}) + \frac{\theta}{1-\theta}\mathbb{E}(1/\widehat{A}), \tag{7}$$

$$\widetilde{P}'_w = \frac{\theta\mathbb{E}(\widetilde{m}'_w)}{1-\theta\mathbb{E}(\widetilde{m}'_P)} = \theta, \tag{8}$$

[5] In the insurers' terminology, the coinsurance rate is sometimes used for $1-\theta$, which is the share of health expenses retained by the policyholder.

[6] Outreville [16] surveys the empirical analysis of socio-demographic variables associated with risk aversion.

Let

$$\widehat{m}(x,\theta,w) \equiv \widetilde{m}(x,\theta,w,\widetilde{P}(\theta,w)) \tag{9}$$

be the health care expense after taking into account the endogenous determination of the insurance premium. We may rewrite the policyholder's expected utility as

$$\mathbb{E}U = \mathbb{E}[u(\widehat{R}(x,\theta))] + \beta\mathbb{E}[x\widehat{m}(x,\theta,w))] + h_0 - \mathbb{E}x, \tag{10}$$

which is a function of $\theta$. The optimal coinsurance rate $\theta$ maximizes $\mathbb{E}U$ in $[0,1]$. Using (1) allows us to write the first-order optimality condition for an interior optimum $\theta \in (0,1)$ as[7]

$$\frac{\partial \mathbb{E}U}{\partial \theta} = \mathbb{E}\left[u'(\widehat{R})(\widehat{m} - \widetilde{P}'_\theta)\right] = 0, \tag{11}$$

where $\widehat{R} \equiv \widehat{R}(x,\theta), \widehat{m} \equiv \widehat{m}(x,\theta,w)$, and $\widetilde{P}'_\theta \equiv \widetilde{P}'_\theta(\theta,w)$. Using (1) and (7) yields

$$\begin{aligned} \tfrac{\partial \mathbb{E}U}{\partial \theta} &= \tfrac{\beta}{1-\theta}\mathbb{E}\left\{X\left[\widehat{m} - \mathbb{E}(\widehat{m}) - \tfrac{\theta}{1-\theta}\mathbb{E}(1/\widehat{A})\right]\right\} \\ &= cov\left(\tfrac{\beta}{1-\theta}X, \widehat{m}(X,\theta,w)\right) - \tfrac{\beta\theta}{(1-\theta)^2}\mathbb{E}(X)\mathbb{E}(1/\widehat{A}) = 0, \end{aligned} \tag{12}$$

which defines the optimal coinsurance rate[8].

Risk aversion affects both terms in Equation (12). The first term is

$$cov\left(\frac{\beta}{1-\theta}X, \widehat{m}(X,\theta,w)\right) = cov(u'(\widehat{R}), \widehat{m}).$$

and it corresponds to the positive effect of an increase in $\theta$ due to the correlation between health care expenses and the marginal utility of wealth. We may intuitively understand the drivers of this correlation by calculating the derivative of $u'(\widehat{R})\widehat{m}$ w.r.t. $x$. Using $\widehat{R}'_x = -(1-\theta)\widehat{m}'_x$ gives

$$\begin{aligned} \tfrac{d}{dx}[u'(\widehat{R})\widehat{m}] &= u'(\widehat{R})\widehat{m}'_x - u''(\widehat{R})\widehat{m}'_x\widehat{m}(1-\theta) \\ &= u'(\widehat{R})\widehat{m}'_x[1 + \widehat{A}\widehat{m}(1-\theta)]. \end{aligned} \tag{13}$$

Hence, for a given trajectory $x \to \widehat{m}(X,\theta,w)$, the larger $\widehat{A}$, the larger $d[u'(\widehat{R})\widehat{m}]/dx$, with a positive effect on $cov(u'(\widehat{R}),\widehat{m})$, and thus a psoitive effect on $\partial\mathbb{E}U/\partial\theta$. This is the standard risk protection effect of insurance: wealth is redistributed toward lower income states, and the index of absolute risk aversion measures the gain from such a redistribution of wealth across states. However, in the present case, risk aversion also affects the trajectory $x \to \widehat{m}(X,\theta,w)$ through an incentive effect. Using (2) allows us to rewrite (13) as

$$\frac{d}{dx}[u'(\widehat{R})\widehat{m}] = \frac{\beta}{\widehat{A}(1-\theta)^2}[1 + \widehat{A}\widehat{m}(1-\theta)],$$

which reverses the sign of the relationship between $\widehat{A}$ and $d[u'(\widehat{R})\widehat{m}]/dx$.

The second term in Equation (12) is another component of the incentive effect of coinsurance. It corresponds to the additional insurance premium induced by the change in the policyholder's behavior caused by a unit increase in $\theta$: for a marginal increase $d\theta$, this additional expected net payment is the difference between the premium increase $\widetilde{P}'_\theta d\theta$ and the increase in the insurer's

[7] Equation (11) is obtained first by substituting $\widehat{R}(x,\theta) = w - (1-\theta)\widehat{m}(x,\theta,w) - \widetilde{P}(\theta,w)$ in $\mathbb{E}U$ and then by observing that, for all $x$, the derivative of $U$ with respect to $m$ vanishes when $m = \widehat{m}(x,\theta,w)$ because of Equation (1). The pointwise derivative of $U$ with respect to $\theta$ is thus written as $u'(\widehat{R}(x,\theta))[\widehat{m}(x,\theta,w) - \widetilde{P}'_\theta(\theta,w)]$. The optimal coinsurance rate cancels the expected value of this pointwise derivative, which gives (11).

[8] Note that $cov\,(X, \widehat{m}(X,\theta,w)) > 0$ because $\widehat{m}(x,\theta,w)$ is increasing w.r.t. $x$.

expected cost for unchanged behavior $\mathbb{E}(\widehat{m})d\theta$. Equation (7) shows that this difference is equal to $\widetilde{P}'_\theta - \mathbb{E}(\widehat{m}) = \theta\mathbb{E}(1/\widehat{A})/(1-\theta)$. Multiplying by expected marginal utility of wealth $\mathbb{E}(u'(\widehat{R})) = \beta\mathbb{E}(X)/(1-\theta)$ provides the second term of Equation (12). The larger $E(1/\widehat{A})$, the larger this net expected cost. Put differently, if the policyholder is very risk averse, she will react to an increase in the coinsurance rate by moderately increasing her health care expenses, and the net expected cost of this adaptation will be small.

All in all, since risk aversion is a determinant of function $\theta \to \widehat{m}(x,\theta,w)$ (i.e., of the incentive effects of insurance coverage), we cannot easily predict whether more risk aversion leads to a larger or lower optimal coinsurance rate. Going further in this direction requires that additional assumptions be made, as we will do in Section 3. Here, with the fact that wealth may be an important driver of risk aversion in mind, we may focus on the relationship between the initial wealth $w$ and the optimal coinsurance coefficient $\theta$. From the implicit theorem, the effect of a change in $w$ on $\theta$ is given by

$$\frac{d\theta}{dw} = -\frac{\partial^2\mathbb{E}U/\partial\theta\partial w}{\partial^2\mathbb{E}U/\partial\theta^2},$$

with $\partial^2\mathbb{E}U/\partial\theta^2 < 0$ at a maximum of $\mathbb{E}U$. Hence, $d\theta/dw$ and $\partial^2\mathbb{E}U/\partial\theta\partial w$ have the same sign. Using (4), (5), and (8) gives $\widehat{m}'_w = \widetilde{m}'_w + \widetilde{m}'_P\widehat{P}'_w = 1$ for all $x, \theta, w$, and thus $\partial[\widehat{m} - \mathbb{E}(\widehat{m})]/\partial w = 0$. Furthermore, (6) gives $\partial E(\widehat{T})/\partial w = 0$. We deduce that $\partial^2\mathbb{E}U/\partial\theta\partial w = 0$, which implies $d\theta/dw = 0$. Hence, an increase initial wealth $w$—-which, for instance, would make the individual less risk averse under DARA preferences—does not affect the optimal coinsurance rate.[9]

## 3. A Computable Example

Let us specify preferences furthermore, by assuming that the individual displays CARA preferences w.r.t. wealth. We write $u(R) = -\exp\{-\alpha R\}$, $\alpha > 0$, where $\alpha$ is the index of absolute risk aversion, and we still assume $v(H) = \beta H, \beta > 0$. In that case, we obtain

$$\widehat{R}(x,\theta) = u'^{-1}(\beta x/(1-\theta)) = \frac{1}{\alpha}\ln\left[\frac{\alpha(1-\theta)}{\beta x}\right], \tag{14}$$

and

$$\begin{aligned} \widetilde{m}(x,\theta,w,P) &= \frac{w - P - \widehat{R}(x,\theta)}{1-\theta} && (15)\\ &= \frac{\alpha(w-P) + \ln\left[\frac{\beta x}{\alpha(1-\theta)}\right]}{\alpha(1-\theta)}. && (16) \end{aligned}$$

Using $\widetilde{P} = \theta\mathbb{E}[\widetilde{m}(x,\theta,w,\widetilde{P})]$ yields

$$\widetilde{P}(\theta,w) = \theta w + \frac{\theta}{\alpha}E\left[\ln\left(\frac{\beta X}{\alpha(1-\theta)}\right)\right], \tag{17}$$

and

$$\begin{aligned} \widehat{m}(x,\theta,w) &= \widetilde{m}(x,\theta,w,\widetilde{P}(\theta,w)) \\ &= w - \tfrac{1}{\alpha}\ln[\alpha(1-\theta)] + \tfrac{\ln(\beta x) - \theta E[\ln(\beta X)]}{\alpha(1-\theta)} \end{aligned} \tag{18}$$

9 The conclusions of Section 2 have been reached for a given value of parameter $\beta$, and the optimal coinsurance rate may depend on $\beta$ as well as on function $u(R)$. Since $MWP = \beta/u'(R)$, we may consider an exogenously-given wealth level $R_0$ as a reference point, and define $MWP_0 = \beta/u'(R_0)$ as the reference $MWP$ of the individual. With this definition, an individual is fully characterized by function $u(R)$, which represents her preferences among financial gambles, by $MWP_0$, which measures her willingness to pay for a better health and by her initial wealth $w$. Our conclusion about the invariance of the optimal coinsurance rate w.r.t. initial wealth holds for unchanged $u(R)$ and $MWP_0$.

By disregarding the constant term $h_0 - \beta E[X]$, (14) and (18) allow us to write the individual's expected utility as

$$\begin{aligned} EU &= -E\left[\exp\{-\alpha\widehat{R}(X,\theta))\}\right] + \beta E[X\widehat{m}(X,\theta,w)] \\ &= -\tfrac{\beta E[X]}{\alpha(1-\theta)} \\ &\quad + \beta\left[E[X]w - \tfrac{E[X]}{\alpha}\ln[\alpha(1-\theta)] + \tfrac{E[X\ln(\beta X)]-\theta E[X]E[\ln(\beta X)]}{\alpha(1-\theta)}\right], \\ &= -\tfrac{\beta E[X]}{\alpha(1-\theta)} \\ &\quad + \beta\left[E[X]w - \tfrac{E[X]}{\alpha}\ln[\alpha(1-\theta)] + \tfrac{E[X\ln(\beta X)]-E[X]E[\ln(\beta X)]}{\alpha(1-\theta)}\right. \\ &\quad \left. + \tfrac{E[X]E[\ln(\beta X)]}{\alpha}\right] \end{aligned} \tag{19}$$

which is maximized with respect to $\theta \in [0,1]$. Let $z = 1/(1-\theta)$. We have

$$\begin{aligned} EU = &-\tfrac{\beta E[X]z}{\alpha} \\ &+ \tfrac{\beta}{\alpha}\{\alpha E[X]w - E[X]\ln(\alpha) + E[X]\ln(z) \\ &+ z[E[X\ln(X)] - E[X]E[\ln(X)]] + E[X]E[\ln(\beta X)\} \end{aligned} \tag{20}$$

Hence, $z$ maximizes

$$V(z) \equiv E[X]\ln(z) + z[\Delta - E[X]], \tag{21}$$

in $[1,+\infty)$, where

$$\begin{aligned} \Delta &= E[X\ln(X)] - E[X]E[\ln(X)] \\ &= cov[X,\ln(X)] > 0. \end{aligned}$$

We have

$$\begin{aligned} V'(z) &= \Delta - E[X] + \frac{E[X]}{z}, \\ V''(z) &= -\frac{E[X]}{z^2} < 0. \end{aligned}$$

and

$$V'(1) = \Delta > 0$$

If

$$\Delta < E[X], \tag{22}$$

then $V(z)$ is maximized over $[1,+\infty)$ when

$$z = \frac{E[X]}{E[X]-\Delta} > 1,$$

that is[10]

$$\theta = \frac{\Delta}{E[X]} = \frac{cov[X,\ln(X)]}{E[X]} \in (0,1). \tag{23}$$

[10] Note that we can also obtain (23) from (12) and (18).

If $\Delta \geq E[X]$, then $\theta = 1$ would be an optimal corner solution, with $m(x) = 1$ for all $x$. Thus (5) is a necessary condition for an optimal interior solution to exist. (4) shows that $m(x)$ is increasing for such an interior solution. Thus, we have $m(x) \in (0,1)$ for all $x \in [a,b]$ if

$$w \in (\underline{w}, \overline{w}), \tag{24}$$

where $\underline{w}$ and $\overline{w}$ are given by (4), $m(a) = 0$, $m(b) = 1$, and $\theta = \Delta/E[X]$, with $\overline{w} > \underline{w}$ if

$$\ln(b/a) < \alpha(1-\theta). \tag{25}$$

In short, under (22), (24), and (25), we have an interior optimal solution $\theta = \Delta/E[X] \in (0,1)$ with $m(x) \in (0,1)$ for all $x$. At this optimal solution, the coinsurance rate $\theta$ is independent of the index of absolute risk aversion $\alpha$ and from parameter $\beta$: it only depends on the probability distribution of the illness severity $X$.[11,12]

## 4. Conclusions

Risk aversion may depend on several parameters, including wealth, age, marital status, and occupation, among others. Consider the case of a background risk, such as business interruption, assumed to be uninsurable and in force for self-employed people, but not for employees. Under risk vulnerability, such a background risk makes the individual more averse to other independent risks, including health care expenditures. If insurance expenses were perfectly monitored by the insurer, then this background risk would increase the coinsurance rate for health care. In other words, everything else given, self-employed people should choose a more complete health insurance than employees. This may not be the case under ex post moral hazard.

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[11] Everything else given, (24) does not hold when $\alpha$ is small enough. In that case, $m(x)$ is equal to 0 or 1 in a sub-interval of $[a,b]$. Thus, strictly speaking, the independence of $\theta$ from $\alpha$ has been established among values of $\alpha$ that are large enough for such corner solutions not to be optimal.

[12] Note that the two terms in Equation (12) may be rewritten as $cov\,(\beta X/(1-\theta), \widehat{m}(X,\theta,w)) = \beta\Delta z/\alpha(1-\theta)$ and $\beta\theta E(X)E(\widehat{T})/(1-\theta)^2 = \beta\mathbb{E}(X)(z-1)/\alpha(1-\theta)$. Hence, in the CARA case, both terms are proportional to the index of absolute risk tolerance $1/\alpha$, so that $\alpha$ does not affect the optimal coinsurance rate. In addition, both terms are independent of $\beta$ (which may not be the case in the more general framework considered in Section 2) and, consequently, the optimal coinsurance rate $\theta$ does not depend on $\beta$.

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