A Recourse-Based Type-2 Fuzzy Programming Method for Water Pollution Control under Uncertainty

Jing Liu 1, Yongping Li 1,*, Guohe Huang 2 and Lianrong Chen 1

1 Department of Environmental Engineering, Xiamen University of Technology, Xiamen 361024, China; wsljing0909@gmail.com (J.L.); iLrchen@hotmail.com (L.C.)
2 Sino-Canada Energy and Environmental Research Center, North China Electric Power University, Beijing 102206, China; guohe.huang@outlook.com

* Correspondence: yongping.li33@gmail.com

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Abstract: In this study, a recourse-based type-2 fuzzy programming (RTFP) method is developed for supporting water pollution control of basin systems under uncertainty. The RTFP method incorporates type-2 fuzzy programming (TFP) within a two-stage stochastic programming with recourse (TSP) framework to handle uncertainties expressed as type-2 fuzzy sets (i.e., a fuzzy set in which the membership function is also fuzzy) and probability distributions, as well as to reflect the trade-offs between conflicting economic benefits and penalties due to violated policies. The RTFP method is then applied to a real case of water pollution control in the Heshui River Basin (a rural area of China), where chemical oxygen demand (COD), total nitrogen (TN), total phosphorus (TP), and soil loss are selected as major indicators to identify the water pollution control strategies. Solutions of optimal production plans of economic activities under each probabilistic pollutant discharge allowance level and membership grades are obtained. The results are helpful for the authorities in exploring the trade-off between economic objective and pollutant discharge decision-making based on river water pollution control.

Keywords: recourse; River Basin; stochastic; type-2 fuzzy; uncertainty; water pollution control

1. Introduction

The trade-off between water pollution control and economic development is of great concern in many basins since it is essential to local sustainable development [1,2]. It is difficult to keep the economy growing under the utilization of water resources and the deterioration of environmental conditions [3,4]. Meanwhile, it is hard to promote human society improvement if the authorities excessively restrict economic development. Under such a contradictory situation, optimization techniques are proper to detect the economic and environmental impacts of alternative pollution control actions from a system point of view, and thus aid the authorities in formulating and adopting cost-effective water pollution plans and policies. However, water pollution control planning is governed by significant sources of uncertainty associated with different variables, and uncertainty is a non-negligible constituent of such a procedure [5]. There are significant uncertainties in not only how the system might develop, but also in how the system is expected to adjust when many system components are altered (e.g., pollutant discharge amount, cost/benefit coefficient and economic activity scale).

In water pollution control problems, values of associated parameters (e.g., cost/benefit coefficients) are usually determined via tests, experiences and expertises, while these methods may fail in determining accurate values, resulting in the parameters being described by fuzzy membership functions. Such deviations in subjective estimations can lead to fuzziness being inherent
in the real-world decision problems (e.g., vagueness and/or impreciseness in the outcomes of a water pollution control sample), neglect of which can cause the solutions of problems deviating greatly from their true values. Fuzzy mathematic programming (FMP), based on fuzzy sets theory, can facilitate the analysis of system associated with uncertainties being derived from vagueness and imprecision \[6,7\]. FMP is capable of handling decision problems under fuzzy goal and constraints and tackling ambiguous coefficients in the objective function and constraints. Previously, a wide range of FMP methods were developed for water pollution control \[8–15\]. For example, Liu et al. \[16\] improved a two-stage fuzzy robust programming model for water pollution control to address fuzzy parameters, which were represented by possibility distributions on the left- and right-hand sides of the constraints. Tavakoli et al. \[17\] developed an interactive two-stage stochastic fuzzy programming method to handle uncertainties expressed as fuzzy boundary intervals (i.e., the lower- and upper-bounds of intervals are presented as possibility distributions). Ji et al. \[18\] enhanced an inexact left-hand-side chance-constrained fuzzy multi-objective programming approach to cope with fuzziness in the constraints and objectives. Generally, the conventional fuzzy programming methods could only tackle fuzzy uncertainty with precise membership grades, which may encounter difficulty when the membership grades are also obtained as fuzzy sets.

In many real-world situations, related data such as unit net benefits of economic activities and pollutant discharge allowances are often highly uncertain, which could not be handled by the conventional fuzzy programming methods. When it is challenging to identify the membership grade of a fuzzy set as crisp values (e.g., unit benefit), type-2 fuzzy sets (T2FS) can effectively determine the membership function through defying membership grades of T2FS are fuzzy sets within \([0,1]\) \[19–22\]. In addition, membership functions cannot express uncertainties featured with randomness. Two-stage stochastic programming with recourse is effective for handling decision-making problems in which an examination of policy levels is desired and the system data is characterized by uncertainty \[23\]. Therefore, as an extension to the existing approaches, a recourse-based type-2 fuzzy programming (RTFP) method incorporating the concepts of type-2 fuzzy programming (TFP) within a two-stage stochastic programming with recourse (TSP) framework will be developed to address the above deficiencies. Then, the RTFP method will be applied to water pollution control in the Heshui River Basin in China. Results of optimal agriculture, industry, forestry, fishery, and livestock husbandry activities will be generated, which will be used for providing insight into the trade-off among system benefit, water pollution control, and sustainability.

### 2. Methodology

In water pollution control decision making problems, uncertainties may arise due to subjective estimation. For instance, decision makers may estimate the unit benefit from planting fruit/vegetable being \([424.1, 575.6] \times 10^3$/km\(^2\) with additional information as the possibility of “the most possible unit benefit of 499.8 $/km^2$ is 0.8, and the possibilities of “there is no possibility that the unit benefit is lower than 424.1 $/km^2$ or higher than 575.6 $/km^2$ are 0.2 and 0.3, respectively. Under such a situation, unit benefit from planting fruit/vegetable should be described as type-2 fuzzy sets (T2FS). Thus, type-2 fuzzy programming (TFP) can be adopted to tackle such uncertainties, which can be presented as \[24\]:

\[
\text{Max } f = \tilde{C}X,
\]

subject to:

\[
\tilde{A}X \leq B,
\]

\[
X \geq 0,
\]
where \( \tilde{C} \in \{ R \}^{1 \times n} \) and \( \tilde{A} \in \{ R \}^{m \times n} \) are vectors of T2FS. A type-2 fuzzy set \( \tilde{C} \) in \( X \) is a fuzzy set in which the membership function is also fuzzy (i.e., type-2 membership function). The \( \tilde{C} \) defined on the universe of discourse \( X \) is represented as [25]:

\[
\tilde{C} = \{ ((x, u), \mu_{\tilde{C}}(x, u)) : \forall x \in X, \forall u \in I_x \subseteq [0, 1] \},
\]

where \( 0 \leq \mu_{\tilde{C}}(x, u) \leq 1 \) is the type-2 membership function, \( I_x \subseteq [0, 1] \) is the primary membership of \( x \in X \), which is the domain of the secondary membership function \( \mu_{\tilde{C}}(x) \) so that all \( u \in I_x \) are the primary membership grades of the point \( x \). The secondary membership function of a triangular \( \tilde{C} = (c_1, c_2, c_3, \theta_1, \theta_r) \) can be defined as:

\[
\mu_{\tilde{C}}(x) = \begin{cases} 
\frac{x-c_1}{c_2-c_1} - \theta_1 \frac{x-c_1}{c_2-c_1}, & \text{if } x \in [c_1, \frac{c_1+c_2}{2}] \\
\frac{x-c_1}{c_2-c_1}, & \text{if } x \in \left( \frac{c_1+c_2}{2}, c_2 \right] \\
\frac{x-c_2}{c_3-c_2} - \theta_1 \frac{x-c_2}{c_3-c_2}, & \text{if } x \in [c_2, \frac{c_2+c_3}{2}] \\
\frac{x-c_2}{c_3-c_2}, & \text{if } x \in \left( \frac{c_2+c_3}{2}, c_3 \right] \\
\frac{x-c_3}{c_3-c_2} - \theta_r \frac{x-c_3}{c_3-c_2} + \theta_1 \frac{x-c_3}{c_3-c_2}, & \text{if } x \in [c_3, c_1+\frac{c_2+c_3}{2}] \\
\frac{x-c_3}{c_3-c_2}, & \text{if } x \in \left( c_3, c_1+\frac{c_2+c_3}{2} \right] 
\end{cases}
\]

where \( c_1, c_2, c_3 \) are real numbers and \( \theta_1; \theta_r \in [0, 1] \) are two parameters representing the spreads of primary membership grades of \( \tilde{C} \).

One of the main limitations of the TFP method remains in its difficulty in coping with uncertainties described as probability distributions when the available historical data is sufficient (e.g., pollutant discharge allowances) [26–28]. Such a problem can be formulated as a two-stage stochastic programming with a recourse (TSP) model [23]. Through incorporating the TFP method within the TSP framework, a recourse-based type-2 fuzzy programming (RTFP) method can be formulated as follows:

\[
\begin{align*}
\text{Max } f &= \sum_{j=1}^{n_1} \tilde{c}_j x_j - \sum_{j=1}^{n_1} s^{\mathsf{T}} \tilde{p}_j y_{\mathsf{s}}, \\
\text{subject to: } &\sum_{j=1}^{n_1} a_{ij} x_j \leq \tilde{b}_r, \quad r = 1, 2, \ldots, m_1, \\
&\sum_{j=1}^{n_1} a_{ij} x_j + \sum_{j=1}^{n_2} a_{ij} y_{\mathsf{s}} \leq \omega_s, \quad t = 1, 2, \ldots, m_2; \quad s = 1, 2, \ldots, v, \\
x_j &\geq 0, \quad j = 1, 2, \ldots, n_2, \\
y_{\mathsf{s}} &\geq 0, \quad j = 1, 2, \ldots, n_2; \quad s = 1, 2, \ldots, v.
\end{align*}
\]

Decision variables are divided into two subsets: those that must be determined before the realizations of random variables are known, and those (recourse variables) that are determined after the realized random variables are disclosed. \( x_j \) is the first-stage decision made before the random variable is observed, \( \omega_s \) is the random variable with a probability level \( p_s \) (i.e., the probability of realization of \( \omega_s \), with \( p_s > 0 \) and \( \sum_{s=1}^{S} p_s = 1 \)) [29], and \( y_{\mathsf{s}} \) is the second-stage adaptive decision, which depends on the realization of the random variable. \( \sum_{j=1}^{n_2} \tilde{c}_j y_{\mathsf{s}} \) denotes the second-stage cost function (\( \tilde{c}_j \) is cost coefficient of \( y_{\mathsf{s}} \)). Inequality (4e) presents the relationship among \( x_j, y_{\mathsf{s}}, \) and \( \omega_s \).

An extra type reduction is needed to convert the output of T2FS into conventional fuzzy sets so that they can be defuzzified to give crisp outputs. Suppose that \( h_1, h_2, \ldots, h_i \) are the value of \( \tilde{c}_{ij} \) (for at least one pair of \( i \neq j \), define \( h_i \neq h_j \)) evaluated by \( t \) different experts. The relative distances of \( h_i \) are used to approximate the center. Values lying closer to the center are considered more important. Generally, the fuzzification of \( \tilde{c}_{ij} \) can be represented as: (1) calculate the relative distance matrix \( D = |d_{ij}|_{1 \times t'} \) where \( d_{ij} = |h_i - h_j| \); (2) calculate the average of relative distances.
\[ \bar{d}_i = \sum_{j=1}^{m} d_{ij} / (t-1); \]

(3) introduce a pair-wise comparison \( p_{ij} (p_{ij} = \bar{d}_j / \bar{d}_i) \), and the pair-wise matrix \( P = [p_{ij}]_{1 \times t} \); (4) obtain the true-importance degree \( w_i \) of \( h_i \) (\( w_i = 1 / \sum_{j=1}^{m} p_{ij} \)); (5) assess the mode of \( m (m = \sum_{i=1}^{t} w_i h_i) \) of the fuzzy number; (6) choose \( s = \sum_{i=1}^{t} w_i h_i - |m| \) to approximate the unknown mean deviation \( \sigma \); (7) acquire the ratio of left spread to the right spread \( \eta = (m - h^l) / (h^r - m) \), with \( h^l = \sum_{i \in A} w_i h_i / \sum_{i \in A} w_i, h^r = \sum_{i \in B} w_i h_i / \sum_{i \in B} w_i \), \( A = \{i | h_i < m, i \in I\} \), \( B = \{i | h_i > m, i \in I\} \) and \( I = \{1, 2, \ldots, t\} \); (8) obtain \( a = m - 3(1 + \eta)\eta \sigma / (1 + \eta^2) \) and \( b = m + 3(1 + \eta)\sigma / (1 + \eta^2) \). The conventional fuzzy number \( F = (a, m, b) \) of \( \bar{c}_{ij} \) can thus be acquired. Then, the defuzzification of T2FS can be conducted according to the critical value (CV)-based reduction method [30].

Suppose that \( \tilde{c} \) is a triangular type-2 fuzzy variable with secondary possibility distribution function \( \mu_\bar{c}(x) \) (which represents a regular fuzzy variable). The method is introducing the CVs as representing values of the regular fuzzy variable \( \mu_\bar{c}(x) \), i.e., \( CV^* [\mu_\bar{c}(x)] \) (optimistic CV), \( CV_s [\mu_\bar{c}(x)] \) (pessimistic CV) and \( CV [\mu_\bar{c}(x)] \). Then, the corresponding fuzzy variables are derived using these CVs of the secondary possibilities:

\[
\tilde{c} = \begin{pmatrix} \bar{a}_1 & \bar{a}_2 & \bar{a}_3 \\ m & 1 & n \end{pmatrix},
\]

where \( \bar{a}_1, \bar{a}_2, \) and \( \bar{a}_3 \) are primary membership grade of \( \tilde{c} \), and \( m, 1, \) and \( n \) are corresponding secondary membership grades.

\[
\text{Pos}(\tilde{c} \geq a) = \sup_{r \geq a} \mu_\bar{c}(r) = \begin{cases} 1 & \text{if } 0 \leq a \leq \bar{a}_2 \\ n & \text{if } \bar{a}_2 < a \leq \bar{a}_3 \\ 0 & \text{if } \bar{a}_3 < a \leq 1 \end{cases}
\]

(12)

\[
\text{Nec}(\tilde{c} \geq a) = 1 - \sup_{r < a} \mu_\bar{c}(r) = \begin{cases} 1 & \text{if } 0 \leq a \leq \bar{a}_1 \\ m & \text{if } \bar{a}_1 < a \leq \bar{a}_2 \\ 0 & \text{if } \bar{a}_2 < a \leq 1 \end{cases}
\]

(13)

\[
\text{Cr}\{\tilde{c} \geq a\} = \frac{1}{2} (\text{Pos}(\tilde{c} \geq a) + \text{Nec}(\tilde{c} \geq a)) = \begin{cases} 1 & \text{if } 0 \leq a \leq \bar{a}_1 \\ \frac{1+3m}{2} & \text{if } \bar{a}_1 < a \leq \bar{a}_2 \\ \frac{3}{2} & \text{if } \bar{a}_2 < a \leq \bar{a}_3 \\ 0 & \text{if } \bar{a}_3 < a \leq 1 \end{cases}
\]

(14)

Then, \( CV^* [\mu_\bar{c}(x)] \), \( CV_s [\mu_\bar{c}(x)] \) and \( CV [\mu_\bar{c}(x)] \) are defined as follows:

\[
CV^* (\tilde{c}) = \sup_{a \in [0, 1]} (a \wedge \text{Pos}\{\tilde{c} \geq a\}),
\]

(15)

\[
CV_s (\tilde{c}) = \sup_{a \in [0, 1]} (a \wedge \text{Nec}\{\tilde{c} \geq a\}),
\]

(16)

\[
CV (\tilde{c}) = \sup_{a \in [0, 1]} (a \wedge \text{Cr}\{\tilde{c} \geq a\}).
\]

(17)

Finally, to obtain crisp values, centroid method (\( \sum_{x} x \mu_\bar{c}(x) / \sum_{x} \mu_\bar{c}(x) \)) is used for these reduced conventional fuzzy variables. Generally, the detailed computational processes for solving the RTFP method can be summarized as follows:

Step 1. Formulate the RTFP model.
Step 2. Discrete probability distribution into several values with each corresponds to one probability.
Step 3. Convert the output of T2FS into conventional fuzzy sets.
Step 4. Conduct defuzzification of T2FS according to the critical value (CV)-based reduction method.
Step 5. Run the RTFP model.
Step 6. Obtain the optimal solutions of the objective function (\( f \)), first-stage decision variable (\( x_i \)), and second-stage decision variable (\( y_{jk} \)).
3. Case Study

Yongxin, a county of Jiangxi Province, is located in the southeast part of China (as shown in Figure 1). It ranges in longitude from 113.83° E to 114.31° E, and in latitude from 26.78° N to 27.23° N. It occupies a total area of around 2194.57 km², and the majority of the county (i.e., approximately 1800 km²) lies within the middle reaches of the Heshui River Basin [31]. The basin features subtropical monsoon humid climatic conditions (abundant rainfall and sunlight, and long frost-free periods) with an average annual precipitation of 1530.7 mm and an average annual temperature of 18.2 °C. The Heshui River has a total length of 225 km, with 77 km flowing within the borders of Yongxin County from west to east. The county mainly relies on the Heshui River to support its agriculture, industry, livestock husbandry, forestry, and fishery [32].

In the county, farmland, orchard, and woodland occupy 15.2%, 0.6%, and 70.1% of the total land, receptively. Grassland accounts for 6.1% of the total area, most of which is in good condition and suitable for feeding a large number of livestock. Land for urban construction, traffic, and water conservancy facilities cover 3.4%, 0.4%, and 0.6% of the total land, respectively. Agriculture is traditionally the primary sector of the County. Paddy soil is the main cultivation soil type, and it is not only suitable for rice planting but can also be used for rapeseeds and other cash crops. Fruit orchards produce a variety of fruits such as pear, peach, orange and plum. Agriculture includes paddy, dry and vegetable/fruit farms. The main types of livestock raised in the County are hogs, cattle and poultry. Pork and beef account for around 75.5 and 13.2% of the total meat production, respectively. More uncertainties associated with the total population was approximately 53,139, and the net income per capita was 8520 Chinese yuan (approximately $1290). One major environmental issue in the study area is water pollution, which is mainly caused by excessive pollutant loadings from agriculture, forestry, fishery, and livestock husbandry, and industry. Water pollution problems pose great obstacles to sustainable development in the county; thus, it is essential to optimize economic structure from a systematic point of view. Based on field investigations and related literature, chemical oxygen demand (COD), total nitrogen (TN), total phosphorus (TP), and soil loss have been selected as the pollutant types to control water pollution, which is mainly caused by excessive pollutant loadings from agriculture, forestry, fishery, and livestock husbandry, and industry.

![Figure 1. The study area.](image-url)
pollution. Five zones (zones 1 to 5) are chosen to control water pollution. After the first-stage decisions (i.e., planning targets of economic activities) are made, associated pollution, the amount of which is in proportion to the economic production, is discharged into the water body. Based on the calculation of discharged wastewater and the measurement of incoming water quality, the mitigation schemes can be determined as the second-stage decisions in order to meet the environmental standards. If the production targets of them are made too high, the additionally generated pollutants will have to be mitigated in a more expensive way or discharged/dranked into the stream (leading to penalties from the government). Conversely, if the production targets are made too low, the system will encounter opportunity losses of economic income, leading to a reduced system benefit. The pollutant discharge allowances are highly uncertain, which are presented as probability distributions. Seven pollutant-discharge allowance levels are generated in association with different probabilities, which are identified as very-low, low, low-medium, medium, medium-high, high, and very-high, respectively. More uncertainties associated with cost/benefit coefficients may come from measurement errors and/or subjective judgment, and they can be expressed as T2FS. Therefore, the developed RTFP method can be adopted to plan water pollution control in the study area. The formulation of the RTFP model is presented as follows:

\[
\text{Max } f^\pm = \sum_{i=1}^{l_f} \sum_{j=1}^{i} \tilde{B}A_{ij} \cdot TA_{ij} (\Delta TA_{ij} + \mu_{ij}) - \sum_{i=1}^{l_f} \sum_{j=1}^{i} H \sum_{h=1}^{K} p_h \cdot PEA_{ih} \cdot \tilde{D}PA_{ijk} \cdot XA_{ihk} + \sum_{i=1}^{l_f} \sum_{j=1}^{i} H \sum_{k=1}^{K-1} \sum_{h=1}^{H} p_h \cdot PEFI_{ik} \cdot \tilde{D}PEI_{ijk} \cdot XFI_{ijk} + \sum_{i=1}^{l_f} \sum_{j=1}^{i} H \sum_{h=1}^{H} p_h \cdot PEL_{ij} \cdot \tilde{D}PEL_{ij} \cdot XL_{ijh} + \sum_{i=1}^{l_f} \sum_{j=1}^{i} H \sum_{h=1}^{H} p_h \cdot PEW_{ij} \cdot \tilde{D}PEW_{ij} \cdot XW_{ijh} + \sum_{i=1}^{l_f} \sum_{j=1}^{i} H \sum_{h=1}^{H} p_h \cdot PAP_{ij} \cdot \tilde{D}PAP_{ij} \cdot XA_{ijh}
\]

(1) COD discharge constraints:

\[
\sum_{i=1}^{l_f} (TF_{ij} - X_{ijh}) \cdot \tilde{C}CO_{ij} \leq PCF_{jih}, \ \forall j, h,
\]  

(19)

\[
\sum_{i=1}^{l_f} (TL_{ij} - XL_{ijh}) \cdot \tilde{D}PL_{ij} \leq PCL_{jih}, \ \forall j, h,
\]  

(20)

\[
\sum_{i=1}^{l_f} (TI_{ij} - XI_{ijh}) \cdot \tilde{D}PI_{ij} \leq PCI_{jih}, \ \forall j, h,
\]  

(21)

\[
\sum_{i=1}^{l_f} (TI_{ij} - XI_{ijh}) \cdot \tilde{D}PI_{ij} \leq PCI_{jih}, \ \forall j, h,
\]  

(22)

(2) Phosphorus discharge constraints:

\[
\sum_{i=1}^{l_f} (TA_{ij} - XA_{ijh}) \cdot (SL_{ij} \cdot AP_{ij} + RA_{ij} \cdot RP_{ij}) \leq PAP_{jih}, \ \forall j, h,
\]  

(23)
\[
\sum_{i=1}^{I} (TF_{ij} - X_{Fijh}) \cdot FN_{ij} \leq PFN_{jh}, \quad \forall j, h, 
\]
(24)

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} (TA_{ij} - XA_{ijh}) \cdot (SL_{ij} \cdot AN_{ij} + RA_{ij} \cdot RN_{ij}) \leq PAN_{jh}, \quad \forall j, h, 
\]
(26)

\[
\sum_{i=1}^{I} (TF_{ij} - X_{Fijh}) \cdot FN_{ij} \leq MPL_{h}, \quad \forall h, 
\]
(27)

(3) Nitrogen discharge constraints:

\[
\sum_{i=1}^{I} (TA_{ij} - XA_{ijh}) \cdot SL_{ij} \leq PSL_{jh}, \quad \forall j, h, 
\]
(29)

\[
\sum_{i=1}^{I} (TW_{ij} - XW_{ijh}) \cdot DPW_{ij} \leq PWS_{jh}, \quad \forall j, h, 
\]
(30)

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} (TA_{ij} - XA_{ijh}) \cdot SL_{ij} \leq MSL_{h}, \quad \forall h, 
\]
(31)

(4) Soil loss constraints:

\[
\sum_{i=1}^{I} (TA_{ij} - XA_{ijh}) \cdot X_{ij} \leq MAX_{Wj}, \quad \forall j. 
\]
(32)

(5) Water supply balance constraint:

\[
TA_{i \min} \leq \sum_{j=1}^{J} TA_{ij} \leq TA_{i \max}, \quad \forall i, 
\]
(33)

\[
TF_{i \min} \leq \sum_{j=1}^{J} TF_{ij} \leq TF_{i \max}, \quad \forall i, 
\]
(34)

\[
TL_{i \min} \leq \sum_{j=1}^{J} TL_{ij} \leq TL_{i \max}, \quad \forall i. 
\]
(35)

(6) Product demand constraints:
\[ TI_{i\min} \leq \sum_{j=1}^{J} TI_{ij} \leq TI_{i\max}, \quad \forall i, \quad (36) \]

\[ TW_{i\min} \leq \sum_{j=1}^{J} TW_{ij} \leq TW_{i\max}, \quad \forall i, \quad (37) \]

(7) Technical constraints:

\[ XA_{ijh}, \ XF_{ijh}, \ XL_{ijh}, \ XI_{ijh}, \ XW_{ijh} \geq 0, \quad \forall i, j, h. \quad (38) \]

The nomenclature of variables and parameters is provided at the end of this article. The imprecise input parameters are investigated according to field surveys, statistical data \cite{31}, government reports \cite{34}, and related literature \cite{32,33}. Table 1 presents unit benefit of agriculture and livestock husbandry, which are expressed as type-2 fuzzy sets. Table 2 provides pollutant allowances and the associated probabilities of occurrence.

### Table 1. Unit benefits of agriculture and livestock husbandry.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Agriculture</th>
<th>Livestock Husbandry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Paddy Farm</td>
<td>Dry Farm</td>
</tr>
<tr>
<td></td>
<td>Unit Net Benefit (10³ $/km²)</td>
<td>Unit Net Benefit ($)/head</td>
</tr>
<tr>
<td>Zone 1</td>
<td>Zone 2</td>
<td>Zone 3</td>
</tr>
<tr>
<td>Zone 1</td>
<td>(201.4, 272.6, 292.3; 0.2, 0.6)</td>
<td>(104.5, 148.4, 163.5; 0.2, 0.6)</td>
</tr>
<tr>
<td>Zone 2</td>
<td>(174.2, 219.6, 269.6; 0.3, 0.5)</td>
<td>(96.9, 130.3, 140.9; 0.3, 0.5)</td>
</tr>
<tr>
<td>Zone 3</td>
<td>(169.6, 231.7, 254.4; 0.4, 0.7)</td>
<td>(87.8, 112.1, 127.2; 0.4, 0.7)</td>
</tr>
<tr>
<td>Zone 4</td>
<td>(187.6, 265.1, 290.8; 0.4, 0.7)</td>
<td>(92.4, 118.1, 142.4; 0.4, 0.7)</td>
</tr>
<tr>
<td>Zone 5</td>
<td>(184.8, 245.4, 275.7; 0.5, 0.7)</td>
<td>(92.4, 122.7, 133.3; 0.5, 0.7)</td>
</tr>
</tbody>
</table>

### Table 2. Pollutant allowances under different levels.

<table>
<thead>
<tr>
<th>Pollutant</th>
<th>Level</th>
<th>Probability</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Zone 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>COD (10³ kg)</td>
<td>Very-low</td>
<td>0.05</td>
<td>2435.3</td>
<td>496.4</td>
<td>3768.4</td>
<td>3284.2</td>
<td>2307.5</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0.10</td>
<td>2469.8</td>
<td>499.3</td>
<td>3780.8</td>
<td>3298.5</td>
<td>2213.3</td>
</tr>
<tr>
<td></td>
<td>Low-medium</td>
<td>0.20</td>
<td>2498.3</td>
<td>500.3</td>
<td>3792.4</td>
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4. Results Analysis and Discussion

Figure 2 represents the benefits under various pollutant discharge allowances and the system benefit, indicating that a higher allowance would correspond to a higher benefit. For instance, when the allowable pollutant discharge changes from very low to very high, the total benefit (i.e., sum of benefit of each activity) would be raised from $172.1 \times 10^6$ to $195.7 \times 10^6$. Decisions at a higher allowable pollutant discharge would lead to a higher system benefit, but the reliability in fulfilling the environmental requirements would decrease; on the contrary, decisions at a lower allowable pollutant discharge would lead to a decreasing risk for violating the pollutant discharge constraints, but with a lower system benefit. It demonstrates a trade-off between environmental requirement violation risk and benefit due to the uncertainties existing in various system components. In practice, when the plan aims to a higher system benefit, the environmental requirements may not be adequately satisfied; contrarily, planning with a lower system benefit may guarantee that the requirements be met. Additionally, the benefit of agriculture activity would take the largest proportion in total benefit and would increase slightly with the raising of pollutant discharge allowances. Moreover, the benefit of fishery would be stable at a low level, approximately occupying 2.1% of the total benefit.

![Figure 2. Benefit of each activity under each pollutant discharge allowance.](image)

The water pollution control problem can be simplified as a recourse-based fuzzy programming (RFP) problem by transforming the membership grades of T2FS into deterministic values. System benefit obtained from RFP ($180.8 \times 10^6$) would be lower than that from RTFP ($191.1 \times 10^6$). This is due to the fact that cost/benefit coefficients are handled by RFP, resulting in higher loss of uncertain information.
than that handled by RTFP. Figure 3 presents the optimal target and standard production scale of each activity under RFP and RTFP. Target of each activity discharge pollutant exceeds standards can be calculated through multiplying excess scale of economic activity (i.e., target—standard) by pollutant discharge rate. Results indicate that the excess planning scale of agricultural activity would be high. This may be attributed to their high crop yields and great selling prices. It is also depicted that the excess feeding size of livestock husbandry activity would also maintain high levels due to its high annual incomes. The excess outputs of industrial activity would also be significant since industry is promoted by the authority to push up the local income. Excess fishery and forestry activities would be low due to their limited planning lands. Furthermore, excess economic activities corresponding to RFP would be higher than that corresponding to RTFP. For instance, target of agricultural activity would be 180 ha in zone 1, while the standard production scale under RTFP and RFP would, respectively, be 156.4 ha and 148.5 ha corresponding to very-low level. Thus, excess agricultural activity would be 23.6 ha and 31.5 ha, respectively. It is revealed that the varied uncertain information would affect the water pollution control plans. Any simplifications may result in unreliable or misleading plans.

Figure 3. Cont.
Figure 3. Optimal target and standard production scale of each activity. (a) Agriculture; (b) Livestock husbandry; (c) Industry; (d) Forestry; (e) Fishery.

Figure 4 displays the excess pollutant discharges under different pollutant discharge allowance levels corresponding to RFP and RTFP. It is indicated that amounts of excess pollutant discharges would reduce with the increased levels. For instance, under RTFP, the amounts of excess soil loss, COD, TP, and TN discharges would be $52.3 \times 10^3$, 152.1, 22.4, and 99.2 t under low pollutant discharge allowance level; in comparison, under high pollutant discharge allowance level, they would respectively decrease to $1.3 \times 10^3$, 8.2, 7.2, 5.4 t. In general, results discover that a more restrictive pollution control would result in a higher excess pollutant discharge while a looser pollution control would bring on a lower excess pollutant discharge.

Figure 4. Excess pollutant discharge under different pollutant allowances (unit: t). (a) soil loss; (b) COD discharge; (c) TP discharge; (d) TN discharge.

Figure 5 displays proportion of excess pollutant discharges under medium-high level. It is indicated that the excess soil loss, TN, and TP discharges of agriculture would account for about 97%,
88%, and 87% of the of the total discharge amounts, respectively. The high excess pollutants discharges are mainly related to its high targeted planning scale and discharge rates. The excess COD discharged from fishery would be high (accounting for 58% of the total excess discharge) due to its high COD discharge rate. In summary, the trade-off between agricultural income and pollution control (i.e., soil loss, TP, and TN discharges) would be of great concern for the local authority; fishery would be ceased due to its low benefit and high pollutant discharges.

![Figure 5. Proportion of pollutant discharges under medium-high level.](image)

Generally, compared with RTFP, RFP that tackles uncertainty in a single fuzzy set is not satisfactory to sufficiently reflect enough uncertain information in the decision-making process. Such a simplification may lead to unreliable or even misleading solutions. For instance, due to loss of the additional design degree of freedom, which is useful in water pollution control systems where input data are highly uncertain, economic coefficients (unit cost and unit benefit) treated by RFP are higher than that treated by RTFP. The higher economic coefficients lead to a lower system benefit, which may present unreliable decision support for local authorities. RTFP can tackle more complex uncertainty in terms of T2FS due to the extra degree of freedom. Moreover, solutions of economic activities scale and excess pollutant discharge also display that RTFP is more capable of balancing economic development and pollutant discharge control in the study area. Thus, RTFP is more enhanced in uncertainty reflection, as well as water pollution control.

5. Conclusions

In this study, a recourse-based type-2 fuzzy programming (RTFP) approach is developed for water pollution control planning. RTFP has incorporated the techniques of type-2 fuzzy programming and two-stage stochastic programming with recourse (TSP) within a general framework. RTFP can handle uncertainties presented as type-2 fuzzy sets (i.e., a fuzzy set in which the membership function is also fuzzy) and probability distributions. RTFP could provide benefit assessment for random pollutant discharge allowance levels through the two-stage framework. The results are helpful for the authorities in exploring the trade-off between economic objective and pollutant discharge decision-making based on river water pollution control.
The developed RTFP method has been demonstrated through its application to water pollution control of Heshui River Basin. Different levels that pollutant discharge allowances are assumed to be random and benefit/cost coefficients are specified as type-2 fuzzy sets have been investigated. Solutions for production scale of agriculture, livestock husbandry, industry, forestry, and fishery are generated. Several findings can be concluded: (i) results reveal that uncertainties in pollutant discharge allowance level have significant effects on the city’s future economic structure; (ii) constrained with low pollutant discharge allowance, the excess pollutant discharge would be high; (iii) the increased restriction of discharge would stimulate the basin taming the pace of economic growth; (iv) results also disclose that the benefit of agriculture activity would take the largest proportion in total benefit and would increase slightly with the raising of pollutant discharge allowances; and (v) system benefit is powerfully impressed by the pollutant discharge allowance level.

It is the first attempt to apply the RTFP method to water pollution control planning, and results indicate that (1) RTFP cannot only handle uncertainty expressed as T2FS, but also effectively cope with uncertainty described as probability distribution; (2) TFCP can help authorities to make trade-offs among environmental violation risk and system benefit. Nevertheless, there are also potential extensions of the proposed method. For example, RTFP has difficulty in addressing uncertainties that cannot be expressed as type-2 fuzzy sets or probability distributions; interval is a proper type to present such a kind of uncertainty. Furthermore, construction of T2FS membership function is usually based on experts’ subjective evaluation, which may also lead to high uncertainty. At present, there is no appropriate evaluation method except experts’ decisions. It is worthwhile to search for more effective methods to improve the method of constructing membership functions for T2FS, which can be helpful to reduce uncertainty.

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Author Contributions: Yongping Li and Jing Liu conceived and designed the experiments; Yongping Li performed the experiments; Guohe Huang analyzed the data; Lianrong Chen contributed reagents/materials/analysis tools; Jing Liu wrote the paper

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclatures

- $i$: index for economic activities; for agricultural activities, $i = 1, 2, \ldots, I_a$; for fishery activities $i = 1, 2, \ldots, I_f$ (e.g., fish and prawn farming); for livestock husbandry activities $i = 1, 2, \ldots, I_l$; for industrial activities $i = 1, 2, \ldots, I_i$ (e.g., manufacturing, mining, architecture, transportation and others); for forestry activities $i = 1, 2, \ldots, I_w$
- $j$: index for zones; $j = 1, 2, \ldots, J$
- $k$: index for pollutants; $k = 1, 2, \ldots, K$ (e.g., COD discharge, TN loss, TP loss, and soil loss)
- $h$: allowable pollutant discharge level; $h = 1, 2, \ldots, H$
- $p_h$: probability of occurrence allowable pollutant discharge level $h$ (%)
- $BA_{ij}$: unit benefit from agricultural activity $i$ in zone $j$ (RMB¥/km$^2$)
- $TA_{ij}$: land area target for agricultural activity $i$ in zone $j$ (km$^2$)
- $PEA_{ik}$: reduction of net benefit from agricultural activity $i$ for excess discharge of pollutant $k$ (RMB¥/kg when $k = 2, 3$; RMB¥/tonne when $k = 4$)
- $DPA_{ijk}$: discharge rate of pollutant $k$ from agricultural activity $i$ in zone $j$ (kg/km$^2$ when $k = 2, 3$; tonne/km$^2$ when $k = 4$)
- $XA_{ijkh}$: decision variables representing amount by which the target of agricultural activity $i$ discharge pollutant $k$ exceeds standards in zone $j$ when level is $h$ (km$^2$)
- $BF_{ij}$: unit benefit from fishery activity $i$ in zone $j$ (RMB¥/km$^2$)
- $TF_{ij}$: land area target for fishery farming activity $i$ in zone $j$ (km$^2$)
PEF_{ij} \quad \text{reduction of net benefit from fishery activity } i \text{ for excess discharge of pollutant } k \text{ (RMB¥/kg)}

DPI_{ijk} \quad \text{discharge rate of pollutant } k \text{ from fishery activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

XF_{ijkh} \quad \text{decision variables representing amount by which target of fishery activity } i \text{ discharge pollutant } k \text{ exceeds standards in zone } j \text{ when level is } h \text{ (kg)}

\overline{BL}_{ij} \quad \text{unit benefit from livestock husbandry activity } i \text{ in zone } j \text{ (RMB¥/head)}

TL_{ij} \quad \text{target for livestock husbandry activity } i \text{ in zone } j \text{ (head)}

\text{PEI}_{ij} \quad \text{reduction of net benefit from livestock husbandry activity } i \text{ for excess discharge of pollutant } (i.e., \text{COD}) \text{ (RMB¥/kg)}

DPI_{ij} \quad \text{discharge rate of pollutant (i.e., \text{COD}) from livestock husbandry activity } i \text{ in zone } j \text{ (kg/\text{head})}

XL_{ij} \quad \text{decision variables representing amount by which target of livestock husbandry activity } i \text{ discharge pollutant (i.e., \text{COD}) exceeds standards in zone } j \text{ when level is } h \text{ (head)}

\text{TL}_{ij} \quad \text{output target for industrial activity } i \text{ in zone } j \text{ (RMB¥)}

\text{PEI}_{i} \quad \text{reduction of net benefit from industrial activity } i \text{ for excess discharge of pollutant (i.e., \text{COD})} \text{ (RMB¥/kg)}

\text{DPW}_{ij} \quad \text{discharge rate of pollutant (i.e., \text{soil loss}) from forestry activity } i \text{ in zone } j \text{ (tonne/km}^2\text{)}

\text{XW}_{ijh} \quad \text{decision variables representing amount by which target of forestry activity } i \text{ discharge pollutant (i.e., \text{soil loss}) exceeds standards in zone } j \text{ when level is } h \text{ (unit)}

\text{BW}_{ij} \quad \text{unit benefit from forestry activity } i \text{ in zone } j \text{ (RMB¥/head)}

\text{TW}_{ij} \quad \text{land area target for forestry activity } i \text{ in zone } j \text{ (unit)}

\text{PEW}_{ij} \quad \text{reduction of net benefit from forestry activity } i \text{ for excess discharge of pollutant (i.e., \text{soil loss})} \text{ (RMB¥/tonne)}

\text{DPW}_{ij} \quad \text{discharge rate of pollutant (i.e., \text{soil loss}) from forestry activity } i \text{ in zone } j \text{ (tonne/km}^2\text{)}

\text{XW}_{ijh} \quad \text{decision variables representing amount by which target of forestry activity } i \text{ discharge pollutant (i.e., \text{soil loss}) exceeds standards in zone } j \text{ when level is } h \text{ (unit)}

\text{COF}_{ij} \quad \text{COD discharge from fishery farming activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

\text{PEF}_{ij} \quad \text{maximum allowable COD discharge for fishery farming activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{PCI}_{j} \quad \text{maximum allowable COD discharge for livestock husbandry activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{PCL}_{jh} \quad \text{maximum allowable COD discharge for industrial activity } i \text{ in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{MCL}_{h} \quad \text{maximum allowable COD discharge from economic activities with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{SL}_{ij} \quad \text{soil loss from agricultural activity } i \text{ in zone } j \text{ (tonne/km}^2\text{)}

\text{AP}_{ij} \quad \text{phosphorous content of soil corresponding to agricultural activity } i \text{ in zone } j \text{ (kg/tonne)}

\text{RA}_{ij} \quad \text{runoff from agricultural activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

\text{RP}_{ij} \quad \text{dissolved phosphorous content of runoff corresponding to agricultural activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

\text{PAP}_{jh} \quad \text{maximum allowable phosphorous loss from agricultural activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{FP}_{ij} \quad \text{dissolved phosphorous loss from fishery farming activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

\text{PFP}_{jh} \quad \text{maximum allowable phosphorous loss from fishery farming activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{MPL}_{h} \quad \text{maximum allowable phosphorous loss from economic activities with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{AN}_{ij} \quad \text{nitrogen content of soil corresponding to agricultural activity } i \text{ in zone } j \text{ (kg/tonne)}

\text{RN}_{ij} \quad \text{dissolved nitrogen content of runoff corresponding to agricultural activity } i \text{ in zone } j \text{ (%)}

\text{PAN}_{jh} \quad \text{maximum allowable nitrogen loss from agricultural activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}

\text{FN}_{ij} \quad \text{dissolved nitrogen loss from fishery activity } i \text{ in zone } j \text{ (kg/km}^2\text{)}

\text{PFN}_{jh} \quad \text{maximum allowable nitrogen loss from fishery farming activities in zone } j \text{ with probability } p_h \text{ of occurrence under level } h \text{ (kg)}
**Symmetry** 2017, 9, 265

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