Financing for a Sustainable PPP Development: Valuation of the Contractual Rights under Exercise Conditions for an Urban Railway PPP Project in Korea

Kangsoo Kim 1, Hyejin Cho 2 and Donghyung Yook 3,*

1 Department of Public Finance and Social Policy, Korea Development Institute, Sejong-si 30149, Korea; kskim@kdi.re.kr
2 Highway & Transportation Research Division, Korea Institute of Civil Engineering and Building Technology, Goyang-si 10223, Korea; hjcho@kict.re.kr
3 National Infrastructure Research Division, Korea Research Institute for Human Settlements, Sejong-si 30147, Korea
* Correspondence: dhyook@krihs.re.kr; Tel.: +82-44-960-0366

Received: 11 February 2019; Accepted: 12 March 2019; Published: 15 March 2019

Abstract: The Minimum Revenue Guarantee (MRG) was designed to mitigate the financial risk of private investors that participate in the transportation project as concessionaire under a public-private partnership (PPP) program. The MRG can pose a significant financial burden to governments especially when the contract revenue is set considerably higher than the actual revenue. This may encourage the concessionaire to inflate the traffic forecast to make the project look as if it will be profitable. In order to mitigate this problem, extra conditions for exercising the MRG can be considered. This study examines how these exercise conditions change the economic value of the MRG using the case study based on the urban railway project in the Republic of Korea. By utilizing the real options analysis, the study identified that the exercise conditions have worked to curtail the expected payment from the government, eventually leading to a reduction in the concessionaire’s expectation of revenue. The value of MRG was at a far lower level compared to the concessionaire’s investment because of the low probability of exercising the MRG when the exercise conditions apply. The findings are expected to contribute to the sustainability of the PPP program by recognizing and quantifying liabilities and risks embedded in the concession agreement in advance.

Keywords: economic value; minimum revenue guarantee; excess revenue sharing; exercise conditions; Public-Private Partnership; urban railway

1. Introduction

The shortage of transportation infrastructure due to the imbalance between demand and supply is a common issue in all countries. Emerging economies that are experiencing rapid economic growth need to provide transportation infrastructure to support the growing demand. Even in countries that passed the period of rapid growth, the occurrence of traffic congestion indirectly suggests the need for building a new transportation infrastructure. However, lack of financial resources is always a problem for governments that need to supply transportation infrastructure in a timely manner.

PPP has been recognized as one of the most effective project delivery systems to address the limitations of financial resources of governments. PPP is defined as the contractual agreement between public agency and the private investors for delivering services or facility to the public [1]. Under the PPP agreement, private investors are not just providing the financial resources, but they have
the opportunities to enhance efficiencies and profitability associated with each phase of the project development. Thus, through the PPP, the government can expect to provide enhanced public services with better efficiency.

When the governments develop a PPP program for delivering transportation infrastructure, their financial support is necessary because it is a high-risk business for private investors which usually finance the capital through interest rates higher than the government. Also, the fact that transportation projects require a huge amount of capital investment for a long period of time necessitates government support, usually in the form of subsidies, grants or guarantees. Among different types of support [2], the MRG has been the popular choice for the government since it typically incurs no immediate cost but only the possibility of future liabilities [3].

However, the MRG could pose a significant financial burden to the government [4,5] when the contract revenue is set considerably higher than the actual revenue. For example, when the PPP agreement is made with a very high contract revenue, and the actual revenue is lower than the contract revenue, the government has to fill the gap through monetary support. In case of the Korean government, they had to make a significant payment every year until recently: 823 billion KRW (686 million USD) in 2013, 868 billion KRW (723 million USD) in 2014, and 970 billion KRW (808 million USD) in 2015. The concessionaire can exploit the downsides of the MRG by inflating the traffic forecasts of the project to make it seem like it will be profitable [6]. According a 2005 survey of international toll roads, bridges and tunnels, traffic forecasts are typically optimistic and tend to be approximately 23% higher than the actual traffic demand on average [7].

To mitigate this problem, the government can consider imposing extra conditions for exercising the MRG. In other words, exercise conditions must be met for the MRG to be paid. For example, if the contract revenue is set to 100 (A) and the actual revenue is identified as 40 (B), the government must pay 60 to the concessioner when there is no condition for exercising the MRG. On the other hand, the government does not pay the MRG if the exercise conditions only allow the MRG to be paid when the ratio of B/A is 50% or higher. With such exercise conditions, private investors which tend to raise the contract revenues at the negotiation table have no reason to inflate the traffic forecasts.

On the behavioral perspective, exercise conditions prevent private investors from inflating the traffic forecasts. However, it is necessary to closely examine the economic value of MRG because the profile of the value of MRG according to exercise conditions can be used as a useful tool for risk sharing. However, not enough research has been conducted yet regarding the effect of these exercise conditions on the valuation of the MRG. This study aims to understand how the MRG values have changed with the addition of exercise conditions under the frame of the real options analysis by examining the case of an urban railway PPP project in Korea. To the best of the authors’ knowledge, whereas research on valuation of options such as the MRG and Excess Revenue Sharing (ERS) are abundant in the literature, previous studies that dealt with exercise conditions for the options are extremely limited.

This paper is organized as follows. Following the introductory Section 1, Section 2 reviews the literature on valuation methods of options associated with PPP programs. Section 3 outlines an urban railway BTO project as a case study and its options embedded in the concession agreement. Section 4 presents the preliminary procedure and associated methodology for estimating the value of options. This is required for the explicit modeling of uncertainty in passenger forecasts, which is the biggest characteristic of transportation investment projects. Section 5 presents the economic values of the options for the case project and analyzes the change when exercise conditions are introduced. Finally, the last section delivers important implications for the terms and conditions that should be considered when preparing PPP agreements, and identifies the limitations of this paper that require further research.

2. Literature Review

Since the option theory for financial assets was expanded into real options analysis (the theoretical origin and historical development of real options theory can be found in [8–13]) for real assets, context
and the types of options also have been diversified. Particularly to transportation projects, several types of options are proposed such as contractual rights including the MRG and ERS, and system flexibilities including the options of delay, expansion, buyout, conditional buyout and even abandonment [1,14]. Recently, the option that limits similar projects nearby the underlying transportation project has been studied by [15]. The variety of the options in different types of industries can be found in [16] where the taxonomy of the options is well documented.

By definition, an option is a right, but not an obligation, to exercise a certain action in the face of uncertainty [17]. The fact that the options are dependent on future conditions which cannot be predicted with certainty requires the option valuation to inherently involve modeling of the uncertainty of the underlying assets. In the case of transportation projects, traffic volume is the most important source of uncertainty which determines a large part of the revenue. Also, the operations and management cost, interest rates, volatility of the markets are major components that characterize the uncertainty in transportation projects.

Because real options are delivered in different forms to represent characteristics of the underlying assets and its contingent nature, a rich repertory of methodologies has been proposed to correctly model and evaluate real options. Broadly speaking, the methodologies can be categorized into three groups: analytical methods, numerical methods, and simulation methods [6]. Except for the analytical approach, one methodology is not necessarily used for one purpose only, but for many other purposes (Figure 1). This is probably due to the fact that real options involve a complicated and dynamic process of real options analysis. For example, the MCS is applied to deal with the uncertainty in traffic forecasts by randomly generating a number of different values, but the binomial lattice plays an identical function in [1]. These binomial lattices are also applied to a methodology for reflecting flexibilities [18,19]. Following is the summary of the methodological development for valuing options, which explicitly considers uncertainty.

![Figure 1. Methodologies for treating options and uncertainty in real options.](image-url)
The analytical approach to value the options begins with the Black–Scholes formula which models the price variation of stocks over time, which is used to determine the price of the European call option. Despite of the limitations of the model due to the lack of realistic assumptions on the risk-free rate and the distribution of the future stock prices [20], the formula still serves as one of the best ways to determine fair prices of options. The Black–Scholes formula has been reformulated under fuzzy set theory by Wu [21] in order to overcome the unrealistic parameter settings of the formula. The following are the recent developments of the analytical model for valuing the real options observed in PPP concession. Pimentel et al. [22] derived a closed form of solution to value high-speed railway (HSR) investment projects in order to deal with the stochastic nature of demand and investment expenditure. This study is succeeded by Couto et al. [23] who extended the previous model by incorporating the positive or negative shocks in the change of demand for HSR. Most recently, the shocks on both HSR demand and investment expenditures were processed by Poisson process [24] which resulted in a consistent outcome presented in [23]. Huang and Chou [25] also developed the option pricing formulas for valuing the combined flexibility of the MRG and the option to abandon.

Most literature takes the numerical approach to value the options because this method is flexible enough to represent the dynamic nature of the real options. Bowe and Lee [26] utilized the log-transformed binomial valuation model developed by Trigeorgis [9] for valuing several compound options associated with the Taiwan HSR project because the method efficiently handles the interactions among different options which generally makes their individual value non-additive. This binomial valuation method is further developed [27] by integrating fuzzy theory in order to reflect the flexibilities of investment decisions such as expansion, extension, or even abandonment of an underlying project. Applications of fuzzy set theory for valuation of real options, particularly to Black–Scholes formula and American put option can be found in [21,28,29] respectively. The risk-neutral valuation method [28,30] is another popular choice for the researcher because it segregates the risk of traffic forecast uncertainty from the traffic market risk in the valuation of the options. Using this feature of the risk-neutral valuation method, Brandao and Saraiva [11] did not treat the revenue of the transportation project as a marketed asset in their minimum traffic guarantee model since the uncertainty in revenue is only dependent on the traffic forecast. The similar risk-neutral valuation method can be found in [1] which emphasizes the merit of the risk-neutral approach for valuing the options in transportation projects against the NPV (Net Present Value). However, the NPV also evolved to account for the uncertainty in traffic forecasts. Ye and Tiong [31] devised the NPV-at-risk model which outputs the NPV at a given confidence level by establishing the cumulative NPV curve assisted by the Monte Carlo Simulation (MCS), making it possible to determine the feasibility of a project with the given confidence level. Later, this NPV-at-risk model is utilized by Kumar et al. [32] for identifying the critical risk factors in thirty real-life PPP projects in India. In the variance model [6,33], uncertainty in traffic forecast is analyzed from two different perspectives which include learning (uncertainty due to lack of knowledge) and increasing uncertainty (intrinsic and natural variation). Modeling the uncertainty by following the properties of two different uncertainty sources would be expected to enhance the estimate of the project’s financial risk.

The MCS has served as a useful tool to tackle the uncertainty issue of transportation projects. The MCS randomly produces a number of values for traffic volumes based on the given distribution, which is the basis for calculating the expected values of options. Several studies [1,17,19,34–38] are dependent on the MCS for dealing with the randomness of traffic forecast. Cheah and Liu [17] adopted the MCS in order to reflect the uncertainty in traffic forecasts which randomly samples the initial traffic and its growth rate to determine the distributions of the total subsidy and the repayment scheme. A similar application of the MCS for handling the uncertainty in traffic forecasts can be found in [6,34] where the technique is modified (Multi-Least Squares Monte Carlo) for structuring the dynamic contract of the Australian option system.

Despite the methodological development for handling uncertainty in transportation projects, the way to deal with the uncertainty in traffic forecasts still needs to be improved. Most studies
quantify uncertainty just by randomizing the traffic volume. However, the traffic volume on a certain roadway section is the result of complicated decisions such as the choice of the route, choice of modes of transportation, and choice of making trips. Taking route choice as an example, travelers usually seek the best route in terms of travel time. If the best route is congested, some portions of travel demand would switch to other routes, and these behavioral patterns will be stabilized to a certain degree. The uncertainty models that consider these behavioral aspects would be different from the ones without.

This study is differentiated from previous studies in that it addresses the uncertainty associated with traffic forecasts based on the in-depth analysis of travel behavior. The study attempts to capture the inherent uncertainty in traffic forecasts in transportation projects through modeling the generation of trips and their behaviors in the transportation networks by adopting the traditional four-step model utilized for transportation demand analysis. The following section describes the details of the methodology.

3. Preliminary Procedure and Methodology for Estimating the Value of Minimum Revenue Guarantee (MRG) and ERS

The purpose of this study is to investigate the effects of exercise conditions on the value of MRG and ERS. In order to correctly value the contractual guarantees under the exercise conditions, the study adopts the risk-neutral valuation method because this method is known to be advantageous for treating the uncertainty in passenger forecasts (so far, traffic forecast has been used for ease of understanding, hereafter, traffic forecast has been replaced with passenger forecast in accordance with the case project) and the market risk in the valuation of the guarantees [1]. The risk-neutral valuation approach is effective when the risk-adjusted probabilities of the underlying asset are available because once the probabilities are identified, the option payoffs are just simply computed by applying the risk-free rate, which implies that the future risk-adjusted discount rate is not necessary. The risk-neutral approach is adequate for achieving the objective of the study where the uncertainty in passenger forecasts is explicitly dealt with risk-adjusted probabilities using the specialized analysis framework. The following describes the preliminary procedure and methodology dedicated to finding the value of MRG and ERS of the underlying project by reflecting the uncertainty in passenger forecast (Figure 2).

![Figure 2. Preliminary procedure and methodology for estimating value of Minimum Revenue Guarantee (MRG) and ERS.](image)

In step 1, the total travel demand is populated with the Poisson regression that models the socio-economic characteristics of the analysis area. Here, the uncertainty in passenger forecasts is quantified through the transit assignment which models the passengers route choices in the given transit network. Then in step 2, the volatility of the passenger forecast is obtained by implementing the MCS which mirrors the probability distribution of the historical data. Finally, the risk premium of the passenger forecast is determined by using the Capital Asset Pricing Model (CAPM) in step 3.
3.1. A Case Project: An Urban Railway BTO Project

A typical urban railway BTO project (A-Line Project) which is currently operating in Seoul, Korea was selected as the case project to evaluate the economic value of MRG and ERS. The A-Line is approximately 18.5 km in length and is designed to pass through six stations.

The total project cost was 1169 billion KRW, where 561.1 billion KRW was subsidized by the government and 607.8 billion KRW was invested by a private Special Purpose Company (SPC). The total project cost only included the expense for the construction and did not include the land acquisition cost which was covered by the Korean government. (See Table 1 for more details)

<table>
<thead>
<tr>
<th>Contents</th>
<th>A-Line Urban Railway PPP Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project Name</td>
<td>Main Line: 18.5 km, Connecting Line: 2.3 km, Six Stations</td>
</tr>
<tr>
<td>Length</td>
<td>80% for the first 5 years, 70% for the 5 years afterwards</td>
</tr>
<tr>
<td>Revenue Guarantee</td>
<td>21 July 2005</td>
</tr>
<tr>
<td>Date of Commencement</td>
<td>31 December 2009</td>
</tr>
<tr>
<td>Date of Completion</td>
<td>1169 billion KRW (private investment: 607.8 billion KRW, construction subsidy: 561.1 billion KRW)</td>
</tr>
<tr>
<td>Total Project Cost</td>
<td></td>
</tr>
</tbody>
</table>

It was agreed that the concessionaire shall have MRG as the put option. The revenue would be guaranteed for the concessionaire at 80% of the contract revenue from the 1st to the 5th year, 70% of the contract revenue from the 6th to the 10th year when the actual revenue is 50% or higher of the contract revenue. In addition, the government shall have the call option for sharing of the excess revenue (ERS). The government and the concessionaire share the revenue when the actual revenue is higher than 120% of the contract revenue for the first 5 years, and 130% from the 6th to the 10th year. The following equations express the MRG and ERS.

$$\text{MRG}_1 = \max(0.8RF_{ta} - RF_{ta}), 0$$

$$\text{RER}_1 = -\max(1.2RF_{ta} - RF_{ta}), 0$$

$$\text{MRG}_2 = \max(0.7RF_{ta} - RF_{ta}), 0$$

$$\text{RER}_2 = -\max(1.3RF_{ta} - RF_{ta}), 0$$

3.2. Modeling the Uncertainty in Passenger Forecasts

Risk analysts and transportation experts deal with the uncertainty associated with passenger forecast in different ways [33]. Risk analysts observe the pattern and change of traffic forecast and seek parameters and events that influence the forecast. On the other hand, traffic analysts make a more fundamental approach to understanding the traffic itself. They identify the uncertainty by analyzing how traffic generation is influenced by socio-economic factors, the origin and destination of the travel, the mode of transport taken, and the route choice. This is done through the analysis of each stage of the four-step model, and the main causes of risk for each stage are shown in the following Table 2. Since the results from each step are calibrated and validated, the outcomes of the four-step model are widely used for testing transportation policies. This study is in line with the four-step model because the uncertainty analysis begins with the first step of the four-step model, trip generation. The subsequent sections address the modeling of uncertainty in passenger forecasts from the stage of trip generation.

Trip generation is represented as a stochastic distribution. Various factors such as closeness to stations, population, number of households, workers, students and the level of services of the competing transportation modes affect the uncertainty in passenger forecast for the A-line.
Table 2. Different perspectives on dealing with uncertainty of forecast.

<table>
<thead>
<tr>
<th>Risk Analysts</th>
<th>Traffic Experts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observe the pattern and change of traffic forecast and seek parameters and events that influence the forecast</td>
<td>Examine the risk factor affecting the traffic forecast under the four-step model</td>
</tr>
<tr>
<td>- construct the portfolio via scenario analysis, sensitivity analysis</td>
<td>- Trip generation: risk of economic and population development</td>
</tr>
<tr>
<td>- focus on the probability of a project’s success or failure</td>
<td>- Trip distribution and mode choice: risk of model selection and data collection</td>
</tr>
<tr>
<td>- consider possible management schemes for dealing with events</td>
<td>- Passenger assignment: risk of competition of transport modes, passengers’ behaviors in the network.</td>
</tr>
</tbody>
</table>

This study reflected the influence of these factors using the Poisson regression model. If the total travel demand for zone $i$, $P_i^*$, is assumed to follow the Poisson distribution with B times of the random numbers, the socio-economic variables play as the determinants for shaping the distribution.

$$\hat{\lambda}(x_i) = \exp(\hat{\beta}_0 + \sum_{j=1}^{p} \hat{\beta}_{ij} x_{ij}), \quad (3)$$

$$P_i^{*+1}, P_i^{*+2}, \ldots, P_i^{*+B} \sim \text{Poisson}(\hat{\lambda}(x_i)) \quad (4)$$

Once the travel demand for zone $i$ is estimated, the travel demand needs to be split into the directional travel demand which originates from zone $i$ and headed to zone $j$, i.e., $y_{ij}^*$. 

$$y_{ij}^{*(1)} = r_{ij} \times P_i^{*(1)}, \ldots, y_{ij}^{*(B)} = r_{ij} \times P_i^{*(B)} \quad (5)$$

where, $r_{ij}$ is the transit mode share for each origin-destination pair associated with zone $i$. The study computed $r_{ij}$ using the existing travel survey data provided by the Korea Transport Database (KTDB). $r_{ij}$ is a simple ratio between the observed $P_i$ and $y_{ij}$.

$$r_{ij} = \frac{y_{ij}}{P_i} = \frac{y_{ij}}{\sum_i y_{ij}} \quad (6)$$

The estimated directional travel demand is the travel demand between two zones which does not show the number of passengers getting on and off at transit stations. It is necessary to know the number of passengers on each transit line and segment. This can be done by the transit assignment that concerns the travelers’ selection of lines when they travel from origins to destinations in transit networks. If there is just one transit line between an origin and a destination, then the travel demand will be identical to the passenger traffic. However, in the urban transit network where several modes of public transportation such as subway, buses, BRT, trams, etc. provide different mobility services, travelers choose the best transit line in consideration of the operation speed, capacity, frequency, and geometric location of the transit lines. Transit assignment involves the modeling of choice behavior with an assumption that travelers would want to minimize their travel time. The passenger traffic processed by the transit assignment is designed to reflect the uncertainty due to the randomized travel demand by Poisson regression.

The transit assignment which represents the travelers’ behavior in the networks where the nodes $i \in I$ are connected by a set of links $a = (i, j) \in A$ can be formulated by the linear optimization problem [39].

$$\text{Min} \sum_{a \in A} c_a v_a + \sum_{i \in I} \omega_i \quad (7)$$

subject to

$$\sum_{a \in A_i^+} v_a = y_{rr}, i \in I v_a \leq f_a \omega_i, a \in A_i^+, i \in I v_a \geq 0, a \in A \quad (8)$$
where, $\omega_i$ is the waiting time at node $i$, $c_a$ is the travel time of the transit segment, $v_a$ is the passenger traffic of link $a$, and $f_a$ is the service frequency of link $a$. $A^+_i$, $A^-_i$ represent the set of links going out of node $i$ and the set of incoming links, respectively.

The study conducted this assignment 30 times deterministically to generate the passenger traffic of the A-line. As a result, the uncertainty in passenger forecasts (i.e., the dispersion) could be measured by calculating the change of the number of passengers getting on and off the trains. The average and standard deviation of the passenger traffic of the A-line following the normal distribution were estimated at 97,455 and 102 passengers per day, respectively.

For modeling the uncertainty in passenger forecast, this study adopted the Geometric Brownian Motion (GBM). The GBM assumption was taken in various literature $[40,41]$ to characterize uncertainty about future passenger traffic. According to the GBM, the passenger forecast of the A-line in year $t$ is expressed as the increasing rate ($\alpha$) of the passengers and its volatility ($\sigma$).

$$\frac{dK_t}{K_t} = \alpha dt + \sigma dw_t$$ \hspace{1cm} (9)

Here, $K_t$ is the passenger traffic of the A-line in year $t$, which follows the normal distribution where the average is 97,455 and the standard deviation is 102. $dw_t$ is standard Winner process which follows the normal distribution where the average is 0 and the dispersion is $1(\sim Z_t \sim N(0,1))$. $\alpha$ is the increasing rate of the passenger traffic and $\sigma$ is its volatility. This study referred to $\alpha$ and $\sigma$ as 5.9% and 22.4% respectively based on the observation in the subway line no. 5, 6, 7, and 8 currently in operation in the Seoul Metropolitan Area (See Table 3).

Table 3. Annual increasing rate and volatility of passenger traffic of the Seoul Subway Lines 5–8.

<table>
<thead>
<tr>
<th>Line</th>
<th>Section</th>
<th>Length (km)</th>
<th>Opening Date</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>Average $\alpha$</th>
<th>Average $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Banghwa-Sangil-Macheon</td>
<td>52.3</td>
<td>30 December 1996</td>
<td>0.049</td>
<td>0.202</td>
<td>0.069</td>
<td>0.224</td>
</tr>
<tr>
<td>6</td>
<td>Eungam-Bonghwason</td>
<td>35.1</td>
<td>9 March 2001</td>
<td>0.060</td>
<td>0.059</td>
<td>0.066</td>
<td>0.177</td>
</tr>
<tr>
<td>7</td>
<td>Jangam-Onsu</td>
<td>46.9</td>
<td>1 August 2000</td>
<td>0.066</td>
<td>0.272</td>
<td>0.066</td>
<td>0.224</td>
</tr>
<tr>
<td>8</td>
<td>Amsa–Moran</td>
<td>17.7</td>
<td>2 July 1999</td>
<td>0.062</td>
<td>0.224</td>
<td>0.062</td>
<td>0.199</td>
</tr>
</tbody>
</table>


3.3. Volatility of the A-Line Project Value

The value of a rail line project can be represented by the net present value which identifies the difference between the revenue and the cost of operation and maintenance of the rail line. Then, the volatility of the A-line Project ($\sigma_p$) can be defined as the change of the cash flow due to the uncertainty in passenger forecasts. Be noted that only the uncertainty in passenger forecasts was taken as a stochastic variable to determine the revenue uncertainty. Since the study deals with the issue with a real case project, the average (97,455) and the standard deviation (102) of the passenger traffic obtained in the previous section enable us to generate random numbers with the MCS. The simulation produces randomized passenger traffic of the A-line Project in year $t$ ($K_t$) then, the subsequent annual passenger traffic was calculated using the increasing rate of 0.059, and its volatility, 0.224. After 1000 times of random number generation, the volatility of the value of the A-line project ($\sigma_p$) during the operation period was estimated at 65%.

3.4. Value of the Passenger Forecasting Risk

The investors of the A-line Project would request a compensation value (risk premium) or an excess expected rate of return to avoid the passenger forecasting risk. To estimate the value of this
risk, this paper applied the CAPM developed by [42] which estimated the beta \( \beta \) from the stock rate of return of companies investing in BTO PPP projects in Korea. They used the stock price data between 2001 and 2011 and the monthly rate of return was calculated based on the adjusted stock price that considered the dividend rate of the companies which invested in BTO PPP projects. The beta value \( \beta \) estimated based on monthly data was 1.2381 and was significant at a confidence level of 99%. As a result, when 5.5% was applied as the risk-free rate of return, the excess expected rate of return (The excess expected rate of return implies the risk premium to avoid risk in predicting the passenger traffic when 5.5% is applied as the risk-free rate of return) was calculated to be 11.04% a year after the conversion of the beta value \( \beta \) by year. The value of the passenger forecasting risk (i.e., risk premium) \( \lambda \) for the A-line Project was analyzed as 17% when the passenger forecasting risk increases by 1%.

4. Valuation of the MRG and ERS with Exercise Conditions

So far, the risk premium of passenger forecasting uncertainty \( \lambda \) has been identified. Since the risk-adjusted probabilities of the case project is identified, application of the risk-free rate would yield the correct valuation of the MRG and ERS. The following formula shows the change in passenger forecasts for the A-line Project under a risk-neutral environment.

\[
\frac{dK_t}{K_t} = (\alpha - \lambda \sigma)dt + \sigma dw_t
\]  

(10)

In order to estimate the amount of government payment and/or share, the number of passengers for year \( t \) is generated, and the expected revenue for year \( t \) is calculated by multiplying the fare to the forecasted passenger traffic. Then, the expected revenue for year \( t \) is compared with the contract revenue for year \( t \) and the amount of government payment and/or share according to the MRG and ERS options are added respectively for 10 years. This process is iterated for 1000 times.

Figure 3 shows the amount of government payment based on the agreement expressed in Equations (1) and (2) and the ratio of expected revenue (Expected revenue refers to the revenue that can be obtained while operating the A-line. The expected revenue is obtained by a stochastic calculation of the number of passengers. The expected revenue is considered as the actual revenue in this paper) to contract revenue (Contract revenue implies the revenue for which the agreement between the concessionaire and the government is made. It becomes the basis of government payments according to the MRG conditions) by operating years.

For the opening years, the ratio of the expected revenue to the contract revenue was estimated at 59% and the expected government payment was analyzed as 7.9 billion KRW. The amount of government payment decreased as the chances that the expected revenue would exceed 50% of the contract revenue (exercise conditions for the MRG) became low. The ratio of the expected revenue to the contract revenue in actual operating years decreased over the 10 years of the operating period.

Figure 4 shows the expected government payment for the MRG with and without the exercise conditions while keeping the revenue guarantee levels and operating period constant.

Note that in the current concession agreement, the revenue would be guaranteed only when the actual revenue is 50% or higher of the contract revenue. For the case of the second year, the ratio of the expected revenue to the contract revenue was estimated at 52% (Figure 3) and the expected government payment was analyzed as 4.4 billion KRW. However, the expected government payment could rise up to 11.3 billion KRW if the project is not required to meet the exercise conditions.
which is the requirement for exercising the MRG, is so low that this reduces the expected government payment. However, the government payment would be much higher without the exercise conditions.

We can see that the annual government payment decreases when exercise conditions are applied for the MRG. The probability that the expected revenue would exceed 50% of the contract revenue, which is the requirement for exercising the MRG, is so low that this reduces the expected government payment. However, the government payment would be much higher without the exercise conditions.

It is highly possible that fairly large MRG payments would have been generated for large-scale PPP projects in the early years that did not have any exercise condition for the MRG. It should be noted that the Korean government had introduced the MRG for reinvigorating PPPs in 1998 without any requirement for exercising the MRG. Exercise conditions for the MRG had been applied in the concession agreement from the year of 2004.

Figure 5 shows the value of MRG and ERS of the A-line Project based on the current exercise conditions while keeping the revenue guarantee levels and operating period constant.

Figure 3. Expected government payment by operating year.

Figure 4. Expected government payment with and without exercise conditions.

We can see that the annual government payment decreases when exercise conditions are applied for the MRG. The probability that the expected revenue would exceed 50% of the contract revenue, which is the requirement for exercising the MRG, is so low that this reduces the expected government payment. However, the government payment would be much higher without the exercise conditions.

It is highly possible that fairly large MRG payments would have been generated for large-scale PPP projects in the early years that did not have any exercise condition for the MRG. It should be noted that the Korean government had introduced the MRG for reinvigorating PPPs in 1998 without any requirement for exercising the MRG. Exercise conditions for the MRG had been applied in the concession agreement from the year of 2004.

Figure 5 shows the value of MRG and ERS of the A-line Project based on the current exercise conditions for the MRG when 5.5% of the risk-free rate of return is applied. The value of MRG is estimated at 20.5 billion KRW and the value of ERS is estimated at 520 million KRW.

The value of MRG is estimated at 3.37% of the total investment (607.8 billion KRW) of the concessionaire. The value of MRG is at a far lower level compared to the concessionaire’s investment since the revenue guarantee is only valid for the first 10 years, and the probabilities that the expected revenue would exceed 50% of the contract revenue during the operating years are very low.

The chances that the ERS option would actually be implemented are very low. The value of ERS is 520 million KRW and it is as small as 0.09% of the government’s construction subsidy (561.1 billion KRW) and this is only about 2.54% of the MRG (20.5 billion KRW).
noted that the Korean government had introduced the MRG for reinvigorating PPPs in 1998 without any requirement for exercising the MRG. Exercise conditions for the MRG had been applied in the concession agreement from the year of 2004.

Figure 5 shows the value of MRG and ERS of the A-line Project based on the current exercise conditions for the MRG when 5.5% of the risk-free rate of return is applied. The value of MRG is estimated at 20.5 billion KRW and the value of ERS is estimated at 520 million KRW.

Figure 5. Value of the change of MRG and ERS according to the risk-free rate of return.

The value of MRG is estimated at 3.37% of the total investment (607.8 billion KRW) of the concessionaire. The value of MRG is at a far lower level compared to the concessionaire’s investment since the revenue guarantee is only valid for the first 10 years, and the probabilities that the expected revenue would exceed 50% of the contract revenue during the operating years are very low. The chances that the ERS option would actually be implemented are very low. The value of ERS is 520 million KRW and it is as small as 0.09% of the government’s construction subsidy (561.1 billion KRW) and this is only about 2.54% of the MRG (20.5 billion KRW).

Figure 6 shows the value of MRG and ERS by changing the amount of revenue guarantee and share of the excess revenue while keeping the current exercise conditions for the MRG. As expected, the value of MRG increases as the level of revenue guarantee increases whilst the value of ERS decreases as the share of the excess revenue increases. As a result, the balancing of the MRG and the ERS conditions does not hold much meaning since it is much more likely that the expected revenue would not exceed 50% of the contract revenue. Considering the value of ERS (520 million KRW), it could have been a fair contract if the range of revenue guarantee was set at a level of less than 55% instead of 70~80%.

Figure 6. Value of MRG and ERS under various levels for MRG and ERS.

Figure 7 shows the change of the value of MRG according to the change of the exercise conditions. Note that the revenue would be guaranteed at 80% for the first 5 years and 70% from the 6th to the 10th year.

Figure 7. Change of the value of revenue guarantee according to the level of exercise conditions.

The value of MRG decreases as the exercise conditions for revenue guarantee are intensified. The value of MRG decreases to 330 million KRW when the exercise conditions for revenue guarantee are intensified up to 70% of the contract revenue. Without the exercise conditions, the value of revenue guarantee would be 232.98 billion KRW, which is 11.36 times higher than the current value of MRG, and this could reach approximately 38.33% of the total private investment (607.8 billion KRW as of 2002).

The figure shows that exercise conditions for the MRG act to reduce the expected financial burden on the government by reducing the expected revenue for the concessionaire. From the government’s perspective, exercise conditions for the MRG can play the role of preventing the possible “strategic” over-estimation of revenue.

Figure 8. Levels of revenue guarantee and ERS share.
Figure 6. Value of MRG and ERS under various levels for MRG and ERS.

Figure 7 shows the change of the value of MRG according to the change of the exercise conditions. Note that the revenue would be guaranteed at 80% for the first 5 years and 70% from the 6th to the 10th year.

Figure 7. Change of the value of revenue guarantee according to the level of exercise conditions. The value of MRG decreases as the exercise conditions for revenue guarantee are intensified. The value of MRG decreases to 330 million KRW when the exercise conditions for revenue guarantee are intensified up to 70% of the contract revenue. Without the exercise conditions, the value of revenue guarantee would be 232.98 billion KRW, which is 11.36 times higher than the current value of MRG, and this could reach approximately 38.33% of the total private investment (607.8 billion KRW as of 2002).

The figure shows that exercise conditions for the MRG act to reduce the expected financial burden on the government by reducing the expected revenue for the concessionaire. From the government’s perspective, exercise conditions for the MRG can play the role of preventing the possible “strategic” over-estimation of revenue.

5. Conclusions

The MRG served as one of the useful options for the government to attract the private investors in the PPP program. If the contract revenue for the MRG has been set in such a way that the government and the private investors can share the risk fairly, then the MRG would be an ideal choice for the PPP program. However, the MRG can also encourage the private investors’ intentional overestimation of traffic forecasts. Exercise conditions for the MRG can be proposed as a tool to prevent this, and in this study, we examined the effect of exercise conditions on the value of MRG and ERS. For this, we selected an urban railway PPP project in the Republic of Korea as a case study and were able to reflect the uncertainty in transportation projects through the explicit modeling of the uncertainty in passenger forecasts.

The study concludes that depending on how the exercise conditions in the MRG option are set, they can sharply curtail the expected financial burden on the government, which can eventually contribute to the sustainability of PPP programs. According to the analysis results, provided that the risk-free rate of return is 5.5% and the exercise conditions are met, the value of the project concessionaire’s right to the MRG was 20.5 billion KRW and that of ERS was 520 million KRW in the A-line Project. The value of the MRG was merely 3.37% of the total investment of the concessionaire. However, when the MRG option does not include the exercise conditions, the value of the MRG increased to 232.98 billion KRW, which is 11 times more than the current option and nearly 38.33% of the total investment of the concessionaire. While, the value of the ERS option granted to the government was as small as 0.09% of the government’s construction subsidy. This is because the chances in the ERS that the revenue sharing option would actually be executed are not high.

This study contributes to the quantitative analysis of the effects of exercise conditions in various options by the consideration of uncertainty. Like the case of Korea, governments of other countries are faced with increasing criticism of the PPP program itself due to the unforeseen government burdens that resulted after promoting the program. The findings of this study can contribute to the sustainability of the PPP program by identifying the financial burden that the government support policy can bring about and suggesting the conditions of contract that can reduce this burden.
This paper assumed that the volatility of the value of a PPP project only exists with the uncertainty in passenger forecasts. However, considering that the value of a PPP project can be changed by the elements of passenger forecast, such elements would need to be investigated in future studies. Furthermore, the uncertainty of passenger forecasts itself needs to be continuously studied along with the efforts to make more accurate estimation in consideration of the construction, management and political risks.

**Author Contributions:** The main idea of the paper and the research framework has been devised by K.K. He contributed to the selection and description of the case project and writing of the introduction and conclusion. H.C. conducted the general analysis and literature review. D.Y. also conducted the literature review and was mainly responsible for the uncertainty modeling and computation of the valuation of the MRG and ERS.

**Funding:** The research received no external funding.

**Acknowledgments:** This paper was supported by the Korea Development Institute’s Transportation Policy Research project (series no. 2013-06).

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**

7. Bain, R. Error and optimism bias in toll road traffic forecasts. *Transport* 2009, 36, 469–482. [CrossRef]


