Article

Advancing towards a Transformational Professional Competence Model through Reflective Learning and Sustainability: The Case of Mathematics Teacher Education

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Abstract: The aim of this study was to begin to characterize the elements of teacher educators’ professional practice that promote the transformation of prior knowledge, experiences, and system beliefs into professional competence, based on reflective learning and education for sustainability. To obtain data, 30 class sessions of a mathematics education teacher were analyzed. Five elements of the lecturer’s practice were identified: (1) presents real action; (2) uncovers students’ values and preconceptions and considers them; (3) systematizes them and contrasts them with the ‘ideal’; (4) helps to understand the perspective offered by mathematical and sustainability concepts; (5) helps students develop the new perspective acquired through grounded and reasoned action plans. A key conclusion of the study is that it is prior knowledge, experiences, and beliefs which are transformed if the two agents involved in the learning process (pre-service teachers and university lecturers) are synchronized.

Keywords: reflective learning; education for sustainability; teacher education; math teachers; knowledge transformation; professional competences

1. Introduction

The amount of scientific research related to teacher education is extraordinary and is focused on multiple unresolved aspects such as educational policies, quality education, or professional development, amongst others.

This article is centered on the professional development of pre-service teachers and, more specifically, on one of its main purposes: to transform prior knowledge, experiences, and belief systems into professional competence through progressive professional development, in order to effectively exercise the teaching profession [1], taking into account sustainability principles. In this paper, we follow Mayer and Lloyd [2] who state that while professional development can be described as the planned activities teachers engage in to improve their practice, professional learning implies how their practice changes. This change can be a result of their professional development, but also of the informal learning that takes place in their everyday classroom work. According to Webster-Wright [3], a prevailing view in this regulation is that there is a stage-based progression in competence from novice to expert. There is an assumption that, through continuing professional learning, professionals will maintain their competence and develop expertise. This assumption is also implicit in current moves seeking to make professional development mandatory for most professions. However, there is an increasing awareness that competence and expertise are context dependent. There is no question that competence development occurs; whether it is stage-based and linear has been challenged by those...
who acknowledge its situated nature. Professionals develop their competences and skills in diverse manners depending on contextual issues influencing their practice, and how they understand practice.

The intention of this article is to offer various keys that empower the professional competence of pre-service teachers [4] in the area of mathematics education for sustainability. From this point of view, and as happens in other highly complex professional contexts, it is assumed that teacher education cannot be rethought based on mere intuition and experience. Progress will only occur if university lecturers, responsible for training pre-service teachers, explicitly incorporate key knowledge and lessons learned provided by research in various fields linked to teacher education. In this paper, we focus on two of these fields: education for sustainability and reflective learning. The first has been endorsed by the international community as a fundamental approach to address sustainable development—one of the most important challenges of our history [5–7]—as well as to move towards a 21st century pedagogy [8]. The second is an integral component of education for sustainability and has been shown to be effective for the development of teachers’ professional competences [9].

This paper analyses and describes the teaching practice of a university lecturer who has embedded education for sustainability principles and reflective learning approaches in the course “Learning Mathematics” of the Degree in Early Childhood Education of the University of Girona. The purpose of the analysis is to identify and characterize key elements that contribute to challenge pre-service teachers’ prior knowledge, experiences, and belief systems into professional competence in the area of mathematics education for sustainability.

1.1. Mathematics Education for Sustainability

For the past 30 years, education has been described as the great hope to create a more sustainable future. Underpinning education for sustainability is a commitment to engaging people and social groups in learning to live in sustainable ways. It encompasses a new vision of education that supports learners to reflect upon preferred futures and define their vision for sustainable development [10]. Education for sustainability is focused on pedagogy as it seeks to equip learners to respond to the complexities and uncertainties of the future, and uses well established and less well-known pedagogical approaches [11], such as futures thinking, learning to change, systems thinking, stakeholder engagement, reflective learning, and participatory learning [10,12,13].

Mainstreaming sustainability in teacher education has been identified as a key priority in authoritative international documentation [5–7]. The education of pre-service teachers plays a vital role in achieving changes in teaching and learning in schools [14], as well as shaping the knowledge and skills of future generations [15]. Bourn and Hunt [16] highlight that education for sustainability amplifies questions about the purpose of teachers in society. They do not only hold the key to promote sustainable development understanding and competences, but also support social justice, equity, and environmental responsibility in our communities.

Much has been written about re-orienting teacher education for sustainability [15–23], stressing the need to rethink content and competences, but also promoting the clarification of sustainability values and the development of reflective practices [6]. There are many experiences documented regarding the embedding of sustainability principles in specific pre-service teacher education subjects such as language or science education [24–26]. However, there is a lack of literature unpacking the connections between mathematics education and sustainability. Most of the documentation consists of reports and booklets describing inquiry-based school projects that have engaged students in using mathematics to explore specific sustainability challenges. There is little research that investigates the fundamental role of mathematics education in supporting learners to design creative ways to meet sustainable development needs in efficient ways, understanding the natural world and our relationship with it, acquiring a critical understanding of progress and technological advance, or solving complex problems using systems approaches, to mention some.

In Spain, competences form the scaffolding around which university education courses are constructed. These competences usually appear as generic, specific, and transferrable skills that
graduates must demonstrate over the course of their degree. The University of Girona has included sustainability as a transferrable competence, and all lecturers are expected to embed education for sustainability in their courses. The Department of Specifics Didactics of the Faculty of Education has worked in this direction for a long time, especially in the areas of science and mathematics education. Many subjects have been rethought in order to tackle the sustainability challenge and help pre-service teachers understand their personal and professional responsibilities regarding sustainable development. First, sustainability competences were embedded using the framework developed by ACES (Higher Education Environmental Curriculum Network) [27] and, later, the guidelines developed by the Conference of Rectors of Spanish Universities (CRUE) Sustainability Group [28] and the UNECE Education for Susatinable Development Competence Framework [29]. The design of the subject “Learning Mathematics” has been informed and shaped by the latter and has implied making an emphasis on holistic thinking, envisioning change, and transforming learning systems, as well as developing learning experiences focused on the pillars of learning to know, learning to do, learning to live together, and learning to be. The professional development experience of this course, therefore, is built on critical reflection and participatory and action learning pedagogical strategies, reflective learning being at the core of the learning experience and being used as a tool for values clarification and action empowerment. This article is interested in analyzing the transformative model of reflective learning in the context of mathematics education and sustainable development. We consider that critical reflective learning should be a key pillar to enhance the professional development of pre-service teachers in the area of mathematics education for sustainable development as it helps learners to clarify and challenge their prior knowledge, experiences, and systems beliefs and transform them to include sustainability criteria.

1.2. Reflective Learning as a Transformative Model of Knowledge, Experiences, and Belief Systems

Reflective learning promotes the integration of people with their experiences as students, with theoretical knowledge, and with their representations of what it is to teach and learn. This approach guides students towards inquiry-based practices within the professional context, in a way that creates new mental structures during the training process through self-regulated learning [30].

Black and Plowright [9] designed a multi-dimensional model of reflective learning for professional development that can be helpful to frame mathematics education for sustainability. In this model, they included the following dimensions: (a) the source of reflection, comprising learning experience and practice experience and known as the experiential process; (b) the target of reflection, namely reflection-on-learning and reflection-on-practice, and, also the levels of reflection in relation to the target; (c) the realization of reflection, through written and internal dialogue with oneself, which is known as the transformational process; and (d) the purpose of reflection, i.e., reflection for learning and reflection for practice, referred to as the developmental process. This model includes reflection on learning for further learning and self-development; reflection on learning for application to professional practice; reflection on professional practice for further learning and self-development; and reflection on professional practice for application to future professional practice [3] (p. 255).

In recent years, there has been an increasing number of studies focusing on analyzing the effects of reflective learning on teacher education and on designing tools aimed at transferring control and awareness of each activity to students, so that they can appropriate the meaning of the knowledge and use it independently through formative and authentic evaluation. A review of the literature has been carried out in relation to the main benefits and obstacles of reflexive learning [31]. The results of this body of research on reflection in teacher education indicate that the main benefits of its application are as follows: it promotes collaborative work among equals [30]; it constructively guides the process of reflection individually and in groups [32]; and it fosters self-regulation processes to promote autonomous learning [33]. Regarding sustainability, reflective learning allows us to understand our own values and abilities and provides strategies to transform our practice; also focusing attention on the process and not the results, thereby helping to better understand the complexity of future professional
practice [34] and sustainable development. Regarding the obstacles, previous studies indicate that reflective learning implies a change in the way teaching, learning, and evaluating are conducted at universities, a challenging task that not everyone is willing to adopt [35]; it requires practice as a starting point for reflection and also an awareness of knowledge, experiences, and beliefs [36]; it can provoke emotional conflicts when contrasting ideas with others and it requires the use of new tools (portfolios, narrative texts, questionnaires, etc.) that can be difficult to develop for students, and challenging for university lecturers to assess [37]. Alsina et al. [38] point out that the use of reflexive learning in the management of discourse and practice during teacher education, together with the use of specific tools, promotes processes of self-regulation and confrontation in pre-service teachers that lead to the deconstruction, co-construction, and reconstruction of knowledge (Figure 1). These processes are key for pre-service teachers to challenge current unsustainable values, practices, and experiences and develop alternative ways of teaching that promote sustainability principles.

![PRE-SERVICE TEACHER](image)

**Figure 1.** Pre-service teacher elements for the transformation of prior knowledge, experiences, and belief systems into professional competence.

“Deconstruction” is meant as a process from which students become aware of their implicit knowledge, values, and beliefs while seeking alternatives to transform them and improve their professional profile in teacher education and sustainability [39]; “co-construction” is conceived as a social and interactive process in which students share their prior knowledge, experiences, and beliefs with the mediation of an expert, to promote professional learning through collective scaffolding, that is, through collective reflection and construction [40]; and finally, “reconstruction” is a process that involves the transformation of prior knowledge, experiences, and belief systems into professional competence [41]. Loughran [42] indicates that all these processes promote socially acquired knowledge, personal opinions, and the reconstruction of social knowledge.

In particular, Alsina et al. [39] identified 12 elements, which have been called self-regulation traits [43], which facilitate the transformation of prior knowledge, experiences, and belief systems about the teaching profession into professional competence. The first five traits foster cognitive processes related to the deconstruction of: prior experiences; beliefs about oneself; beliefs about the way the class functions; implicit disciplinary knowledge; and implicit didactic knowledge. The remaining seven traits promote processes related to the co-construction and reconstruction of professional competence: interaction with context I (at the school); interaction with context II (the university); interaction with oneself; interaction with peers; interaction with experts; interaction with theory; and critical professional knowledge. According to Larrivee [44], these elements are fundamental for the training of reflective teachers who are critical and capable of learning throughout their lives. Thus, they are fundamental to promote sustainability across the schooling system.
In addition, other elements that have been defined from the results of educational research on teaching practices also intervene in this transformation process. Due to its important role, we want to highlight the element called “agentic engagement”, which Reeve and Tseng [45] (p. 258) define as follows:

“Agentic engagement is students’ constructive contribution into the flow of the instruction they receive. What this new concept captures is the process in which students intentionally and somewhat proactively try to personalize and otherwise enrich both what is to be learned and the conditions and circumstances under which it is to be learned.”

The above definition includes emotional and cognitive elements. Thus, we must consider that in addition to the conflicts and contradictions indicated in Figure 1, other elements also participate in the construction of new knowledge and experiences that take into account sustainability. These are elements of an emotional and cognitive nature, such as intentionality and proactivity or, more generically, attitudes towards learning and sustainable development.

2. Method

A case study of a descriptive type has been designed based on a qualitative approach [46]. Stake [47] described case studies as “the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances” (p. xi). To obtain the data, the professional practice of a mathematics education university lecturer at the University of Girona (Spain) was observed. Specifically, the analysis was made in the subject “Learning Mathematics” of the Degree in Early Childhood Education, which includes 30 class sessions (10 theoretical and 20 practical). It is the first subject of mathematics education that students attend and it is located in the first semester of the 2nd year, after students have received basic training in psychopedagogical bases. In total, there are 48 students enrolled.

All class sessions were recorded on video, since this technique provides data from a more objective perspective; allows the analysis of a large quantity of data from a variety of perspectives and with different analytical frameworks; and also allows the integration of microanalysis and macroanalysis [48], among other advantages.

An audio recorder was attached to the teacher’s neck and a video camera was used to record the class sessions. The camera was located in a corner of the room so that the blackboard and as many students as possible could be included in the frame. During the recording, the person filming was attentive at all times to what was occurring, to be able to move the camera if necessary or zoom in to better record what was happening. The camera constantly followed the teacher (who is the focus of the study), covering as many students as possible and zooming in to record interactions related to the reflective dialogues, or to focus on the written productions resulting from the shared experience.

For the analysis, MediaNotes was used, which is a software-based video annotation system [49] that allows users to upload, tag, and annotate segments of the video. According to ref. [50], video annotation software systems provide users with the ability to synchronously or asynchronously watch and code (or tag) the video. In some of the video annotation software programs, the user can pre-define the criteria for tagging (analysis), whereas in other examples, the user is provided with pre-established tags. In most of the video annotation systems, the user can highlight certain parts of the video and add commentaries. MediaNotes was chosen over other video analysis tools because the coding and search options make it easier to data mine [51,52]. In addition, it has also been used as a video-based method of observation to promote teacher reflection [50,53].

To analyze the data, the authors used a deductive application of categories. Deductive category application works with prior formulated, theoretical derived aspects of analysis, bringing them in connection with the text. The qualitative step of analysis consists of a methodological controlled assignment of the category to a passage of text. Even if several procedures of text analysis are processing that step, it is still poorly described [54]. More specifically, the authors used the twelve self-regulation
traits described by Alsina et al. [38]. As indicated in the section above, these traits are the pre-service teacher elements that promote the transformation of prior knowledge, experiences, and belief systems about teaching into scientific and professional competence (see Figure 1). To conduct the analysis, MediaNotes’ tagging features were used to mark specific segments according to our own framework and the commenting features used to provide detailed insight that provided evidence of the university lecturer’s practice.

To complete the analysis and investigate the lecturer’s practice and its effect on student performance in more detail, the authors also analyzed the lecturer’s evaluation forms completed by students enrolled on the “Learning Mathematics” subject. In a recent study, Ayllon, Alsina, and Colomer [55] assessed the effect of these on the three dimensions of need-supportive teaching (NST): autonomy support, structure and involvement [56], and students’ self-efficacy [57], in order to gain new knowledge about students’ achievement in higher education. In this context, and within the Self-determination Theory (SDT) framework [56], NST is a powerful instrument to motivate students and help them achieve better results. From this point of view, these authors analyzed 86,038 complete evaluation forms (27,216 for 2014; 29,946 for 2015; and 28,876 for 2016) and confirmed their ability to inform higher education teaching and learning through the university lecturer’s teaching practice. Thus, this paper considers that it is important to analyze these forms that can provide complementary data about how the lecturer’s teaching practice can contribute to the professional development of pre-service teachers in the area of mathematics education for sustainable development, taking into account the vision that students have about their lecturer’s teaching practice. The analysis of the forms also allowed the triangulation of results, thus enhancing the research validity.

The process for students to complete the evaluation forms is as follows: three weeks before the end of classes, students answer a brief online questionnaire via the Moodle platform: this means that students do not know their final mark when completing the questionnaire. The questionnaire is not compulsory, but students receive messages encouraging them to answer. They can do so at any time of day during the seven days that the questionnaire is online, and it is completely anonymous. In total, the lecturer obtained 12 complete evaluation forms, which represents 25% of the total number of students who attend class.

The evaluation form consists of two main parts. In Part A, students are asked the following six questions (in this order):

1. This lecturer has introduced the course syllabus and the evaluation criteria clearly.
2. With this lecturer I learn.
3. This lecturer motivates me to make an effort and to learn by myself.
4. The course support material that the lecturer provides me with, helps.
5. The evaluation procedure allows me to show my knowledge.
6. This lecturer helped me with my doubts when I consulted him/her. Questions 1 and 4 focus on structure, that is to say, they refer to the amount and clarity of the information that the lecturer has provided to students about what is expected and how they can fulfil these expectations (such as, for example, setting rules and providing feedback). Question 3 focuses on autonomy support, as students find themselves more engaged in the process of learning when the lecturer fosters relevance by identifying the value of tasks, lessons, materials, and activities. Question 6 focuses on lecturer involvement and to what extent the lecturer is available to all students and committed to their learning. Finally, Questions 2 and 5 refer to students’ self-efficacy or feelings of competence in relation to their cognitive judgement of their personal capacity to learn. Finally, Part B simply asks,
7. I evaluate this lecturer’s performance as positive. Answers to all questions are on a scale from 1 to 5, where 1 indicates “strong disagreement” and 5 “strong agreement.”

The authors are aware that through their researcher positionality and as qualitative researchers, they can influence the research they do and shape the knowledge produced [58]. For this reason,
reflexivity has been an integral aspect of this study and has been put into practice through informal descriptions, memo-ing, and an iterative process of questioning the methods used and results emerged. Following Bourdieu [59], when conducting the research and analyzing the data, the authors took into account their social origins (class, gender, background, etc.), the position they occupy in their academic fields (maths education and sustainability), and their intellectual bias associated with how they see and interpret the world.

Regarding the ethical considerations, before initiating the study, the informed consent of the lecturer and the students was obtained. In addition, the lecturer agreed to provide the results of the evaluation forms, but all data that could identify the lecturer was intentionally deleted.

3. Results

This section is organized according to the self-regulation and confrontation processes described in Figure 1 (deconstruction, co-construction, and reconstruction), and considering the 12 self-regulation traits defined by ref. [38]. It is considered that pre-service teachers’ comments and productions are the result of their way of acting, which is why such evidence is indispensable to identify teacher practice and extract the main elements that characterize it. Although all sessions were recorded, a teaching sequence of one of the parts of “Learning Mathematics” is presented and analyzed as an example: algebra and logical-mathematical reasoning in Early Childhood Education.

3.1. Phase 1. Deconstruction of Prior Knowledge, Experiences, and Belief Systems

In this initial phase, the university lecturer puts into practice some procedures and skills to establish a relational climate that encourages the active participation of students. In our case, the lecturer uses previously thought-out questions and formulates challenges to the students. On the one hand, the questions that are posed to pre-service teachers when starting the topic of logical-mathematical reasoning are: “What do you think is logical-mathematical reasoning?”; “what is it for?”; and “what benefits do you think it contributes to effective performance in today’s unsustainable context?” On the other hand, a structured logical material (Dienes’ Logical Blocks) is presented to the students and they are asked to find out what knowledge related to algebra, children from 3–6 years old could learn with this material. These questions and challenges are formulated to reveal pre-service teachers’ prior knowledge, experiences, and beliefs. The questions are linked to the socio-cultural context so that students can begin to understand the role that mathematics education can play as an agent of change in accordance with sustainable development.

The procedure for discussing student responses takes into account the guidelines of ref. [60] to promote reflective dialogue: students are grouped into small groups (3–4 students) and debate the answers for a limited time, not exceeding 15 min. Afterwards, each group writes the agreements and a spokesperson communicates the answers to the others. While the lecturer writes and organizes the answers on the board, he does not make evaluative comments, makes sure that what is written matches what the students say, accepts all interventions, and maintains close contact with the participants through non-verbal language (gestures, looks, movement in the room, etc.). Table 1 presents some evidence about the prior knowledge, experiences, and beliefs revealed by students.

In the evidence provided in Table 1, it is observed that, through the questions formulated, the university lecturer has helped to uncover some pre-service teachers’ intuitive knowledge of logical-mathematical reasoning. In summary, this knowledge shows that students know some of the general functions of logical-mathematical reasoning (organize and structure thought, enable thinking, internalize strategies, etc.), but do not specify the main content associated with logical-mathematical reasoning or algebra in the early ages, such as sorting, classifying, and ordering objects by size, number, and other properties; recognizing, describing, and extending patterns, such as sequences of sounds and shapes or simple numerical patterns, and translating from one representation to another; or analyzing how both repeating and growing patterns are generated [61]. The data also shows that pre-service teachers realize that math can be used to understand better the socio-cultural context and environment
where they live, but do not generally explicitly see its connections with sustainability, such as predicting possible effects or searching alternative ways of thinking and acting.

Table 1. Prior knowledge, experiences, and beliefs systems on logical-mathematical reasoning.

| Prior Experiences                                                                 | "I was not aware of the possibilities offered by the socio-cultural context and environment to work on mathematics. Therefore, I have realized that my previous learning experiences have not been relevant to me."
|-----------------------------------------------------------------------------------| "First I thought that doing a worksheet or two a day was little because I had learned math in schools where they did more."
|                                                                                  | "When I reflected more as when we remember the games of our childhood, the impressions we had. I realized that by enjoying the game, we used mathematical reasoning."
| Beliefs about Oneself                                                            | "I haven’t liked mathematics since ESO (compulsory secondary education). I now like it thanks to engaging with other methodologies. Before I only saw numbers and math operations, now I see many other themes, such as sustainability, that I have never dealt with before."
|                                                                                  | "I noticed that I had never thought if students were capable of keeping up with the pace I asked for in the activities. And that I acted according to my preconceptions."
|                                                                                  | "Will I be able to change traditional educational practices and carry out innovative actions and dynamics that favor logical-mathematical reasoning and sustainable development in an experiential learning way?"
| Beliefs about How the Class Functions                                            | "Before I used to think that it was more difficult to work mathematical concepts with little children, but I have learned that there are many resources to do this and that it is important to work on it because mathematics is everywhere."
|                                                                                  | "Having noticed how difficult communication between the two tutors of the same course can be, I considered that managing the same class group between two tutors would be equally or more complicated to manage."
| Implicit Disciplinary Knowledge                                                  | "Logical-mathematical reasoning is a type of mathematical knowledge that allows us to use different strategies to solve problems."
|                                                                                  | "This is a series of strategies to develop thinking in general and mathematical thinking in particular."
|                                                                                  | "Logic serves to learn to reason and structure the mind."
|                                                                                  | "Logical-mathematical reasoning is a part of mathematics that is not contemplated in the Curriculum as a block of content, but that is given an instrumental role."
| Implicit Didactical Knowledge                                                    | "Before I saw maths as something that we had to learn through books, and that it served little in everyday life. I had a wrong idea about maths since I thought it was not in our daily lives."
|                                                                                  | "Previously it did not relate with a capacity to experiment and manipulate with logical-mathematical reasoning."
|                                                                                  | "I considered that algebraic reasoning was not necessary until high school."

3.2. Phases 2 and 3. Co-Construction and Reconstruction of Knowledge

After deconstructing pre-service teachers’ knowledge, experiences, and system beliefs, the lecturer takes the pre-service teachers to the co-construction phase. First, he makes an anchor between prior knowledge and sustainability science knowledge, providing students with theoretical documentation in the form of the specific documents provided, and supported by their own bibliographical searches. In this context, he uses a type of neo-Vygotskian instruction called “Concept-based instruction” (C-BI), which is carried out through a series of stages that can be organized and implemented for a specific educational context, as determined by the educator in accordance with the specific context [62–65]. Generally, however, the initial stage from which all others emerge is the orientation stage, which determines the overall quality of a particular approach to a given situation. This stage begins with the students’ pre-understanding of a certain topic. Hershkowitz and Schwarz [60] referred to pre-understanding as the Orienting Basis of Action, or OBA, because it is assumed that students base their actions, including language action, on their OBA. This knowledge can come from everyday experiences or from previous instruction, especially of the traditional type. The new conceptual knowledge is first explained and then represented imagistically to learners as a Schema for the
Orienting Basis of Action, or SCOBA. According to ref. [65], the remaining stages of the Gal’perin [64] educational model are designed to promote student internalization of the knowledge represented in a SCOBA, and to use that knowledge in practical activities, including spoken and written communication.

Based on the lecturer’s guidelines, the pre-service teachers share their conceptual expansion or SCOBA, following the same procedure as in the first phase of the teaching sequence. Table 2 presents some evidence of the co-construction phase in relation to algebra, logical-mathematical reasoning, and sustainability, obtained from their pre-service teachers’ portfolios.

| Interaction with Context II (the University) | “The methodology has been very different from the one we are used to... contrasting opinions with colleagues, building our own knowledge has a very important added value.”
| Interaction with Peers | “Discussing and sharing experiences is always better than sitting on a chair and the teacher dictating something you often do not even listen to”.
| Interaction with the Expert | “The reflective dialogues with the lecturer have made me see that mathematical logic reasoning offers some basis to think critically, to understand the world, and analyze things in a different way.”
| Interaction with Theory | “Reasoning in general, and logical-mathematical reasoning in particular, helps to structure the mind; to develop children’s ability to reason; and, above all, it helps to progressively internalize various elementary capacities to build other mathematical knowledge (quantities, geometry, measurement, etc.), as well as non-mathematical knowledge”.

As it can be observed in Table 2, using CB-Instruction, the lecturer not only encourages pre-service teachers to acquire disciplinary knowledge, but also to learn strategies and resources to carry out effective teaching in considering sustainable development. According to ref. [61], effective teaching means identifying what students already know and what they need to learn, and then stimulating and helping them to learn it well. The association of American mathematics teachers develops this idea with the following three requirements: (1) teacher effectiveness requires mathematical knowledge and awareness that students are learning and must have adequate access to pedagogical strategies; (2) effective teaching requires a supportive and stimulating learning environment; and (3) effective teaching requires constantly striving to improve.

From this perspective, the SCOBA designed by the lecturer in the co-construction phase of the teaching sequence has allowed students to learn methodological resources and appropriate ways of acting according to education for sustainability principles in order to promote the learning of algebra and logical-mathematical reasoning at early ages.
In the next stage, the reconstruction phase, the lecturer encourages students to contrast their previous knowledge with new perspectives, understanding contrast to be a process that starts from the experience of each member of the group [60]. The lecturer shows different structured logical materials and asks students to work in groups (3–4 students) to analyze them and design some activities based on a learning guide. In each activity, students have to describe: (1) the level and contents; (2) the management of the activity; (3) the solution of the activity; and (4) the mathematical language and other aspects associated with communication, such as the questions that children are asked to promote understanding, etc.

The activities designed are presented to the other students and, after the lecturer has mediated and consensus has been reached by everyone through a process of interaction, negotiation, and reflective dialogue, activities are then implemented in a school. Specifically, each group of students applies the activities to a group of about 10–15 students from 3 to 6 years old. With this task, the teacher enables pre-service teachers to apply the theoretical knowledge acquired and to obtain conclusions regarding their own practice in a real situation. Considering education for sustainability criteria, the lecturer establishes links between the university and the community while creating a context that encourages the development of professional competences in this area. Table 3 shows the evidence collected from this phase.

**Table 3.** Reconstruction of knowledge, experiences and belief systems.

| Interaction with Context I (at the School); | “At the school we were able to practice and then we reflected on what had gone well and what had gone wrong. In short, we did a self-assessment of our own practice. With this reflective practice, we observed that theory was fundamental to our design and implementation of the activities”.
| “Knowing examples about mathematics and sustainable development and seeing real situations that occur in schools has made me realize the important role of the educator, his methodology and the great amount of work he has previously done to reach these results.”
| “The fact of creating ourselves a teaching material for children to learn mathematics (and then put it into practice in a school) has allowed me to learn to ask questions, as well as to see that the use of materials is a suitable methodology for children to learn.”
| “By observing real practices on mathematics and sustainability, I have realized that, by allowing children to act freely, a more relaxed and more adequate environment for their global development is created. I have also understood that it is a methodology that offers the results that every teacher wishes, that students acquire good learning.” |
| Interaction with Oneself | “I have observed that you learn much more from experience and autonomous learning than through the introduction of theoretical content.”
| “My attitude towards mathematics in general has changed: I have adopted a more reflective attitude of analysis and, above all, of counteracting my own opinion with that of others.”
| “Throughout the course I have learned several things and have applied some that have been useful to interact with students, and I think that I will continue to use them in my day to day as a future teacher.” |
| Critical Professional Knowledge | “Manipulating, experimenting since children create mental patterns of knowledge from direct action with objects. This aspect is fundamental, so it is important to have many different materials, both commercialized and self-built.”
| “Presenting activities in reverse order, according to age”.
| “Performing activities from simulated environments (computers, etc.), after having sufficiently guaranteed direct action with manipulatives”.
| “Progressively introducing symbolism in logic games”.

The evidence presented in Table 3 reveals the starting point from which students began, the process they followed, and the transformation experienced throughout the process. This contrast has caused some cognitive conflicts, according to ref. [32] (p.108) who state that “critically reflective learning is, in itself, disturbing, but also stimulating and demanding, potentially”. In addition, students identify what they need to rethink about how to teach and learn in the area of mathematics education for sustainable development. This idea links with Esteve and Alsina [30], who point out that these
cognitive conflicts are necessary to begin to self-regulate one’s knowledge, and also to seek answers individually and collectively in order to advance, improve, and learn.

From this perspective, these cognitive conflicts, managed successfully by the lecturer and the students themselves, help to reconstruct prior knowledge, experiences, and belief systems and to build new professional knowledge in a collective and consensual way. This is when it becomes clear that the pre-service teachers in our study have understood that knowledge and skills in mathematics and sustainable development cannot be acquired only through the simple transmission of information by the lecturer, and that they must assimilate it from their own practice.

If the evidence of Table 3 is analyzed in more detail, we can also observe that the experience has enabled endogenous, collective, and cooperative work, key to education for sustainability. Students come to the conclusion that everyone can learn from everyone: the students from the lecturer, the lecturer from the students and, above all, the students from themselves.

Finally, and in order to complement the data presented, we considered the results of the lecturer’s evaluation forms and compared the results with the means of the department and of the degree in Early Childhood Education. For the seven questions posed, the lecturer received a better mark than the means of the department and the degree. He was marked with the highest rate in five of the questions (5 out of 5), including the final one that asks the students to evaluate the overall lecturer’s performance. The analysis of students’ qualitative comments shows that what they valued the most is the practical nature of the subject and the lecturer’s teaching performance.

4. Discussion and Conclusions

In this article, a link was established between reflexive learning and education for sustainability, and this was integrated within a teaching-learning process of the professional didactic and disciplinary knowledge needed to teach mathematics.

In accordance with Sanmartí, Jorba, and Ibáñez [66], when training teachers, we think it is necessary to teach them how to learn from within the discipline itself, if we want the teaching process to be successful. This is not so much because it is difficult for pre-service teachers to apply general learning strategies to the learning of specific didactic knowledge—in our case referring to mathematics education—but also because teachers must design their didactic practice so that students can implement these strategies to learn.

More specifically, the initial data obtained tends to show that the teaching practice described proved to be an effective tool to promote the professional development of future mathematics teachers. In particular, we were able to extract various elements of the teacher’s practice, typical of reflective learning and education for sustainability. For example, the formulation of questions and challenges when starting a teaching practice, which are established as a fundamental tool to encourage reflective dialogue and activate students’ prior knowledge in maths and sustainable development; or the fact that encouraging interaction with others, with oneself, and with theory promotes the construction of meanings and the learning of knowledge, in addition to the contrast between the new perspective and the starting point, according to Hershkowitz and Schwarz [65]. The lecturer uses education for sustainability pedagogical approaches such as participatory and action learning, as well as promotes closer links with schools in the community to work on mathematical and sustainability matters. Strategies are used to help train critical professionals who are willing to act and are prepared to adapt to different situations, based on the pillars of learning to know, learning to do, learning to live together, and learning to be.

In addition, the data from the evaluation forms confirmed that the lecturer’s practice has had positive effects on various aspects of the students. In particular, the elements of reflective learning and sustainability seem to have influenced the three dimensions of need-supportive teaching (NST): structure, autonomy, support, and involvement [67,68], and also student self-efficacy [69,70].
From this perspective, Figure 2 shows the elements of the teacher’s professional practice that have contributed to the transformation of prior knowledge, experiences and belief systems into professional competence, using reflexive learning in the framework of sustainability.

Figure 2. Lecturer elements for the transformation of prior knowledge, experiences, and beliefs into professional competence in teacher education.

In order for the university lecturer management to be effective, the next element is to systematize this prior knowledge, experiences, and belief systems, contrasting it with pre-service teachers’ sustainability ideals, which can result in the emergence of some conflicts and contradictions. In order to manage emotions, it is important to build an anchor between prior knowledge and the students’ ideals, which is to say that the value that intuitive knowledge has in the construction of one’s own teacher profile must be acknowledged. As indicated, it is through this anchoring that pre-service teachers progressively incorporate and understand new concepts through Concept-Based Instruction [62–65]. Finally, in the last phase of the training, the lecturer incorporates new action plans, in the form of new methods of action that allow the co-construction and reconstruction of prior knowledge, experiences, and beliefs into professional competence. This allows the students to put into practice mathematics education for sustainable development in schools and reflect on their performance.

An important consideration to note is that, in this final phase, many students started to speak as a collective. This may mean that, through education for sustainability approaches and reflective learning, some of the students were able to leave behind a unidirectional vision of the teaching–learning process, in which the teacher transmits knowledge and the student receives it passively [31,69,70]. This statement is, however, a risky interpretation, since it is difficult to determine with exactitude if there has been a change in ways of teaching mathematical knowledge with the analysis of a single teaching sequence. However, some transcripts suggest that the inquiry-based and collective scaffolding that the lecturer promoted may have generated this transformation in some students, in line with ref. [67].

From the data obtained in this study on teacher elements that promote the transformation of prior knowledge, experiences, and belief systems into professional competence, together with the student elements indicated in Figure 1 [34], a first definition is proposed to advance towards a transformational
knowledge model in teacher education based on education for sustainability principles and reflective learning approaches (Figure 3).

Figure 3. Elements for the transformation of prior knowledge, experiences, and belief systems into professional competence in teacher education.

Figure 3 shows, first of all, that one of the main purposes of teacher education and sustainability is to progressively transform students’ prior knowledge, experiences, and beliefs into competences that contribute to their professional development. A second issue that is essential to consider is that knowledge is transformed into various phases that must necessarily be developed symmetrically in the two agents involved: pre-service teachers and lecturers. This is a matter of great importance, because if pre-service teachers and lecturers are not synchronized during a teaching sequence, it is very difficult for teacher education to contribute to the transformation of students’ knowledge and values.

Some of the main limitations of the study have been the following: first, the data was obtained from the analysis of a single didactic sequence, which is why it is difficult to determine with exactitude if future teachers transform their knowledge about ways of teaching mathematics with sustainability criteria in mind. In addition, the teaching practice of a single teacher was analyzed, so the results cannot be generalized; second, we consider that it is not possible to achieve the effective transformation of university teaching and learning practice from a single subject. Therefore, it seems necessary that the university should adopt a whole-institutional approach to sustainability; third, some students indicated that the continued use of the techniques mentioned in our study, for example those aimed at promoting reflective dialogues, may cause monotony; and, finally, the data of the teaching evaluation is limited, since only 25% of the students answered, so it is not possible to obtain firm conclusions about the effect of the lecturer’s practice on the three dimensions of need-supportive teaching (NST) and student self-efficacy. Therefore, in the future it would be useful to design new studies with larger samples to refine the transformational knowledge model described, and to incorporate other instruments and techniques to continue advancing towards teacher education based on education for sustainability principles and reflective learning approaches, with the purpose of improving the professional development of pre-service teachers.
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References


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