An Efficient Approach for Coordination of Dual-Channel Closed-Loop Supply Chain Management

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Abstract: In this paper, a closed-loop supply chain composed of dual-channel retailers and manufacturers, a dynamic game model under the direct recovery, and an entrusted third-party recycling mode of the manufacturer is constructed. The impact of horizontal fairness concern behavior is introduced on the pricing strategies and utility of decision makers under different recycling models. The equilibrium strategy at fair neutrality is used as a reference to compare offline retail sales. Research shows that in the closed-loop supply chain of dual-channel sales, whether in the case of fair neutrality or horizontal fairness concerns, the manufacturer’s direct recycling model is superior to the entrusted third-party recycling, and the third-party recycling model is transferred by the manufacturer. In the direct recycling model, the horizontal fairness concern of offline retailers makes two retailers in the positive supply chain compete to lower the retail price in order to increase market share. Manufacturers will lower the wholesale price to encourage competition, and the price will be the horizontal fairness concern coefficient, which is negatively correlated. In the reverse supply chain, manufacturers increase the recycling rate of used products. This pricing strategy increases the utility of manufacturers and the entire supply chain system compared to fair neutral conditions, while two retailers receive diminished returns. Manufacturers, as channel managers to encourage retailers to compete for price cuts, can be coordinated through a three-way revenue sharing contract to achieve Pareto optimality.

Keywords: dual channel; supply chain management; competitive advantage; game theory; price dependent demand

1. Introduction

Product quality and pricing are the key factors in attracting consumers and the supply chain decisions on such factors that would influence the cooperative relationship between them. However, while rapidly increasing sales, the entire supply chain system also faces cost pressures and the recycling of used products [1–4]. Online sale channels are formed by a combination of online and offline marketing models. Due to the difference in operating costs between online and offline sales channels, the increased competition caused by the coexistence of online and offline retailers will lead to channel conflicts. The problem of a sales channel management faced by manufacturers is becoming more and more serious [5]. Therefore, building a recycling and profit closed-loop supply chain has become a problem of rational choice to solve costs and channel optimization issues. Then, how does
the sense of unfair concern affect the dual-channel closed-loop supply chain? Or the pricing strategy of each decision maker? How does one maximize their profits? How do the dominant players resolve channel conflicts to achieve a closed-loop supply chain? In order to solve the above problems, it is necessary to directly recycle the dual-channel closed-loop supply chain under the third-party recycling model. The fair concern of the seller is the establishment of a suitable contract to achieve closure optimization of the ring supply chain to achieve Pareto optimality.

The existing research [5–20] does not consider the horizontal fairness between retailers of the dual-channel closed-loop supply chain, which will lead to an inadequate understanding of pricing issues.

The main contribution of this paper is to provide a comprehensive and innovative performance framework to compare the direct and third-party recovery mode in a dual-channel supply-chain management system. This study provides the performance comparison of supply chain members between two different dual-channel supply chain structures. This paper divides the profit maximization of each decision-maker into the decision-making goal. A dual-channel sales game model based on direct recycling from manufacturers and third-party recycling is established to consider the impact of offline fairness concerns of offline retailers on the pricing and utility of decision-makers. Research shows that in the closed-loop supply chain of dual-channel sales, whether in the case of fair neutrality or horizontal fairness concerns, the manufacturer’s direct recycling model is superior to the entrusted third-party recycling, and the third-party recycling model is transferred by the manufacturer.

The contribution of this paper is mainly reflected in the following:

1. Under the condition of horizontal fairness, the offline retailers have extended the traditional dual-channel supply chain to the dual-channel closed-loop supply chain of recycling and remanufacturing based on the manufacturer’s direct recycling model and the entrusted third-party recycling model, respectively.
2. The impact of horizontal fairness concern behavior is introduced in the pricing strategies and utility of decision makers under different recycling models, which has enriched previous studies that have only considered vertical fairness concerns among the various entities in the supply chain.
3. In view of the impact of horizontal fairness concerns on the pricing and utility of decision makers, an improved three-way revenue sharing contract is proposed to achieve optimal coordination of the entire closed-loop supply chain when there is horizontal fairness concern among retailers.

The rest of the paper is organized as follows. Section 2 provides the literature review. Section 3 presents the problem statement and basic assumptions in dual-channel closed-loop supply chain management. Section 4 describes the pricing strategies for fair and neutral policymakers. Section 5 provides the pricing strategies of decision makers under the horizontal fairness concern behavior of offline retailers. Section 6 discusses the supply chain coordination based on a direct recycling mode of manufacturing under horizontal fairness concern. Section 7 gives the numerical simulations results and analysis. Section 8 concludes the paper.

2. Literature Review

Not every aspect of social life is fair, and people are paying more and more attention to social inequalities. For example, when the media reported that fruit farmers prefer to let fruit rot or not sell, one reason is that the profit distribution is too low. Fairness, to a certain extent, is the result of psychological preference. Some management practitioners believe that fairness is an important factor in the supplier’s ability to maintain channel relationships, and fairness preferences also have a significant impact in multi-channel supply chains. Scholars have introduced fairness preferences into the field of supply chain contract research and analyzed the impact of fairness preferences on the value of contract parameters, coordination, and efficiency of supply chain operations. At present, fairness preference has become an important factor in the research of supply chain contracts, which can provide
a solid microbehavior basis for supply chain optimization. Top-level journals at home and abroad, including Management Science and Journal of Management Science, contain many academic papers. It can be seen that the study of supply chain contract coordination based on fairness has become a significant topic in management research.

Since the 1990s, based on the pressure of environmental protection legislation and the huge profits brought about by recycling, more and more enterprises have begun to implement closed-loop supply chain management strategies, such that closed-loop supply chain management has become the focus of academic and business circles. In recent years, many domestic and foreign scholars have done a lot of research on the pricing and coordination of closed-loop supply chain systems and many achievements have been made. Savaskan et al. [6] studied the channel efficiency of closed-loop supply chain systems under three different recycling modes based on the complete rational hypothesis modeling of supply chain members. Later, Savaskan and Waseenhove et al. [7] studied the design of supply chain reverse channels under the competition of multiple retailers, and it was found that the effectiveness of the recovery model is to some extent dependent on the level of competition of retailers but is still based on the full rational assumptions of participating members. Ferrer and Swarminathan [8] studied two-cycle and multi-cycle pricing models in terms of the differential pricing of new products and remanufactured products, and recycling pricing of used products. The pricing strategy for new products and remanufactured products in the case of manufacturers and third-party remanufacturing under monopoly was studied. On this basis, Ferguson and Toktay [9] further studied the differentiated pricing strategy of new products and remanufactured products of manufacturers under heterogeneous consumer groups. However, the literature [6–9] does not consider the influence of the player’s behavioral tendencies (loss avoidance, fairness, and other cognitive biases) on decision-making. Most of the current research on the dual-channel sales model in the supply chain are based on the game models established by foreign scholars Webb et al. [10] and Park et al. [11]. Research indicates that dual-channel sales are beneficial to the overall utility of manufacturers and supply chains, but the effectiveness of retailers is diminishing. This creates a conflict of utility between the retailer and the manufacturer. This conflict stems from the pricing strategy of the channels in the supply chain. Regarding how to solve the conflict problem of channel pricing, Kurata et al. [12] pointed out that it is impossible to realize supply chain coordination using a wholesale price contract alone, but Tsay et al. [13] show that it can be realized through an income-sharing contract and a transfer-payment contract. With the deepening of research, Nagurneya et al. [14] extended the dual-channel sales model to the closed-loop supply chain, established the expected profit model of each vendor based on the stochastic market demand, and studied the network equilibrium problem of the dual-channel closed-loop supply chain. In addition, for the study of the dual-channel closed-loop supply chain, the government is a factor that cannot be ignored. Considering the government’s incentives for the recycling of waste products in the closed-loop supply chain, the work of Ma et al. [15] and Saha et al. [16] is based on consumer alignment. The difference in sale channel preferences between upper and lower lines studies the impact of government subsidy behavior on the dual-channel closed-loop supply chain pricing strategy and the changes in profit before and after subsidies. Domestic scholars have expanded on the basis of studying and learning from scholars. First, Zhen-Zhang et al. [17] and Dai et al. [18] studied the influencing factors of the pricing problem of the dual-channel closed-loop supply chain and found that the proportion of defective products and carbon emission indicators in the production process will affect the decision-making in the entire closed-loop supply chain pricing and balancing strategies. Second, considering that some companies are profiting from the dual-channel sales model, they will also lead to increased supply chain competition and conflict. In response to issues, such as how to alleviate channel conflicts and the coordination of closed-loop supply chains, Huang et al. [19], based on the dual-channel sales and dual-channel recycling model of the closed-loop supply chains, considered the sale and recycling channels under conflict through an improved two department’s fee-based contract that realizes the coordinated optimization of the dual-channel closed-loop supply...
chain. On this basis, Zhou et al. [20] considered manufacturers and retailers when the business is the leader of the game and the pricing of the decision-maker under two different market forces. The above literature assumes that the decision-maker is completely rational. However, in real life, when the profit is lower than other manufacturers, it will produce unfair disgust, and the utility will be reduced, which will affect the pricing strategy. Therefore, the introduction of fair concerns into the closed-loop supply chain can more clearly reflect real decision-making. To this end, Liu et al. [21] and Li et al. [22] considered the equilibrium pricing strategy and utility of the dual-channel supply chain under the three conditions of only the manufacturer’s fair concern, the retailer’s fair concern, and the fairness of both parties. They showed that fair concern affects the utility level of decision-makers, and the revenue sharing contract is designed to achieve coordination. However, the study only considered the impact of fair concerns on the decision makers in the traditional dual-channel supply chain. On this basis, Yi et al. [23] found that the problem of fairness in the dual-channel closed-loop supply chain still exists, and it can better reflect the behavior of decision-makers under centralized and decentralized decision-making. The study only involved vertical fairness preferences between upstream and downstream. With the deepening of research, scholars have found that there are not only vertical fair concerns between upstream and downstream, but also horizontal fair concerns among retailers. Therefore, Ho et al. [24] and Qiu-Xiang et al. [25], based on the existence of competitive behavior among retailers, considered the traditional dual-channel supply chain under the horizontal and vertical fair concerns of retailer’s equilibrium strategy.

However, the existing research does not involve the introduction of horizontal equity between retailers into the pricing problem of the dual-channel closed-loop supply chain, which will lead to an insufficient understanding of pricing issues and strategy choices for decision-makers throughout the closed-loop supply chain.

3. Dual-Channel Closed-Loop Supply Chain Problem Description and Basic Assumptions

This article considers two retailers by one manufacturer for a closed-loop supply chain consisting of quotients. Manufacturers can choose either a direct recycling model (shown in Figure 1 below) or a third-party recycling model (shown in Figure 2). Among them, the manufacturer is in an oligopoly. It is the leader who determines the market price. It can ensure the coordination among the channel members through incentive channels and channel decision-making. Retailers and third-parties are price followers.

The specific assumptions and parameters are described below.

1. Assuming that the unit cost of the new product is \( c_1 \) and the unit cost of the remanufactured product is \( c_2 \), the production cost of the new product is higher than that of the remanufactured product, that is, \( c_1 > c_2 \), and \( \delta = c_1 - c_2 \) represents the unit cost of producing the remanufactured product.

2. Under normal circumstances, according to the research of Huang et al. [19], the online sales cost is lower than the offline sales. Assume that the offline retailer unit sales cost is \( c_s \) and the online retailer sales cost is zero.

3. Referring to the assumptions of Savaskan et al. [7], when waste products are passed back, there is no difference in quality and function between remanufacturing and new products, and consumers have the same level of acceptance and manufacturers have no price discrimination for independent online retailers and offline retailers, and the wholesale price is \( \omega \).

4. Assume that the market recovery rate \( t \) satisfies \( 0 < t < 1 \), and all recovered products are used in the remanufacturing process, according to the research of Savaskan et al. [7]. The average unit production cost of the manufacturer can be expressed as: \( \bar{c} = (1-t)c_1 + tc_2 = c_1 - t(c_1 - c_2) = c_1 - \delta t \). The recycling cost of waste products is \( C(t) = kt^2 \), where \( k \) is the scale parameter \( (k > 0) \). It can be seen that the higher the recovery rate, the higher the recovery cost, and when the third party recycles, the manufacturer’s transfer payment cost is \( A \).
(5) Considering the online and offline dual-channel competition sales model, refer to Qiu-Xiang et al. [25], in which it is assumed that the demand function for offline sales is \( D_1 = Q - P_1 + \beta P_2 \) \((D_1 > 0)\), where \( Q \) is the potential market capacity, and \( \beta \) is the degree of substitution for online and offline sales \((0 < \beta < 1)\). Moreover, the two retailers wholesale goods to the manufacturer according to market demand, and the inventory cost is zero.

(6) Decision makers are not completely rational, and they will have a fair preference in the face of uneven distribution of profits.

Therefore, the relationship and decision of each vendor in the dual-channel closed-loop supply chain are shown in Figures 1 and 2.

In Figure 1, the dual-channel supply chain (DCSC) model consists of one manufacturer and two retailers. The manufacturer produces new products by using new material and uses waste products for remanufacturing. The retailers are responsible for recycling waste products and then sells them to the manufacturer. In order to help consumers to distinguish new products from remanufactured products, the manufacturer sells them through a direct channel and the retailer channel, respectively. The consumer can buy products through these two channels according to their own needs. The first retailer (\( R_1 \)) is responsible for the offline retailing of products while the second retailer (\( R_2 \)) is responsible for the online retailing of manufactured products.

In general, the manufacturer uses special manufacturing processes to make remanufactured products as good as new products in quality and performance. Therefore, the products sold through the two channels are homogeneous and substitutes. This leads to an inevitable price competition between channels.

On the other hand, Figure 2 illustrates the case in which a third-party is involved in the remanufacturing of products and then hand over to the manufacturer. This adds one more player and this adds additional competition and double marginalization in the entire process. The third-party determines the recovery rate that impacts the transfer payment of the manufacturer. The need of a third-party is to evaluate the relative influence of third-party recycling mode over direct recycling mode. This approach clearly adds more risk of losses in the DCSC system.

\[ \begin{align*}
D_1 &= Q - P_1 + \beta P_2 \\
&> 0 \\
D_2 &= \beta P_2
\end{align*} \]

Moreover, the two retailers wholesale goods to the manufacturer according to market demand, and the inventory cost is zero.

\[ D_1 = \beta P_2 \]

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**Figure 1.** Dual-channel closed-loop supply chain decision under direct recycling mode.
Figure 2. Dual-channel closed-loop supply chain decision under third-party recycling mode.

4. Pricing Strategies for Fair and Neutral Decision Makers

Assuming that the manufacturer and the two retailers are fair and neutral, they do not care about the fairness of profit distribution, and they all play a dynamic game with the goal of maximizing their own profits.

4.1. Pricing Strategy in the Manufacturer’s Direct Recycling Mode

The dynamic game sequence of each manufacturer is as follows: First, the manufacturer determines the wholesale price \( \omega \) and the recovery rate \( t \). Second, the two retailers determine their retail prices \( P_1 \) and \( P_2 \), respectively.

The profit functions of offline retailers, online retailers, manufacturers, and the entire supply chain system can be expressed as:

\[
\Pi_{R_1} = (P_1 - \omega - c_s)D_1 \quad (1)
\]

\[
\Pi_{R_2} = (P_2 - \omega)D_2 \quad (2)
\]

\[
\Pi_M = (\omega - c_1)D_1 + (\omega - c_1)D_2 + \delta t(D_1 + D_2) - kt^2 \quad (3)
\]

\[
\Pi_T = (P_1 - c_1 - c_s)D_1 + (P_2 - c_1)D_2 + \delta t(D_1 + D_2) - kt^2 \quad (4)
\]

From the inverse induction method [1–16,18] and using Mathematica (Wolfram Research, Champaign, IL, USA) software, we can know:

\[
\frac{\partial \Pi_{R_1}}{\partial P_1} = Q - P_1 + \beta P_2 - (P_1 - \omega - c_s) = 0 \quad (5)
\]

\[
\frac{\partial \Pi_{R_2}}{\partial P_2} = Q - P_2 + \beta P_1 - (P_2 - \omega) = 0 \quad (6)
\]

This gives:

\[
\begin{align*}
P_1 &= \frac{(2 + \beta)(Q + \omega) + 2c_s}{4 - \beta^2} \\
P_2 &= \frac{(2 + \beta)(Q + \omega) + \beta c_s}{4 - \beta^2}
\end{align*} \quad (7)
\]
Substituting Equation (7) into the manufacturer’s profit function gives:

$$\Pi_M = (\omega - c_1) \left[ \frac{Q}{2} - \frac{(1-\beta)\omega}{2-\beta} - \frac{(2-\beta^2)c_1}{4-\beta^2} \right] + (\omega - c_1) \left[ \frac{Q}{2} - \frac{(1-\beta)\omega}{2-\beta} + \frac{\beta c_1}{4-\beta^2} \right] + \frac{\delta t}{2} \left[ \frac{Q}{2} - \frac{(1-\beta)\omega}{2-\beta} - \frac{(2-\beta^2)c_1}{4-\beta^2} \right] - kt^2$$  \hspace{1cm} (8)

The optimal wholesale price and optimal recovery rate are obtained by taking the partial derivatives $\frac{\partial \Pi_M}{\partial \omega} = 0$ and $\frac{\partial \Pi_M}{\partial \omega} = 0$, and solving using Mathematica gives [12–16,18]:

$$\omega^* = \frac{2(1-\beta)k - (1-\beta)^2\beta^2}{2k(1-\beta)(2-\beta) - \beta^2(1-\beta)} + \frac{2k(2-\beta)c_1 - [k(2-\beta) - \beta^2(1-\beta)]c_s}{4k(2-\beta) - 2\beta^2(1-\beta)}$$  \hspace{1cm} (9)

$$t^* = \frac{\delta Q}{2k(2-\beta) - \beta^2(1-\beta)} - \frac{\delta(1-\beta)(c_s + 2c_1)}{4k(2-\beta) - 2\beta^2(1-\beta)}$$  \hspace{1cm} (10)

Substituting Equations (9) and (10) into Equations (7) and (8) to find the optimal sales price and sales volume of offline and online retailers gives:

$$P_1^* = \frac{(3 - 2\beta)Q}{2(1-\beta)(2-\beta)} + \frac{2c_1 - 2\delta t^*}{4(2-\beta)} + \frac{(6 - \beta)c_s}{4(4 - \beta^2)}$$  \hspace{1cm} (11)

$$P_2^* = \frac{(3 - 2\beta)Q}{2(1-\beta)(2-\beta)} + \frac{2c_1 - 2\delta t^*}{4(2-\beta)} + \frac{(3\beta - 2)c_s}{4(4 - \beta^2)}$$  \hspace{1cm} (12)

$$D_1^* = \frac{Q}{2(2-\beta)} - \frac{(1-\beta)(2c_1 - 2\delta t^*)}{4(2-\beta)} + \frac{(3\beta^2 - \beta - 6)c_s}{4(4 - \beta^2)}$$  \hspace{1cm} (13)

$$D_2^* = \frac{Q}{2(2-\beta)} - \frac{(1-\beta)(2c_1 - 2\delta t^*)}{4(2-\beta)} - \frac{(\beta^2 - 3\beta - 2)c_s}{4(4 - \beta^2)}$$  \hspace{1cm} (14)

Furthermore, the optimal profits of offline retailers, online retailers, and manufacturers are [16–20,22]:

$$\Pi_{R_1} = \left\{ \frac{Q}{2(2-\beta)} - \frac{(1-\beta)(2c_1 - 2\delta t^*)}{4(2-\beta)} + \frac{(3\beta^2 - \beta - 6)c_s}{4(4 - \beta^2)} \right\}^2$$  \hspace{1cm} (15)

$$\Pi_{R_2} = \left\{ \frac{Q}{2(2-\beta)} - \frac{(1-\beta)(2c_1 - 2\delta t^*)}{4(2-\beta)} + \frac{(-\beta^2 + 3\beta + 2)c_s}{4(4 - \beta^2)} \right\}^2$$  \hspace{1cm} (16)

$$\Pi_M^* = \frac{Q^2}{2(1-\beta)(2-\beta)} + \frac{(\delta t - c_1)Q}{2-\beta} + \frac{(1-\beta)[4c_1^2 + 2c_1 + (2\delta t - c_1^2)]}{8(2-\beta)} + \frac{(1-\beta)c_1}{2(2-\beta)} - kt^2$$  \hspace{1cm} (17)

When the decision-making body is fair and neutral, the optimal recovery rate and the optimal wholesale price determined by the manufacturer in the direct recycling mode are $t^*$ and $\omega^*$, respectively, and the prices are then determined by offline and online retailers. The optimal retail prices are $P_1^*$ and $P_2^*$, respectively. At this time, the optimal returns of each manufacturer are $\Pi_{R_1}$, $\Pi_{R_2}$, and $\Pi_M$. Analysis of the above expression shows that since $0 < \beta < 1$, $3\beta^2 - \beta - 6 < -\beta^2 + 3\beta + 2$, then $\Pi_{R_1} \leq \Pi_{R_2}$.

**Conclusion 1.** When the manufacturers are fair and neutral, under the manufacturer’s direct recycling pricing strategy, $\Pi_{R_1} \leq \Pi_{R_2}$; that is, offline retailers’ profits are lower than online retailers, which is at a disadvantage in the competition.

4.2. Pricing Strategy in Third-Party Recycling Mode

The game order of each manufacturer:

1. The manufacturer determines the wholesale price $\omega$ and the transfer payment price $A$;
2. The two retailers determine the retail prices $P_1$ and $P_2$, respectively;
(3) The third party determines the recovery rate \( t \).

The profit function of offline retailers, online retailers, manufacturers, third parties, and the entire supply chain system can be expressed as:

\[
\begin{align*}
\Pi_{R_1} &= (P_1 - \omega - c_s)D_1 \\
\Pi_{R_2} &= (P_2 - \omega)D_2 \\
\Pi_M &= (\omega - c_1)D_1 + (\omega - c_1)D_2 + t(\delta - A)(D_1 + D_2) \\
\Pi_t &= At(D_1 + D_2) - kt^2 \\
\Pi_T &= (P_1 - c_1 - c_s)D_1 + (P_2 - c_1)D_2 + \delta t(D_1 + D_2) - kt^2
\end{align*}
\]

Using the inverse inductive method [12–14,16,18]:

\[
t = \frac{A(D_1 + D_2)}{2k}
\]

Substituting Equations (23) and (24) into the profit function of the manufacturer, using \( \frac{\partial \Pi_M}{\partial \omega} = 0 \) and \( \frac{\partial \Pi_M}{\partial A} = 0 \), the optimal wholesale price and transfer payment price can be determined using Mathematica [12–16,18]:

\[
\overline{\omega}^* = \frac{\left[2\beta^2(\beta - 1) + 4(2 - \beta)k\right]Q + 4k(2 - \beta)(1 - \beta)\bar{c}_t}{8k(2 - \beta)(1 - \beta - 2(1 - \beta)\beta^2)}
\]

\[
\bar{A}^* = \frac{\delta}{2}
\]

From this, the optimal retail prices of the two retailers are \( \overline{P}_1^* \) and \( \overline{P}_2^* \), and the optimal recovery rate of the third party is \( \overline{t}_1^* \). The optimal profits of each member of the channel are \( \Pi_{R_1}^*, \Pi_{R_2}^*, \Pi_M^*, \) and \( \Pi_T^* \).

When the decision-making bodies are fair and neutral, the manufacturer commissions in the tripartite recycling mode, where the manufacturer first determines the optimal wholesale price and transfer payment price as \( \overline{\omega}^* \) and \( \overline{A}^* \), respectively. Subsequently, the optimal retail price determined by the offline and online retailers is \( \overline{P}_1^* \) and \( \overline{P}_2^* \), and the third-party optimal recovery is \( \overline{t}_1^* \). The profit of each manufacturer is \( \Pi_{R_1}^*, \Pi_{R_2}^*, \Pi_{R_1}^*, \) and \( \Pi_{R_M}^* \). Analysis of the above expression shows that \( \Pi_{R_1}^* - \Pi_{R_1}^* > 0, \Pi_{R_2}^* - \Pi_{R_2}^* > 0, \) and \( \Pi_{M}^* - \Pi_{M}^* > 0 \).

**Conclusion 2.** When the manufacturers are fair and neutral, under the pricing decision of the manufacturer to entrust the third-party to recycle, the profit of the offline retailer is still lower than that of the online retailer, namely \( \Pi_{R_1}^* < \Pi_{R_1}^* \), and the profits of each manufacturer are lower than the direct recycling of the manufacturer, that is, \( \Pi_{M}^* < \Pi_{M}^* \).

5. **Pricing Strategies of Decision Makers under the Horizontal Fair Concern Behavior of Offline Retailers**

In reality, non-fully rational decision makers will only be disadvantageous to themselves. The unfairness creates a sense of disgust and does not arouse the unfairness of its own favorableness [26–28]. Since offline retailers are less profitable than online retailers, unbalanced offline retailers in the face of profit distribution will have a horizontal equity concern that will affect
their effectiveness. Referring to the practice of Zhang et al. [29], the fairness concern coefficient \( \lambda (\lambda \geq 0) \) is introduced, where when \( \lambda = 0 \), it means fair neutrality.

5.1. Pricing Strategy Based on Manufacturer’s Direct Recycling Mode under Horizontal Fair Concerns

The utility functions of offline retailers, online retailers, manufacturers, and the entire supply chain system are:

\[
U_{R_1} = \Pi_{R_1} - \lambda ((\Pi_{R_2} - \Pi_{R_1})) = (P_1 - \omega - c_s)D_1 - \lambda [(P_2 - \omega)D_2 - (P_1 - \omega - c_s)D_1]
\] (27)

\[
U_{R_2} = \Pi_{R_2} = (P_2 - \omega)D_2
\] (28)

\[
U_M = \Pi_M = (\omega - c_1)D_1 + (\omega - c_1)D_2 + \delta t(D_1 + D_2) - k\lambda
\] (29)

\[
U_T = (P_1 - c_1 - c_s)D_1 + (P_2 - c_1)D_2 + \delta t(D_1 + D_2) - k\lambda[(P_2 - \omega)D_2 - (P_1 - \omega - c_s)D_1]
\] (30)

Solving using the inverse inductive method [16–20,22] in Mathematica:

\[
\frac{\partial U_{R_1}}{\partial P_1} = Q - P_1 + \beta P_2 - P_1 + \omega + c_s - \lambda [\beta(P_2 - \omega) - (Q - P_1 + \beta P_2) + P_1 - \omega - c_s] = 0
\] (31)

\[
\frac{\partial U_{R_2}}{\partial P_2} = Q - P_2 + \beta P_1 - P_2 + \omega = 0
\] (32)

Joint solution:

\[
P_1 = \frac{(2 + 2\lambda + \beta)Q + (2 + \beta + 2\lambda + 2\beta\lambda)\omega + 2(1 + \lambda)c_s}{4 + 4\lambda - \beta^2}
\] (33)

\[
P_2 = \frac{(\lambda + 1)(\beta + 2)Q + [(2 + \beta)(\lambda + 1) + \beta^2\lambda] \omega + \beta(1 + \lambda)c_s}{4 + 4\lambda - \beta^2}
\] (34)

Substituting the Equations (33) and (34) into the utility function of the manufacturer, from \( \frac{\partial U_M}{\partial \omega} = 0 \) and \( \frac{\partial U_M}{\partial \delta} = 0 \), the optimal wholesale price and recovery rate of the manufacturer when the offline retailer has horizontal fairness concern is [12–16,18]:

\[
\omega^{**} = \frac{f(\lambda)}{g(\lambda)}
\] (35)

\[
\delta^{**} = \frac{h(\lambda)}{g(\lambda)}
\] (36)

The expressions \( f(\lambda), g(\lambda), \) and \( h(\lambda) \) are shown in Appendix A. In turn, the optimal selling prices of the two retailers can be found as \( P_1^{**} \) and \( P_2^{**} \), respectively, and the sales volume is \( D_1^{**} \) and \( D_2^{**} \), the utility is \( U_{R_1}^{**} \) and \( U_{R_2}^{**} \), the manufacturer’s utility is \( U_M^{**} \), and the utility of the entire supply chain is \( U_T^{**} \).

5.2. Pricing Strategy Based on Third-Party Recycling Mode under Horizontal Fairness Concerns

The utility functions of offline retailers, online retailers, manufacturers, a third party, and the entire supply chain system are:

\[
U_{R_1} = (P_1 - \omega - c_s)D_1 - \lambda [(P_2 - \omega)D_2 - (P_1 - \omega - c_s)D_1]
\] (37)

\[
U_{R_2} = \Pi_{R_2} = (P_2 - \omega)D_2
\] (38)

\[
U_M = (\omega - c_1)D_1 + (\omega - c_1)D_2 + t(\delta - A)(D_1 + D_2)
\] (39)
\[ \Pi_\tau = At(D_1 + D_2) - kl^2 \]  
\[ U_\tau = (P_1 - c_1 - c_0)D_1 + (P_2 - c_1)D_2 + \delta t(D_1 + D_2) - kl^2 \]

Using the inverse induction method [12–16,18] in Mathemtica:

\[ t = \frac{A(D_1 + D_2)}{2k} \]  
\[ P_1 = \frac{(2 + 2\lambda + \beta)Q + (2 + \beta + 2\lambda + 2\beta\lambda)\omega + 2(1 + \lambda)c_0}{4 + 4\lambda - \beta^2} \]  
\[ P_2 = \frac{(\lambda + 1)(\beta + 2)Q + [(2 + \beta)(\lambda + 1) + \beta^2\lambda]\omega + \beta(1 + \lambda)c_0}{4 + 4\lambda - \beta^2} \]

Substituting Equations (42)–(44) into the manufacturer’s utility function, \( \frac{\partial U_M}{\partial \omega} = 0 \) and \( \frac{\partial U_M}{\partial t} = 0 \), the optimal retail price and transfer payment price for the offline retailer’s horizontal fairness concern, respectively, obtained from Mathemtica are:

\[ \overline{\omega}^{**} = \frac{F(\lambda)}{G(\lambda)} \]  
\[ \overline{A}^{**} = \frac{\delta}{2} \]

The expressions of \( F(\lambda) \) and \( G(\lambda) \) are given in Appendix A. In turn, the optimal selling price of the two retailers is \( \overline{P}_1^{**} \overline{P}_2^{**} \), the sales volume is \( \overline{D}_1^{**} \overline{D}_2^{**} \), the utility is \( \overline{U}_{R_1} \overline{U}_{R_2} \), the utility \( \overline{U}_M \) of the manufacturer, the utility of the third-party is \( \overline{U}_t \), and the utility of the entire supply chain is \( \overline{U}_r \).

**Conclusion 3.** Under the third-party recycling mode, the optimal transfer payment price \( A \) determined by the manufacturer is independent of the retailer’s fair concern behavior and is not affected by the fairness concern coefficient, which is equal to \( \frac{\delta}{2} \).

**6. Supply Chain Coordination Based on the Manufacturer’s Direct Recycling Mode under Horizontal Fair Concern Behavior**

Through the comparison of assignments and Conclusion 2, it is known that the manufacturer is fair. In the direct recycling mode, the utility of each manufacturer is higher than the third-party recycling mode. The manufacturer whom is the leader of the game driven by profit maximization will choose the direct recycling mode. At this time, when offline retailers have horizontal fairness concerns, the utility is higher than the fairness and neutrality. Therefore, the manufacturer needs to provide a reasonable contract to achieve coordination. To this end, based on the research of Yi et al. [23], the traditional revenue-sharing contract was improved, and the \( \varphi_1 \) of the manufacturer’s income increase \( (\overline{U}_M - \overline{\Pi}_M) \) was allocated to the offline retailer \( R_1 \), and the income increase amount was \( \varphi_2 \) was assigned to the online retailer \( R_2 \). The manufacturer shares the remaining \( (1 - \varphi_1 - \varphi_2) \), where \( 0 < \varphi_1 < 1, 0 < \varphi_2 < 1 \), and \( 0 < \varphi_1 + \varphi_2 < 1 \) are to be satisfied.

\[ U_{M}^{*} = (1 - \varphi_1 - \varphi_2)(U_{M}^{*} - \overline{\Pi}_M^{*}) + \Pi_M^{*} \geq \Pi_M^{*} \]  
\[ U_{R_1}^{*} = U_{R_1}^{*} + \varphi_1(U_{M}^{*} - \overline{\Pi}_M^{*}) \geq \Pi_{R_1}^{*} \]  
\[ U_{R_2}^{*} = U_{R_2}^{*} + \varphi_2(U_{M}^{*} - \overline{\Pi}_M^{*}) \geq \Pi_{R_2}^{*} \]

Solutions must satisfy:

\[ \varphi_1 + \varphi_2 < 1 \]
ϕ₁ ≥ Π^{*}_{R₁} - U^{*}_{R₁} \over U^{*}_{M} - Π^{*}_{M} \tag{51}

ϕ₂ ≥ Π^{*}_{R₂} - U^{*}_{R₂} \over U^{*}_{M} - Π^{*}_{M} \tag{52}

**Conclusion 4.** When ϕ₁ changes within the range of [ϕ₁, ϕ₁] and ϕ₂ changes within the range of [ϕ₂, ϕ₂], this revenue sharing contract can realize the coordination of the dual-channel closed-loop supply chain. Among them.

ϕ₁ = Π^{*}_{R₁} - U^{*}_{R₁} \over U^{*}_{M} - Π^{*}_{M} \tag{53}

ϕ₂ = Π^{*}_{R₂} - U^{*}_{R₂} \over U^{*}_{M} - Π^{*}_{M} \tag{55}

ϕ₁ = 1 - Π^{*}_{R₁} - U^{*}_{R₁} \over U^{*}_{M} - Π^{*}_{M} \tag{54}

ϕ₂ = 1 - Π^{*}_{R₂} - U^{*}_{R₂} \over U^{*}_{M} - Π^{*}_{M} \tag{56}

Analysis of Conclusion 4 shows that the smaller ϕ₁ and ϕ₂ are, the more favorable it is to the manufacturer, and the larger the advantage is to the two retailers. The larger the interval, the larger the bargaining space between the manufacturer and the retailer, but the value of ϕ₁ and ϕ₂ depends on the ability of two retailers, both offline and online, to bargain for manufacturers.

7. Numerical Simulations

The results derived from the model are complex and cannot be visually divided for analysis and comparison. In order to make the conclusion clearer, numerical simulations were used to discuss the value of each variable with or without fairness and the sensitivity analysis of each variable with respect to the coefficient of fairness. It was assumed that Q, c₁, c₂, and k are exogenous variables, which are all given by the system. Refer to the study by Ho et al. [24]. The specific parameter assignments are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Q</td>
</tr>
<tr>
<td>2</td>
<td>c₁</td>
</tr>
<tr>
<td>3</td>
<td>c₂</td>
</tr>
<tr>
<td>4</td>
<td>k</td>
</tr>
<tr>
<td>5</td>
<td>cₛ</td>
</tr>
<tr>
<td>6</td>
<td>β</td>
</tr>
</tbody>
</table>

The numerical values of each variable when the fairness of each manufacturer is obtained by numerical simulations are shown in Table 2.

It can be clearly found from Table 2, under the different recovery mode, that there is the phenomenon of the online retailer below the retailer’s profit down the line, and when a third-party manufacturer’s recovery increased wholesale and retail prices, the profits of the manufacturers are reduced accordingly. Therefore, manufacturers will choose the direct recovery model in order to maximize profits, thus verifying Conclusions 1 and 2.
Table 2. Values for variables when Fair and Neutral.

<table>
<thead>
<tr>
<th>Recycling Method</th>
<th>Direct Recycling</th>
<th>Third-Party Recycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Prices</td>
<td>135.0</td>
<td>142.0</td>
</tr>
<tr>
<td>Transfer payment price</td>
<td>-</td>
<td>25</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>0.50</td>
<td>0.22</td>
</tr>
<tr>
<td>Offline retail price</td>
<td>162.0</td>
<td>166.7</td>
</tr>
<tr>
<td>Online retail price</td>
<td>158.0</td>
<td>162.7</td>
</tr>
<tr>
<td>Offline retailer profit</td>
<td>289.0</td>
<td>215.3</td>
</tr>
<tr>
<td>Online retailer profit</td>
<td>529.0</td>
<td>427.4</td>
</tr>
<tr>
<td>Third-party profit</td>
<td>-</td>
<td>97.6</td>
</tr>
<tr>
<td>Manufacturer profit</td>
<td>1900.0</td>
<td>1679.1</td>
</tr>
<tr>
<td>Supply chain system profit</td>
<td>2718.0</td>
<td>2419.5</td>
</tr>
</tbody>
</table>

Under the decision-making mode of direct recycling by manufacturers, referring to the research of Yi et al. [23], the sensitivity analysis of the variation of the horizontal fairness concern coefficient in the interval [0, 1] for each variable is considered below.

7.1. The Relationship between Retail Prices $P_1$ and $P_2$, and the Horizontal Fairness Concern

Looking at Figure 3, the manufacturer chooses the direct recycling model. Since the profit of offline retailers is lower than that of online retailers under fair neutral conditions, the fairness concerns drive offline retailers to take measures to reduce the income gap. Offline retailers hope to lower retail prices and increasing sales to ensure a market share, thereby reducing negative effects. Faced with the price reduction measures of offline retailers, online retailers will follow the pace and cut prices in time to prevent the decline in sales. As a result, there will be a competitive price cut in the market, resulting in increased competition. On the other hand, with the price cuts of offline retailers, online retailers should increase the price cuts to ensure their sales. In other words, consumers will be more sensitive to offline price reduction strategies, thereby increasing online purchases. Online retailers must make price cuts that are more satisfying to consumers in order to maintain market share in the fierce competition.

Figure 3. Sensitivity analysis of $P_1$ and $P_2$ versus the fairness concern coefficient $\lambda$.

Conclusion 5. Under the manufacturer’s direct recycling model, when offline retailers have horizontal fairness concerns, the retail prices of offline retailers and online retailers are lower than the retail price
of fair and neutral operations for each manufacturer, and horizontal fairness. The concerning factor is negatively correlated and the magnitude of the $P_2$ decline is greater than $P_1$.

7.2. The Relationship between the Recovery Rate $t$, the Wholesale Price, and the Horizontal Fairness Concern Factor Determined by the Manufacturer

By using horizontal fairness concern, and observing Figures 4 and 5, combined with Conclusion 5, the manufacturer chooses under the direct recycling mode. The retail prices of online and offline products are reduced, and the market demand increases. Manufacturers will recycle more used products in order to reduce the average production cost per unit. The continuous increase in recycling rate has gradually reduced the manufacturer’s unit production cost. Manufacturers encourage the two retailers to compete to lower prices to increase market demand and obtain long-term benefits, which will reduce the wholesale price accordingly.

![Figure 4](image4.png)

**Figure 4.** Sensitivity analysis of recovery rate $t$ versus the fairness concern coefficient $\lambda$ in direct recovery mode.

![Figure 5](image5.png)

**Figure 5.** Sensitivity analysis of wholesale price $\omega$ versus the fairness concern coefficient $\lambda$ in direct recovery mode.
Conclusion 6. Under the manufacturer’s direct recycling mode, when offline retailers have horizontal fairness concerns, the recovery rate of used products is higher than that of each manufacturer’s fair neutrality, and it positively correlated with the horizontal fairness concern coefficient. Furthermore, the wholesale price of products is low. The fair neutrality situation is negatively correlated with the horizontal fairness concern coefficient.

7.3. The Relationship between Offline Retailers Utility, Online Retailers Utility, and Horizontal Fairness Concerns

Looking at Figure 6 and Conclusion 5, the manufacturer directly recycles the model. Next, with the increasing level of horizontal fairness concerns, consumers are more sensitive to offline price cuts during the competitive price cuts of the two retailers, making online retailers’ prices lower than offline retailers, resulting in lower wholesale prices and the increased advantage of sales cannot make up for the loss caused by the reduction of the retail price. This situation will slowly evolve into a malicious withdrawal from the market. On the contrary, with the increase of horizontal fairness concern, when \( \lambda \approx 0.6 \), the huge loss of online retailers makes the offline retailer’s utility rebound and achieve the purpose of price reduction. However, in real life, online retailers cannot always passively accept the price reduction measures of offline retailers and make corresponding price cuts. When their own utility is lower than offline retailers, they will also have a fair tendency. Therefore, when the horizontal fairness concern coefficient is about 0.95, the utility curves of the two retailers intersect to reach the equilibrium point. However, the utility of the two retailers’ curve intersections is lower than the fairness and neutrality of each manufacturer. Therefore, the retailer’s horizontal fairness concern will not improve the effectiveness of both parties. The fierce market competition will only lead to the situation of “two losses”.

Conclusion 7. Under the manufacturer’s direct recycling mode, when the offline retailer \( R_1 \) has horizontal fairness concerns, with the increase of the horizontal fairness concern coefficient, the utility
of offline retailers first declines and then rises slowly, while the effectiveness of online retailers is lower than the fairness and neutrality of each manufacturer. The fairness and neutrality of each manufacturer are negatively correlated with the horizontal fairness concern coefficient, and the loss is higher than an offline retailer.

7.4. The Relationship between the Utility of the Manufacturer and the Utility of the Entire Supply Chain System and the Horizontal Fairness Concern

Looking at Figure 7, we can see that manufacturers are fair and neutral. The manufacturers income from the direct recovery model is higher than the revenue from the entrusted third party (Conclusion 2 is verified), the revenue under the direct recovery model increases with the increase in the horizontal fairness concern factor, and the revenue under the third-party recycling model decreases, such that the manufacturer direct recycling mode is chosen for maximizing revenue. Looking at Figure 8, we can see that in the direct recycling mode, in the process of two retailers competing to reduce the price to obtain market share, the demand for products will continue to increase, and the sales volume of manufacturers will increase, benefiting from market leaders. Mostly, the income is increased. At the same time, the utility of the entire closed-loop supply chain system has increased as manufacturers have benefited the most, even far beyond the losses of the two retailers. It can be seen that when offline retailers have horizontal fairness concerns, manufacturers and society as a whole will benefit from it.

**Figure 7.** Sensitivity analysis of $U_M$ versus the fairness concern coefficient $\lambda$ in different recovery mode.

**Conclusion 8.** In the case of fair neutrality or horizontal fairness concern, the manufacturer’s direct recovery mode is superior to the entrusted third-party recovery mode, and when the manufacturer chooses the direct recovery mode when retailer $R_1$ has horizontal fairness concerns, the manufacturer utility quotient and the utility of the entire supply chain system are higher than the fairness of each vendor and are positively correlated with the horizontal fairness concern factor.
Figure 8. Sensitivity analysis of $U_M$ and $U_T$ versus the fairness concern coefficient $\lambda$ in direct recovery mode.

7.5. The Relationship between the Revenue Sharing Ratios of Offline and Online Retailers $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$, and the Horizontal Fairness Concern Coefficient

By observing Figure 9 and Conclusions 7 and 8, we see that in the manufacturer direct-recovery mode, when offline retailers have horizontal fairness concerns, offline and online retailers are competing to lower prices, resulting in lower utility than fair neutrality. Manufacturers are improving their own, consumer, and social utility. A three-way revenue sharing contract is needed to achieve coordination, and as the horizontal fairness concern coefficient increases, the online retailer’s losses are greater than the offline retailer, and the manufacturer balances the two retailers’ profits by increasing the online profit distribution ratio, which has reached the Pareto optimum.

Figure 9. Sensitivity analysis of revenue sharing ratio versus the fairness concern coefficient $\lambda$. 
Conclusion 9. Under the manufacturer’s direct recovery mode, with the increase of the horizontal fairness concern coefficient, the upper and lower limits of the offline retailer’s revenue sharing ratio become lower and lower, while the online retailer’s revenue sharing ratio is getting higher and higher.

8. Conclusions

With the development of e-commerce, the closed-loop supply chain is online and offline. The phenomenon of competition among retailers is becoming more and more serious. The online sales model has even seriously impacted the offline sales model. Unfair disgust has forced offline retailers to take measures, thus introducing fair concerns into the dual-channel closed-loop supply chain. The chain is more reflective of real decision-making. This paper divides the profit maximization of each decision-maker into the decision-making goal. A dual-channel sales game model based on direct recycling from manufacturers and third-party recycling was established to consider the impact of offline fairness concerns of offline retailers on the pricing and utility of decision-makers. Research shows that in the closed-loop supply chain of dual-channel sales, whether in the case of fair neutrality or horizontal fairness concerns, the manufacturer’s direct recycling model is superior to the entrusted third-party recycling, and the third-party recycling model is transferred by the manufacturer. The payment price is always equal to half of the unit cost savings of remanufactured products. Under the direct recycling model, the horizontal fair concern behavior of offline retailers has caused the two retailers to compete in price reduction and recovery rate, which has improved the manufacturer and the entire closed-loop supply chain system utility; but the utility of offline and online retailers has caused a “two losses” situation in the competition. Moreover, the limitation of the proposed research is that it lacks a trade-off capability between the online and offline retailers’ revenue-versus-fairness concern. In order to improve resource utilization and improve their own and overall social benefits, manufacturers have to formulate appropriate revenue sharing contracts to transfer some profits to achieve coordination across the supply chain. The proposed approach has effective practical implications that will impact the real-life seller–retailer relationship and coordination. Such an approach will also influence the supply–demand ratio of DCSC management.

Supply channel conflicts are getting more and more serious, but they are not one-of-a-kind harm. The development of enterprises requires competition to provide motivation, but the rational adjustment of pricing strategies between retailers alone cannot optimize the entire system. In order to grasp the degree of goodness and prevent malicious competition, channel managers need to intervene to coordinate. With the increasing price competition caused by fairness, demand, and supply increasing, this will inevitably lead to the problem of excessive use and waste of resources. Future work for this research may be extended in different supply chain configurations, such as divergent supply chain. Mutual interrelations and a comprehensive view of selecting different problems suggest several future directions in problem classifications and opportunities. Furthermore, the proposed research can be extended to consider a CLSC system consisting of a manufacturer and multiple competitive retailers.

Author Contributions: M.A. conceived the idea, performed the experiments, and wrote the paper; Q.S.K. provided extensive technical support throughout the research work; J.L. provided extensive support in the theoretical analysis and supervision of the research; and A.L. provided the results validation and conclusion.

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Conflicts of Interest: The authors declare no conflict of interest.
Appendix A

\[ f(\lambda) = (\beta^2 \lambda + 3\beta \lambda + 4\lambda + 2\beta + 4) [(8k + 8k\lambda - 2\beta^2k + \delta^2(\beta^3 \lambda + 2\beta^2 \lambda + \beta \lambda - 4\lambda + 2\beta^2 + 2\beta - 4)]Q \\
- (8k + 8k\lambda - 2\beta^2k) (\beta^3 \lambda + 2\beta^2 \lambda + 2\beta^2 + 2\beta - 2) c_1 \\
+ (8k + 8k\lambda - 2\beta^2k) (\lambda + 1)(\beta + 2)(1 - \beta) c_1 \\
+ \delta^2(\lambda + 1)(\beta + 2) (\beta - 1) (\beta^3 \lambda + 2\beta^2 \lambda + \beta \lambda - 4\lambda + 2\beta^2 + 2\beta - 4)(\beta^2 \lambda + \beta \lambda - 2\lambda + \beta^2 + \beta - 2)c_s \\
- 4)(\beta^2 \lambda + \beta \lambda - 2\lambda + \beta^2 + \beta - 2)c_s \]

\[ g(\lambda) = [(-\beta^3 - 2\beta^2 - \beta + 4) \lambda + (-2\beta^2 - 2\beta + 4)] [16k - 4\beta^2k + (2\beta^2 + 2\beta - 4)\delta^2 \\
+ (\beta^3 \delta^2 + 2\beta^2 \delta^2 + \beta \delta^2 - 4\delta^2 + 16k) \lambda] \]

\[ h(\lambda) = \delta(\beta^2 \lambda + 3\beta \lambda + 4\lambda + 2\beta + 4)(-\beta^3 \lambda - 2\beta^2 \lambda - \beta \lambda + 4\lambda - 2\beta^2 - 2\beta + 4)Q \\
- \delta(\beta^3 \lambda + 2\beta^2 \lambda - 2\lambda + \beta^2 + \beta - 2)(\beta^3 \lambda + 2\beta^2 \lambda + \beta \lambda - 4\lambda + 2\beta^2 + 2\beta - 4)c_1 \\
+ \delta(\lambda + 1)(\beta + 2)(\beta - 1)(-\beta^3 \lambda - 2\beta^2 \lambda - \beta \lambda + 4\lambda - 2\beta^2 - 2\beta + 4)c_s \]

\[ F(\lambda) = 4k[(16 + 8\beta - 4\beta^2 - 2\beta^3) + (32 + 20\beta - 3\beta^2 - \beta^4) \lambda \\
+ (16 + 12\beta + 4\beta^2) \lambda^2]Q \\
+ 2\lambda^2 [(16 + 8\beta + 12\beta^2 - 2\beta^3 - 2\beta^4) \\
+ (32 + 12\beta + 20\beta^2 + 3\beta^3 - 2\beta^4 - \beta^5) \lambda \\
+ (16 + 4\beta + 8\beta^2 + 4\beta^3) \lambda^2]Q \\
+ \delta^2 [(4 + 2\beta + 2\beta^2)(-4 + 2\beta + 2\beta) \\
+ (-4 + 2\beta + 2\beta^2)(-8 + 4\beta + 3\beta^2 + \beta^3) \lambda \\
+ (-4 + \beta + 2\beta^2 + \beta^3)(-4 + 3\beta + \beta^2) \lambda\lambda^2]Q \\
+ 4k[(8 - 4\beta - 6\beta^2 + \beta^3 + \beta^4) \\
+ (16 - 4\beta - 10\beta^2 - 4\beta^3 + \beta^4 + \beta^5) \lambda + (8 - 4\beta^2 - 4\beta^3) \lambda^2]c_1 \\
+ 4k[(8 - 4\beta - 6\beta^2 + \beta^3 + \beta^4) + (16 - 8\beta - 10\beta^2 + \beta^3 + \beta^4) \lambda \\
+ (8 - 4\beta - 4\beta^2) \lambda^2] (c_1 - c_s) \\
+ \delta^2 [(4 + 2\beta + 2\beta^2)(-2 + \beta + \beta^2) \\
+ (-2 + \beta + \beta^2)(-8 + 3\beta + 4\beta^2 + \beta^3) \lambda \\
+ (-2 + \beta + \beta^2)(-4 + \beta + 2\beta^2 + \beta^3) \lambda^2]c_s \]

\[ G(\lambda) = 8k[(16 - 8\beta - 12\beta^2 + 2\beta^3 + 2\beta^4) + (32 - 12\beta - 20\beta^2 - 3\beta^3 + 2\beta^4 + \beta^5) \lambda] \\
+ (16 - 4\beta - 8\beta^2 - 4\beta^3) \lambda^2 \\
+ \delta^2 [(4 - 2\beta - 2\beta^2)(-4 + 2\beta + 2\beta^2) \\
+ 2(-4 + 2\beta + 2\beta^2)(4 - 2\beta^2 - 2\beta^3) \lambda \\
+ (-4 + \beta + 2\beta^2 + \beta^3)(4 - \beta - 2\beta^2 - \beta^3) \lambda^2] \]

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