Alleviating Financing Constraints of SMEs through Supply Chain

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Abstract: Small and medium-sized enterprises play a crucial role in sustaining economic development in both developed economies and developing economies, however, many of them suffer from chronic and structural difficulties in accessing external financing. In this paper, we develop a theoretical framework to illustrate how information structures work in the strategic interactions between banks and firms in a supply chain, and why the transaction information in a supply chain may serve to reduce information asymmetry and improve SMEs’ access to external financing. We find that under incomplete information, the transactions between SMEs and suppliers can serve as signals for banks, which may help banks access the private information of SMEs, thus reducing information asymmetry between them. To maximize profit, banks should dynamically adjust both their interest rate policies and risk management strategies when providing financing services to SMEs, according to the structure of the financial market. The improvement of the external financial environment for SMEs may benefit the entire supply chain, thus facilitating its sustainable development and the growth of SMEs. Our framework sheds light on how SMEs in a supply chain may enhance their survivability and facilitate their development through appropriate strategies to improve business performances and manage credit risks.

Keywords: SMEs; financing constraints; supply chain; information asymmetry

JEL Classification: C72; D82; G21; O10

1. Introduction

Small and medium-sized enterprises (Henceforth, SMEs) are the backbone of economies around the world, ensuring sustainable economic growth [1], innovation, job creation [2], and social integration in both developed economies and developing economies [3,4]. In the US, SMEs make up 99% of all firms, employ over 50% of private sector employees, and create 65% of net new private sector jobs in the US. SMEs account for over half of US non-farm GDP, and represent 98% of all exporters and 34% of export revenue in the US (“State of SME Finance in the United States in 2015”, http://www.tradeupfund.com/uploads/2/6/0/4/26048023/state_of_sme_finance_in_the_united_states_2015.pdf). In the EU, 20 million European SMEs account for more than 86 million employees (66.5% of the total) and 57.6% of the gross value added generated by the private and non-financial sectors (“Entrepreneurship and Small and medium-sized enterprises (SMEs)”,

They represent 99% of all businesses in the EU and have created, during the past five years, around 85% of new jobs in the EU (“Entrepreneurship and Small and medium-sized enterprises (SMEs)”, https://ec.europa.eu/growth/smes_en). In the UK, for example, SMEs contribute up to 60% of total private sector employment. They account for about 99% of all private sector businesses and 47% of total private sector turnover [5]. According to the China Statistical Yearbook, SMEs made up about 97.9% of all registered firms in China in 2015. They contribute to more than 60% of the GDP, employ about 82% of the total employees, and create nearly 75% of the new jobs every year in China.

According to the statistics of SME sector in 76 countries by Ayyagari et al. [6], SMEs contribute 42% to a country’s gross domestic product and provide work for 54% of its labor force on average. Memili et al. [7] found a positive relationship between the economic growth and SMEs share in the market. Yazdanfar and Ohman [8] state that SMEs promote economic growth, through creating jobs and facilitating competition and innovation in an economy. The growth of SMEs increases household income and helps reduce poverty [9–11], playing a critical role in facilitating sustainable socioeconomic development.

Despite SMEs’ great importance in an economy, many of them suffer from a chronic and structural difficulty in accessing external financing [12,13], which is one of the most important obstacles to the growth of SMEs. There are two main factors contributing to the financing constraints on SMEs: information asymmetry [14–16] and a lack of trading history [17]. In this paper, we will introduce a theoretical framework to illustrate how the trading information within a supply chain may help to reduce the information asymmetry between banks and SMEs, thus alleviating the financing constraints on SMEs.

SMEs can raise funds by selling shares of equities or by applying for loans. For SMEs, however, consistent with the financial growth cycle model [16], it is more feasible to satisfy their financial needs by selling shares of equities instead of by applying for loans due to the problem of information asymmetry and other concerns. However, Reid [18] argues that additional shares of equities issued to satisfy firms’ financial needs may lead to a dilution in the ownership and the control power of the original equity holders. In addition, the tax shield benefit also encourages SMEs to rely on loans instead of other financing resources.

There has been a large number of studies on the debt financing of SMEs. Following Ayyagari et al. [19], many researchers distinguish bank finance (formal finance) from nonbank finance (informal finance). According to Apoga and Degryse et al. [20,21], banks are the main external capital providers for SMEs in both developed and developing countries. However, many SMEs have great difficulties in getting loans from banks. Beck [22] and Degryse, Lu and Ongena [23] emphasize the importance of informal finance for the development of SMEs, and they find that SMEs depend heavily on informal financial channels. According to the Federation of Small Businesses, half of the financial institutions that lend to SMEs are informal ones. In developing countries, informal financial institutions play a more important role in supporting the growth of SMEs. Both formal financial institutions and informal ones are important channels for SMEs to raise funds.

There are some mechanisms and techniques developed and adopted by banks to deal with the problem of information asymmetry between banks and SMEs, such as factoring [24] and relationship lending [25]. Many researchers examine the role of the supply chain background of SMEs in accessing bank financing. They argue that the operational status and transaction background of SMEs in a supply chain may reduce information asymmetry, helping SMEs access external financing [26,27]. These studies, mostly empirical works, provide important insights into how the supply chain background may help SMEs access external financing. Nonetheless, there are limited discussions on the strategic interactions between banks and firms in a supply chain. Moreover, studies on supply chain finance (henceforth, SCF) have not yet provided enough explanations on why and how the supply chain background can help relieve information asymmetry and improve SMEs’ access to external financing.
In this study, we develop a framework to illustrate how information structures work in the strategic interactions between banks and firms in a supply chain, and why the transaction information in a supply chain may serve to reduce information asymmetry and improve SMEs’ access to external financing. Our framework incorporates important aspects of the financing issues of SMEs in the real world, including the characteristics of SMEs and the risk control strategy of banks. It extends the boundary of SCF beyond simply financial solutions into the strategic interactions between banks and firms in a supply chain. Our framework provides an alternative perspective to deal with SMEs financing problem, based on their supply chain backgrounds. Moreover, our framework shed light on how to enhance the survivability and facilitate the development of a supply chain by adopting appropriate strategies to improve business performances and manage credit risks.

The remainder of this paper is organized as follows. Section 2 reviews the literature related to our study with regards to three aspects. Section 3 develops our theoretical framework and analyzes the interactions under complete information and incomplete information, respectively. Section 4 discusses and concludes this study.

2. Literature Review

In this section, we review the literature related to our study with regards to three aspects: SMEs and their financial constraints, information asymmetry, and supply chain finance and sustainability.

2.1. SMEs and Their Financial Constraints

The definition of SMEs is important for access to finance and support programs targeted specifically at these enterprises, however, it varies greatly across different economies. In the US, Small and medium-sized enterprises (SMEs) refer to those firms with fewer than 500 employees (“What is an SME?”, http://ec.europa.eu/growth/smes/business-friendly-environment/sme-definition_en). According to the European Commission, SMEs are defined as those enterprises having less than 250 persons employed, and having an annual turnover of up to EUR 50 million, or a balance sheet total of no more than EUR 43 million (“Entrepreneurship and Small and medium-sized enterprises (SMEs)”, https://ec.europa.eu/growth/smes_). In most OECD (Organization for Economic Cooperation and Development) economies, an SME is defined mainly through its number of employees. By contrast, its definition in China is much more complex. The categorizing is based on the SME Promotion Law of China published in 2003, and the classification of SME depends on the industry category in terms of the number of employees (the upper bound ranging from 100 to 3000), their sales (the upper bound ranging from 40 million to 400 million RMBs), and assets (the upper bound ranging from 30 million to 300 million RMBs) (“Small And Medium Enterprises (SMEs) In China” (Richard Hoffmann, 4 July 2017), https://ecovis-beijing.com/investment/smes-china/). In this paper, SMEs refer to those enterprises which have little power in affecting the market equilibrium as an individual market player.

Accessing external financing is of crucial importance for the survival and the sustainable growth of SMEs. External financing promotes the growth and development of existing businesses, facilitates the creation of new businesses and nurtures the innovations in an economy [28]. Both formal financial institutions and informal ones are important channels for SMEs to raise funds. However, compared with large enterprises, SMEs face greater difficulties in accessing external financing due to the many inherent features of SMEs, such as an information asymmetry [16], a lack of trading history [17], a high risk of failure [29], and a low portion of tangible assets in their total assets [30]. Many studies find that it is more difficult for SMEs to access external financing than larger enterprises, both in developing countries and developed countries [31,32]. The lack of capital is one of the most important obstacles to the growth of SMEs and most SMEs still suffer from chronic and structural difficulties in accessing external financing [12,13].
2.2. Information Asymmetry

In the area of financing services, information asymmetry refers to the disparity between the information available to businesses seeking capital and suppliers of capital who are typically assumed to be at an informational disadvantage with respect to the insiders of businesses [33,34]. Information asymmetry often results in “lemon market” and can potentially lead to a breakdown in the functioning of the capital market [35]. As argued by Spence [36], ex-ante information asymmetry will lead to adverse selection, while ex-post information asymmetry will result in moral hazards. There are two important solutions to deal with information asymmetry. One is to find the optimal contract that provides incentives for the full disclosure of private information, thus mitigating the problem of adverse selections. Another one is to work out the regulations that require information intermediaries with private information, such as government and rating agencies, to publicize certain information. Numerous previous studies have investigated the major causes of information asymmetry in different circumstances and provided a series of solutions for specific problems of adverse selection or moral hazards [37,38].

The principal-agent framework has been widely adopted to deal with adverse selection and moral hazards resulting from information asymmetry. Wilson [39], Spence and Zeckhauser [40], and Ross [41] introduced the state-space formulation, which provides a critical foundation for the principal-agent framework. Mirrlees [42] and Holmstrom [43] developed the parameterized distribution formulation and introduced a formal principal-agent framework. Jensen and Meckling [44] applied the principal-agent framework in analyzing the mechanisms to deal with interest conflicts between different parties. Their work greatly promotes the application of the principal-agent framework in various areas, including financial studies [45], management science [46], political science [47] and sociology [48]. Dai et al. [49] argued that almost all information asymmetry problems can be analyzed by the principal-agent framework.

Stiglitz and Weiss [33] developed a theoretical framework to analyze the credit rationing issues in markets with imperfect information, which later became the workhorse framework for many researchers studying the credit rationing issues for SMEs. Most researchers agree that information asymmetry is a major factor leading to financing difficulties for SMEs. Forkuoh et al. [50] argued that it is hard for SMEs to access external finances due to information asymmetry and their inability to credibly signal the external financial institutions their real capacity to repay debts. Minard [51] and Nanyondo et al. [52], through their empirical studies, found that when SMEs can credibly signal unobserved firm quality to the external financial institutions, this will reduce information asymmetry and improve their access to finance.

Berger and Udell [53] discussed the channels and mechanisms of a supply chain that can be used to solve the problem of information asymmetry between banks and SMEs. Later, many researchers analyze how to use the operational and transaction information of SMEs in a supply chain to reduce information asymmetry between SMEs and financial institutions, thus alleviating the financing constraints on SMEs [27,54]. However, there are limited theoretical discussions on how the transaction information of a supply chain can be utilized by external financial institutions, to design the optimal credit policies for SMEs. In this paper, we will address this issue in our theoretical framework.

2.3. Supply Chain Finance and Sustainability

Another branch of related literature focuses on Supply Chain Finance (Henceforth SCF). SCF is introduced to increase financing efficiencies and decrease costs [55]. Although the history of SCF research can be traced back to the 1970s [56], the formal definition of SCF was first proposed by Stemmler in 2002 [57]. Stemmler stated that SCF is an essential part of supply chain management, and its primary characteristic is the integration of financial flows into the physical supply chain. According to Pfohl and Gomn [58], SCF refers to inter-company optimization of financing and the integration of financing processes with customers, suppliers, and service providers, to increase the value of all participating companies. Most of the previous studies on supply chain finance can be categorized
into two classes based on their research perspectives. One class is supply chain-oriented studies, focusing on the impact of special forms of financing mechanisms on operational decisions of the supply chain [59,60]. The other class is finance-oriented studies, emphasizing the financial aspects and exploring the roles of financing service providers in the supply chain [59,61].

Many supply chain-oriented studies analyze the impact of supply chain finance on decisions of the classical inventory model [60,62]. They are all based on the classical economic order quantity (EOQ) or economic production quantity (EPQ) models and examine different forms of supply chain finance policies, such as permissible delays in payment, discount cash flows effect on the inventory decisions, uncertain opportunity and risky storage [63,64]. Some researchers extend the classical EOQ or EPQ models by incorporating new elements into the model, such as introducing more complex scenarios or relaxing existing assumptions [65,66]. For example, Mahata and Goswami [67] considered holding costs, ordering costs, and purchasing costs as fuzzy numbers in the EOQ model. In Chang et al. [68], the EPQ model is discussed in the circumstance of two-level trade credit instead of one-level trade credit. These extensions bring the study on SCF into a broader scope.

The finance-oriented studies investigate different mechanisms of supply chain financing services, such as trade credit, reverse factoring, and also discuss how supply contracts vary with these different mechanisms [69,70]. A series of comparative analyses were conducted by a group of researchers, including Chen [71] and Andrieu et al. [72]. Fabbri and Klapper [73] investigated the impact of supplier’s bargaining power on SCF solution using a firm-level database of Chinese firms. Caniato et al. [74] provided a case study of Italian firms to show the effects of different factors on the adoption of various SCF solutions.

Some articles in the literature focused on the issues related to sustainable SCF. Tseng et al. [75] argued that SCF and its sustainable development require a deeper discussion to address the theoretical and managerial gaps. They develop a model of sustainable supply chain finance under uncertainty to identify the existing problems and deficiencies of financing patterns. Lozano [76] illustrated the gap between social and economic dimensions in sustainable SCF and proposed a dynamic framework to fill this gap and help to make appropriate SCF decisions. As a matter of fact, the effective working capital management is, to a large extent, a critical determinant in the sustainability of SCF [77,78]. For the entire supply chain, the weakest links of working capital management are often the SME client firms of the supplier [79]. Kot [80] found that all the sustainability areas are very important in the supply chain management practices of SMEs. Randall and Farris [81] argued that SCF bring new opportunities for all firms in the supply chain, especially for SMEs, which is supported by Nienhuis et al. [82].

SCF facilitates financing supports to the members of a supply chain, especially for SMEs, promoting their development and improving the sustainability of a supply chain [83,84]. However, there are a few articles in the literature exploring why the operational status and the transaction background of a supply chain can entitle SMEs new financing opportunities, based on a theoretical framework. In this paper, we are going to address this issue in our theoretical model.

3. Theoretical Framework

We introduce a theoretical framework to illustrate the interactions between the four players in an economy: the supplier, the SME (the retailer), the bank and the informal financial institution (Henceforth, IFI). The supplier and the SME are in the same supply chain of a final good. The SME relies on both the bank and the IFI in their debt financing, therefore, in order to explore the mechanism of SCF, we incorporate both the bank and the IFI in our framework.

3.1. The Basic Settings

The SME faces a competitive final market, and there is a constant market price \( p > 0 \), for the final good. In this paper, the final good market is described as a “newsboy” market [85], in which the SME is a price taker. The SME is either an \( H \) type or \( L \) type based on its stochastic market demand.
Assumption 1 (A1): The stochastic market demand for type \( i, i \in \{H, L\} \), is \( \varepsilon / \theta_i \), where \( \varepsilon \sim U[0, a] \) and it is a random value with a distribution function \( F(x) \) and a density function \( f(x), \theta_i \in \{ \theta_H, \theta_L \} \), and \( \theta_L > \theta_H > 0 \).

Here, \( \theta_L > \theta_H > 0 \) implies the distribution of market demand on the final good of \( H \) type SME is always greater than that of \( L \) type SME [86].

\[
\frac{\varepsilon}{\theta_H} \overset{\text{FSD}}{\geq} \frac{\varepsilon}{\theta_L}
\]

where, FDS means the first-order stochastic dominance (FSD). The above inequality implies that under the market disturbance \( \varepsilon \), the \( H \) type SME always has a higher market demand than \( L \) type one.

The interactions between the supplier and the SME is described as a Stackelberg game, in which the supplier is a leader, while the SME is a follower. First, the supplier sets a wholesale price \( w_i \) for the type \( i \) SME, where \( i \in \{H, L\} \). After observing \( w_i \), the type \( i \) SME sets its order quantity \( q_i \) to maximize its profit. The unit cost of the supplier to produce for the SME is denoted by \( c \).

Assumption 2 (A2): The supplier has complete information on the SME’s type, because of the longstanding collaborative relationship between supplier and the SME [87].

This assumption implies that the supplier has an information advantage over external financial institutions, regarding the type of the SME. Without affecting the main analytical results, we assume that all the working capital of the SME comes from external financing sources.

Assumption 3 (A3): The bank can only provide \( K > 0 \) units of the loan at an interest rate \( r, 0 < r < 1 \), which is less than the working capital required by the SMEs.

This is to capture the notion that SMEs usually have very limited fixed assets that can be used as collaterals, hence, the bank is often unwilling to meet all the financing needs of SMEs. A smaller \( K \) represents a less credit authorization for SMEs. \( K \) can also be regarded as a proxy for the SME’s financing environments. A larger \( K \) implies a more favorable financial market for the SME, while a smaller \( K \) means a less desirable financial environment. The SME faces the same \( K \), despite its type, while \( r \) may vary with its type. \( r_i \) represents the interest rate on the SME of type \( i \), where \( i \in \{H, L\} \). In order to fill the gap between the loans and the working capital required, SMEs have to resort to the IFI for more loans [61]. We denote the financial cost from the IFI as \( \lambda \), and assume that \( r \ll \lambda \). This is to capture the notion that the financing cost to borrow from the IFI is much greater than that to borrow from the bank [88]. We further assume that \( c(1 + \lambda) < w_i(1 + \lambda) < p \). Here, \( c(1 + \lambda) < w_i(1 + \lambda) \) implies that the informal financing would not destabilize the supply chain, and \( w_i(1 + \lambda) < p \) means that there are enough incentives for the SME to borrow from the IFI.

The downside risk strategy is adopted by banks to control the credit risk of the SME. The parameter \( \beta, 0 < \beta < 1 \) represents the maximum ruin probability that the bank can bear, and a smaller \( \beta \) implies a tighter control strategy. Without affecting the main analytical results, the cost of financial institutions is normalized to zero. Here, we assume that unlike the bank, the IFI has no effective strategy to control the credit risk.

Assumption 4 (A4): All the players are independent entities and there is no collusion between the SME and the supplier against the bank.

Given this assumption, we can focus on the mechanism of a sustainable SCF that may help to alleviate the financing constraints of SMEs.

Figure 1 shows the timeline of interactions between the supplier, the SME, and the bank.
while it will declare bankruptcy if

\[ K \in \{ H, L \} \]

its expected profit can be expressed as

\[ \pi_i = p \int_{0}^{q_i} \frac{x}{\theta_i} f(x) dx + p q_i \int_{q_i}^{\theta_i} \frac{x}{\theta_i} f(x) dx - (K(1 + r_i) + (w_i q_i - K)(1 + \lambda))(1 - F(e_i^0)) \]
From Equation (4), we can derive

$$\pi_i^* = pq_i[1 + F(q_i\theta_i) - F(\epsilon_i)] - \frac{P}{\theta_i}[\epsilon_i + \int_{\epsilon_i}^{q_i\theta_i} F(x)dx]$$ (5)

For the supplier, it does not bear any risk caused by the fluctuation in the final good market, therefore, it is clear that the expected profit of the supplier is given by

$$\pi_i^* = (w_i - c)q_i$$ (6)

Since the supplier knows the SME's type, we can use the converse computing algorithm to solve this game. The response function of the type i SME, given the wholesale price $w_i$, can be obtained by the first-order condition (FOC) of Equation (5), $\partial \pi_i^* / \partial q_i = 0$, which gives

$$pa(1 - p) + K\theta_i(r_i - \lambda)\sqrt{p - w_i(1 + \lambda)} = q_i\theta_i[p + w_i(1 + \lambda)]^2$$ (7)

It is easy to show that

$$q_i = \frac{pa(1 - p) + K\theta_i(r_i - \lambda)\sqrt{p - w_i(1 + \lambda)}}{\theta_i[p + w_i(1 + \lambda)]^2}$$ (8)

Based on the above response function of the type i SME, the supplier sets the wholesale price $w_i$ to maximize its expected profit. By substituting Equation (8) into the supplier's profit-maximizing FOC, $\partial \pi_i^* / \partial w_i = 0$, we can obtain:

$$w_i^* = \frac{(p + 2c\phi)(p\xi_i + \delta)}{\phi(2p\xi_i + 2c\phi\xi_i + \delta)}$$ (9)

where $\phi = 1 + \lambda$, $\xi_i = K\theta_i(r_i - \lambda) < 0$, and $\delta = pa(1 - pa)$. By substituting Equation (9) into Equation (8), we obtain:

$$q_i^* = \frac{(2p\xi_i + 2c\phi\xi_i + \delta)(p\xi_i + \delta)^2}{(2p^2\xi_i + 2p\delta + 4cp\phi\xi_i + 2c\phi\delta)^2}$$ (10)

Here, $w_i^*$ represents the optimal wholesale price of supplier and $q_i^*$ stands for the optimal order quantity of type i SME in equilibrium. Unlike those traditional Stackelberg supply chain frameworks [89], the equilibrium order quantity and the wholesale price in our framework incorporate the impacts of external financial institutions. Since $\partial \phi / \partial r_i = 0$, $\partial \delta / \partial r_i = 0$, $\partial \xi_i / \partial r_i = K\theta_i$ and $\xi_i < 0$, it is easy to show that $\partial w_i^* / \partial r_i > 0$, $\partial q_i^* / \partial r_i < 0$.

**Theorem 1:** If the bank increases its interest rate $r_i$, where $i = H, L$, the supplier will jack up its optimal wholesale price $w_i^*$, and both types of SMEs will consequently reduce their optimal order quantity $q_i^*$.

Theorem 1 shows that the interest rate for the SME to borrow from a bank has a significant impact on the operational status and transactions in a supply chain. This result is consistent with those in Tang and Musa [90]. In our framework, the rising the financing costs of the SME may directly lower its optimal order quantity, disregarding its type. The rationale is that an increase in the financing cost for the SME raises its risk of bankruptcy, hence, a more conservative ordering strategy would be adopted by the SME to deal with the increased risk of bankruptcy. As a result, facing a decrease in order quantity, the supplier increases its wholesale price to maximize its profit.

Based on Equation (9) and Equation (10), it is easy to show that $\partial w_i^* / \partial K > 0$ and $\partial q_i^* / \partial K < 0$. This indicates that an improvement in the financing environment for the SME not only improves the unit profit margin of the supplier, but also reduces the SME’s order quantity. Therefore, an improvement in the financing environment results in two different effects for the entire supply chain: “profit effect”
and “risk effect”. Profit effect means that an improvement in the financing environment for the SME increases the wholesale price set by the supplier, while at the same time, decreases the financing cost for the SME, which enhances the total profit of the entire supply chain. While the risk effect implies that an improvement in the financing environment lowers the order quantity of the SME, which reduces the total profit of the entire supply chain. A numerical analysis is conducted in our study to investigate the relationship between the external financial environment and the benefits of the entire supply chain. Figure 2 shows this relationship.

\[ q^*_j = p a (1 - p) + K \theta_j (r_j - \lambda) [p - w_j (1 + \lambda)] + \theta_j [p + w_j (1 + \lambda)]^2 \]

Figure 2. The relationship between the financing environment and the total profit of the supply chain.

From Figure 2, we can find that there is a non-linear positive relationship between the financing environment and the total profit of the supply chain. The positive profit effect always dominates the negative risk effect when there is an improvement in the financing environment, hence, their aggregate effect on the total profit of the supply chain is positive. However, we can also observe that this aggregate positive effect becomes weaker and weaker beyond a certain level of the financing environment. The above analysis indicates that an improvement in the financing environment for SMEs facilitates the development of the supply chain, which is supported by Beck et al. [91].

Given the downside risk management strategy, the bank selects an interest rate to maximize its expected profit, based on its expectation of the credit risk of the SME. Under complete information, the bank knows the type of the SME, hence, its optimization problem can be represented as

\[
\max_{r_i} K (1 + r_i) \\
\text{s.t.} \quad \Pr [K (1 + r_i) \geq \frac{p a}{\theta_i}] \leq \beta
\]

By solving the above optimization problem, we obtain

\[
r^*_i = \frac{a p \beta}{K \theta_i} - 1
\]

Under complete information, the bank chooses the optimal interest rate \( r^*_i \) for the type \( i \) SME in equilibrium. Recall: \( \theta_L > \theta_H > 0 \), hence, given Equation (12), it is obvious \( r^*_L < r^*_H \), i.e., the optimal interest rate on type \( H \) SME is higher than that for type \( L \) SME.

**Theorem 2**: Given the same interest rate, the type \( L \) SME has higher credit risk, compared to the type \( H \) SME.

**Proof**: Given the same interest rate \( r \), the default probability of the SME of type \( i, i \in \{ H, L \} \) can be obtained as

\[
\Pr [\varepsilon \leq \frac{K \theta_i (1 + r)}{p}]
\]
Following Assumption 1, \( \theta_H < \theta_L \) implies:

\[
\Pr \left[ \epsilon \leq \frac{K\theta_H(1+r)}{p} \right] < \Pr \left[ \epsilon \leq \frac{K\theta_L(1+r)}{p} \right]
\]  

which proves Theorem 2.

Given the first order stochastic dominance condition of the market demand, the type \( H \) SME, compare with the type \( L \) SME, is less vulnerable to the fluctuation in the market demand. Therefore, compared with the type \( L \) SME, the type \( H \) SME has lower default risk under the same interest rate. According to Lafferty and Goldsmith [92], the credit risk of enterprises would affect their market performance. However, in this paper, we find the opposite causal relationship—the market performance of the enterprises would affect their credit risk.

**Corollary 1:** Compared with the type \( L \) SME, there are a higher order quantity and a lower wholesale price for the type \( H \) SME in equilibrium.

Proof: According to the optimal wholesale price in equilibrium, shown in Equation (9), we can obtain:

\[
\frac{\omega_H^*}{\omega_L^*} = \frac{(p\xi_H + \delta)(2p\xi_L + 2c\phi\xi_L + \delta)}{(p\xi_L + \delta)(2p\xi_H + 2c\phi\xi_H + \delta)}
\]

Given \( \theta_H < \theta_L \), (12) and (15), we can derive that \( \xi_L < \xi_H \), and \( w_H^* < w_L^* \). Substituting \( \xi = K\theta(r - \lambda) \) into (10), we can obtain:

\[
q_i^* = \frac{(4p\beta N - 2p\beta L + \delta)(2p^2 \beta + \delta - \psi)}{(2p^2 \beta M + 2\delta N - pM\psi)^2}
\]

where \( N = p + c\beta, M = 3p + 4c\beta \), and \( \psi = K(1 + \lambda)\theta_i \).

The parameter \( \psi \) represents the risk-adjusted substitution cost to borrow from the IFI, and it is obvious that the optimal order quantity \( q_i^* \) varies with the type of the SME. Hence, we can denote \( q_i^* \) as \( q^*(\theta_i) \). Since \( \partial q^*(\theta_i)/\partial \theta_i > 0 \) and \( \partial q^*(\theta_i)/\partial \psi_i < 0 \), given Equation (16), it is easy to show that \( \partial q^*(\theta_i)/\partial \theta_i < 0 \). Given \( \theta_H < \theta_L \), it is clear that \( q_H^* > q_L^* \).

The proof of Corollary 1 is completed.

Substituting Equation (9) into Equation (5) and Equation (10) into Equation (6), given \( \xi_H > \xi_L \) and \( \theta_H < \theta_L \), we can derive: \( \pi_H^* > \pi_L^* \) and \( \pi_H^* < \pi_L^* \). These results indicate that the supplier can get a higher profit from the transaction with the type \( H \) SME than that with the type \( L \) SME. The rationale is that even though the supplier gets a lower unit profit margin from the transaction with the type \( H \) SME, the order quantity from the type \( H \) SME is much higher. The positive effect of the higher order quantity on the total profit dominates the negative effect of the low unit profit margin.

### 3.3. Interactions under Incomplete Information

Under complete information, as shown in Section 3.1, the bank can identify the type of the SME and introduce a corresponding interest rate to maximize its expected profit, given the downside risk management strategy. Nevertheless, the type of a specific SME is usually hard to be observed by banks directly due to information asymmetry between the bank and the SME. In this subsection, we are going to focus on the case when there is incomplete information.

Under incomplete information, we assume that the supplier knows the type of the SME because of the longstanding cooperation between them, but the bank cannot observe the type of the SME directly due to information asymmetry. For the bank, the prior probability of the SME being type \( i, i \in \{ H, L \} \) is denoted by \( \mu_i \), where \( \mu_H + \mu_L = 1 \), and \( \mu_H > \mu_L \geq 0 \). We further assume that the bank can observe \( \omega \) set by the supplier, however, it is too costly for the bank to know the \( q \) chosen by the SME.
Under incomplete information, the following is the stage game between the supplier, the SME and the bank:

Stage 1: Nature decides the type of the SME, either type H or type L.
Stage 2: After observing the type of the SME, the supplier decides its wholesale price \( w \) for the type \( i \) SME, where \( i \in \{ H, L \} \).
Stage 3: After observing \( w \), the type \( i \) SME, where \( i \in \{ H, L \} \), decides its order quantity \( q_i \).
Stage 4: After observing \( w \), the bank selects \( r \) for the SME.

We use \( a(i|w) \) to represent the adjusted probability of the SME being type \( i \), \( i \in \{ H, L \} \) after the wholesale price \( w \) is observed by the bank, and \( \rho_i(w) \) denotes the probability of the supplier to set the wholesale price \( w \) for the SME when its type is \( i \), \( i \in \{ H, L \} \). The following equation represents the Bayesian updating process for the bank with regards to the type of the SME after observing \( w \):

\[
a(i|w) = \frac{\rho_i(w)\mu_i}{\sum_{j=H,L} \rho_j(w)\mu_j}
\]

(17)

The wholesale price can be seen as a signal of the order quantity to the bank. Through this signal, the bank forms an updated belief on the order quantity and the corresponding type of the SME. Recall that \( r^*_L < r^*_H \) under complete information, i.e., the optimal interest rate chosen by the bank for the type \( H \) SME is higher than that for the type \( L \) SME. Since the bank will always set a higher interesting rate when the SME is identified as type \( H \), there is an incentive for the type \( H \) SME to hide its type. Therefore, the bank may adopt the following mixed strategy to set interest rate:

\[
r = r^*_L a(L|w) + r^*_H a(H|w)
\]

(18)

Given the interest rate \( r \), from Equation (8), we can express the order quantity as a function of the wholesale price for the SME of type \( i \), \( i \in \{ H, L \} \):

\[
q = \frac{pa(1-p) + K\theta_i(r - \lambda)[p - w(1 + \lambda)]}{\theta_i[p + w(1 + \lambda)]^2}
\]

(19)

Since \( w > 0 \), it is easy to show that \( \partial q / \partial w < 0 \).

Lemma 1: When \( a - ap(1 - 2\beta) > 2K\theta_i\phi \), the wholesale price is uniquely determined by the order quantity chosen by the SME under a given interest rate \( r \) selected by the bank. There is a negative relationship with the wholesale price \( w \) and the order quantity \( q \).

Here, \( K\theta_i\phi \) can be seen as the risk-adjusted substitution cost to borrow from the IFI, and the inequality \( a - ap(1 - 2\beta) > 2K\theta_i\phi \) can be treated as a structural constraint on the financial market, which is necessary to derive the pooling Bayesian subgame perfect equilibrium for the stage game under incomplete information. Here, we assume that \( a - ap(1 - 2\beta) > 2K\theta_i\phi \).

The above analysis shows that the wholesale price for a specific type of SME can be seen as a function of the interest rate, the type of this SME, and the order quantity. For this reason, the wholesale price for a specific type SME can be denoted by \( w(r; \theta_i; q) \) when the substitution cost of informal financial institutions is small enough. It is clear that the wholesale price \( w(r; \theta_i; q) \) is strictly monotonically decreasing in the order quantity \( q \), i.e., \( \partial w(r; \theta_i; q) / \partial q < 0 \).

Then we consider the incentive constraint for the type \( H \) SME. If type \( H \) SME pretends to be type \( L \), the wholesale price and order quantity in equilibrium can be derived from the Stackelberg framework since the supplier knows the type of the SME. In this case, we have:

\[
\hat{w}_H = \frac{(p + 2c\phi)\delta_H + (p + 3c\phi)p\theta_H\tau_L}{\phi\delta_H + 3(p - c\phi)\phi\theta_H\tau_L}
\]

(20)
There is an order quantity \( \tilde{q}_{HI} \) given by:

\[
\tilde{q}_{HI} = \frac{(p + 2c\phi)\delta{\theta}_L + (p + 3c\phi)p{\theta}_H{\tau}_L}{\theta_H[(p + p\phi + 2c\phi)\delta{\theta}_L + (p + 3p\phi - 3c\lambda\phi)p{\theta}_H{\tau}_L]^2}
\]  \hspace{1cm} (21)

where \( \tau_L = a\beta - K\phi{\theta}_L \). From Equation (21) and Equation (10), it is easy to show that

\[\tilde{q}_{HI} > q_H\]  \hspace{1cm} (22)

**Theorem 3:** There is an order quantity \( \tilde{q} < \tilde{q}_{HI} \) such that the expected profit for the type \( H \) SME is the same, no matter the interest rate \( r^*_H \) or \( r^*_L \).

Proof: According to Theorem 1, given the same interest rate, the type \( H \) SME gets a higher profit, compared to the type \( L \) SME. Suppose the interest rate is \( r = r^*_L \) for both types of SMEs, we have: \( \pi'_H(q_H; \hat{w}; r^*_L) > \pi'_L(q_L; w_L; r^*_L) \), where \( \pi'_H(q_H; \hat{w}; r^*_L) \) and \( \pi'_L(q_L; w_L; r^*_L) \) represent the maximum profit for the type \( H \) SME and type \( L \) SME, respectively, given the same interest rate \( r^*_L \).

According to Corollary 1, under complete information, the profit for the type \( L \) SME is greater than that for the type \( H \) SME in equilibrium, i.e., \( \pi'_H(q_H; \hat{w}; r^*_L) > \pi'_L(q_H; w_H; r^*_L) \). Therefore, we can infer \( \pi'_H(q_H; \hat{w}; r^*_L) > \pi'_L(q_H; w_H; r^*_L) \). In addition, since \( \pi'_H \) is a continuous function of the order quantity, it is obvious that when the order quantity is 0, we have \( \pi'_H(0; w_H; r^*_L) = 0 \). Hence, according to Roll’s Lemma, there must be an order quantity \( q \) such that \( \pi'_H(q_H; \hat{w}; r^*_L) = \pi'_L(q_H; w_H; r^*_L) \).

The proof of Theorem 3 is completed.

Based on Theorem 3, we can infer that under incomplete information, if the type \( H \) SME deviates from the order quantity \( \tilde{q}_{HI} \) and chooses a lower quantity, the supplier will raise the wholesale price to maximize its profit. There are two different effects on the SME resulting from the adjustment in the wholesale price by the supplier. One effect is that this adjustment reduces the SMEs’ profit from selling its final goods, because of a decrease in the order quantity and an increase in its unit cost. Another effect is that this adjustment serves as a signal for the bank, and induces the bank to select a lower interest rate for the type \( H \) SME, thus lower its financial cost to borrow from the bank. According to Theorem 3, their aggregate effect improves the overall profit position of the SME. Therefore, there is an incentive for the type \( H \) SME to pretend to be type \( L \). Figure 3 illustrates the relationship between the profit of the type \( H \) SME and its order quantity.

![Figure 3. The order quantity and margins.](image)

**Corollary 2:** When the condition \( \tilde{q} < q_L \) holds, there is a separating Perfect Bayesian Equilibrium given by:

\[
\alpha(H|w) = 1, \text{ if } w > \hat{w}(r^*_L; \theta_H; \tilde{q}) \\
\alpha(L|w) = 1, \text{ if } w \leq \hat{w}(r^*_L; \theta_H; \tilde{q})
\]  \hspace{1cm} (23)

Corollary 2 shows that when \( \tilde{q} < q_L \), there is a separating Perfect Bayesian Equilibrium (henceforth PBE) depending on the threshold value of \( \hat{w}(r^*_L; \theta_H; \tilde{q}) \). If the bank observes a wholesale price \( w > \hat{w}(r^*_L; \theta_H; \tilde{q}) \), then the SME is identified as type \( H \) and the bank selects the interest rate \( r^*_H \) for the
SME. However, if the bank observes a wholesale price \( w \leq w(r_H^i; \theta_H; \tilde{q}) \), then the SME is identified as type \( L \) and the bank chooses the interest rate \( r_H^i \). In a multi-period equilibrium, this interest rate policy adopted by the bank will finally induce the type \( i \) SME to choose the order quantity \( q_i \), \( i \in \{ H, L \} \) in the long run equilibrium. When \( \rho_{1i}[w(q_i)] = 1, i \in \{ H, L \} \), there may exist other separating PBEs, according to Cho and Kreps [93]. When \( \tilde{q} < q_L \), there may exist pooling PBEs, i.e., the SME may always choose the order quantity \( q_L \), regardless of its type. Based on Lemma 1, the homogeneously adjusted wholesale price \( w(q_L) \) is observed by the bank. Denoting \( a(i|w) = \mu_i, i \in \{ H, L \} \), the bank will adopt a mixed strategy for their interest rate policy, \( r = \mu_i r_L^i + \mu_H r_H^i \). We must note that there is no incentive for the type \( L \) SME to deviate from the order quantity \( q_L \).

According to Figure 3, when \( \tilde{q} < q_L \), the order quantity of the type \( H \) SME is higher than that of the type \( L \) one. In our framework, corollary 2 implies that there exists a separating PBE if and only if the type \( H \) SME’s business performance is much better than that of the type \( L \) one.

From the perspective of the bank, the problem is whether the bank can distinguish the type \( i \) SME from the type \( L \) one through differentiated interest rate policies. In other words, it means whether the incentive constraint of the SME can be satisfied. According to the Spence–Mirrlees condition proposed by Spence [38], we can derive the following theorem:

**Theorem 4**: The bank can identify the SME’s type through differentiated interest rate policies: \( \pi_i, i \in \{ H, L \} \), when the following inequality holds:

\[
\zeta_i + q_i w_i \theta_i \phi > \zeta_H + \tilde{q}_i \bar{w}_i \theta_H \phi
\]

where \( \zeta_i = K \theta_i (\bar{T}_i - \lambda), \zeta_H = K \theta_H (\bar{T}_H - \lambda), i, j \in \{ H, L \} \) and \( i \neq j \).

\((q_i, w_i)\) and \((\tilde{q}_i, \bar{w}_i)\) represent the optimal response of the order quantity and the wholesale price corresponding to interest rate \( \pi_i \) and \( \pi_H \) respectively.

**Proof**: According to the Stackelberg framework in our study, we have

\[
q_i = \frac{(2p \zeta_i + 2c \phi \zeta_i + \delta)(p \zeta_i + \delta)^2}{(3p^2 \zeta_i + 2p \delta + 4c p \phi \zeta_i + 2c \phi \delta)^2}
\]

\[
w_i = \frac{(p + 2c \phi)(p \zeta_i + \delta)}{\phi(2p \zeta_i + 2c \phi \zeta_i + \delta)}
\]

\[
\bar{\pi}_i = \frac{(2p \zeta_i + 2c \phi \zeta_i + \delta)(p \zeta_i + \delta)^2}{(3p^2 \zeta_i + 2p \delta + 4c p \phi \zeta_i + 2c \phi \delta)^2}
\]

\[
\bar{w}_i = \frac{(p + 2c \phi)(p \zeta_i + \delta)}{\phi(2p \zeta_i + 2c \phi \zeta_i + \delta)}
\]

By substituting the above four equations into Equation (5), we can derive the profit of the type \( i \) SME given interest rates \( \pi_i \) and \( \pi_H \), respectively. We further denote the profit of the type \( i \) SME’s given interest rate \( \pi_i \) as \( \pi_i'(q_i; w_i; \pi_i) \) and the given interest rate \( \pi_H \) as \( \pi_i'(\bar{\pi}_i; \bar{w}_i; \pi_H) \). If Inequality (24) holds, the following incentive constraint for the SME is satisfied.

\[
\pi_i'(q_i; w_i; \pi_i) > \pi_i'(q_i; w_i; \pi_H)
\]

Given the above constraint, there is no incentive for the type \( H \) SME or the type \( L \) SME to pretend to be the other type. The SME will reveal its true type, given the interest rate policy \( (\bar{\pi}_H, \bar{T}_H) \) by the bank.

The proof of Theorem 4 is completed.

Theorem 4 can be seen as a refinement of Corollary 2. The condition \( \zeta_i + q_i w_i \theta_i \phi > \zeta_H + \tilde{q}_i \bar{w}_i \theta_H \phi \) in Equation (24) is a stronger constraint compared to the condition \( \tilde{q} < q_L \). It rules out the possibility of a pooling equilibrium and limits the separating PBE to the unique and optimal solution for the
Theorem 4 indicates that when $\zeta_i + q_i w_i \theta_i \varphi > \zeta_i + q_i w_i \theta_i \varphi$, the bank can use differentiated interest rate policies to identify the SME’s type. Nevertheless, the interest rates policy that helps to identify the SME’s may not necessarily be the global optimal solution for the bank (recall the payoff maximization problem for the bank in Equation (11)). In spite of this, Theorem 4 shows that there is at least a locally optimal solution for banks in deciding their interest rate policies under incomplete information. If banks have access to the information of the operational status and transactions in a supply chain, this may help reduce information asymmetry between banks and SMEs. Consequently, SMEs will have more access to external financing, thus alleviating the financing constraints on them.

4. Discussion and Conclusions

SMEs are the backbone of economies around the world, ensuring sustainable economic growth, innovation, job creation, and social integration in both developed economies and developing economies. Despite their great importance for an economy, many of them suffer from a chronic and structural difficulty in accessing external financing, which greatly hinders their development. There are two main factors contributing to the financing constraints on SMEs: information asymmetry and a lack of trading history. Many researchers examine the role of the supply chain background of SMEs accessing to bank financing and point out that the operational status and transaction background of SMEs in a supply chain may reduce information asymmetry, helping SMEs access external financing. These studies provide important insights into how the supply chain background may help SMEs access external financing. However, there are limited discussions on the strategic interactions between banks and firms in a supply chain. Moreover, studies on supply chain finance (henceforth, SCF) have not yet provided enough explanations on why and how the supply chain background can help relieve information asymmetry and improve SMEs’ access to external financing.

In this paper, we develop a theoretical framework to illustrate how information the structure works in the strategic interactions between banks and firms in a supply chain, and why the transaction information in a supply chain may serve to reduce information asymmetry and improve SMEs’ access to external financing. In our framework, we treat the bank as an external financing source for the SME in a supply chain, exploring the optimal interest rate policies for the bank with a downside risk management strategy under complete information and incomplete information, respectively. There are some important findings, based on the analytical results of our framework.

Firstly, consistent with the findings in many empirical studies, the improvement of the external financial environment for SMEs may benefit the entire supply chain, thus facilitating its sustainable development. Therefore, an improvement in the financing environment results in two different effects for the entire supply chain: “profit effect” and “risk effect”. Profit effect enhances the total profit of the entire supply chain, but risk effect reduces its total profit. Our static analysis shows that the positive profit effect always dominates the negative risk effect when there is an improvement in the financing environment, hence, their aggregate effect on the total profit of the supply chain is positive.

Secondly, under incomplete information, the transactions between SMEs and suppliers can serve as signals for banks, which may help banks access the private information of SMEs, thus reducing information asymmetry between banks and SMEs. Under certain conditions, the bank can use differentiated interest rate policies to identify the SME’s type, and there is at least a locally optimal solution for banks in deciding their interest rate policies under incomplete information. If banks have access to the information of the operational status and transactions in a supply chain, this may reduce information asymmetry between banks and SMEs and improve their access to external financing.

Lastly, even though limited by the structure of a financial market, there may not be an optimal interest rate policy for the bank under incomplete information, there exist suboptimal solutions for the bank, such as a separating PBE or a pooling PBE. These suboptimal solutions may also alleviate the information asymmetry between banks and SMEs to some extent, and improve their access to external financing. In order to maximize profit, banks should dynamically adjust both their interest rate policies.
and risk management strategies when they are providing financing services to SMEs based on the structure of the financial market.

This paper contributes to the literature on supply chain finance (SCF) and the literature on SMEs financing. Our framework incorporates important aspects of the financing issues of SMEs in the real world, including the characteristics of SMEs and the risk control strategy of banks. We provide an alternative perspective to deal with SMEs financing problem, based on their supply chain backgrounds. Moreover, our framework sheds light on how to enhance the survivability and facilitate the development of a supply chain by adopting appropriate strategies to improve the business performances and manage credit risks.

There are some limitations in our framework. Firstly, we simplified the transactions in a supply chain and the risk management strategy of banks, which limits the generalization of the analytical results in our framework. Secondly, we did not consider the collusion between the SMEs and the Supplies in getting the financing from banks. Lastly, we assumed that the supply chain consists of only two players, a supplier and an SME. This simplification helps us construct a concise theoretical framework, but it is not able to capture some potentially important features of a more complex supply chain consisting of many supplies and many SMEs.

Author Contributions: Y.Y. and H.F. conceived the study; Y.Y. and J.G. constructed and analyzed the mathematical framework; Y.Y. and J.G. wrote the draft; X.C. edited the manuscript; All authors have approved the final manuscript.

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References
34. Shavell, S. Moral Hazard and Insurance. *Q. J. Econ.* 1979, 93, 541–562. [CrossRef]
42. Mirrlees, J. The Optimal Structure of Authority and Incentives within an Organization. *Bell J. Econ.* 1976, 7, 105–131. [CrossRef]
43. Holmstrom, B. Moral Hazard and Observability. *Bell J. Econ.* 1979, 10, 74–91. [CrossRef]
60. Liao, J. An EOQ Model with Noninstantaneous Receipt and Exponentially Deteriorating Items under Two-Level Trade Credit. *Int. J. Prod. Econ.* 2008, 2, 852–861. [CrossRef]


87. Li, W.; Chen, J. Backward Integration Strategy in a Retailer Stackelberg Supply Chain. *Omega* **2018**, *75*, 118–130. [CrossRef]


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