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Warranty Decision Model and Remanufacturing Coordination Mechanism in Closed-Loop Supply Chain: View from a Consumer Behavior Perspective

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Abstract: The remanufacturing warranty strategy has become an effective mechanism for reducing consumer risk and stimulating market demand in closed-loop supply chain management. Based on the characteristics of consumers' behavior of purchase decisions, this paper studies the warranty decision model of remanufacturing closed-loop supply chain under the Stackelberg game model. The present study discussed and compared the decision variables, including remanufacturing product pricing, extended warranty service pricing, warranty period and supply chain system profit. The research shows that consumers' decision-making significantly affirms the dual marginalization effect of the supply chain system while significantly affecting the supply chain warranty decision; the improved revenue sharing contract and the two charge contracts respectively coordinates the manufacturer-led and retail-oriented closed-loop supply chain system, which effectively implements the Pareto improvement of the closed-loop supply chain system with warranty services. In the present study, the model is verified and analyzed by numerical simulation.

Keywords: remanufacturing; closed-loop supply chain; warranty decision; contract coordination

1. Introduction

The recycling and reuse of electronic waste is currently a global concern, and remanufacturing provides an effective solution for used electronic products. However, due to the slow development of the market entities of remanufactured products in China, the development of the remanufacturing industry has been seriously hindered. Manufacturers (including original manufacturers and remanufacturers) and retailers have been using price leverage to attract customers and promote sales performance. However, in addition to price factors, consumer brand loyalty is increasingly built rather on the quality of the (service of) remanufactured products. The product warranty strategy has become a new round of competition hotspots besides traditional price competition. The warranty strategy refers to the obligation or warranty provided by the manufacturer or the retailer, distributor, etc., to the consumer in terms of technical performance, product effect and maintenance of the product during the sales process. The consumer can predict the product quality based on the product warranty (Boulding and Kirmani, 1993) [1]. Warranty has become an effective mechanism to reduce consumer risk and stimulate market demand, while also being linked to corporate social responsibility, which is a key feature of corporate sustainability and business sustainability (Wei et al. 2015 [2]; Ahi and Searcy, 2013 [3]). In closed loop supply chain management, the length of the remanufactured product warranty period (related to remanufactured failure rate) and extended warranty pricing play a key role in determining the total cost of the product (Shafiee and Chukova, 2013 [4]). A satisfactory warranty policy will increase consumers' willingness to purchase remanufactured products while contributing to production sustainability and resource efficiency (Song et al. 2018 [5]). However, the supply chain must balance between the warranty inputs and outputs to maximize benefits.

This paper is primarily related to the research in three streams: the operation of the warranty strategy (which mainly focusing on the warranty period and the product life cycle), the supply chain pricing strategies and the consumer behavior theory. In the following, we review the literature in these three streams.

At present, scholars have done preliminary research on the operation of warranty strategies, mainly focusing on the warranty period and the product life cycle. Li et al. (2016a) [6] studied the impact of the warranty period on the closed-loop supply chain system from the perspective of product warranty period. Lan et al. (2014) [7] explored the impact of product price and quality on the development of warranty strategies under the fuzzy supply chain based on three types of warranties. Du et al. (2016) [8] accurately estimated product warranty costs based on product endogenous variables (product loss rate, etc.) and gave cost calculation methods. Arabi et al. (2017) [9] determined the the best warranty period from the perspective of the manufacturer and consumer to minimize the total cost of use and increase the service life. Chen et al. (2017) [10] considered product warranty as an economic compensation for consumers in the event of product failure and compared the impact of the extended warranty provided by the manufacturer or distributor on the overall market. Li et al. (2016b) [11] discussed issues related to the manufacturer's pricing strategy in two supply chains, including one manufacturer and two competing retailers with warranty-related requirements. Mai et al. (2017) [12] analyzed the optimal warranty strategy given the original product warranty provided by the manufacturer and the additional product warranty provided by the seller. Sabbaghi and Behdad (2012) [13] studied the impact of product characteristics, sales environment and market size on consumer purchasing decisions in the context of manufacturers providing warranty services. Xu et al. (2018) [14] studied the issue of bundled pricing of durable consumer goods with warranty services under the two market structures of monopoly and duopoly. Based on the research above on warranty decision-making, it is found that there are few existing studies on the coordination of supply chain systems when strong retailers provide extended warranty services.

In terms of supply chain pricing strategies, Savaskan et al. (2004) [15] used manufacturers and distributors as recycling bearers to explore how to select recycling channels and determine pricing decisions for remanufactured products. Fleischmann et al. (1997) [16] emphasized the coordination of price decision in the reverse channel of the closed-loop supply chain through qualitative analysis. Xu et al. (2018) [17] studied the impact of the retailer's overconfidence on supply chain pricing and performance in the duopoly market environment of demand uncertainty and concluded that the overconfident retailers tend to give higher pricing. Gong and Jiang (2018) [18] studied the optimal decision of the supply chain system recycling model for the four mixed recycling models, and determined the optimal pricing strategies in different situations. Kaya et al. (2014) [19] did empirical analysis of pricing decisions and incentives for remanufacturing closed-loop supply chains in uncertain environments. Luo et al. (2017) [20] studied two different brand manufacturers selling products through unified distributors by building the pricing decision model under horizontal and vertical competition, and then analyzed the impact of different power structures on product pricing. Yoo and Kim (2016) [21] built a three-tier supply chain consisting of manufacturers, distributors and refurbished processors, by considering multiple supply chain process combinations, introducing five different supply chain structures, and comparing the models in pricing and performance of each product. Gan et al. (2017) [22] constructed a closed-loop supply chain pricing model for short-lived products consisting of manufacturers, distributors, and recyclers. In this model, new products are sold through traditional means, and remanufactured products are passed through manufacturers. The study found that the degree of acceptance and the direct sales channel preferences affect supply chain pricing. However, existing research on the coordinated pricing of closed-loop supply chains is still relatively rare.

In addition, the current study also investigates consumer behavior theory. Consumer behavior directly affects the remanufacturing closed-loop supply chain sales performance of warranty services. Chen et al. (2012) [23] provided a variety of optional warranty services based on consumer heterogeneity risk preferences and moral hazard decisions in different market periods. Gaur et al. (2017) [24] conducted research on the influencing factors of consumer purchase and perceived behavior, and analyzed the pricing decision problem. In terms of factors affecting consumer purchasing behavior, there is an uncertainty in the duration of consumers holding their own goods (Joshi and Rahman, 2015 [25]), loss aversion and other influencing factors. Gallego et al. (2014) [26] and Su and Wang (2016) [27] designed flexible warranty services and tapped flexible warranty strategies to create higher profits. Li et al. (2018) [28] conducted an exploratory study of consumer assurance perspectives and consumer risk preferences based on the consumer perception dimension. Zhu and Yu (2018) [29] studied the closed-loop supply chain pricing decision-making and coordination mechanism based on the differentiated willingness to pay of consumers. Zhou et al. (2017) [30] studied the pricing strategies and service cooperation of manufacturers under uncertain consumer demand. However, the study of warranty decisions for remanufacturing closed-loop supply chains has less introduced consumer behavior theory.

Different from previous research, by considering a re-manufacturing closed-loop supply chain system with warranty services, where market demand is disturbed by consumer behavior, this paper studies the retail as the warranty subject, the optimal decision of supply chain for warranty pricing and warranty period and the contract coordination problem of supply chain. In addition, our article focuses on three different game models, namely the retailer-led Stackelberg game, the retailer-led Stackelberg game, and the centralized decision model, and consider how manufacturers and retailers can optimize the cooperation contract under full information conditions.

The main contributions of this paper are as follows. First, establish five decentralized decision models by considering a remanufacturing closed-loop supply chain system of two-tier extended warranty services consisting of one manufacturer and one retailer, and improve the closed-loop supply chain coordination contract with warranty services, and supplemented the current related research; secondly, through the use of game theory, the equilibrium wholesale price, retail price, warranty price and warranty period are obtained and analyzed in five decentralized decision models; third, through numerical simulation, We have gained some valuable management insights. For example, firstly, the increase in consumer preference will increase the profit of the node enterprises and play a positive incentive role. Secondly, the increase of consumer preference has aggravated the loss of decision-making efficiency, and, at the same time, widened the gap between the total profit of supply chain system in centralized and decentralized decision-making, amplifying the effect of "double marginalization". Thirdly, the traditional revenue-sharing contract based on commodity sales revenues fails to coordinate the closed-loop supply chain system with extended warranty services. The cooperation between manufacturers and retailers can be improved by adopting improved revenue sharing contracts.

The rest of the paper is organized as follows: in the second section, we introduce the problem description and symbolic assumptions. The third section describes the constructed warranty decision model in detail, and gives the model analysis of the properties in the fourth section. The fifth section sets up the coordination contract to improve the double marginalization effect generated by the supply chain system. The sixth section presents and verifies the decision model in the form of numerical simulation. Finally, we summarize our results and propose implications for future research.

2. Problem Description and Assumptions

2.1. Problem Description

This paper considers a single-stage closed-loop supply chain system consisting of a single manufacturer and a single retailer. First, the consumer determines whether to purchase the remanufactured product and its extended warranty service. The manufacturer then determines the wholesale price of the remanufactured product. Finally, the retailer determines the product price and provides the extended warranty service for the market after buying remanufacturing products from the manufacturer via wholesale. This paper discusses three market-predictive decision models, as shown in Figure 1: centralized decision-making models (manufacturers and retailers see decision-making as a whole), manufacturer-led decentralized decision models (i.e., M-R models), and the retailer-led decentralized decision-making model (i.e., the R-M model) corresponding to the practical cases under the business models of Gree Group (Zhuhai, China), Apple Inc. (Cupertino, CA, USA) and Suning Tesco (Nanjing, China), respectively.

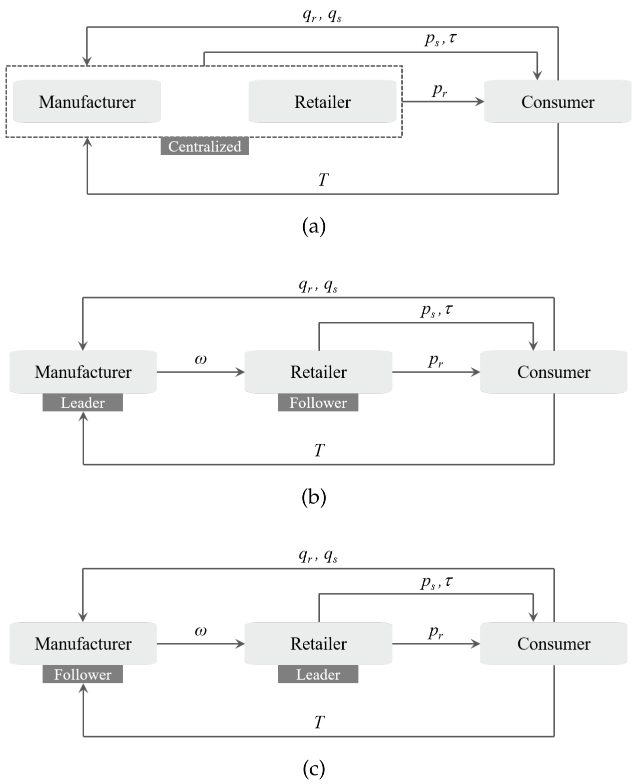

Figure 1. (**a**) centralized decision model; (**b**) M-R decision model; (**c**) R-M decision model.

2.2. Basic Assumptions and Parameters

Assumption 1: *Manufacturers and retailers in decentralized decision-making have a Stackelberg game relationship. They are risk-neutral and in the game of complete information.*

Assumption 2: *To be specific, we assume that there are only remanufactured products in the market, and the retail price of the product is p_r, the extended warranty price is p_s, the unit wholesale price of the remanufactured product is ω, the remanufactured unit cost is c_r, and the extended warranty service unit cost is c_s.*

Assumption 3: *The fixed cost of remanufacturing is T; τ is the warranty period of remanufactured products. According to the research results of Li et al. (2012)* [31], *the failure rate of remanufactured products during the warranty period is $\mu\tau^2$ (μ is a constant and $\mu > 0$), $\iota > 0$ is the average cost of the repair or replacement of the unit product that the manufacturer has failed, and the fixed service cost of the remanufactured product warranty.*

For $\frac{1}{2}\eta\tau^2$, *where* $\eta = \mu\iota > 0$ *represents the final cost factor for the retailer to provide extended warranty service; the profit function is* Π_j, *where* $j \in \{S, R, M\}$, *and S represents the supply chain system, R the retailer, and M the manufacturer, respectively.*

Assumption 4: *Let the production quantity (i.e., demand) of the remanufactured product be* q_r. *Let the market be Q the consumer's Willingness To Pay (WTP) is* α *and subject to a uniform distribution of* $[0, 1]$, *and its distribution function is* $F(\alpha) = \alpha$. *If the consumer's WTP for the remanufactured product is* α_r, *the utility function of the consumer to purchase the remanufactured product is* $\mu = \alpha_r - p_r$, *and the consumer decides whether to purchase remanufacturing products by comparing the size of the consumer utility function. When the consumer utility function satisfies* $\mu = \alpha_r - p_r > 0$, *the consumer chooses to purchase the remanufactured product. At this time, under the influence of the consumer remanufactured product preference intensity, the remanufactured product demand function is represented as:*

$$q_r^N = \int_{p_r}^{Q} F'(\alpha_r)\, d\alpha_r + \theta q_r^e = \int_{p_r}^{Q} 1\, dF(\alpha_r) + \theta q_r^e, \tag{1}$$

where q_r^N *represents the market demand function,* q_r^e *represents the user's expected consumer size for remanufactured products, and* $\theta \in (0, 1)$ *represents the consumer preference over remanufactured products.*

To simplify the discussion, assuming $q_r^N = q_r^e$, the remanufactured product demand function is converted to:

$$q_r = \frac{\int_{p_r}^{Q} 1\, dF'(\alpha_r)}{1 - \theta} = \frac{Q - p_r}{1 - \theta}. \tag{2}$$

Assumption 5: *Remanufactured product extended warranty service demand is* q_s, *remanufactured product extended warranty service demand is affected by extended warranty service price* p_s, *warranty period* τ *and product demand:*

$$q_s = q_r - \beta p_s + \gamma\tau. \tag{3}$$

Among them, β indicates the consumer's sensitivity to the price of the extended warranty service, γ is the coefficient of influence of the warranty period on the product demand, and $\beta > 0, \gamma > 0$. In particular, when $\beta p_s > \gamma\tau$, some consumers who purchase remanufactured products choose to purchase extended warranty services, hereinafter referred to as partial extensions.

Note: In the following, the superscripts "c", "d1" and "d2" in the variables represent centralized decision, decentralized M-R decision and decentralized R-M decision in closed-loop supply chain decision, respectively; superscript "r1", "r2" and "r3" respectively represent the revenue sharing contract under the coordination mechanism, the improved revenue sharing contract and the two charge contracts. The superscript "*" indicates the optimal decision result.

3. Closed-Loop Supply Chain Decision

3.1. Centralized Decision Making (Model C)

Under the centralized decision model, manufacturers and retailers form a unified joint decision to maximize the overall profit of the supply chain system (e.g., Gree's centralized business strategy), and its profit function is expressed as:

$$\begin{aligned} \max_{p_r, p_s, \tau} \Pi_S^c &= q_r\,(p_r - c_r) + q_s\,(p_s - c_s) - \frac{1}{2}\eta\tau^2 - T \\ s.t \quad & \beta p_s \geqslant \gamma\tau. \end{aligned} \tag{4}$$

Sections 1 and 2 of Model (4) are the proceeds from the sale of remanufactured products and sales extension services, respectively, and the third part is the fixed fee required for the remanufacturing and sales process. When $\beta p_s > \gamma\tau$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase the remanufactured product extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve model (4) and get Proposition 1.

Proposition 1: *Under the centralized decision, the optimal selling price of the remanufactured product, the optimal extended warranty price of the remanufactured product, the optimal warranty period, the remanufactured product output and the warranty service sales are respectively:*

$$p_r^{c*} = \frac{(\theta - 1)\left(\beta\eta c_s - c_r\left(\gamma^2 - 2\beta\eta\right)\right) + Q\left(\eta(2\beta(\theta - 1) + 1) - \gamma^2(\theta - 1)\right)}{\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)}, \tag{5}$$

$$p_s^{c*} = \frac{\eta c_r + c_s\left(\eta(2\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)\right) + \eta(-Q)}{\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)}, \tag{6}$$

$$\tau_s^{c*} = \frac{\gamma\left(c_r - 2\beta(\theta - 1)c_s - Q\right)}{\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)}, \tag{7}$$

$$q_r^{c*} = \frac{\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) + \beta\eta c_s}{\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)}, \tag{8}$$

$$q_s^{c*} = \frac{\beta\eta\left(c_r - 2\beta(\theta - 1)c_s - Q\right)}{\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)}. \tag{9}$$

At this point, under the centralized decision, the optimal profit of the supply chain system is:

$$\Pi_S^{c*} = -\frac{-\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right)^2 + 2\beta\eta c_s\left(c_r - Q\right) - 2\beta^2\eta(\theta - 1)c_s^2}{2\left(\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)\right)}. \tag{10}$$

Proof: To build a centralized decision Lagrangian function:

$$L^c(p_r, p_s, \tau, \lambda) = q_r\left(p_r - c_r\right) + q_s\left(p_s - c_s\right) - \frac{1}{2}\eta\tau^2 + \lambda\left(\beta p_s - \gamma\tau\right). \tag{11}$$

A centralized decision-making profit function for the Hessian Matrix of p_r, p_s, τ:

$$\mathbf{H^c} = \begin{bmatrix} \frac{2}{\theta - 1} & \frac{1}{\theta - 1} & 0 \\ \frac{1}{\theta - 1} & -2\beta & \gamma \\ 0 & \gamma & -\eta \end{bmatrix}. \tag{12}$$

If and only if the order master subroutine $H_1 = -\frac{2}{1-\theta} < 0$, $H_2 = -\frac{4\beta(\theta - 1) - 1}{(1-\theta)^2} > 0$, $H_3 = \frac{2(2\eta\beta - \gamma^2)(1-\theta) - \eta}{(1-\theta)^2} < 0$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, the Hessian Matrix is negative, and the centralized decision profit function Π_S^{c*} is a strict concave function for p_r, p_s, τ so there is an optimal solution.

Solve the first-order partial derivative of p_r, p_s, τ for L^c and make it 0:

$$\frac{\partial L^c}{\partial p_r} = \frac{c_r + c_s - 2p_r - p_s + Q}{1 - \theta} = 0, \tag{13}$$

$$\frac{\partial L^c}{\partial p_s} = \beta\lambda + \beta c_s + \gamma\tau + \frac{p_r}{\theta - 1} - 2\beta p_s + \frac{Q}{1 - \theta} = 0, \tag{14}$$

$$\frac{\partial L^c}{\partial \tau} = -\gamma\lambda - \gamma c_s - \eta\tau + \gamma p_s = 0. \tag{15}$$

The simultaneous decision equations can be obtained by substituting Equation (4), and Proposition 1 is proved. □

3.2. Decentralized Decision

3.2.1. M-R Decision (Model D1)

In the M-R decision-making(e.g., Apple Inc. and its affiliates), the manufacturer as the market leader first considers the retailer's optimal response function to determine the wholesale price. Then, the retailer determines the remanufactured product sales price and the extended warranty pricing decision according to the manufacturer's decision. The decision model is:

$$\begin{aligned} &\max_{p_r,p_s,\tau} \Pi_M^{d1} = q_r\left(\omega - c_r\right) - T, \\ s.t. \quad &\max_{p_r,p_s,\tau} \Pi_R^{d1} = q_r\left(p_r - \omega\right) + q_s\left(p_s - c_s\right) - \frac{1}{2}\eta\tau^2, \\ &\beta p_s \geqslant \gamma\tau. \end{aligned} \tag{16}$$

The first part of the objective function in model (16) is the gain from the manufacturer selling the remanufactured product, and the second part is the remanufactured fixed fee. The constraint is the retail price selected by the retailer under the maximization of the objective function and the price of the extended service. When $\beta p_s > \gamma\tau$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase the remanufactured product's extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve the model (16). Proposition 2 is available.

Proposition 2: *Under the decentralized M-R decision, the optimal selling price of the remanufactured product, the optimal extended warranty price of the remanufactured product, the optimal warranty period, the remanufactured product output and the warranty service sales are respectively:*

$$p_r^{d1*} = -\frac{(\theta-1)\left(c_r\left(\gamma^2 - 2\beta\eta\right) - \beta\eta c_s\right) + Q\left(3\gamma^2(\theta-1) - 2\eta(3\beta(\theta-1)+1)\right)}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)}, \tag{17}$$

$$p_s^{d1*} = -\frac{-\eta c_r + c_s\left(4\gamma^2(\theta-1) - \eta(4\beta(\theta-1) + \eta\phi\beta + 2)\right) + \eta Q}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)}, \tag{18}$$

$$\tau^{d1*} = \frac{\gamma\left(c_r + \beta c_s(-4\theta + \phi\eta + 4) - Q\right)}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)}, \tag{19}$$

$$\omega^{d1*} = -\frac{-\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) - \beta\eta c_s}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)}, \tag{20}$$

$$q_r^{d1*} = \frac{\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) + \beta\eta c_s}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}, \tag{21}$$

$$q_s^{d1*} = \frac{\beta\eta\left(\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) - \beta c_s\left(\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)\right)\right)}{2\left(\gamma^2 - 2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)}. \tag{22}$$

Note: To simplify the decision model, make $\phi = \frac{1}{\gamma^2 - 2\beta\eta}$.

At this point, under the decentralized M-R decision, the optimal profits of manufacturers and retailers are:

$$\Pi_M^{d1*} = -\frac{\left(\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) + \beta\eta c_s\right)^2}{4\left(\gamma^2 - 2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)}, \tag{23}$$

$$\Pi_R^{d1*} = \frac{-\left(\gamma^2-2\beta\eta\right)^2\left(Q-c_r\right)^2-2\beta\eta c_s\left(\gamma^2-2\beta\eta\right)\left(Q-c_r\right)+\beta^2\eta c_s^2\left(\eta(16\beta(\theta-1)+3)-8\gamma^2(\theta-1)\right)}{8\left(\gamma^2-2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)}. \quad (24)$$

Proof: The Lagrangian function to build retailer profits is:

$$L^{d1}(p_r,p_s,\tau,\lambda) = q_s\left(p_s-c_s\right)-\frac{1}{2}\eta\tau^2+q_r\left(p_r-\omega\right)+\lambda\left(\beta p_s-\gamma\tau\right). \quad (25)$$

A decentralized M-R decision profit function for the Hessian Matrix of p_r, p_s, τ:

$$\mathbf{H^{d1}} = \begin{bmatrix} \frac{2}{\theta-1} & \frac{1}{\theta-1} & 0 \\ \frac{1}{\theta-1} & -2\beta & \gamma \\ 0 & \gamma & -\eta \end{bmatrix}. \quad (26)$$

If and only if the order master subroutine $H_1 = -\frac{2}{1-\theta} < 0$, $H_2 = -\frac{4\beta(\theta-1)-1}{(1-\theta)^2} > 0$, $H_3 = \frac{2(2\eta\beta-\gamma^2)(1-\theta)-\eta}{(1-\theta)^2} < 0$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1-\frac{\eta}{2\eta\beta-\gamma^2}$ the Hessian Matrix is negative, and the decentralized M-R decision profit function Π_R^{d1*} is a strict concave function for p_r, p_s, τ, so there is an optimal solution.

Solve the first-order partial derivative of p_r, p_s, τ for L^{d1} and make it 0:

$$\frac{\partial L^d}{\partial p_r} = \frac{c_s-2p_r-p_s+Q+\omega}{1-\theta} = 0, \quad (27)$$

$$\frac{\partial L^d}{\partial p_s} = \beta\lambda+\beta c_s+\gamma\tau+\frac{p_r}{\theta-1}-2\beta p_s+\frac{Q}{1-\theta} = 0, \quad (28)$$

$$\frac{\partial L^d}{\partial \tau} = -\gamma\lambda-\gamma c_s-\eta\tau+\gamma p_s = 0. \quad (29)$$

The simultaneous equations solve for $p_r^{d1}(\omega), p_s^{d1}(\omega), \tau(\omega)$ for ω,and substitute Π_M^{d1} can get ω^*. Finally, by substituting the above results into the model (16), we get Π_M^{d1*}, Π_R^{d1*}, so Proposition 2 is proved. □

3.2.2. R-M Decision (Model D2)

In the R-M decision-making (e.g., Wal-Mart), the retailer, as the market leader, prioritizes its own objective function and the manufacturer's optimal response function to determine the sales price. Then, the manufacturer determines the remanufactured product sales price and the extended warranty pricing decision based on the retailer's decision. The model is:

$$\begin{aligned} &\max_{p_r,p_s,\tau} \Pi_R^d = q_r\left(p_r-\omega\right)+q_s\left(p_s-c_s\right)-\frac{1}{2}\eta\tau^2, \\ s.t. \quad &\max_{p_r,p_s,\tau} \Pi_M^d = q_r\left(\omega-c_r\right)-T, \\ &\beta p_s \geqslant \gamma\tau, \\ &p_r = \omega+Z. \end{aligned} \quad (30)$$

The first part of the model (30) objective function is the revenue earned by the retailer from remanufacturing the product, and the second part is the gain from the sales extension service. The constraint is the wholesale price chosen by the manufacturer under the maximization of the objective function. When $\beta p_s > \gamma\tau$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase remanufactured product's extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve model (26). Proposition 3 is then available.

Proposition 3: *Under the decentralized R-M decision, the optimal selling price of the remanufactured product, the optimal extended warranty price of the remanufactured product, the optimal warranty period, the remanufactured product output, and the warranty service sales are respectively:*

$$p_r^{d2*} = \frac{(\theta-1)\left(\beta\eta c_s - c_r\left(\gamma^2 - 2\beta\eta\right)\right) + Q\left(\eta(6\beta(\theta-1)+1) - 3\gamma^2(\theta-1)\right)}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}, \tag{31}$$

$$p_s^{d2*} = \frac{\eta c_r + c_s\left(\eta(4\beta(\theta-1)+1) - 4\gamma^2(\theta-1)\right) - \eta Q}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}, \tag{32}$$

$$\tau^{d2*} = \frac{\gamma\left(-c_r + 4\beta(\theta-1)c_s + Q\right)}{\eta(-8\beta(\theta-1)-1) + 4\gamma^2(\theta-1)}, \tag{33}$$

$$q_r^{d2*} = \frac{\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}, \tag{34}$$

$$q_s^{d2*} = \frac{\beta\eta\left(c_r - 4\beta(\theta-1)c_s - Q\right)}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}. \tag{35}$$

At this point, under the decentralized R-M decision, the optimal profits of manufacturers and retailers are:

$$\Pi_M^{d2*} = -\frac{(\theta-1)\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)^2}{\left(\eta(-8\beta(\theta-1)-1) + 4\gamma^2(\theta-1)\right)^2}, \tag{36}$$

$$\Pi_R^{d2*} = -\frac{-\left(\gamma^2 - 2\beta\eta\right)(Q - c_r)^2 + 2\beta\eta c_s(c_r - Q) - 4\beta^2\eta(\theta-1)c_s^2}{2\left(\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)\right)}. \tag{37}$$

Proof: The Lagrangian function to build retailer profits is:

$$L^{d2}(p_r, p_s, \tau, \lambda) = q_s(p_s - c_s) - \frac{1}{2}\eta\tau^2 + q_r(p_r - \omega) + \lambda(\beta p_s - \gamma\tau). \tag{38}$$

A decentralized M-R decision profit function for the Hessian Matrix of p_r, p_s, τ:

$$\mathbf{H^{d2}} = \begin{bmatrix} \frac{2}{\theta-1} & \frac{1}{\theta-1} & 0 \\ \frac{1}{\theta-1} & -2\beta & \gamma \\ 0 & \gamma & -\eta \end{bmatrix}. \tag{39}$$

If and only if the order master subroutine $H_1 = -\frac{2}{1-\theta} < 0$, $H_2 = -\frac{4\beta(\theta-1)-1}{(1-\theta)^2} > 0$, $H_3 = \frac{2(2\eta\beta-\gamma^2)(1-\theta)-\eta}{(1-\theta)^2} < 0$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta-\gamma^2}$, the Hessian Matrix is negative, and the decentralized R-M decision profit function $\Pi_M^{d2*}$is a strict concave function for p_r, p_s, τ, so there is an optimal solution.

Solve the first-order partial derivative of ω for L^{d2}, introduce the artificial variable and make it 0:

$$\omega = c_r - p_r + Q. \tag{40}$$

Substituting Π_R^{d2} and solving the first-order partial derivatives of p_r, p_s, τ and making it 0, yields:

$$\frac{\partial L^d}{\partial p_r} = \frac{c_r + c_s - 4p_r - p_s + 3Q}{1-\theta} = 0, \tag{41}$$

$$\frac{\partial L^d}{\partial p_s} = \beta c_s + \frac{\gamma(\theta-1)\tau + p_r - Q}{\theta - 1} - 2\beta p_s = 0, \tag{42}$$

$$\frac{\partial L^d}{\partial \tau} = -\gamma c_s - \eta\tau + \gamma p_s = 0. \tag{43}$$

The simultaneous equations can be solved by deriving $p_r^{d2*}, p_s^{d2*}, \tau^{d2*}$, substituting Π_M^{d2}. Finally, the result is substituted into the model (30) to obtain Π_M^{d2*}, Π_R^{d2*}, and Proposition 3 is proved. □

In order to facilitate the analysis below, the optimal decision results of the closed-loop supply chain are collated, as shown in Table 1.

Table 1. Optimization decision summary table.

Model C	$q_r = \frac{(\gamma^2 - 2\beta\eta)(Q - c_r) + \beta\eta c_s}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$q_s = \frac{\beta\eta(c_r - 2\beta(\theta-1)c_s - Q)}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$p_r = \frac{(\theta-1)(\beta\eta c_s - c_r(\gamma^2 - 2\beta\eta)) + Q(\eta(2\beta(\theta-1)+1) - \gamma^2(\theta-1))}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$p_s = \frac{\eta c_r + c_s(\eta(2\beta(\theta-1)+1) - 2\gamma^2(\theta-1)) + \eta(-Q)}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$\tau = \frac{\gamma(c_r - 2\beta(\theta-1)c_s - Q)}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$\Pi_S^c = -\frac{-(\gamma^2 - 2\beta\eta)(Q - c_r)^2 + 2\beta\eta c_s(c_r - Q) - 2\beta^2\eta(\theta-1)c_s^2}{2(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1))}$
Model D1	$q_r = \frac{(\gamma^2 - 2\beta\eta)(Q - c_r) + \beta\eta c_s}{\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)}$
	$q_s = \frac{\beta\eta((\gamma^2 - 2\beta\eta)(Q - c_r) - \beta c_s(\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)))}{2(\gamma^2 - 2\beta\eta)(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1))}$
	$p_r = -\frac{(\theta-1)(c_r(\gamma^2 - 2\beta\eta) - \beta\eta c_s) + Q(3\gamma^2(\theta-1) - 2\eta(3\beta(\theta-1)+1))}{2(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1))}$
	$p_s = -\frac{-\eta c_r + c_s(4\gamma^2(\theta-1) - \eta(4\beta(\theta-1) + \eta\phi\beta + 2)) + \eta Q}{2(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1))}$
	$\tau = \frac{\gamma(c_r + \beta c_s(-4\theta + \phi\eta + 4) - Q)}{2(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1))}$
	$\Pi_M^{d1} = -\frac{((\gamma^2 - 2\beta\eta)(Q - c_r) + \beta\eta c_s)^2}{4(\gamma^2 - 2\beta\eta)(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1))}$
	$\Pi_R^{d1} = \frac{-(\gamma^2 - 2\beta\eta)^2(Q - c_r)^2 - 2\beta\eta c_s(\gamma^2 - 2\beta\eta)(Q - c_r) + \beta^2\eta c_s^2(\eta(16\beta(\theta-1)+3) - 8\gamma^2(\theta-1))}{8(\gamma^2 - 2\beta\eta)(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1))}$

Table 1. *Cont.*

Model D2	$q_r = \frac{\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}$
	$q_s = \frac{\beta\eta\left(c_r - 4\beta(\theta-1)c_s - Q\right)}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}$
	$p_r = \frac{(\theta-1)\left(\beta\eta c_s - c_r\left(\gamma^2 - 2\beta\eta\right)\right) + Q\left(\eta(6\beta(\theta-1)+1) - 3\gamma^2(\theta-1)\right)}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}$
	$p_s = \frac{\eta c_r + c_s\left(\eta(4\beta(\theta-1)+1) - 4\gamma^2(\theta-1)\right) - \eta Q}{\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)}$
	$\tau = \frac{\gamma\left(-c_r + 4\beta(\theta-1)c_s + Q\right)}{\eta(-8\beta(\theta-1)-1) + 4\gamma^2(\theta-1)}$
	$\Pi_M^{d2} = -\frac{(\theta-1)\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)^2}{\left(\eta(-8\beta(\theta-1)-1) + 4\gamma^2(\theta-1)\right)^2}$
	$\Pi_R^{d2} = -\frac{-\left(\gamma^2 - 2\beta\eta\right)(Q - c_r)^2 + 2\beta\eta c_s(c_r - Q) - 4\beta^2\eta(\theta-1)c_s^2}{2\left(\eta(8\beta(\theta-1)+1) - 4\gamma^2(\theta-1)\right)}$

4. Analysis of Decision-Making Properties of Closed-Loop Supply Chain

Property 1: *In model C, D1, D2 decisions, extended warranty service price p_s, warranty period τ, remanufactured product output q_r, extended warranty service sales q_s, and supply chain system profit Π increase as consumer preference θ increases, and remanufactured product sales price p_r decreases as consumer preference θ increases, indicating that consumers prefer in view of supply chain member companies* that *θ has a positive incentive for the revenue of member companies in the supply chain.*

Proof: Taking the decentralized MR decision as an example, when $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, the optimal solution is established:

$$\frac{\partial p_r^{d1*}}{\partial\theta} = \frac{\eta\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)^2} < 0, \quad (44)$$

$$\frac{\partial p_s^{d1*}}{\partial\theta} = \frac{\eta\left(-\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) - \beta\eta c_s\right)}{\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)^2} > 0, \quad (45)$$

$$\frac{\partial \tau^{d1*}}{\partial\theta} = \frac{\gamma\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\gamma\eta c_s}{\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)^2} > 0, \quad (46)$$

$$\frac{\partial q_r^{d1*}}{\partial\theta} = \frac{\left(\gamma^2 - 2\beta\eta\right)\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)}{\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)^2} > 0, \quad (47)$$

$$\frac{\partial q_s^{d1*}}{\partial\theta} = \frac{\beta\eta\left(-\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) - \beta\eta c_s\right)}{\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)^2} > 0, \quad (48)$$

$$\frac{\partial \Pi_M^{d1*}}{\partial\theta} = \frac{\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)^2}{2\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)^2} > 0, \quad (49)$$

$$\frac{\partial \Pi_R^{d1*}}{\partial\theta} = \frac{\left(\left(\gamma^2 - 2\beta\eta\right)(Q - c_r) + \beta\eta c_s\right)^2}{4\left(\eta(4\beta(\theta-1)+1) - 2\gamma^2(\theta-1)\right)^2} > 0. \quad (50)$$

The same can be proved in the centralized decision-making and decentralized R-M decision. □

Property 2: *Decentralized M-R and decentralized RM decision, retailers and manufacturers each make their own interests to maximize the goal of decision-making, resulting in double marginalization effect. The closed-loop supply chain system is not optimal, specifically in decentralized decision-making. The optimal selling price is greater than the optimal selling price under the centralized decision, and the optimal production quantity and the optimal warranty period are less than the optimal production quantity and warranty period under the centralized decision, namely:*

1. $\tau^{c*} > \tau^{d1*}, \tau^{c*} > \tau^{d2*}$,
2. $p_r^{d1*} > p_r^{c*}, p_r^{d2*} > p_r^c; p_s^{d1*} > p_s^c, p_s^{d2*} > p_s^c$,
3. $q_r^{d1*} > q_r^c, q_r^{d2*} > q_r^c; q_s^{d1*} > q_s^c, q_s^{d2*} > q_s^c$.

In particular, as consumer preference for the degree of θ increases, the degree of differentiation between decentralized decision-making and centralized decision-making increases.

Proof: Take the decentralized M-R decision as an example, let:

$$Z_{pr} = p_r^{c*} - p_r^{d1*} = -\frac{(\theta-1)\left(-\left(\gamma^2-2\beta\eta\right)(Q-c_r)-\beta\eta c_s\right)}{2\left(\eta(4\beta(\theta-1)+1)-2\gamma^2(\theta-1)\right)}, \tag{51}$$

$$Z_{ps} = p_s^{c*} - p_s^{d1*} = \frac{\eta\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)}{2\left(\gamma^2-2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)}, \tag{52}$$

$$Z_{\tau} = \tau^{c*} - \tau^{d1*} = \frac{\gamma\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\gamma\eta c_s}{2\left(\gamma^2-2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)}. \tag{53}$$

Solving a one-stage partial guide on θ,

$$\frac{\partial Z_{pr}}{\partial\theta} = -\frac{\eta\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)}{2\left(\eta(4\beta(\theta-1)+1)-2\gamma^2(\theta-1)\right)^2}, \tag{54}$$

$$\frac{\partial Z_{ps}}{\partial\theta} = -\frac{\eta\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)}{\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)^2}, \tag{55}$$

$$\frac{\partial Z_{\tau}}{\partial\theta} = -\frac{\gamma\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\gamma\eta c_s}{\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)^2}. \tag{56}$$

Since $\gamma^2 < \eta\beta$, i.e.,$\gamma^2 - 2\beta\eta > 0, \frac{\partial Z_{pr}}{\partial\theta} > 0, \frac{\partial Z_{ps}}{\partial\theta} > 0, \frac{\partial Z_{\tau}}{\partial\theta} > 0$, Property 2 is certified. The same is true for re-centralized decision-making and decentralized RM decisions. □

Property 3: *Under decentralized decision-making, the optimal total profit of the supply chain system is lower than the optimal profit of the centralized decision system, i.e.,* $\Pi_S^{c*} > \Pi_R^{d1*} + \Pi_M^{d1*}; \Pi_S^{c*} > \Pi_R^{d2*} + \Pi_M^{d2*}$. *In addition, from the perspective of consumer behavior, the increase in consumer preference θ will be magnified under decentralized decision-making. The "dual marginal effect" exacerbates the loss of efficiency of decentralized decision-making systems.*

Proof: Taking the decentralized MR decision as an example, when $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, the optimal solution is established:

$$Z_{\Pi} = \Pi_S^{c*} - (\Pi_M^{d1*} + \Pi_R^{d1*}) = -\frac{\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)^2}{8\left(\gamma^2-2\beta\eta\right)\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)} > 0 \tag{57}$$

$$\frac{\partial Z_{\Pi}}{\partial\theta} = \frac{\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)^2}{4\left(\eta(4\beta(\theta-1)+1)-2\gamma^2(\theta-1)\right)^2} > 0 \tag{58}$$

Property 3 is proved. Similarly, the re-centralized decision-making and the decentralized R-M decision are also established. □

5. Coordination Mechanism Design

Because the decentralized M-R and R-M decisions have double marginal effects, the system is not optimal, and with the increase of consumer preference, the double marginalization effect is gradually expanded. Therefore, for the M-R model, the income sharing contract will be adopted. For the R-M model, two charge contracts will be adopted to improve the total system profit of the supply chain under the decentralized decision approach or to achieve the total system profit of centralized decision-making, and make each member company of the supply chain implement Pareto improvements.

5.1. M-R Decision: Revenue Sharing Contract (S-1 Model)

According to Cachon and Lariviere, (2005) [32], revenue sharing contracts can significantly increase the overall benefits of the supply chain: manufacturers give retailers lower wholesale prices, while retailers give manufacturers a certain amount of revenue. The sharing ratio is δ, and the corresponding share of the revenue earned by the manufacturer is $1-\delta$, and the decision model is:

$$\begin{aligned} &\max_{p_r,p_s,\tau} \Pi_M^{r1} = q_r\left(\omega - c_r\right) + (1-\delta)p_r q_r - T, \\ s.t. \quad &\max_{p_r,p_s,\tau} \Pi_R^{r1} = q_r\left(\delta p_r - \omega\right) + q_s\left(p_s - c_s\right) - \frac{1}{2}\eta\tau^2, \\ &\beta p_s \geqslant \gamma q_s. \end{aligned} \tag{59}$$

The first part of the model (50) objective function is the gain from the manufacturer selling the remanufactured product, and the second part is the share-sharing share. The constraint is the retail price selected by the retailer under the maximization of the objective function and the price of the extended service. When $\beta p_s > \gamma\tau$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase the remanufactured product's extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve model (50), and get Proposition 4.

Proposition 4: *The optimal retail price, optimal extended warranty price and optimal warranty period under the revenue sharing contract coordination M-R model are respectively:*

$$p_r^{r1*} = \frac{\eta\left(\eta(2\beta(\delta+1)(\theta-1)+1) - \gamma^2(\delta+1)(\theta-1)\right)\left(2\beta(\theta-1)c_s + Q\right)}{\eta - 2\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right)} + (\theta-1)c_r\left(\gamma^2 - 2\beta\eta\right), \tag{60}$$

$$p_s^{r1*} = \frac{c_r\left(\gamma^2 - 2\beta\eta\right)\left(2\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right) - \eta\right)}{\eta - 2\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right)} + \eta Q\left(\gamma^2 - 2\beta\eta\right)\left[2\delta\left(\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right) - \eta\right)\right], \tag{61}$$

$$\tau^{r1*} = \frac{c_r\left(\gamma^2 - 2\beta\eta\right)\left(2\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right) - \eta\right)}{2\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right) - \eta} + Q\left(\gamma^2 - 2\beta\eta\right)\left[2\delta\left(\delta(\theta-1)\left(\gamma^2 - 2\beta\eta\right) - \eta\right)\right]. \tag{62}$$

Proof: The Lagrangian function to build retailer profits is:

$$L^{r1}(p_r, p_s, \tau, \lambda) = q_s\left(p_s - c_s\right) + q_r\left(\delta p_r - \omega\right) - \frac{1}{2}\eta\tau^2 + \lambda\left(\beta p_s - \gamma\tau\right). \tag{63}$$

Under the revenue sharing contract coordination mechanism, the profit function about the Hessian Matrix of p_r, p_s, τ is:

$$\mathbf{H^{d1}} = \begin{bmatrix} \frac{2}{\theta-1} & \frac{1}{\theta-1} & 0 \\ \frac{1}{\theta-1} & -2\beta & \gamma \\ 0 & \gamma & -\eta \end{bmatrix}. \tag{64}$$

If and only if the order master subroutine $H_1 = -\frac{2}{1-\theta} < 0$, $H_2 = -\frac{4\beta(\theta-1)-1}{(1-\theta)^2} > 0$, $H_3 = \frac{2(2\eta\beta-\gamma^2)(1-\theta)-\eta}{(1-\theta)^2} < 0$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta-\gamma^2}$, the Hessian Matrix is negative. At this time, the decentralized revenue sharing contract decision profit function Π_R^{r2*} is a strict concave function for p_r, p_s, τ, so there is an optimal solution.

Solve the first-order partial derivative of p_r, p_s, τ for L^{r1} and make it 0:

$$\frac{\partial L^{r1}}{\partial p_r} = \frac{c_s - 2\delta p_r - p_s + \delta Q + \omega}{1-\theta} = 0, \tag{65}$$

$$\frac{\partial L^{r1}}{\partial p_s} = \beta\lambda + \beta c_s + \gamma\tau + \frac{p_r}{\theta-1} - 2\beta p_s + \frac{Q}{1-\theta} = 0, \tag{66}$$

$$\frac{\partial L^{r1}}{\partial \tau} = -\gamma\lambda - \gamma c_s - \eta\tau + \gamma p_s = 0. \tag{67}$$

The simultaneous equations solve for $p_r^{r1}(\omega), p_s^{r1}(\omega), \tau(\omega)$ for ω, and by substituting Π_M^{r1} we can get ω^*. Finally, substituting the results above into the model (59) will result in Π_M^{r1*}, Π_R^{r1*},and Proposition 4 will be proved. □

Property 4: *The traditional revenue sharing contract cannot coordinate the remanufacturing closed-loop supply chain system with warranty services. The specific implementation is as follows: under the coordination of revenue sharing contract, the overall benefit of the decentralized M-R decision system is improved, but the Pareto improvement is not achieved. The coordinated retailer's revenue is less than that before the coordination, i.e.,* $\Pi_R^{d1*} > \Pi_R^{r1*}$. *At this time, the retailer refuses to accept the contract coordination, and the revenue sharing contract is invalid.*

Proof: When $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta-\gamma^2}$, the optimal solution is established. At this point, let $Z_\Pi^1 = \Pi_M^{d1*} + \Pi_R^{d1*}, Z_\Pi^2 = \Pi_M^{R1*} + \Pi_R^{r1*}$, then:

$$Z_\Pi^1 - Z_\Pi^2 = \frac{(\delta-1)(\theta-1)\left(2\eta(\beta(3\delta+1)(\theta-1)+1) - \gamma^2(3\delta+1)(\theta-1)\right)\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)^2}{8\left(\eta(-4\beta(\theta-1)-1)+2\gamma^2(\theta-1)\right)\left(\eta(-2\beta(\delta+1)(\theta-1)-1)+\gamma^2(\delta+1)(\theta-1)\right)^2} > 0, \tag{68}$$

$$\frac{\partial Z_\Pi^2}{\partial \theta} = \frac{\left[\gamma^2(\delta+1)(2\delta+1)(\theta-1) - \eta(2\beta(\delta+1)(2\delta+1))\right)\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right]^2}{4\left(\eta(-2\beta(\delta+1)(\theta-1)-1)+\gamma^2(\delta+1)(\theta-1)\right)^3} > 0, \tag{69}$$

$$\Pi_R^{d1*} - \Pi_R^{r1*} = \frac{(\delta-1)^2(\theta-1)^2\left(2\beta\eta-\gamma^2\right)\left(\left(\gamma^2-2\beta\eta\right)(Q-c_r)+\beta\eta c_s\right)^2}{8(\eta(-4\beta(\theta-1))+2\gamma^2(\theta-1))(\eta(-2\beta(\delta+1)(\theta-1))+\gamma^2(\delta+1)(\theta-1))^2} > 0. \tag{70}$$

At this point, it can be judged that $Z_\Pi^1 > Z_\Pi^2, \Pi_R^{d1*} > \Pi_R^{r1*}$, and Property 4 is proved. □

5.2. M-R Decision: Improved Revenue Sharing Contract (S-2 Model)

Under the manufacturer-led Stackelberg game model environment, we assume that the manufacturer is trying to motivate the retailer to sell. It will give the retailer a lower wholesale price and allow the retailer to retain a portion of the F, while requiring a margin over the retained profit. Part of the profit sharing, with a retailer retention ratio of δ the manufacturer is divided into $1-\delta$, and its decision model is:

$$\begin{aligned} &\max_{p_r,p_s,\tau} \Pi_M^{r2} = q_r(\omega - c_r) + (1-\delta)\left[q_r(p_r-\omega) + q_s(p_s-c_s) - \frac{\eta\tau^2}{2} - F\right], \\ s.t.\quad &(p_r,p_s,\tau) \in argmax\Pi_R^{r2} = \delta\left[q_r(p_r-\omega) + q_s(p_s-c_s) - \frac{\eta\tau^2}{2} - F\right] + F, \\ &T \geqslant \Pi_R^{c*}. \end{aligned} \tag{71}$$

The first part of the objective function in model (71) is the gain from the manufacturer selling the remanufactured product, and the second part is the share-sharing share obtained by the manufacturer. The constraints are the retailer's incentive compatibility constraints and the retailer's participation constraints. When $\beta p_s > \gamma\tau$, i.e.,$\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase the remanufactured product extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve model (71), and get Proposition 5.

Proposition 5: *The optimal retail price, optimal extended warranty price, optimal wholesale price and optimal warranty period by improving the revenue sharing contract coordination under M-R are respectively:*

$$p_r^{r2*} = \frac{(\theta-1)c_r\left(\gamma^2 - 2\beta\eta\right) - \beta\eta(\theta-1)c_s + Q\left(\gamma^2(2\delta+1)(\theta-1) - \eta(2\beta(2\delta+1)(\theta-1) + \delta+1)\right)}{(\delta+1)\left(\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right)}, \tag{72}$$

$$p_s^{r2*} = \frac{\eta\left(-c_r + \delta\eta\phi\beta c_s + \delta(-Q)\right) + c_s\left(\eta(-2\beta(\theta-1)-1) + 2\gamma^2(\theta-1)\right) + \eta Q}{\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)}, \tag{73}$$

$$\omega^{r2*} = \frac{\delta c_r\left(\gamma^2 - 2\beta\eta\right) + \beta\delta^2\eta c_s + \delta^2 Q\left(\gamma^2 - 2\beta\eta\right)}{\delta(\delta+1)\left(\gamma^2 - 2\beta\eta\right)}, \tag{74}$$

$$\tau^{r2*} = \frac{\gamma\left(\delta\left(\eta\phi\beta c_s + Q\right) - c_r + 2\beta(\theta-1)c_s + Q\right)}{\eta(-4\beta(\theta-1)-1) + 2\gamma^2(\theta-1)}. \tag{75}$$

Proof: The Lagrangian function to build retailer profits is:

$$L^{r2}(p_r,p_s,\tau,\lambda,A) = \delta\left[q_s(p_s-c_s) - F - \frac{\eta\tau\tau}{2} + q_r(p_r-\omega)\right] + F + \lambda(\beta p_s - \gamma\tau) + A\left(T - \Pi_R^{c*}\right). \tag{76}$$

Under the improved revenue sharing contract coordination mechanism, the profit function about the Hessian Matrix of p_r, p_s, τ is:

$$\mathbf{H^{d1}} = \begin{bmatrix} \frac{2}{\theta-1} & \frac{1}{\theta-1} & 0 \\ \frac{1}{\theta-1} & -2\beta & \gamma \\ 0 & \gamma & -\eta \end{bmatrix}. \tag{77}$$

If and only if the order master subroutine $H_1 = -\frac{2}{1-\theta} < 0$, $H_2 = -\frac{4\beta(\theta-1)-1}{(1-\theta)^2} > 0$, $H_3 = \frac{2(2\eta\beta-\gamma^2)(1-\theta)-\eta}{(1-\theta)^2} < 0$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \frac{\eta}{2\eta\beta-\gamma^2}$, the Hessian Matrix is negative. At this time, the decentralized revenue sharing contract decision profit function Π_R^{r2*} is a strict concave function for p_r, p_s, τ, so there is an optimal solution.

Solve the first-order partial derivative of p_r, p_s, τ for L^{r1} and make it 0:

$$\frac{\partial L^{r2}}{\partial p_r} = -\frac{\delta\left(c_s - 2p_r - p_s + Q + \omega\right)}{\theta-1} = 0, \tag{78}$$

$$\frac{\partial L^{r2}}{\partial p_s} = \beta\lambda + \delta\left[\beta c_s + \frac{\gamma(\theta-1)\tau + p_r - Q}{\theta - 1} - 2\beta p_s\right] = 0, \tag{79}$$

$$\frac{\partial L^{r2}}{\partial \tau} = -\gamma\lambda + \gamma\delta\left(p_s - c_s\right) - \delta\eta\tau = 0. \tag{80}$$

The simultaneous equations solve for $p_r^{r2}(\omega), p_s^{r2}(\omega), \tau(\omega)$ for ω, and, by substituting Π_M^{r2}, we can get ω^*. Finally, substituting the results above into the model (59) will result in Π_M^{r2*}, Π_R^{r2*}, and Proposition 5 will be proved. □

Property 5: *The improved revenue sharing contract will effectively coordinate the remanufacturing closed-loop supply chain system with warranty services, which is embodied in the following: improving the total profit of the system under the coordination of the revenue sharing contract is greater than the total profit of the traditional revenue sharing contract supply chain system, and each member company of the supply chain has achieved a Pareto improvement, namely* $Z_\Pi^1 > Z_\Pi^3 > Z_\Pi^2$. *(Note: Let* $Z_\Pi^3 = \Pi_M^{r2*} + \Pi_R^{r2*}$*).*

Proof: When $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \dfrac{\eta}{2\eta\beta - \gamma^2}$, the optimal solution is established, then:

$$Z_\Pi^3 - Z_\Pi^2 = \frac{(\delta-1)\eta\left[\delta\left(\eta(8\beta(\delta+1)(\theta-1)) + 4\gamma^2(\delta+1)(\theta-1)\right) + \eta\right]\left[\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right) + \beta\eta c_s\right]^2}{8(\delta+1)^2\left(\gamma^2 - 2\beta\eta\right)\left(\eta(-4\beta(\theta-1)) + 2\gamma^2(\theta-1)\right)\left[\eta(-2\beta(\delta+1)(\theta-1)) + \gamma^2(\delta+1)(\theta-1)\right]^2} > 0, \tag{81}$$

$$\Pi_R^{d1*} - \Pi_R^{r2*} = 0. \tag{82}$$

It shows that the improved revenue sharing contract can realize the decision-making efficiency of the remanufacturing closed-loop supply chain system, and the two parties in the supply chain system receive the contract, improve the overall profit of the supply chain system and realize the Pareto improvement. At this point, how to determine the revenue sharing ratio of δ is particularly important, directly affecting the supply chain coordination effect. □

5.3. R-M Decision: Two Charging Contracts (T Model)

In the retail-led Stackelberg game model environment, in the market transaction process, the retailer will use the bargaining advantage to charge the manufacturer a certain channel fee of S. Therefore, this paper will use two charge contracts to coordinate the retailer-led closed-loop supply chain (Cachon, 2003 [33]). Its decision model is:

$$\begin{aligned} &\max_{p_r, p_s, \tau} \Pi_R^{r3} = q_r\left(p_r - \omega\right) + q_s\left(p_s - c_s\right) - \frac{1}{2}\eta\tau^2 - S, \\ s.t.\quad &\left(p_r, p_s, \tau\right) \in argmax\Pi_M^{r3} = q_r\left(\omega - c_r\right) - T + S, \\ &\Pi_M^{r3} > \Pi_M^{d3}, \\ &\beta p_s \geqslant \gamma q_s, \\ &p_r = \omega + Z. \end{aligned} \tag{83}$$

In model (83), the first part of the objective function is the revenue obtained by the retailer for remanufacturing the product, the second part is the profit obtained by the retailer to sell the extended warranty service, and the third part is the channel fee paid by the retailer and the fixed service cost. The constraint is the manufacturer's participation constraint. When $\beta p_s > \gamma\tau$, i.e., $\beta > 1, \gamma^2 < \eta\beta, 0 < \theta < 1 - \dfrac{\eta}{2\eta\beta - \gamma^2}$, some consumers choose to purchase the remanufactured product's extended warranty service. At this time, there is an optimal solution for the objective function. Combine Equation (2) and Formula (3) to solve the model (83), and get Proposition 6.

Proposition 6: *The two charging contracts can coordinate the R-M decision model, and the parameters must satisfy* $\omega^{r3*} = p_r^{r3*} = p_r^{c*}, p_s^{r3*} = p_s^{c*}$,

$$S^* = -\frac{-\left(\gamma^2 - 2\beta\eta\right)\left(Q - c_r\right)^2 + 2\beta\eta c_s\left(c_r - Q\right) - 2\beta^2\eta(\theta - 1)c_s^2}{2\left(\eta(4\beta(\theta - 1) + 1) - 2\gamma^2(\theta - 1)\right)}. \tag{84}$$

Proof: For Π_M^{r3}, it is easy to know that $\Pi_M^{r3}(\omega, p_r)$ is a strict concave function for ω, p_r, and there is a unique optimal solution. If the two charging contract can coordinate the closed-loop supply chain, it must satisfy $p_r^{r3*} = p_r^{c*}, \omega^{r3*} = \omega^{c*}$, and substitute Π_R^{r3}; by deriving, we can see that $\Pi_R^{r3}(p_r, p_s, \tau)$ is a strict concave function on p_r, p_s, τ, and there is a unique optimal solution, and the value of S^* can be obtained with the two constraints and the retailer's target profit function. The verification is completed. □

6. Numerical Simulation

In order to reflect the conclusions obtained more clearly, the results of the models above are numerically simulated below. Referring to the parameter setting of Zhou et al. (2017) [34], let $Q = 1000, c_r = 100, c_s = 10, \eta = 1.3, \beta = 4, \gamma = 2, T = 50,000$. According to Zhu and Yu, (2018) [29], when the revenue sharing ratio $\delta \in [0.249, 0.512]$, the supply chain realized the income optimization. This paper assumes that $\delta = 0.5$, make model C, model D1 Model D2, S-1 model, S-2 model, and T model's remanufactured product's optimal retail price, optimal extended warranty service price, optimal warranty period, remanufacturing output, warranty service sales, and the profit of member companies of the supply chain and supply chain system's total profit comparison charts, respectively, and coordinate optimization of the model.

6.1. Remanufacturing Closed-Loop Supply Chain Warranty Decision Model

(1) Under the decision of each game model, consider the comparison of consumer preference θ for remanufactured product retail price p_r and output q_r.

From Figure 2, under each decision model, the retail price p_r decreases as the consumer preference θ increases, and the remanufactured output q_r increases with the increase of θ. It shows that, with the increase of consumers' preference, retailers can increase the sales volume of products by lowering the price; that is, the "small profits but quick turnover" model, and, at the same time, the number of potential consumers who can expand the subsequent warranty services.

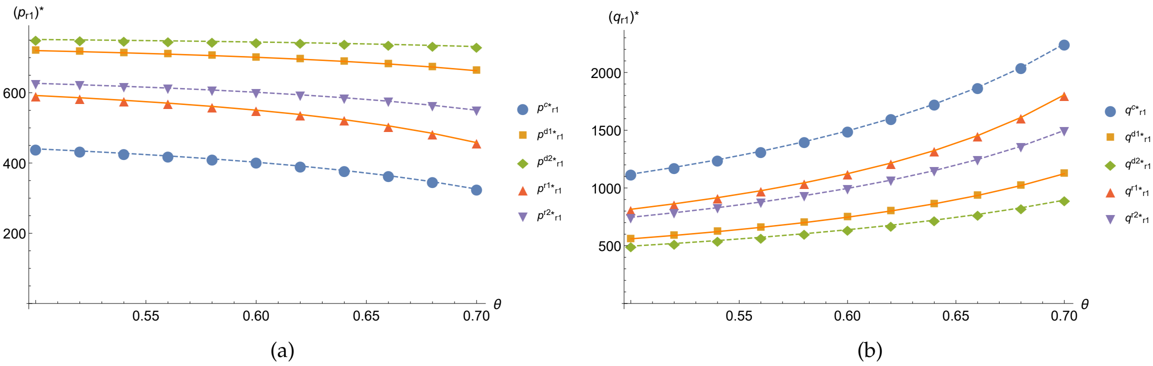

Figure 2. (**a**) the optimal retail price; (**b**) optimal yield.

(2) Under the decision of each game model, consider the comparison of consumer preference θ for remanufactured product extended service price p_s, warranty period τ and output q_s.

From Figure 3, under each decision model, the remanufactured product extension service price p_s, warranty period τ and production q_s all increase with the increase in consumer preference θ. It shows that, with the increase of consumer preference, retailers can increase the performance of extended service by attracting more consumers to choose the remanufactured product warranty service by increasing the service policy of the extended warranty period.

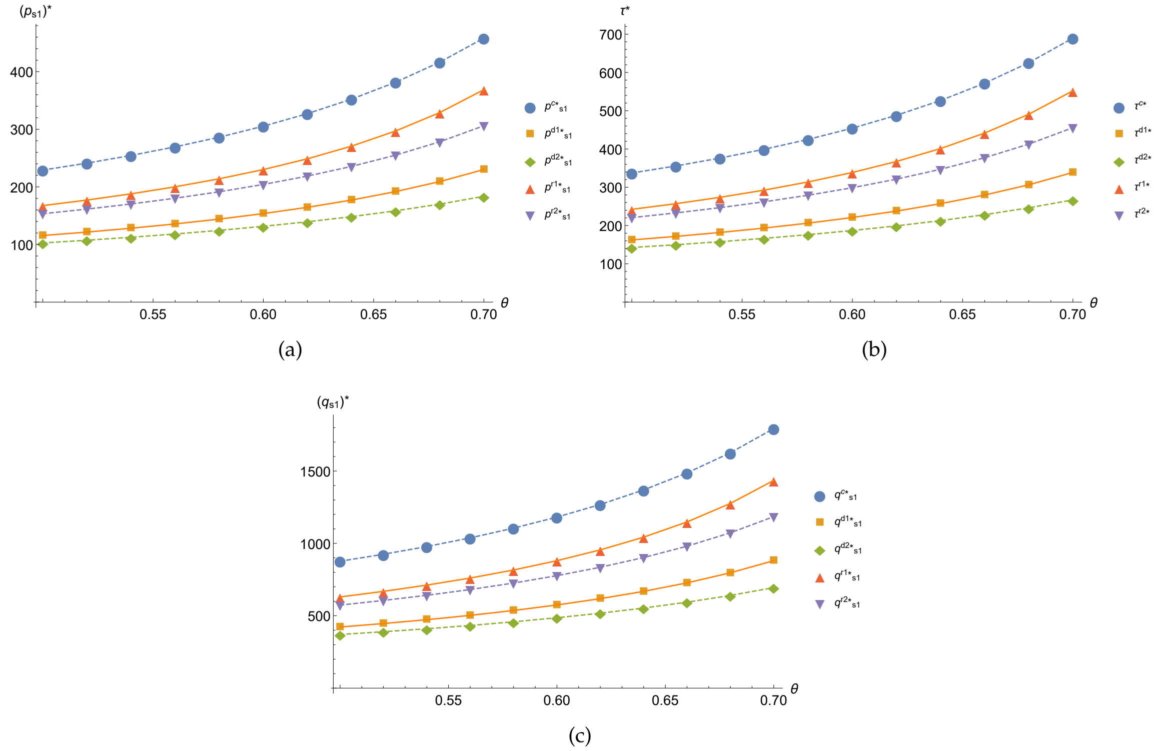

Figure 3. (**a**) optimal extended warranty price; (**b**) optimal warranty period; (**c**) optimal extended warranty sales.

(3) Under the decision of each game model, consider the consumer preference θ for the comparison of the income of each member company in the supply chain and the total profit of the supply chain system.

As shown in Table 2, firstly, as the consumer's preference θ increases, the profits of manufacturers, retailers, and supply chain systems increase. Secondly, the retailers under the R-M decision have the best profit, and, under the M-R decision, the manufacturers have the best profit, indicating that the leaders of the supply chain will gain greater profits with greater bargaining power. The system profit of M-R decision-making is greater than the system profit of R-M decision-making, indicating that the double marginal benefit of the manufacturer-led extended service supply chain is less than the retailer-oriented system. Moreover, under the R-M decision, the optimal retail price, optimal extended warranty price, optimal warranty period and production volume of remanufactured products are less than the optimal level under the M-R decision. Finally, as the degree of consumer preference increases, the gap in decision-making between models increases.

Table 2. The impact of the change in the parameter θ on the total profit of different decision models, manufacturers, retailers and closed-loop supply chains.

θ	Centralized Decision	Decentralized M-R Decision			Decentralized R-M Decision		
	Π_S^{c*}	Π_M^{d1*}	Π_R^{d1*}	$\Pi_M^{d1*}+\Pi_R^{d1*}$	Π_M^{d2*}	Π_R^{d2*}	$\Pi_M^{d2*}+\Pi_R^{d2*}$
0.3	332,462	166,150	83,237.3	249,387.3	82,566.5	153,316	235,882.5
0.4	399,130	199,484	99,904.3	299,388.3	98,889.1	181,202	280,091.1
0.5	499,263	249,550	124,938	374,488	123,181	221,503	344,684
0.6	666,501	333,169	166,747	499,916	163,062	284,884	447,946

6.2. Remanufacturing Closed-Loop Supply Chain Coordination Mechanism

Substituting the simulation data into the optimal profit results of the coordination mechanism model can reveal the optimal profit changes of manufacturers, retailers and supply chain systems under different game models. For the M-R decision model (see Table 3), the traditional revenue sharing contract cannot coordinate the supply chain system with warranty services, namely $\Pi_M^{r1*} + \Pi_R^{r1*} > \Pi_M^{d1*} + \Pi_R^{d1*}, \Pi_M^{r1*} > \Pi_M^{d1*}, \Pi_R^{r1*} < \Pi_R^{d1*}$, and through an improved revenue sharing contract the system can get Pareto improvement, i.e., $\Pi_M^{r3*} + \Pi_R^{r3*} > \Pi_M^{d1*} + \Pi_R^{d1*}, \Pi_M^{r3*} > \Pi_M^{d1*}, \Pi_R^{r3*} = \Pi_R^{d1*}$, effectively implementing supply chain coordination. For the R-M decision model (see Table 4) combined with decentralized M-R decision data, it is shown that $\Pi_M^{r3*} = \Pi_M^{d2*}, \Pi_R^{r3*} > \Pi_R^{d2*}$, indicating that the retailer is the market leader in the R-M decision model. The entire excess profit after coordination, and the manufacturer still retains the optimal profit before coordination.

Table 3. Profit comparison of the supply chain system of the M-R decision model under revenue sharing contract.

Decentralized M-R Decision			Revenue Sharing Contract			Revenue Sharing Contract (Improved)		
Π_M^{d1*}	Π_R^{d1*}	$\Pi_R^{d1*} + \Pi_M^{d1*}$	Π_M^{r1*}	Π_R^{r1*}	$\Pi_M^{r1*} + \Pi_R^{r1*}$	Π_M^{r3*}	Π_R^{r2*}	$\Pi_M^{r2*} + \Pi_R^{r2*}$
166,150	83,237.3	249,387.3	212,817	69,047.8	281,864.8	182,334	83,237.3	265,571.3
199,484	99,904.3	299,388.3	263,360	81,419.7	344,779.7	226,779	99,904.3	326,683.3
249,550	124,938	374,488	341,630	98,862.1	440,492.1	293,535	124,938	418,473
333,169	166,747	499,916	479,065	124,442	603,507	405,027	166,747	571,774

Table 4. Profit comparison of the R-M decision model supply chain system under two charge contracts.

Decentralized M-R Decision			Two Charge Contracts		
Π_M^{d2*}	Π_R^{d2*}	$\Pi_R^{d2*} + \Pi_M^{d2*}$	Π_M^{r3*}	Π_R^{r3*}	$\Pi_M^{r3*} + \Pi_R^{r3*}$
82,566.5	153,316	235,882.5	82,566.5	248,955	331,521.5
98,889.1	181,202	280,091.1	98,889.1	290,762	389,651.1
123,181	221,503	344,684	123,181	271,168	394,349
163,062	284,884	447,946	163,062	378,543	541,605

7. Conclusions

This paper considers the consumer's purchasing behavior, and analyzes the warranty decision model and coordination problem of remanufactured products under the three types of decisions: centralized, decentralized manufacturer-oriented and retailer-led. Based on the consumer behavior decision theory, the remanufactured product pricing, extended warranty service pricing, warranty period and system profit of each member of the supply chain are discussed. With strong bargaining power, the dominant players tend to earn more than their followers. For the dual marginal effects of decentralized manufacturer-led and retailer-led decision-making models, the first is to use commodity-based revenue sharing. The contract and the improved revenue sharing contract coordinate the M-R decision model, and then use the two toll system contracts to optimize the supply chain for the R-M decision model. Finally, the numerical simulation is used for further analysis.

The results show that: firstly, the increase of consumer preference can effectively increase the output of remanufactured products (i.e., demand), the extended service sales volume and the profit of each member of the supply chain. According to the different value ranges of consumer preferences, this paper derives five different optimal dynamic pricing and production strategies for manufacturers and retailers, which can provide practical guidance for corporate warranty decision-making. Secondly, the increase in consumer preference magnifies the dual margins of supply chain members. The effect of the process has aggravated the loss of decision-making efficiency. Finally, the improved revenue sharing contract and the two charging system contracts can effectively coordinate the closed-loop supply chain system with warranty services, so that the system can be improved by Pareto, which provides a decision-making reference for cooperation among member companies in the supply chain.

Several extensions to this article are possible. Firstly, this paper only considers the secondary supply chain consisting of a single manufacturer and a single retailer, and does not consider the supply chain warranty system under the market competition environment. Secondly, the three decision models only consider the supply chain. The system's warranty service is provided by the retailer, but, as the market capacity of the extended warranty service continues to expand, many large manufacturers and third-party guarantors (e.g., Safeware (Dublin, Ireland), etc.) are beginning to enter the warranty market, and the future can be studied from the perspective of the entire supply chain.

Finally, in addition to the design of the warranty entity combination model, in the actual operation, the guaranteed closed-loop supply chain system also has key warranty factors such as warranty quality, government regulation, sales efforts, etc., which will affect consumers' purchasing behavior decision-making. This will also become a further research direction of the article.

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