



# *Article* **Carbon-Regulated EOQ Models with Consumers' Low-Carbon Awareness**

**Zhimiao Tao**  $1,2,*$  $1,2,*$  **and Jiuping Xu**  $1,2$  $1,2$  **D** 

- <sup>1</sup> Uncertainty Decision-Making Laboratory, Sichuan University, Chengdu 610064, China; xujiuping@scu.edu.cn
- <sup>2</sup> Low Carbon Technology and Economy Research Center, Sichuan University, Chengdu 610064, China
- **\*** Correspondence: taozhimiao@scu.edu.cn; Tel.: +86-1308-663-8708

Received: 9 January 2019; Accepted: 13 February 2019; Published: 15 February 2019



**Abstract:** In the context of a low-carbon economy, firms must make positive responses in their operation management, including inventory management. Carbon-emission regulation policies have marked their influence on the optimization of low-carbon inventory systems. In addition to regulation policies, consumers' low-carbon awareness can also influence inventory systems by affecting demand. This study investigates the influence of regulation policies and consumers' low-carbon awareness on optimal order size, emission levels, and total costs. Two widely used regulation policies, i.e., the carbon-tax mechanism and cap-and-trade mechanism, are incorporated into the classical Economic Order Quantity (EOQ) model. Analytical conclusions were obtained by optimization methods to indicate the influences of regulation policies and consumers' low-carbon awareness. Our study implies that inventory systems under different regulation policies perform similarly except with regard to total cost. Numerical examples provide more support for these analytical conclusions. Some managerial insights can be derived from the analytical conclusions and numerical examples.

**Keywords:** EOQ model; carbon tax; cap-and-trade; low-carbon awareness

# **1. Introduction**

As reported by the Fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC), greenhouse gas (GHG) emissions from human production and consumption have been the main drivers of climate warming since the mid-20th century [\(https://www.ipcc.ch\)](https://www.ipcc.ch). Among these greenhouse gases, carbon dioxide  $(CO<sub>2</sub>)$  has the most significant effect on global warming, accounting for about 75% of the total effect [\[1\]](#page-14-0). Global warming leads to climate change, which leads to a series of consequences (floods, droughts, sea level rise, etc.). These consequences may have a serious impact on the human economy and society. According to the Fourth National Climate Assessment [\(https://nca2018.globalchange.gov\)](https://nca2018.globalchange.gov), with continued growth in emissions at historic rates, annual losses in some economic sectors are projected to reach hundreds of billions of dollars by the end of the century—more than the current gross domestic product (GDP) of many US states.

Facing these challenges, governments use regulation policy instruments to curb carbon emission. Carbon-tax and cap-and-trade mechanisms are the two main regulation policies [\[2,](#page-14-1)[3\]](#page-14-2). Regulators levy carbon tax on firms' carbon emission. Carbon tax is a form of carbon pricing and causes cost pressure to emission firms. In 2010, the European Commission considered implementing a pan-European minimum carbon tax on emission permits purchased [\(https://www.nytimes.com/2010/06/23/](https://www.nytimes.com/2010/06/23/business/energy-environment/23carbon.html?ref=cap_and_trade) [business/energy-environment/23carbon.html?ref=cap\\_and\\_trade\)](https://www.nytimes.com/2010/06/23/business/energy-environment/23carbon.html?ref=cap_and_trade). Cap-and-trade, or emissions trading, is a market-based approach to controlling emissions by providing economic incentives for achieving reductions in the emissions. Under the cap-and-trade policy, carbon emissions are

tradable through a system. The European Union Emissions Trading System (EU ETS) is the first large greenhouse gas emissions trading scheme in the world.

Under regulation policies, firms must implement various measures to reduce carbon emissions. It is the conventional thinking that firms can invest in green (low-carbon) technologies achieving "physical emission reduction", such as replacing equipment with more energy-efficient equipment, using cleaner energy, and running more environmentally friendly manufacturing processes [\[4](#page-14-3)[–6\]](#page-14-4). The clean development mechanism (CDM) under the Kyoto Protocol makes it possible for developing countries to introduce green (low-carbon) technology investment from developed countries. China has evolved into the world's largest supplier of CDM projects [\[7\]](#page-14-5). For example, Shenzhen Nantian Electric Power Co., Ltd. cooperates with Climate Corporation Emissions Trading GmbH in CDM project to reduce carbon emissions through fuel substitution technology. The project is estimated to reduce emissions by 206,479 tons [\(http://cdm.ccchina.org.cn/NewItemList.aspx\)](http://cdm.ccchina.org.cn/NewItemList.aspx). Without the emission reduction investment, firms can still achieve carbon emission reduction by adjusting their operations [\[8\]](#page-14-6). For example, Wal-Mart launched Project Gigaton that is an initiative to avoid one billion metric tons (a gigaton), of greenhouse gas emissions from the global value chain by 2030 [\(https://www.walmartsustainabilityhub.com/project-gigaton/emissions-targets\)](https://www.walmartsustainabilityhub.com/project-gigaton/emissions-targets). With the increasing low-carbon awareness, people are willing to pay higher prices for low-carbon products. Firms are also beginning to use carbon labels to stimulate consumer demand for low-carbon products [\[9\]](#page-14-7).

Inventories play an important role in the operations and the profitability of a company. This study develops inventory models concerning emission-regulation policies with consumers' low-carbon awareness, providing decision support to relevant managers. We focus on the following questions: (1) How regulation parameters affect order decision, total cost, and emission? (2) What are the similarities and differences between the influences of different regulation policies?

The remainder of this paper is organized as follows. Section [2](#page-1-0) reviews the literature closely related to this study. In Section [3,](#page-3-0) the extended EOQ models with consumers' low-carbon awareness are formulated under carbon-tax regulation and cap-and-trade regulation. We derived the optimal order sizes under different regulations and developed a serial of theorems to indicate the influences of the exogenous variables (tax rate, low-carbon awareness level, carbon price, and cap) on the optimal order sizes, the corresponding emission levels, and total costs. We also made a comparison analysis for the models under different regulations in this section. Section [4](#page-10-0) presents numerical examples that illustrate the theoretical results in Section [3.](#page-3-0) Some concluding remarks and future research directions are presented in the final section.

#### <span id="page-1-0"></span>**2. Literature Review**

This study is closely related to three streams of the literature, that is, the general low-carbon management of operations, the extensions of EOQ models and the customers' low-carbon awareness (environmental awareness).

An increasing number of research works on operation management considering emission-regulation policies can be found in the literature. Zhang and Xu [\[10\]](#page-14-8) study the multiitem production-planning problem under carbon cap-and-trade regulation. They obtain the optimal policy of production and carbon-trading decisions and analyze the impact of carbon price, carbon cap on the shadow price of the common capacity, production decisions, carbon emission, and total profit. Palak et al. [\[11\]](#page-14-9) investigate the influences of carbon regulations on replenishment decisions in a biofuel supply chain. Their research suggests that carbon regulations have a significant impact on replenishment schedules, costs, and emissions in the supply chain. Chang et al. [\[12\]](#page-14-10) propose two profit-maximization models for the independent demand market and the substitutable demand market under cap-and-trade regulation. They found carbon price is more effective in controlling production and emissions compared with the carbon cap in both types of market. Xu et al. [\[13\]](#page-14-11) investigated the production and emission abatement decisions of a make-to-order supply chain under cap-and-trade regulation. They concluded that the manufacturer can reduce unit product carbon emission by using green technology, with the

co-operation of a retailer by certain contracts. Tseng and Huang [\[14\]](#page-14-12) propose a strategic decision-making model considering both the operational costs and social costs caused by the carbon dioxide emissions and use the model to evaluate carbon dioxide emissions and operational costs under different scenarios in an apparel manufacturing supply chain network. Zakeri et al. [\[15\]](#page-14-13) present an analytical supply chain planning model that can be used to examine the supply chain performance at the tactical/operational planning level under these two policy schemes. Lee [\[16\]](#page-14-14) and Cachon [\[17\]](#page-14-15) also contributes to supply chain design and management concerning carbon emissions. A comprehensive review on the very recent development on the environment management in green supply chain operations and management is given by Reference [\[18\]](#page-15-0).

As an important part of operation management, inventory management under emission-regulation policies has aroused attention in recent years. Hua et al. [\[19\]](#page-15-1) extended the classical EOQ model in the cap-and-trade scheme, and discussed the impact of carbon price and carbon cap on order decisions, carbon emissions, and total cost. Under a variety of emission regulations, Chen et al. [\[20\]](#page-15-2) analyze the effects of operational adjustment of the inventory system on emission decrement and cost increment. They proposed conditions under which relative reduction in emissions is greater than the relative increase in cost. Battini et al. [\[21\]](#page-15-3) explored the factors affecting the environmental impact within the traditional EOQ model and developed a Sustainable EOQ Model, and the environmental impact of transportation and inventory is involved in the model and investigated from an economic point of view. In Reference [\[22\]](#page-15-4), Toptal et al. incorporate green technology investment into EOQ models under carbon regulations. They presented an analytical comparison between various investment opportunities and compared different emission-regulation policies in terms of costs and emissions. Bozorgi et al. [\[23\]](#page-15-5) developed nonlinear, noncontinuous cost, and emissions functions for a new inventory model in which temperature-controlled items need to be stored at a certain, non-ambient temperature. Kazemi et al. [\[24\]](#page-15-6) investigate the impact of emission costs on the replenishment order sizes and the total profit of the EOQ models with imperfect quality from a sustainable point of view. It is noteworthy that some studies take emissions into account but do not consider environmental regulations. For instance, Bouchery et al. [\[25\]](#page-15-7) take sustainability concerns into account and then reformulate the classical EOQ model as a multiobjective problem. Tiwari et al. [\[26\]](#page-15-8) develop an integrated single-vendor single-buyer inventory model for deteriorating items with the imperfect quality considering carbon emission. Reference [\[27](#page-15-9)[,28\]](#page-15-10) study the extended EOQ models considering environmental factors from different perspectives.

In the context of carbon regulations, customers' low-carbon awareness (environmental awareness) has considerable influence on operation management. Many studies illustrate that consumers with higher environmental awareness have more willingness to buy low-carbon products. Shuai et al. [\[29\]](#page-15-11) used Dunnett's T3 test approach for single-factor variance analysis to find consumers' willingness to pay for low-carbon products. Customers' low-carbon awareness (environmental awareness) impacts the parameters related to operation management and, consequently, changes their decisions. Liu et al. [\[30\]](#page-15-12) found that in two-stage supply chains, retailers and manufacturers with superior ecofriendly operations benefit as consumers' environmental awareness increases. Xia and He [\[31\]](#page-15-13) concluded that consumers' low-carbon awareness was to enhance consumer utility and decrease the profits of supply chain firms without regulation. Cheng et al. [\[32\]](#page-15-14) studied how a carbon-labeling scheme could be integrated into operational decision-making for manufacturers and retailers. Reference [\[32\]](#page-15-14) shows that a carbon-labeling scheme can significantly reduce the overall carbon emission supply chain and have an initially negative impact on the manufacturer and retailer's profits. Zhang et al. [\[33\]](#page-15-15) study the impact of consumer environmental awareness on order quantities and channel coordination within a one-manufacturer and one-retailer supply chain. Yu et al. [\[34\]](#page-15-16) develop an optimization model under oligopolistic competition considering consumer environmental awareness, with the objective of profit maximization for the manufacturers. Reference [\[34\]](#page-15-16) show that an increase of consumer environmental awareness will incentivize manufacturers to produce more green products with higher green levels, but this does not necessarily lead to higher profits for the manufacturers.

Most of the literature that incorporates consumers' low-carbon awareness into operation management assumes the demand is independent of consumers' low-carbon awareness (environmental awareness). To our best knowledge, there are only a few studies available that directly consider the emissions in demand function, especially in inventory systems. Hovelaque and Bironneau [\[35\]](#page-15-17) take into account the emission-sensitive demand within a mixed regulation framework. However, the mixed regulation framework does not exist in reality. The main goal of our study is to investigate how regulation policies (carbon tax and cap-and-trade) and consumers' low-carbon awareness influence order decisions, emission levels, and total costs for inventory systems. The results proposed in this study are expected to provide managerial insights for managers of inventory systems. Regulators can also get inspiration from these results.

## <span id="page-3-0"></span>**3. Models and Analysis**

In this study, we focus on the classical economic order quantity model or, for short, the EOQ model. The classical EOQ model is developed to determine the optimal order size to minimize the total cost of the inventory system per unit time. The total cost consists of fixed ordering cost, variable ordering cost and holding cost. There are several assumptions for the classical EOQ model listed as follows:

- Shortages are not allowed;
- When the inventory level drops to zero, the inventory system can be replenished instantaneously;
- The demand is continuous and uniform;
- The order size is unchanged every time;
- The fixed cost per order is dependent of order size;
- The cost for producing/purchasing each unit and holding cost per unit per unit of time held in inventory are constants.

<span id="page-3-1"></span>The inventory level the under the assumptions above can be shown by Figure [1.](#page-3-1) More details about the model can be referred to [\[36\]](#page-15-18).



**Figure 1.** The inventory level as a function of time for the classical EOQ model (*Q*: order size; *D*: actual demand rate).

Prior to presenting the mathematical models, we summarize the parameters and decision variables needed in Table [1.](#page-4-0)

<span id="page-4-0"></span>

Model Parameters	
A	fixed cost per order
h	holding cost per unit per unit of time held in inventory
C	unit cost for producing/purchasing each unit
Â	carbon emission per order
ĥ	carbon emission per unit held in inventory per unit time
ĉ	variable carbon emission per unit purchased/produced
	carbon emission rate
p	market price for per unit emission
	carbon-emission cap
$D_0$	potential demand rate
K	consumers' low-carbon awareness level (LCAL)
D	actual demand rate that is influenced by K
F.	emission level per unit time
ТC	total cost for the classical EOQ model per unit time
$TC_1$	total cost under carbon-tax regulation per unit time
TC <sub>2</sub>	total cost under carbon-and-trade regulation per unit time
Decision variable	
	order size

**Table 1.** Notations for parameters and variables.

Based on the assumptions and notations above, the classical EOQ model is formulated as follows:

$$
\min_{Q} TC = \frac{AD}{Q} + \frac{hQ}{2} + cD. \tag{1}
$$

Carbon emissions occur in the stages of ordering, inventory holding, and purchasing/producing. Thus, the emission level per unit time of the inventory system is formulated as:

<span id="page-4-2"></span>
$$
E = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D.
$$
 (2)

In this study, the influence of consumers' low-carbon awareness level (LCAL) can influence demand, which is depicted as

<span id="page-4-1"></span>
$$
D = D_0 - KE. \tag{3}
$$

As shown in Formula [\(3\)](#page-4-1), the actual demand is decreasing with increasing LCAL, implying the relatively high emission level and LCAL can reduce the actual demand. Equations [\(2\)](#page-4-2) and [\(3\)](#page-4-1) indicate that emission level and demand interact with each other. By solving the equation group consisting of Equations  $(2)$  and  $(3)$ , we have

<span id="page-4-3"></span>
$$
\begin{cases}\nE = \frac{\hat{h}Q^2 + 2\hat{c}D_0Q + 2\hat{A}D_0}{2((K\hat{c} + 1)Q + \hat{A}K)} \\
D = \frac{Q(2D_0 - K\hat{h}Q)}{2((K\hat{c} + 1)Q + \hat{A}K)}\n\end{cases}
$$
\n(4)

Please note that this is not sensible when the demand is negative. Thus, order size *Q* is required to satisfy  $0 < Q < \frac{2D_0}{K\hat{h}}$ .

#### *3.1. Carbon-Tax Regulation*

Under carbon-tax regulation, carbon emissions from the inventory system are levied by the regulator with tax rate *t*. Total cost under carbon-tax regulation *TC*<sup>1</sup> consists of two parts, operation cost and the tax levied on carbon emissions. Thus,  $TC_1$  is formulated as

$$
TC_1 = \frac{AD}{Q} + \frac{hQ}{2} + cD + tE
$$
\n<sup>(5)</sup>

Combining the constraint  $Q \leq \frac{2D_0}{K \hat{h}}$ , the inventory-optimization model under carbon-tax regulation is formulated as

<span id="page-5-0"></span>
$$
\begin{cases}\n\min TC_1 = \frac{AD}{Q} + \frac{hQ}{2} + cD + tE \\
s.t. \ 0 < Q \le \frac{2D_0}{Kh}\n\end{cases}\n\tag{6}
$$

where *D* and *E* are defined by Equation [\(4\)](#page-4-3).

For a given *K* and *t*, the second-order derivative of  $TC_1$  in *Q* is

$$
\frac{d^2TC_1}{dQ^2} = \frac{\left(\hat{A}\hat{h}K^2 + 2D_0cK + 2D_0\right)(A + \hat{A}t + AK\hat{c} - \hat{A}Kc)}{\left(Q + \hat{A}K + KQ\hat{c}\right)^3} \tag{7}
$$

Since we need to minimize *TC*<sub>1</sub>, it is reasonable to assume that  $A + \hat{A}t + AK\hat{c} - \hat{A}Kc \geq 0$ . In this case,  $TC_1$  is convex in *Q*, and solution  $Q_1^*$  to minimize  $TC_1$  must satisfy  $\frac{dTC_1}{dQ}|_{Q=Q_1^*}=0$ . Solving for  $Q_1^*$  in  $\frac{dTC_1}{dQ}|_{Q=Q_1^*}=0$  leads to

$$
Q_1^* = \frac{1}{(K\hat{c} + 1)} \left( -\hat{A}K + \frac{\sqrt{2A_1D_1h_1}}{h_1} \right)
$$
(8)

 $A_1 = A + \hat{A}t + A\hat{c}K - \hat{A}cK$ ,  $D_1 = \frac{1}{2} (\hat{A}\hat{h}K^2 + 2D_0\hat{c}K + 2D_0)$ , and  $h_1 = h + \hat{h}t + h\hat{c}K - \hat{h}cK$ . It is necessary to assume that  $h_1 > 0$  to ensure  $2A_1D_1h_1 > 0$ .

**Remark 1.** When  $K = 0$ ,  $t = 0$ ,  $Q_1^*$  equals  $\sqrt{\frac{2AD}{h}}$  and satisfies  $0 < Q_1^* \leq \frac{2D_0}{Kh} (= +\infty)$ . This indicates *that Model [\(6\)](#page-5-0) is a reasonable extension of the classical EOQ model when carbon-tax regulation and LCAL are concerned. If*  $Q_1^* \leq \frac{2D_0}{K\hat{h}}$ , then  $Q_1^*$  is the optimal solution of Model [\(6\)](#page-5-0). Otherwise,  $\frac{2D_0}{K\hat{h}}$  is the optimal solution of *Model [\(6\)](#page-5-0) due to the convexity of TC*<sub>1</sub> *in Q. In what follows, we denote*  $\frac{2D_0}{K\hat{h}}$  *by*  $Q_0^*$ .

<span id="page-5-2"></span>**Theorem 1.** Given a fixed K, (1)  $Q_1^*$  is increasing in t if  $\frac{A}{h}<\frac{\hat{A}}{\hat{h}}$ ; (2)  $Q_1^*$  is decreasing in t if  $\frac{A}{h}>\frac{\hat{A}}{\hat{h}}$ ; (3)  $Q_1^*$  is *not affected by t if*  $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$ *.* 

**Proof.** Results are derived from the expression of the first-order derivative of  $Q_1^*$  on *t* as follows:

$$
\frac{dQ_1^*}{dt} = \frac{\left(\hat{A}h - A\hat{h}\right)D_1}{\sqrt{2A_1D_1h_1}h_1}.
$$

 $\Box$ 

When  $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$ , order size  $Q_1^*$  simultaneously minimizes operation cost  $\frac{AD}{Q} + \frac{hQ}{2} + cD$  and emission level *E*. Thus,  $Q_1^*$  is independent of tax rate *t*. When  $\frac{A}{h} < \frac{\hat{A}}{h}$ , the emission level due to order is relatively large, the manager of the inventory system should reduce the order times to lessen the levied tax. The reduction of order times means the increment of order size. When  $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ , the opposite analysis deduces that  $Q_1^*$  is decreasing in *t*.

The derivative of  $Q_1^*$  in *K* has a very complex form. However, the following theorem provides a sufficient condition, such that  $Q_1^*$  is monotonically increasing in *K*.

<span id="page-5-1"></span>**Theorem 2.** *Given a fixed t, if A*ˆ*h* − *Ah*ˆ < 0*, then Q*<sup>∗</sup> 1 *is decreasing in K when*

$$
K(\hat{A}\hat{h} - D_0\hat{c}^2) < D_0\hat{c} \tag{9}
$$

**Proof.** We rewrite  $Q_1^*$  as

$$
Q_1^* = \frac{-\hat{A}K}{(K\hat{c}+1)} + \frac{\sqrt{2A_1D_1h_1}}{(K\hat{c}+1)h_1}.
$$
\n(10)

Obviously, the first item  $\frac{-\hat{A}K}{(K\hat{c}+1)}$  is decreasing in *K*. The theorem is proved as long as we can verify that  $\frac{\sqrt{2A_1D_1h_1}}{(K_1^2+1)h_1}$  $\frac{\sqrt{2}A_1D_1h_1}{(K\hat{c}+1)h_1}$  is decreasing in *K*. The first-order derivative of  $\frac{\sqrt{2}A_1D_1h_1}{(K\hat{c}+1)h_1}$  $\frac{\sqrt{2A_1D_1n_1}}{(\kappa\hat{c}+1)h_1}$  is calculated and arranged as

<span id="page-6-0"></span>
$$
\frac{1}{\sqrt{2A_1h_1D_1}(K\hat{c}+1)^2}(A_1h_1(K(\hat{A}\hat{h}-D_0\hat{c}^2)-D_0\hat{c})+D_1(A\hat{h}-\hat{A}h)(K\hat{c}+1))
$$
(11)

Under the assumptions of  $A\hat{h}-\hat{A}h< 0$  and  $K(\hat{A}\hat{h}-D_0\hat{c}^2)< D_0\hat{c}$ , Formula [\(11\)](#page-6-0) is negative, i.e., √ 2*A*1*D*1*h*<sup>1</sup>  $\frac{\sqrt{2}A_1D_1h_1}{(K\ell+1)h_1}$  is decreasing in *K*. Consequently,  $Q_1^*$  is decreasing in *K*.

Theorem [2](#page-5-1) indicates that  $Q_1^*$  is decreasing with respect to *K* in any non-negative interval when initial market potential  $D_0$  is large enough such that  $\hat{A}\hat{h}-D_0\hat{c}^2 < 0$  as long as the condition  $A\hat{h}-\hat{A}h < 0$  holds. It should be noted that Theorem [2](#page-5-1) provides a strong sufficient condition, such that *Q*<sup>∗</sup> 1 is decreasing in *K*. The condition for the theorem to be held is actually more relaxed, which is shown by a numerical example in the remainder of this paper. For  $Q_0^*$ , it is decreasing in *K* without additional conditions.

When the optimal order size is  $Q_1^*$ , the corresponding emission level is

$$
E(Q_1^*) = \frac{\hat{h}(Q_1^*)^2 + 2\hat{c}D_0Q_1^* + 2\hat{A}D_0}{2((K\hat{c} + 1)Q_1^* + \hat{A}K)}.
$$
\n(12)

The following theorem states the influence of *t* on  $E(Q_1^*)$ .

<span id="page-6-1"></span>**Theorem 3.** Given a fixed K, the emission level  $E(Q_1^*)$  is decreasing in t if  $\hat{A}h \neq A\hat{h}$ ; otherwise,  $E(Q_1^*)$  is *independent of t.*

**Proof.** The first-order derivative of  $E(Q_1^*)$  in *t* is calculated as

$$
\frac{dE(Q_1^*)}{dt} = -\frac{\left(A\hat{h} - \hat{A}h\right)^2 D_1^{\frac{1}{2}}}{\left(2A_1h_1\right)^{\frac{3}{2}}}
$$
(13)

The theorem is derived from the negativity of  $-\frac{(A\hat{h}-\hat{A}h)^2D_1^{\frac{1}{2}}}{3}$  $\frac{(2A_1h_1)^{\frac{3}{2}}}{(2A_1h_1)^{\frac{3}{2}}}$ 

Theorem [1](#page-5-2) states that different comparisons of  $\frac{A}{h}$  and  $\frac{\hat{A}}{\hat{h}}$  generate three influences of *t* on  $Q_1^*$ . For the emission level, as shown in Theorem [3,](#page-6-1) *t* has two only two possible influences on  $E(Q_1^*)$ .  $E(Q_1^*)$ is decreasing in *t*, regardless of whether  $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$  or  $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ . Theorem [3](#page-6-1) also indicates that the carbon tax is effective on curbing the emission level only if  $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$ . Since  $Q_0^*$  is not affected by *t*,  $E(Q_0^*)$  is independent of *t*.

Although it is difficult to determine the sign of  $\frac{dE(Q_1^*)}{dk}$  for a fixed *t* due to its complex form, this following theorem indicates that  $\frac{dE(Q_1^*)}{dk}$  and  $\frac{dQ_1^*}{dk}$  have the same sign under some conditions.

<span id="page-6-2"></span>**Theorem 4.** For a fixed t, if  $Q_1^*$  is decreasing in K, then  $E(Q_1^*)$  is also decreasing in K when  $\hat{A} > -\frac{dQ_1^*}{dK}$ .

**Proof.** The first-order derivative of  $E(Q_1^*)$  with respect to *K* is calculated as

$$
\frac{dE(Q_1^*)}{dK} = \frac{(-\hat{c}\hat{h})(Q_1^*)^3 + (\hat{h}(K\hat{c}+1)\frac{dQ_1^*}{dK} - \hat{A}\hat{h} - 2D_0\hat{c})) (Q_1^*)^2 + 2(\hat{A}\hat{h}K\frac{dQ_1^*}{dK} - 2D_0\hat{A}\hat{c})Q_1^* + 2\hat{A}D_0(-\hat{A} - \frac{dQ_1^*}{dK})}{2((K\hat{c}+1)Q_1^* + \hat{A}K)^2}
$$
(14)

 $\frac{dQ_1^*}{dK}$  is negative because  $Q_1^*$  is decreasing in *K*. Thus, all coefficients of  $(Q_1^*)^3$ ,  $(Q_1^*)^2$ ,  $Q_1^*$  are negative. Furthermore,  $\frac{dE(Q_1^*)}{dK}$  is negative when  $-\hat{A} - \frac{dQ_1^*}{dK} < 0$ , i.e.,  $\hat{A} > -\frac{dQ_1^*}{dK}$ . The proof is complete.

Three negative items are moved out during the proof of Theorem [4.](#page-6-2) Therefore, Theorem [4](#page-6-2) proposes a relatively strong sufficient condition, such that  $E(Q_1^*)$  is decreasing in *K*. In fact, the necessary and sufficient condition such that  $E(Q_1^*)$  is decreasing in *K* would be more relaxed, which is illustrated by a numerical example in the remainder of the paper.

If  $Q_0^*$  is the optimal solution of Model [\(6\)](#page-5-0), the corresponding emission level is

$$
E(Q_0^*) = \frac{D_0\left(2D_0 + 2D_0\hat{c} + \hat{A}\hat{h}\right)}{2D_0 + 2D_0\hat{c}K + \hat{A}\hat{h}K}
$$

Obviously,  $E(Q_0^*)$  is decreasing in *K*.

The following theorems present the influences of *t* and *K* on cost  $TC_1(Q_1^*)$ .

<span id="page-7-0"></span>**Theorem 5.** For a fixed K, the optimal cost  $TC_1(Q_1^*)$  is increasing in t.

**Proof.** When  $\hat{A}h = A\hat{h}$ , as shown in Theorems [1](#page-5-2) and [3,](#page-6-1)  $Q_1^*$  and  $E(Q_1^*)$  are dependent of *t*. Since  $TC_1(Q_1^*) = \frac{AD}{Q_1^*} + \frac{hQ_1^*}{2} + cD(Q_1^*) + tE(Q_1^*)$ ,  $TC_1(Q_1^*)$  is (linearly) increasing in *t*. In what follows, we assume that  $\hat{A}h \neq A\hat{h}$ . Let  $\tilde{t} > t$ , then

$$
TC_1(Q_1^*(t)) = \frac{(A+\hat{A}t)D(Q_1^*)}{Q_1^*(t)} + \frac{(h+\hat{h}t)Q_1^*(t)}{2} + (c+\hat{c}t)D(Q_1^*(t))
$$
  
\n
$$
\leq \frac{(A+\hat{A}t)D(Q_1^*(\tilde{t}))}{Q_1^*(\tilde{t})} + \frac{(h+\hat{h}t)Q_1^*(\tilde{t})}{2} + (c+\hat{c}t)D(Q_1^*(\tilde{t}))
$$
  
\n
$$
\leq \frac{(A+\hat{A}t)D(Q_1^*(\tilde{t}))}{Q_1^*(\tilde{t})} + \frac{(h+\hat{h}\tilde{t})Q_1^*(\tilde{t})}{2} + (c+\hat{c}\tilde{t})D(Q_1^*(\tilde{t}))
$$
  
\n
$$
= TC_1(Q_1^*(\tilde{t})).
$$

The first inequality holds because  $Q_1^*(t)$  is the order size to minimize  $TC_1$ . Setting  $\tilde{t} > t$  results in the second inequality. Hence,  $TC_1(Q_1^*)$  is increasing in *t*.

Clearly,  $TC_1(Q_0^*)$  is linearly increasing in *t* as  $Q_0^*$  is the optimal solution of Model [\(6\)](#page-5-0). Combining Theorem [3](#page-6-1) with Theorem [5,](#page-7-0) the carbon-tax regulation has two influences on the inventory system: curbing emission level but increasing the total cost.

<span id="page-7-1"></span>**Theorem 6.** For a given t, if  $Q_1^*$  is decreasing in K, then  $TC_1(Q_1^*)$  also decreases with K when  $\min\{A,\hat{A}\} > -\frac{dQ_1^*}{dK}$ .

**Proof.** Let us denote  $\frac{AD}{Q} + \frac{hQ}{2} + cD$  by  $TC_0$ .  $TC_0$  and *E* have the same form with respect to *Q* except for the coefficients. Similar to Theorem [4,](#page-6-2)  $TC_0(Q_1^*)$  decreases in *K* when  $A > -\frac{dQ_1^*}{dK}$ . Combining the result with Theorem [4,](#page-6-2) the proof is complete.  $\Box$ 

Theorem [6](#page-7-1) proposes a strong sufficient condition, such that  $TC_1(Q_1^*)$  is decreasing in *K*. In fact, the necessary and sufficient condition that  $TC(Q_1^*)$  decreases in *K* would be more relaxed, which is shown by a numerical example in the remainder of the paper.

If  $Q_0^*$  is the optimal solution of Model [\(6\)](#page-5-0), the corresponding cost is

$$
TC_1(Q_1^*) = D_0(2D_0h + A\hat{h}^2 + 2D_0c\hat{h} + 2D_0\hat{h}t + \hat{A}\hat{h}^2t + 2D_0\hat{c}\hat{h}t + \hat{h}(\hat{A}h - A\hat{h})K + 2D_0(\hat{A}h - A\hat{h})K.
$$

Thus,  $TC_1(Q_0^*)$  linearly increases (decreases) in *K* if  $\hat{A}h - A\hat{h} > 0 \le 0$  and  $\hat{A}h - A\hat{h} > 0 \le 0$ .

#### *3.2. Cap-And-Trade Regulation*

Under the cap-and-trade regulation, the manager of inventory systems buys (sells) a carbon-emission permit when the actual carbon-emission level is more (less) than cap *C*, allocated by a regulator (e.g., Development and Reform Commission in China). For simplicity, we assume the market price *p* for per unit emission is fixed regardless of buying or selling. Thus, the total cost under the cap-and-trade regulation is formulated as

$$
TC_2 = \frac{AD}{Q} + \frac{hQ}{2} + cD + p(E - C)
$$
\n(15)

where *D* and *E* are defined by [\(4\)](#page-4-3). Taking constraint  $0 < Q < \frac{2D_0}{K\hbar}$  into account, the inventory-optimization problem under cap-and-trade regulation is modeled as

<span id="page-8-0"></span>
$$
\begin{cases}\n\min TC_2 = \frac{AD}{Q} + \frac{hQ}{2} + cD + p(E - C) \\
s.t. 0 \le Q \le \frac{2D_0}{K\hbar}\n\end{cases}
$$
\n(16)

The second-order derivative of  $TC_2$  with respect to  $Q$  is

$$
\frac{d^2TC_2}{dQ^2} = \frac{\left(\hat{A}\hat{h}K^2 + 2D_0\hat{c}K + 2D_0\right)\left(A + \hat{A}p + AK\hat{c} - \hat{A}Kc\right)}{\left(Q + \hat{A}K + KQ\hat{c}\right)^3} \tag{17}
$$

Since we need to minimize *TC*<sub>2</sub>, it is reasonable to assume that  $A + \hat{A}p + AK\hat{c} - \hat{A}Kc \ge 0$ . In this case,  $TC_2$  is convex in *Q*, and the solution  $Q_2^*$  to minimize  $TC_2$  must satisfy  $\frac{dTC_2}{dQ}|_{Q=Q_2^*}=0$ . Solving for  $Q_2^*$  in  $\frac{dTC_2}{dQ} = 0$  leads to

$$
Q_2^* = \frac{1}{(K\hat{c}+1)} \left( -\hat{A}K + \frac{\sqrt{2A_2D_2h_2}}{h_2} \right)
$$
(18)

where  $A_2=A+\hat{A}\,p+A\hat{c}K-\hat{A}\,cK$  ,  $D_2=\frac{1}{2}\left(\hat{A}\hat{h}K^2+2D_0\hat{c}K+2D_0\right)(=D_1)$  , and  $h_2=h+\hat{h}\,p+h\hat{c}K-\hat{A}\,cK$  $\hat{h}$ cK. It is necessary to assume that  $h_2 > 0$  to ensure  $2A_2D_2h_2 > 0$ . If  $Q_2^*$  satisfies  $Q_2^* \leq Q_0^*$ , then  $Q_2^*$  the optimal solution of model [\(16\)](#page-8-0). If  $Q_2^* > Q_0^*$ , then the optimal solution of model (16) is  $Q_0^*$ . It is worth noting that both of  $Q_2^*$  and  $Q_0^*$  are not affected by the cap *C*.

*Q*<sup>∗</sup></sup><sub> $2$ </sub><sup> $\frac{1}{2}$  and *Q*<sup>\*</sup><sub>1</sub><sup> $\frac{1}{2}$ </sub> have the same form except for *p*. As we replace *t* by *p* in Theorem [1,](#page-5-2) it indicates the</sup></sup> influence of  $p$  on  $Q_2^*$  $Q_2^*$  $Q_2^*$  for a fixed *K*. The influence of *K* on  $Q_2^*$  for a fixed  $p$  is derived from Theorem 2 as *t* is replaced by *p*.

Both of  $Q_2^*$  and  $Q_0^*$  are independent of cap *C*, which is the same as [\[19](#page-15-1)[,20,](#page-15-2)[22\]](#page-15-4). In real market case, the price of goods is influenced by the supply of goods. i.e., more supply quantity implies lower price. It is reasonable to assume that *p* is a decreasing function of *C*, denoted by *p*(*C*). The influence of *C* on *Q*∗ 2 is stated by the following theorem.

**Theorem 7.** *Given a fixed K, if the carbon price p is a decreasing function of cap C, then (1) Q*<sup>∗</sup> 2 *is decreasing* in C if  $\frac{A}{h}<\frac{\hat{A}}{\hat{h}}$ ; (2)  $Q_2^*$  is increasing in C if  $\frac{A}{h}>\frac{\hat{A}}{\hat{h}}$ ; (3)  $Q_2^*$  is not affected by C if  $\frac{A}{h}=\frac{\hat{A}}{\hat{h}}$ .

**Proof.** The theorem is easily derived from Theorem [1](#page-5-2) (*p*  $\rightarrow$  *t*,  $Q_1^* \rightarrow Q_2^*$ ) and  $\frac{dQ_2^*}{dC} = \frac{dQ_2^*}{dp(C)}$  $\frac{dp(C)}{dC}$ .

If the optimal solution of  $(16)$  is  $Q_2^*$ , the corresponding emission level is

$$
E(Q_2^*) = \frac{\hat{h}(Q_2^*)^2 + 2\hat{c}D_0Q_2^* + 2\hat{A}D_0}{2((K\hat{c} + 1)Q_2^* + \hat{A}K)}.
$$
\n(19)

If  $E(Q_2^*) > C$ , then the manager needs to buy a carbon permit to offset the emission gap; otherwise, the manager can sell the surplus carbon permit for extra profit.

The influences of *p* and *K* on  $E(Q_2^*)$  are the same as the influences *t* and *K* on  $E(Q_1^*)$  shown in Theorems [3](#page-6-1) and [4.](#page-6-2) If we substitute  $p$  and  $Q_2^*$  for  $t$  and  $Q_1^*$ , respectively, in Theorem [3,](#page-6-1) the theorem states the influence of  $p$  on  $E(Q_2^*)$  for a given  $K$ .

If carbon price *p* is regarded as a decreasing function of the cap *C*, the influence of *C* on emission level is stated by the following theorem:

**Theorem 8.** *For a fixed K, if carbon price p is regarded as a decreasing function of cap C, then E*(*Q*<sup>∗</sup> 2 ) *increases in* C if  $\hat{A}h \neq A\hat{h}$ ; otherwise,  $E(Q_2^*)$  is independent of p.

**Proof.** The theorem is easily derived from Theorem [3](#page-6-1) ( $p \to t$ ,  $Q_1^* \to Q_2^*$ ) and  $\frac{dQ_2^*}{dC} = \frac{dQ_2^*}{dp(C)}$  $\frac{dp(C)}{dC}$ .

<span id="page-9-0"></span>**Theorem 9.** For fixed K and C, (1) If  $C < E(Q_2^*)$ , then  $TC_2(Q_2^*)$  is increasing in p; (2) If  $C > E(Q_2^*)$ , *then*  $TC_2(Q_2^*)$  *is decreasing in p.* 

**Proof.** When  $\hat{A}h = A\hat{h}$ ,  $Q_2^*$  and  $E(Q_2^*)$  are not affected by *p*, then  $TC_2(Q_2^*)$  is linearly increasing (decreasing) in *p* when  $E(Q_2^*) > C(E(Q_2^*) > C)$ .

When  $\hat{A}h \neq A\hat{h}$ , the first-order derivative of  $TC_2(Q_2^*)$  with respect to  $p$  is  $E(Q_2^*)$  – C. Hence,  $TC_2(Q_2^*)$  increases (decreases) in *p* if  $C < E(Q_2^*)(C > E(Q_2^*))$ . This proof is complete.

As indicated in Theorem [9,](#page-9-0) the influence of  $p$  on  $TC_2(Q_2^*)$  depends on the allocated cap. Total cost increases if the cap allocated by the regulator is less than the actual emission level. The reason lies in the manager of the inventory system with to purchase a carbon-emission permit to compensate the emission gap, which generates the additional cost. On the other hand, the manager can sell the redundant permit to obtain profit if the actual emission level is less than the allocated cap. For a fixed  $p$  and *K*, it is clear that  $TC_2(Q_2^*)$  decreases in *C* from the expression of  $TC_2(Q_2^*)$ .

If the optimal solution of Model [\(16\)](#page-8-0) is  $Q_0^*$ , the similar results hold, i.e., (1) for a fixed *K* and *C*, if  $C < E(Q_0^*)(C \ge E(Q_0^*))$ , then  $TC_2(Q_0^*)$  increases (decreases) in  $p$ ; (2) for a fixed  $p$  and  $K$ ,  $TC_2(Q_0^*)$ decreases in *C*.

If *p* is regarded as a decreasing function of *C*, the following corollary states the influences of *C* on  $TC_2(Q_2^*)$ .

**Corollary 1.** For a fixed K, let p be a decreasing function of C. (1) If  $C < E(Q_2^*)$ , then  $TC_2(Q_2^*)$  decreases in *C*; (2) If  $C > E(Q_2^*)$ , then  $TC_2(Q_2^*)$  increases in C.

If  $Q_0^*$  is the optimal solution of Model [\(16\)](#page-8-0), let  $p$  be a decreasing function of *C*. The influence of *C* on *TC*<sup>2</sup> for fixed *K* is stated by the following theorem:

<span id="page-9-1"></span>**Theorem 10.** For a fixed K, let  $p$  be a decreasing function of C.  $TC_2(Q_0^*)$  decreases (increases) in C if and only if

$$
\frac{dp}{dC} (E(Q_0^*) - C) - p < 0 > 0)
$$

*holds.*

**Proof.** Since  $Q_0^*$  is not influenced by *p*, the first-order derivative of  $TC_2(Q_0^*)$  is calculated as

$$
\frac{dTC_2(Q_0^*)}{dC} = \frac{dp}{dC} \left( E(Q_0^*) - C \right) - p.
$$

Thus, the theorem is proved.  $\square$ 

**Remark 2.** In Theorem [10,](#page-9-1)  $C \leq E(Q_0^*)$  is a sufficient condition, such that  $TC_2(Q_0^*)$  decreases in C because p is a decreasing function of C. However, if  $C>E(Q_0^*)$ , there is no deterministic conclusion that can be derived.

Since *C* and *p* are independent of *K*, Theorem [6](#page-7-1) can state the influence of *K* on  $TC_2(Q_2^*)$  if *t*,  $Q_1^*$ and  $TC_1$  are replaced by  $p$ ,  $Q_1^*$ , and  $TC_2$ , respectively.

If  $Q_0^*$  is the optimal solution of Model [\(16\)](#page-8-0), the corresponding cost is

 $TC_2(Q_0^*) = D_0(2D_0h + A\hat{h}^2 + 2D_0c\hat{h} + 2D_0\hat{h}t + \hat{A}\hat{h}^2t + 2D_0\hat{c}\hat{h}t + \hat{h}(\hat{A}h - A\hat{h})K + 2D_0(\hat{A}h - A\hat{h})K) - pC.$ 

Thus,  $TC_2(Q_0^*)$  linearly increases (decreases) in *K* if  $\hat{A}h - A\hat{h} > 0 \le 0$  and  $\hat{A}h - A\hat{h} > 0 \le 0$ .

## <span id="page-10-0"></span>**4. Numerical Examples**

In this section, we present a series of numerical examples to illustrate the theoretical results in Section [3.](#page-3-0)

**Example 1.** *The influence of carbon-tax rate t on optimal order size Q*<sup>∗</sup> 1 *under carbon-tax regulation (Theorem [1\)](#page-5-2) is illustrated by the example in the form of Figure [2.](#page-10-1) As shown in Figure [2a](#page-10-1), the curve of Q*<sup>∗</sup> <sup>1</sup> *with respect to t increases and is convex (approximately linear), which is because higher emission level per order leads to fewer order times and more order size to reduce the levied tax. The opposite situation is shown in Figure [2c](#page-10-1). Figure [2b](#page-10-1) illustrates a special case, that is, the cost-optimal order size and the emission-optimal order size are the same.*

<span id="page-10-1"></span>

**Figure 2.** Influence of *t* on  $Q_1^*$  (*K* = 2, *D*<sub>0</sub> = 600, *A* = 120, *c* = 3,  $\hat{A}$  = 12,  $\hat{h}$  = 1,  $\hat{c}$  = 1).

**Example 2.** *The influence of K on optimal order size Q*<sup>∗</sup> 1 *under carbon-tax regulation (Theorem [2\)](#page-5-1) is illustrated by the example in the form of Figure [3.](#page-11-0) As shown in Figure [3,](#page-11-0) Q*<sup>∗</sup> 1 *decreases and is convex in K, which means that Q*<sup>∗</sup> 1 *initially decreases fast and tends to be flat as K increases further.*

<span id="page-11-0"></span>

**Figure 3.** Influence of *K* on  $Q_1^*$  (*D*<sub>0</sub> = 600, *A* = 120, *h* = 12, *c* = 3,  $\hat{A}$  = 12,  $\hat{h}$  = 1,  $\hat{c}$  = 1,  $t$  = 5).

<span id="page-11-3"></span>**Example 3.** *Influence of t on emission level E*(*Q*<sup>∗</sup> 1 ) *(Theorem [3\)](#page-6-1) is illustrated by the example in the form of Figure* [4.](#page-11-1) As shown in Figure [4,](#page-11-1) emission level decreases with increasing t regardless of whether  $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$  or  $\frac{A}{h}<\frac{\hat{A}}{\hat{h}}.$  This means that carbon-tax regulation does reduce emission level as long as  $\frac{A}{h}\neq\frac{\hat{A}}{\hat{h}}.$ 

<span id="page-11-1"></span>

**Figure 4.** Influence of *t* on  $E(Q_1^*)$  ( $K = 5$ ,  $D_0 = 600$ ,  $A = 120$ ,  $h = 10$ ,  $c = 3$ ,  $\hat{h} = 1$ ,  $\hat{c} = 1$ ).

<span id="page-11-2"></span>**Example 4.** *The influence of*  $K$  *on emission level*  $E(Q_1^*)$  (*Theorem 4*) *is illustrated by the example in the form of Figure [5.](#page-11-2) As shown in Figure [5,](#page-11-2) emission level E*(*Q*<sup>∗</sup> 1 ) *rapidly declines in the initial stage. With the further increase of K, the reduction of E*(*Q*<sup>∗</sup> 1 ) *gradually gently trends. This observation indicates that enhancing customers' LCAL in the initial stage can play a significant role in reducing emission level for an inventory system. If the regulator or the manager of the inventory system needs to reduce the emission level, it is feasible to solely enhance K. However, the effectiveness of emission reduction decreases with the increase of K.*



**Figure 5.** Illustration of Theorem  $4 (t = 20, D_0 = 600, A = 120, h = 24, c = 3, \hat{A} = 10, \hat{h} = 1, \hat{c} = 1)$  $4 (t = 20, D_0 = 600, A = 120, h = 24, c = 3, \hat{A} = 10, \hat{h} = 1, \hat{c} = 1)$ .

**Example 5.** The influence of *t* on the total  $TC_1(Q_1^*)$  (Theorem [5\)](#page-7-0) is illustrated by the example in the form of *Figure [6.](#page-12-0)*

<span id="page-12-0"></span>

**Figure 6.** Influence of *t* on total cost  $TC_1(Q_1^*)$  ( $K = 2$ ,  $D_0 = 600$ ,  $A = 120$ ,  $h = 12$ ,  $c = 3$ ,  $\hat{h} = 1$ ,  $\hat{c} = 1$ ).

*As shown in Figure [6,](#page-12-0) total cost TC*1(*Q*<sup>∗</sup> 1 ) *increases with respect to t for a fixed K regardless of whether or not*  $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$ . Increasing *t* not only reduces the emission level as shown in Example [3,](#page-11-3) but also leads to more cost. *Too much increasing cost is not conducive to a firm's operation. Therefore, when the regulator needs to determine the carbon-tax rate, they must make a trade-off between emission level and cost.*

<span id="page-12-1"></span>**Example 6.** Influences of *K* on the total  $TC_1(Q_1^*)$  (Theorem [6\)](#page-7-1) is illustrated by the example in the form of *Figure [7.](#page-12-1) As shown in Figure [7,](#page-12-1) total cost falls very fast with the increasing K initially and the curve tends to be flat as K increases further. This means that enhancing LCAL is more efficient on the cost reduction in the* initial stage. Putting Figures [5](#page-11-2) and [7](#page-12-1) together, we get an interesting observation: both of  $E_1(Q_1^*)$  and  $TC_1(Q_1^*)$ *are decreasing in K under the same parameter setting. The carbon tax reduces emission level but increases cost. In this sense, enchanting LCAL is preferred over introducing carbon-tax regulation.*



**Figure 7.** The influence of *K* on the total cost  $TC_1(Q_1^*)$  ( $D_0 = 600$ ,  $A = 120$ ,  $h = 12$ ,  $c = 3$ ,  $\hat{h} = 1$ ,  $\hat{c} = 1, t = 5$ .

The influences of *p* and *K* on  $Q_2^*$ ,  $E_2(Q_2^*)$  and the influences of *t* and *K* on  $Q_1^*$ ,  $E_2(Q_2^*)$  are the same, while *C* does not directly affect  $Q_2^*$ ,  $E_2(Q_2^*)$ . Therefore, we only present the example to illustrate the influences of the exogenous variables on total cost  $TC_2(Q_2^*)$ .

**Example 7.** The influence of  $p$  on the total  $TC_2(Q_2^*)$  (Theorem [9\)](#page-9-0) is illustrated by the example in the form of *Figure [8.](#page-13-0)*

<span id="page-13-0"></span>

**Figure 8.** Influence of *p* on the total cost with different emission caps ( $D_0 = 600$ ,  $A = 120$ ,  $h = 12$ ,  $c = 3, \hat{A} = 12, \hat{h} = 1, \hat{c} = 1, K = 5$ .

*As shown in Figure* [8](#page-13-0)*, emission level decreases from 106.72 to 106.69 (two decimal place) as p decreases from 0 to 30. When*  $C = 106$  (<106.69), according to Theorem [9,](#page-9-0) total cost  $TC_2(Q_2^*)$  is increasing in  $p$  as  $p$  lies in the interval [0, 30]. While  $C = 107$  (>106.69), total cost  $TC_2(Q_2^*)$  decreases in  $p$  as  $p$  lies in the interval [0, 30].

#### **5. Conclusions**

To have a positive response to carbon-emission regulation policies, firms worldwide should implement operational modes to adapt the regulation. In this study, we focused on inventory management under two common carbon-regulation policies: carbon-tax regulation and cap-and-trade regulation. We solved optimal order sizes under the two regulation policies. We derived some analytic results about the influences of carbon-tax rate, LCAL, carbon price, and the cap allocated on the optimal order sizes, the emission levels, and total costs. A series of numerical examples illustrates the theoretical results and presents some interesting observations.

In this study, we found that the allocated cap does not affect the optimal order size under cap-and-trade regulation. As optimal order size and the corresponding emission level are concerned, tax rate and carbon price play the same role. The allocated cap only affects total cost. For a fixed LCAL, the influences of tax rate and carbon price on optimal order size and corresponding emission level depend on the relationship between  $\frac{\hat{A}}{\hat{h}}$  and  $\frac{A}{h}$ . When the carbon price is regarded as a function of the cap, the cap under the cap-and-trade regulation affects the optimal order size and the corresponding emission level. The main difference between the two regulation policies is the influences of parameters on total costs. Under carbon-tax regulation, total cost increases in tax rate. Under cap-and-trade regulation, the influence of carbon price on total cost is related to the the relationship between actual emission levels and the allocated cap. Compared to other carbon-regulated EOQ models, this study concerns customers' low-carbon awareness. We analyzed the influences of LCAL on optimal order size, corresponding emission level, and total cost under different regulation policies. The role of LCAL under different regulation policies are similar because it is independent of the allocated cap. The first-order derivative of the optimal order size, emission level, and total cost with respect to LCAL is so complex that it is hard to determine the signs. However, we derived sufficient conditions under which the optimal order size, the corresponding emission levels, and total costs decreased in LCAL under relatively strong assumptions. These sufficient conditions are relaxed in numerical simulations. From numerical simulations, we found that enhancing customers' LCAL could simultaneously reduce emission level and total cost. This reduction effect was more pronounced in the early stage of enhancing customers' LCAL, which inspires regulators to enhance customers' LCAL through publicity and education.

This study is a reasonable extension of the classical EOQ model. However, this study is subject to some limitations which implies the research directions. First, we assume that LCAL has linear influence on demand. More realistic function structure in LCAL could be conducted based on regression analysis. Second, only one decision variable is considered in this study. In reality, managers may invest in low-carbon technologies to reduce the emission. A possible extension of this study is incorporating joint decisions on order size and emission reduction investment based on the current models. In addition, some features of inventory systems, such as backorder and deteriorating items could also be incorporated in future research.

**Author Contributions:** Z.T. conceived and designed the models. J.X. conducted the numerical experiment. Both authors read and approved the final manuscript.

**Funding:** This research was funded by the National Natural Science Foundation of China (Grant No. 71401114) and the Fundamental Research Funds for the Central Universities (Funded by Sichuan University, Grant No. skqy201524).

**Conflicts of Interest:** The authors declare no conflict of interest.

# **References**

- <span id="page-14-0"></span>1. Stern, N. The Economics of Climate Change. *Am. Econ. Rev.* **2008**, *98*, 1–37. [\[CrossRef\]](http://dx.doi.org/10.1257/aer.98.2.1)
- <span id="page-14-1"></span>2. Huang, M.; Chen, Y.; Zhang, Y. Assessing Carbon Footprint and Inter-Regional Carbon Transfer in China Based on a Multi-Regional Input-Output Model. *Sustainability* **2018**, *10*, 4626. [\[CrossRef\]](http://dx.doi.org/10.3390/su10124626)
- <span id="page-14-2"></span>3. Carl, J.; Fedor, D. Tracking global carbon revenues: A survey of carbon taxes versus cap-and-trade in the real world. *Energy Policy* **2016**, *96*, 50–77. [\[CrossRef\]](http://dx.doi.org/10.1016/j.enpol.2016.05.023)
- <span id="page-14-3"></span>4. Nemet, G.F.; Jakob, M.; Steckel, J.C.; Edenhofer, O. Addressing policy credibility problems for low-carbon investment. *Glob. Environ. Chang. Hum. Policy Dimens.* **2017**, *42*, 47–57. [\[CrossRef\]](http://dx.doi.org/10.1016/j.gloenvcha.2016.12.004)
- 5. Markandya, A.; Arto, I.; Gonzalezeguino, M.; Roman, M.V. Towards a green energy economy? Tracking the employment effects of low-carbon technologies in the European Union. *Appl. Energy* **2016**, *179*, 1342–1350. [\[CrossRef\]](http://dx.doi.org/10.1016/j.apenergy.2016.02.122)
- <span id="page-14-4"></span>6. Omer, A.M. Focus on low carbon technologies: The positive solution. *Renew. Sustain. Energy Rev.* **2008**, *12*, 2331–2357. [\[CrossRef\]](http://dx.doi.org/10.1016/j.rser.2007.04.015)
- <span id="page-14-5"></span>7. Zhang, B.; Lai, K.; Wang, B.; Wang, Z. The clean development mechanism and corporate financial performance: Empirical evidence from China. *Resour. Conserv. Recycl.* **2018**, *129*, 278–289. [\[CrossRef\]](http://dx.doi.org/10.1016/j.resconrec.2017.10.004)
- <span id="page-14-6"></span>8. Benjaafar, S.; Li, Y.; Daskin, M. Carbon Footprint and theManagement of Supply Chains: Insights from Simple Models. *IEEE Trans. Autom. Sci. Eng.* **2013**, *10*, 99–116. [\[CrossRef\]](http://dx.doi.org/10.1109/TASE.2012.2203304)
- <span id="page-14-7"></span>9. Zhao R.; Geng, Y.; Liu, Y.; Tao, X.; Xue, B. Consumers' perception, purchase intention, and willingness to pay for carbon-labeled products: A case study of Chengdu in China. *J. Clean. Prod.* **2018**, *171*, 1664–1671. [\[CrossRef\]](http://dx.doi.org/10.1016/j.jclepro.2017.10.143)
- <span id="page-14-8"></span>10. Zhang, B.; Xu, L. Multi-item production planning with carbon cap and trade mechanism. *Int. J. Prod. Econ.* **2013**, *144*, 118–127. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2013.01.024)
- <span id="page-14-9"></span>11. Palak, G.; Eksioglu, S.D.; Geunes, J. Analyzing the impacts of carbon regulatory mechanisms on supplier and mode selection decisions: An application to a biofuel supply chain. *Int. J. Prod. Econ.* **2014**, *154*, 198–216. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2014.04.019)
- <span id="page-14-10"></span>12. Chang, X.; Xia, H.; Zhu, H.; Fan, T.; Zhao, H. Production decisions in a hybrid manufacturing-remanufacturing system with carbon cap and trade mechanism. *Int. J. Prod. Econ.* **2015**, *162*, 160–173. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2015.01.020)
- <span id="page-14-11"></span>13. Xu, X.; He, P.; Xu, H.; Zhang, Q. Supply chain coordination with green technology under cap-and-trade regulation. *Int. J. Prod. Econ.* **2017**, *183*, 433–442. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2016.08.029)
- <span id="page-14-12"></span>14. Tseng, S.C.; Hung, S.W. A strategic decision-making model considering the social costs of carbon dioxide emissions for sustainable supply chain management. *J. Environ. Manag.* **2014**, *133*, 315–322. [\[CrossRef\]](http://dx.doi.org/10.1016/j.jenvman.2013.11.023)
- <span id="page-14-13"></span>15. Zakeri, A.; Dehghanian, F.; Fahimnia, B.; Sarkis, J. Carbon pricing versus emissions trading: A supply chain planning perspective. *Int. J. Prod. Econ.* **2015**, *164*, 197–205. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2014.11.012)
- <span id="page-14-14"></span>16. Lee, K.H. Integrating carbon footprint into supply chain management: The case of Hyundai Motor Company (HMC) in the automobile industry. *J. Clean. Prod.* **2011**, *19*, 1216–1223. [\[CrossRef\]](http://dx.doi.org/10.1016/j.jclepro.2011.03.010)
- <span id="page-14-15"></span>17. Cachon, G.P. Retail store density and the cost of greenhouse gas emissions. *Manag. Sci.* **2014**, *60*, 1907–1925. [\[CrossRef\]](http://dx.doi.org/10.1287/mnsc.2013.1819)
- <span id="page-15-0"></span>18. Bhattacharya, A.; Dey, P.K.; Ho, W. Green manufacturing supply chain design and operations decision support. *Int. J. Prod. Econ.* **2015**, *53*, 1–5. [\[CrossRef\]](http://dx.doi.org/10.1080/00207543.2015.1065021)
- <span id="page-15-1"></span>19. Hua, G.; Cheng, T.C.E.; Wang, S. Managing carbon footprints in inventory management. *Int. J. Prod. Econ.* **2011**, *132*, 178–185. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2011.03.024)
- <span id="page-15-2"></span>20. Chen, X.; Benjaafar, S.; Elomri, A. The carbon-constrained EOQ. *Oper. Res. Lett.* **2013**, *41*, 142–149. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2012.09.019)
- <span id="page-15-3"></span>21. Battini, D.; Persona, A.; Sgarbossa, F. A sustainable EOQ model: Theoretical formulation and applications. *Int. J. Prod. Econ.* **2014**, *149*, 145–153. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2013.06.026)
- <span id="page-15-4"></span>22. Toptal, A.; Ozlu, H.; Konur, D. Joint decisions on inventory replenishment and emission reduction investment under different emission regulations. *Int. J. Prod. Res.* **2014**, *52*, 243–269. [\[CrossRef\]](http://dx.doi.org/10.1080/00207543.2013.836615)
- <span id="page-15-5"></span>23. Bozorgi, A.; Pazour, J.A.; Nazzal, D. A new inventory model for cold items that considers costs and emissions. *Int. J. Prod. Econ.* **2014**, *155*, 114–125. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2014.01.006)
- <span id="page-15-6"></span>24. Kazemi, N.; Abdulrashid, S.H.; Shekarian, E.; Zanoni, S. Economic order quantity models for items with imperfect quality and emission considerations. *Int. J. Syst. Sci.* **2018**, *5*, 99–115. [\[CrossRef\]](http://dx.doi.org/10.1080/23302674.2016.1240254)
- <span id="page-15-7"></span>25. Bouchery, Y.; Ghaffari, A.; Jemai, Z.; Dallery, Y. Including Sustainability Criteria into Inventory Models. *Eur. J. Oper. Res.* **2012**, *222*, 229–240. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2012.05.004)
- <span id="page-15-8"></span>26. Tiwari, S.; Daryanto, Y.; Wee, H.M. Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *J. Clean. Prod.* **2018**, *192*, 281–292. [\[CrossRef\]](http://dx.doi.org/10.1016/j.jclepro.2018.04.261)
- <span id="page-15-9"></span>27. Letmathe, P.; Balakrishnan, N. Environmental Considerations on the Optimal ProductMix. *Eur. J. Oper. Res.* **2005**, *167*, 398–412. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2004.04.025)
- <span id="page-15-10"></span>28. Bouchery, Y.; Ghaffari, A.; Jemai, Z.; Tan, T. Impact of coordination on costs and carbon emissions for a two-echelon serial economic order quantity problem. *Eur. J. Oper. Res.* **2017**, *260*, 520–533. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2016.12.018)
- <span id="page-15-11"></span>29. Shuai, C.; Ding, L.; Zhang, Y.; Guo, Q.; Shuai, J. How consumers are willing to pay for low-carbon products?-Results from a carbon-labeling scenario experiment in China. *J. Clean. Prod.* **2014**, *83*, 366–373. [\[CrossRef\]](http://dx.doi.org/10.1016/j.jclepro.2014.07.008)
- <span id="page-15-12"></span>30. Liu, Z.; Anderson, T.; Cruz, J.M. Consumer environmental awareness and competition in two-stage supply chains. *Eur. J. Oper. Res.* **2012**, *218*, 602–613. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2011.11.027)
- <span id="page-15-13"></span>31. Xia, L.; He, L. Game theoretic analysis of carbon emission reduction and sales promotion in dyadic supply chain in presence of consumers low-carbon awareness. *Discret. Dyn. Nat. Soc.* **2012**. [\[CrossRef\]](http://dx.doi.org/10.1155/2014/837376)
- <span id="page-15-14"></span>32. Cheng, Y.; Sun, H.; Jia, F.; Koh, L. Pricing and Low-Carbon Investment Decisions in an Emission Dependent Supply Chain under a Carbon Labelling Scheme. *Sustainability* **2018**, *10*, 1238. [\[CrossRef\]](http://dx.doi.org/10.3390/su10041238)
- <span id="page-15-15"></span>33. Zhang, Z.; Wang, W.; You, J. Consumer environmental awareness and channel coordination with two substitutable products. *Eur. J. Oper. Res.* **2015**, *241*, 63–73. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ejor.2014.07.043)
- <span id="page-15-16"></span>34. Yu, Y.; Han, X.; Hu, G. Optimal production for manufacturers considering consumer environmental awareness and green subsidies. *Int. J. Prod. Econ.* **2016**, *182*, 397–408. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2016.09.014)
- <span id="page-15-17"></span>35. Hovelaque, V.; Bironneau, L. The carbon-constrained EOQ model with carbon emission dependent demand. *Int. J. Prod. Econ.* **2015**, *164*, 285–291. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ijpe.2014.11.022)
- <span id="page-15-18"></span>36. Hillier, F.S.; Lieberman, G.J. (Eds.) *Introduction to Operations Research*; McGraw-Hill: New York, NY, USA, 2010; pp. 833–843.



 c 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license [\(http://creativecommons.org/licenses/by/4.0/\)](http://creativecommons.org/licenses/by/4.0/.).