An Emergency Decision Making Method for Different Situation Response Based on Game Theory and Prospect Theory

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Abstract: Because of the continuous burst of emergency events (EEs) recently, emergency decision making (EDM) has become an active research topic due to its crucial role in relieving and reducing various losses and damages (property, lives, environment, etc.) caused by EEs. Current EDM studies based on prospect theory (PT) have considered decision maker’s (DM’s) psychological behavior, which is very important in the EDM process because it affects DM’s decision behavior directly, particularly under the uncertainty decision environment. However, those studies neglected an important fact that different emergency situations should be handled by different measures to show the pertinence and effectiveness of the emergency response in the real world, which has been taken into consideration in EDM studies based on game theory (GT). Different behavior experiments show that DMs usually have limited rationality when involved in risk and an uncertain decision environment, in which their psychological behavior has distinct impacts on their decision choice and behavior. Nevertheless, the existing studies of EDM based on GT build on an assumption that DMs are totally rational; however, it is obvious that such an assumption is unreasonable and far from the real-world situation. Motivated by these limitations pointed out previously, this study proposes a novel EDM method combining GT and PT that considers not only the DM’s psychological behavior, but also takes different situations’ handling for EEs into account, which is closer to the EDM problems in reality. An example and comparison with other methods are provided to demonstrate the validity and rationality of the proposed method for coping with real-world EDM problems.

Keywords: emergency response; prospect theory; game theory; situation-response

1. Introduction

The definition of emergency event (EE) is [1] “events which suddenly take place causing or having the possibility to cause intense death and injury, property loss, ecological damage and social hazards”, such as landslides, earthquakes, terrorist attacks, etc. In the World Disaster Report 2016, there were 6090 disasters that took place between 2006 and 2015 in the world. In these disasters, 771,911 people had been killed, 1,917,557 thousands people had been affected and the economic damage had reached 1,424,814 million dollars [2]. From such ghastly statistics, it is necessary to take some strategies to reduce such kinds of losses and impacts on mankind’s daily life and socio-economic development. Fortunately, emergency decision making (EDM) is one such kind of strategy, which is defined as a process in which a decision maker (DM) selects the optimal alternative to respond to or control the EE in order that life and property protection and political and social stability can be achieved [3]. Because of the important role in reducing the losses and impacts caused by EEs, EDM has become an active research field in recent years [4–8].
The EDM problem is usually complex and dynamic because the EDM environment is full of risk and uncertainty [9]. Different behavior studies prove that DMs have limited rationality under an environment with risk and uncertainty, and the psychological behavior of DM is an important factor in the EDM process due to its direct influence on decision behavior and outcomes. Hence, some researchers pay close attention to DM’s psychological behavior by means of prospect theory (PT), proposed by Kahneman and Tversky in 1979 [10], in the EDM process because of its greatest influence among different behavior theories (such as regret theory [11], disappointment theory [12], third-generation PT [13], etc.) and having achieved fruitful results [1,14–18].

All the research with and without PT has made important contributions to EDM; however, both of them have limitations that they do not take into consideration about the different emergency situations, which are caused by the dynamic evolution and uncertainty of EEs, nor do they consider DM’s psychological behavior. Each emergency situation should be considered and be handled by proper measures because of the limited resources in the real world and the importance of DM’s psychological behavior in the decision process.

Game theory (GT) is a useful tool for providing a mathematical process to select the optimal strategy for one player with respect to all possible strategies of the other ones throughout the game [19]. Thus, theoretically speaking, GT can help DM select proper measures to deal with different situations that may occur in real-world EEs. The EDM problem is a typical noncooperation game if we regard the EE and DM as the game players [20], in which the emergency situations and the measures are regarded as the strategies of EE and DM, respectively. Therefore, the EDM problems can be solved from the perspective of game theory.

In recent years, some EDM methods based on GT have been studied, which have taken into account different emergency situations dealt with by different measures [20–24]. However, it is necessary to point out that existing EDM methods based on GT build on an assumption that the player (decision maker) has total rationality [24,25]. Nevertheless, different studies [19,26,27] have shown that DMs have limited rationality under an environment with risk and uncertainty, and the DM’s psychological behavior is very important to the decision process in EDM problems and must be considered.

To manage the limitations mentioned above, this study proposes a novel EDM method based on GT and PT that takes into account DM’s psychological behavior by means of PT and different situations handled by using different measures based on GT.

The outline of this paper is as follows: Section 2 provides a brief introduction of PT and GT that will be utilized in our proposal together with a brief review of related works highlighting the importance of this study. A novel EDM method will be presented in Section 3 that considers both DM’s psychological behavior and coping with different emergency situations. Section 4 offers a case study on a typhoon emergency and a comparison with existing studies. Section 5 provides the conclusions and future works of this paper.

2. Preliminaries

In this section, GT and PT will be briefly reviewed so that unfamiliar readers can understand our proposed method easily. In addition, some related works to illustrate the importance and necessity of this research are reviewed.

2.1. Game Theory in Emergency Decision Making

GT is a useful tool to solve decision making problems in which the situations either have conflict or cooperation and sometimes both [23]. These situations may happen when there are two or more players (DMs) involved in a same system and they attempt to achieve their own objectives using the same resources [28]. As a branch of mathematical analysis, GT provides a scientific process to choose the best strategies for each possible situation throughout the game [19]. Such a characterization of GT is suitable for EDM problems, in which the DM usually needs to have a corresponding response with respect to different emergency situations.
Generally, if a game has \( n \) players, it will be denoted as \( G = \{(S_i; P_i), i = 1, 2, \ldots, n\} \), where \( S_i \) and \( P_i \) denote the strategies and payoffs of the \( i \)-th player, respectively. In the game process of EDM problems, there are usually two players, i.e., the EE and DM, in which the EE is a special player because it is unconscious about the benefits or costs. Thus, the game between EE and DM can be denoted as \( G = \{(S_i; P_i)\} \), where \( i = 1, 2 \).

The game can be classified according to the relationship among the players [29]: if the relationship among the players is competitive, the game is a noncooperation game; otherwise, if the players are cooperative, it is a cooperation game. Obviously, the relationship between EE and DM in the game is noncooperation, so the game in EDM problems can be assumed as a typical noncooperation game, the zero-sum game, i.e., \( P_1 + P_2 = 0 \), which means if the DM gains \( \triangle_i \), the EE loses \( \triangle_i \), otherwise, the EE gains \( \triangle_i \), while the DM loses \( \triangle_i \).

Three basic notions of GT for the EDM problem are briefly introduced as follows:

1. **Players**: Players are always denoted by \( i = 1, 2, \ldots, n \) and at least \( i \geq 2 \); this means that there are at least two players in one game. In EDM, there are two players, who are the decision maker (DM) and the EE. Thus, in the emergency game \( G = \{(S_i; P_i)\} \), \( i = 1, 2 \), where 1 denotes the DM and 2 refers to EE.
2. **Strategies**: Let \( S_i = \{S_{ik}\} \) be the set of action strategies of the \( i \)-th player who has \( k_i \) strategies. In EDM, \( S_1 = \{S_{1\delta}\} \) refers to the set of different alternatives of DM, in which \( S_{1\delta} \) denotes the \( \delta \)-th alternative, \( \delta = 1, 2, \ldots k_1 \). \( S_2 = \{S_{2\theta}\} \) refers to the set of different situations of EE, where \( S_{2\theta} \) denotes the \( \theta \)-th possible situation of EE, \( \theta = 1, 2, \ldots k_2 \).
3. **Payoffs**: Let \( P_i(S_i) \) be the payoffs of the \( i \)-th player, where \( P_1(S_1) + P_2(S_2) = 0 \).

The game can also be classified according to the action sequence among players [29]: if the players take the action simultaneously or the players do not know the exact information of the other player's action, the game is a static game; if not, the game is a dynamic one. The dynamic one is also called the extensive from game (EFG) [29]. Obviously, in the EDM problems, the player EE always takes the action firstly, so the game between DM and EE is an EFG problem. However, in the real world, because of the imprecise and incomplete information of the EE, which strategy the EE will take the DM does not know. Thus, when this situation occurs, the EFG problem can be regarded as the static one, and its game tree is shown in Figure 1.

![Figure 1](image_url). The game tree between an emergency event (EE) and decision maker (DM).

Based on the presentation mentioned above, since the EDM problem is a static game, therefore the payoff matrix of EE and DM can be simplified into Table 1 according to Figure 1.
Emergency events presented in Kahneman and Tversky’s study in 1979 [10] and was developed by them in 1992 [30].

As was mentioned in the Introduction, DM’s psychological behavior is a key and important factor in the EDM process especially, when DM is under pressure. However, such an important issue is neglected in the current EDM approaches based on GT; thus, it will be taken into account in this proposal by using PT.

PT is a useful tool to consider human being’s psychological behavior issues, which was firstly presented in Kahneman and Tversky’s study in 1979 [10] and was developed by them in 1992 [30] as an economic behavior theory. In the proposal of Kahneman and Tversky, they provided a simple and clear computation process to describe the psychological behavior using reference points (RPs), losses, gains and overall prospect values, which are important concepts in PT. Since PT has a simple calculation process and a clear logic, it has been widely applied in the field of decision making to solve the problems considering human being’s psychological behavior [13, 15, 30–32]. Therefore, the PT will be utilized to address the DM’s psychological behavior in our proposal.

Based on Figure 1 and Table 1, the game process between EE and DM can be described as shown in Figure 2. In our proposal, we assume that the EE chooses its strategy randomly.

The assumption presented in current EDM studies based on GT [20–24] in which the DM is completely rational is not fully reasonable. Due to the importance of the psychological behavior of DM, it will be taken into account in the phase of determining payoffs and will be introduced in detail in the third section of this proposal.

2.2. Prospect Theory in Emergency Decision Making

As was mentioned in the Introduction, DM’s psychological behavior is a key and important factor in the EDM process especially, when DM is under pressure. However, such an important issue is neglected in the current EDM approaches based on GT; thus, it will be taken into account in this proposal by using PT.

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Generally, in the process of decision making, PT was distinguished as three phases [30]:

1. An editing phase, in which the gains and losses can be calculated according to the RPs provided by DM.
2. An evaluation phase: in this phase, the prospect values can be obtained by a value function, then the overall prospect values will be calculated on the foundation of prospect values and the weighting vector.

Table 1. The payoff matrix of emergency event (EE) and decision maker (DM).

<table>
<thead>
<tr>
<th>EE</th>
<th>( S_{21} )</th>
<th>( S_{22} )</th>
<th>( S_{23} )</th>
<th>( S_{24} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{11} )</td>
<td>( (P_1(S_{21}, S_{11}), P_2(S_{21}, S_{11})) )</td>
<td>( (P_1(S_{22}, S_{11}), P_2(S_{22}, S_{11})) )</td>
<td>( (P_1(S_{23}, S_{11}), P_2(S_{23}, S_{11})) )</td>
<td>( (P_1(S_{24}, S_{11}), P_2(S_{24}, S_{11})) )</td>
</tr>
<tr>
<td>( S_{12} )</td>
<td>( (P_1(S_{21}, S_{12}), P_2(S_{21}, S_{12})) )</td>
<td>( (P_1(S_{22}, S_{12}), P_2(S_{22}, S_{12})) )</td>
<td>( (P_1(S_{23}, S_{12}), P_2(S_{23}, S_{12})) )</td>
<td>( (P_1(S_{24}, S_{12}), P_2(S_{24}, S_{12})) )</td>
</tr>
<tr>
<td>( S_{13} )</td>
<td>( (P_1(S_{21}, S_{13}), P_2(S_{21}, S_{13})) )</td>
<td>( (P_1(S_{22}, S_{13}), P_2(S_{22}, S_{13})) )</td>
<td>( (P_1(S_{23}, S_{13}), P_2(S_{23}, S_{13})) )</td>
<td>( (P_1(S_{24}, S_{13}), P_2(S_{24}, S_{13})) )</td>
</tr>
</tbody>
</table>

Figure 2. The game process between EE and DM.
3. A selection phase, in which the alternative with the highest overall prospect value will be selected as the best one to deal with the given decision problem.

According to PT, human beings are usually more sensitive to losses than the same gains, and their psychological behavior shows risk-seeking for losses and risk-aversion for gains [26]. Thus, PT can be depicted by means of an S-shaped value function that shows a concave shape in the loss domain and a convex shape in the gain domain, respectively (see Figure 3). The value function of PT is related to RPs and expressed by a power law presented as below [10]:

\[
v(x) = \begin{cases} 
  x^\alpha, & x \geq 0 \\
  -\lambda(-x)^\beta, & x < 0 
\end{cases}
\]  

(1)

where \( \alpha \) is the parameter with respect to gains, while \( \beta \) is the parameter associated with losses, \( 0 \leq \alpha, \beta \leq 1 \). \( x \) means gains with \( x \geq 0 \), and losses with \( x < 0 \). \( \lambda \) denote the parameter of risk aversion, \( \lambda > 1 \). The values of parameters \( \alpha, \beta \) and \( \lambda \) are determined through experiments [26,33–35].

![Figure 3. S-shaped value function of prospect theory (PT).](image)

2.3. Related Works

In order to demonstrate the importance and necessity of this study, several important studies in the literature are briefly reviewed that are close to our research.

The DM’s psychological behavior has been addressed in existing EDM studies by different researchers. For example, Fan et al. [14] proposed a risk decision analysis method for emergency response that addressed DM’s psychological behavior in the decision process by employing PT. Wang et al. [16] developed an EDM method that considered not only DM’s psychological behavior in the decision process by using PT, but also the dynamic evolution feature of EE. Due to the uncertainty information about EEs in real-world situations, it is usually a big challenge for DM to estimate possible losses by using crisp values that are employed in existing EDM studies [14,16,36]. Wang et al. [18] presented an EDM method based on PT considering DM’s psychological behavior with interval values, which not only extended the scope of PT for dealing with interval values, but also made the EDM method close to the real world. With the increasing complexity of EEs in the real world, one DM alone [14,16,18,36] cannot make comprehensive judgments and proper decisions; therefore, Wang et al. [17] proposed a group EDM method for emergency situations by using group wisdom to support DM making a decision that takes into account experts’ psychological behavior in the decision process by using PT. Due to the fact that there are various types of information about EEs in the real world, such as crisp values [14,16,36], interval values [18], linguistic information [37], and so on, none of the proposals considers various types of information at the same time; to do that, Wang et al. [38] proposed a group EDM method for not only considering various types of information at the same time, but also together with experts’ psychological behavior and hesitation in qualitative contexts. Motivated
by [38], Zhang et al. [39] presented an EDM method based on PT and hesitant fuzzy sets considering not only experts’ psychological behavior, but also experts’ hesitation in quantitative contexts.

Despite existing EDM studies based on PT having achieved fruitful results [14,16–18,36–40], they neglect an important fact that different emergency situations should be handled by using different measures because of the limited resources and dynamic evolution of EEs.

Nevertheless, to address such an important issue in the real world, GT has been employed in existing EDM studies. For example, Yang and Xu [20] proposed an engineering model based on sequential games considering different situations coping with a flood eruption EDM problem. Chen et al. [41] provided a game theory-based approach for evaluating possible terrorist attacks and corresponding deployment of emergency responses. Gupta et al. [23] proposed a game-theoretic EDM method for considering the optimal allocation solutions of resources to different situations of the EEs, particularly when the available resources are limited. Cheng and Zheng [42] proposed a game-theoretical analysis method considering possible solutions of emergency evacuation for different emergency cases. Rezazadeh et al. [43] presented a security risk assessment method based on game theory for considering the possible terrorist attacks on oil and gas pipelines. Gao et al. [44] proposed an approach for considering different scenarios coping with corporate environment risk based on game theory. Wu [45] presented two game theoretic models for search-and-rescue resource allocation and selection of an acceptable plan for different districts after devastating tsunamis.

Although the existing EDM studies based on GT have obtained remarkable results regarding the different situations coping with the problems of EEs, they build on an assumption that DM is totally rational in the decision process. However, different behavior studies [19,26,27] have proven that DM has limited rationality and his/her psychological behavior can affect the decision behavior directly, especially under a risk and uncertainty environment, and must be considered because of its importance in the decision process.

To overcome the limitations pointed out above and highlight the significance and importance of our research, this study combines the merits of PT and GT to propose a novel EDM method based on GT and PT that considers not only the different situations of coping with problems, but also DM’s psychological behavior in the EDM process, which is introduced in detail in Section 3.

3. Emergency Decision Making Method Based on Game Theory and Prospect Theory

As previously mentioned, the proposed EDM method based on GT and PT is introduced in this section. The general framework of our proposal is illustrated in Figure 4, and it consists of three main phases:

1. Definition framework: this part introduces the basic notations and related terminology that are employed in this proposal.
2. Computation of overall prospect values: in this part, the value function will be used to compute the overall prospect values according to gains and losses.
3. Selecting the optimal alternative based on payoffs: the payoffs of DM including his/her psychological behavior and the payoffs of EE will be determined. Based on the payoffs, the optimal alternative will be selected to respond to corresponding emergency situation.
3.1. Definition Framework

Due to the information about EE usually being inadequate or incomplete, especially in the early stage in a real-world situation, and related emergency situations become more and more complicated with the dynamic evolution of EE across time, it is hard for DM to describe the EE using just one type of information; thus, for convenience, different types of information will be used to describe the situation of EE and emergency response alternatives [15,16]. Thus, in our proposal, both interval and numerical values are employed, in which the interval values are used to estimate the damages or losses caused by EE and numerical values are used to describe the cost of alternatives.

The following notations that will be used in our proposal are defined below:

- $S_1 = \{S_{1\delta}\}$: refers to the set of different alternatives, in which $S_{1\delta}$ denotes the $\delta$-th alternative, $\delta = 1, 2, \ldots, k_1$.
- $S_2 = \{S_{2\theta}\}$: refers to the set of different situations, in which $S_{2\theta}$ denotes the $\theta$-th situations, $\theta = 1, 2, \ldots, k_2$.
- $X = \{X_m\}$: refers to the set of criteria, in which $X_m$ represents the $m$-th criterion, $m = 1, 2, \ldots, M$.
- $W_{X_m} = (w_{X_{1m}}, \ldots, w_{X_{Mm}})$: refers to the weighting vector, in which $w_{X_{m}}$ represents the weight of the $m$-th criterion. The weighting vector is usually provided by the DM satisfying $\sum_{m=1}^{M} w_{X_{m}} = 1$, $w_{X_{m}} \in [0, 1]$, $m = 1, 2, \ldots, M$.
- $C_{\delta}$: refers to the cost of the $\delta$-th available emergency alternative, $\delta = 1, 2, \ldots, k_1$.
- $R_{\theta m} = [R_{L_{\theta m}}^{L}, R_{H_{\theta m}}^{H}]$, $R_{L_{\theta m}}^{H} > R_{H_{\theta m}}^{L}$: refers to the values of RPs, in which $R_{L_{\theta m}}^{L}$ and $R_{H_{\theta m}}^{H}$ represent the lower and upper limits of RP provided by DM for the $m$-th criterion in the $\theta$-th situation, respectively, $m = 1, 2, \ldots, M$, $\theta = 1, 2, \ldots, k_2$.
- $E_{\delta m} = [E_{L_{\delta m}}^{L}, E_{H_{\delta m}}^{H}]$, $E_{L_{\delta m}}^{H} > E_{H_{\delta m}}^{L}$: refers to the value of the pre-defined effective control scope [18], in which $E_{L_{\delta m}}^{L}$ and $E_{H_{\delta m}}^{H}$ represent the lower and upper limits of losses’ protection scope from EE with respect to the $\delta$-th alternative concerning the $m$-th criteria, respectively. $E_{\delta m}$ is usually determined by the local government, $\delta = 1, 2, \ldots, k_1$, $m = 1, 2, \ldots, M$.

3.2. Calculation of Gains and Losses

When an EE occurs, it may have different possible emergency situations. The DM needs to collect related information about possible situations and losses to make a decision. According to the collected
information, DM forms the corresponding RP, \( R_{\theta m} \), of the \( m \)-th criterion \( X_m \) in the \( \theta \)-th situation \( S_\theta \).

Gains and losses can be determined on the basis of the RPs \( R_{\theta m} \) and the pre-defined effective control scope \( E_{\delta m} \) of different alternatives. Because both the RPs and the pre-defined effective control scopes are expressed in the form of interval values, the relationship between the interval values \( R_{\theta m} \) and \( E_{\delta m} \) should be analyzed before determining the gains and losses. To simplify, the relationship between \( R_{\theta m} \) and \( E_{\delta m} \) and the computation formulas for obtaining gains and losses taken from Wang et al. [17] will be utilized in our proposal.

The positional relationship between \( R_{\theta m} \) and \( E_{\delta m} \) is summarized in Table 2. Tables 3 and 4 provide the computation formulas of gains and losses for all possible relationships between \( R_{\theta m} \) and \( E_{\delta m} \), in which Tables 3 and 4 are for cost criteria and benefit criteria, respectively.

Based on the computation formulas of gain and loss provided in Tables 3 and 4, the gain matrix \( GM_\theta \) and the loss matrix \( LM_\theta \) can then be formed. Afterwards, the overall prospect values can be calculated by the value function on the basis of the gain and loss matrix \( GM_\theta, LM_\theta \).

### Table 2. Positional relationship between interval values \( R_{\theta m} \) and \( E_{\delta m} \) [17].

<table>
<thead>
<tr>
<th>Cases</th>
<th>Positional Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( E^H_{\delta m} &lt; R^L_{\theta m} )</td>
</tr>
<tr>
<td>Case 2</td>
<td>( R^H_{\theta m} &lt; E^L_{\delta m} )</td>
</tr>
<tr>
<td>Case 3</td>
<td>( E^L_{\delta m} &lt; R^L_{\theta m} &lt; E^H_{\delta m} &lt; R^H_{\theta m} )</td>
</tr>
<tr>
<td>Case 4</td>
<td>( R^L_{\theta m} &lt; E^L_{\delta m} &lt; R^H_{\theta m} &lt; E^H_{\delta m} )</td>
</tr>
<tr>
<td>Case 5</td>
<td>( E^L_{\delta m} &lt; R^L_{\theta m} &lt; R^H_{\theta m} &lt; E^H_{\delta m} )</td>
</tr>
<tr>
<td>Case 6</td>
<td>( R^L_{\theta m} &lt; E^L_{\delta m} &lt; E^H_{\delta m} &lt; R^H_{\theta m} )</td>
</tr>
</tbody>
</table>

### Table 3. Computation formulas of gain and loss for cost criteria [17].

<table>
<thead>
<tr>
<th>Cases</th>
<th>Gain ( G_{\delta m} )</th>
<th>Loss ( L_{\delta m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( E^H_{\delta m} &lt; R^L_{\theta m} )</td>
<td>( R^L_{\theta m} - 0.5(E^L_{\delta m} + E^H_{\delta m}) )</td>
</tr>
<tr>
<td>Case 2</td>
<td>( R^H_{\theta m} &lt; E^L_{\delta m} )</td>
<td>0</td>
</tr>
<tr>
<td>Case 3</td>
<td>( E^L_{\delta m} &lt; R^L_{\theta m} &lt; E^H_{\delta m} &lt; R^H_{\theta m} )</td>
<td>0.5( (R^L_{\theta m} - E^L_{\delta m}) )</td>
</tr>
<tr>
<td>Case 4</td>
<td>( R^L_{\theta m} &lt; E^L_{\delta m} &lt; R^H_{\theta m} &lt; E^H_{\delta m} )</td>
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<td>( E^L_{\delta m} &lt; R^L_{\theta m} &lt; R^H_{\theta m} &lt; E^H_{\delta m} )</td>
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</tr>
<tr>
<td>Case 6</td>
<td>( R^L_{\theta m} &lt; E^L_{\delta m} &lt; E^H_{\delta m} &lt; R^H_{\theta m} )</td>
<td>0</td>
</tr>
</tbody>
</table>
3.3. Computation of Overall Prospect Values

Assume that the gain matrix of the $\theta$-th situation is denoted by $GM_{\theta} = (G_{\theta}\delta m)_{\delta \times m}$, and similarly, the loss matrix and value matrix of the $\theta$-th situation are denoted by $LM_{\theta} = (L_{\theta}\delta m)_{\delta \times m}$ and $VM_{\theta} = (V_{\theta}\delta m)_{\delta \times m}$, respectively.

\[ v_{\theta}\delta m = G_{\theta}\delta m^a + \left[-\lambda \left(-L_{\theta}\delta m \right)^\beta \right], \quad \delta = 1, 2, \ldots, k_1; \quad \theta = 1, 2, \ldots, k_2; \quad m = 1, 2, \ldots, M \quad (2) \]

where $v_{\theta}\delta m$ means the value with respect to the alternative $S_{\theta}\delta$, concerning criterion $X_{\theta}m$, in the situation $S_{2\theta}$. According to [30], the parameters $a$, $\beta$ and $\lambda$ can employ different values. In this proposal, the following ones will be employed, i.e., $a = \beta = 0.88, \lambda = 2.25$. According to PT, Equation (2) is usually utilized to measure the degree of gains and losses, in which different feelings of DM towards gains and losses are reflected by using prospect values; the greater $v_{\theta}\delta m$, the more DM satisfies, which denotes that the DM satisfies his/her decisions; otherwise, he/she regrets or feels depressed about his/her decisions. In this way, the DM’s psychological behavior can be described clearly and comprehensively.

Due to $v_{\theta}\delta m$ not usually having the same units, a normalization process for removing the effect of units is needed. The normalized value matrix $\overline{VM}_{\theta} = (\overline{v}_{\theta}\delta m)_{\delta \times m}$ can be obtained by using:

\[ \overline{v}_{\theta}\delta m = \frac{v_{\theta}\delta m}{\overline{v}_{\theta}}, \quad \delta = 1, 2, \ldots, k_1; \quad \theta = 1, 2, \ldots, k_2; \quad m = 1, 2, \ldots, M \quad (3) \]

where $v_{\theta}\delta^a = \max_{m \in M} \left| v_{\theta}\delta m \right|$.

On the basis of the normalized value matrix $\overline{VM}_{\theta}$ and the weighting vector $W_{X\theta}$ provided by DM, the overall prospect values of alternative $S_{\theta}\delta$ can be calculated by using the following equation,

\[ O_{\theta\delta} = \sum_{m=1}^{M} \overline{v}_{\theta}\delta m w_{X\theta m}, \quad \delta = 1, 2, \ldots, k_1; \quad \theta = 1, 2, \ldots, k_2; \quad m = 1, 2, \ldots, M \quad (4) \]

3.4. Selecting Optimal Alternative Based on Payoffs

In this section, the payoffs of EE and DM will be determined on the basis of the overall prospect values, $O_{\theta\delta}$, obtained above. Then, according to the payoffs of EE and DM, the optimal alternative can be selected as the proper response regarding different emergency situations.

3.4.1. Determining the Payoffs of the Players

Due to the fact that the game between EE and DM is a zero-sum game and EE is unconscious of the benefits or costs that it will get or lose, just determining the payoffs of the DM is adequate for the emergency response.

Because $O_{\theta\delta}$ is a comprehensive value that reflects the DM’s psychological behavior, it is regarded as the part of the payoffs of DM. Since each alternative has its own cost, it is more reasonable to
consider the prospect values of per unit cost rather than the overall prospect values. The payoffs of DM are determined as follows:

\[ P_1(S_1) = f(O_{\theta\delta}, C_{\delta}) = \frac{O_{\theta\delta}}{C_{\delta}}, \quad \delta = 1, 2, \cdots k_1; \quad \theta = 1, 2, \cdots k_2 \]  

(5)

Then, the payoffs of EE can be obtained as:

\[ P_2(S_2) = -P_1(S_1) \]  

(6)

From Equations (5) and (6), the selection process of the optimal alternative can be determined in the coming subsection.

3.4.2. Selection of the Optimal Alternative with Respect to Each Emergency Situation

As mentioned previously, the game between EE and DM is a zero-sum game, and EE is a special player, which has no consciousness about the real world, so it is adequate to determine the optimal strategy of the DM.

The equation for selecting the optimal strategy of DM with respect to each possible emergency situation goes as follows:

\[ P_1(S_{2\theta}, S_{1\delta}) = \max_{\delta \in k_1} P_1(S_{2\theta}, S_{1\delta}), \quad \theta = 1, 2, \cdots k_2 \]  

(7)

The vector strategy \((S_{2\theta}, S_{1\delta})\) means if the EE has taken \(S_{2\theta}\) as its strategy, the best response for the DM is the strategy \(S_{1\delta}\). In other words, the strategy \(S_{1\delta}\) will be the optimal strategy of DM to deal with the emergency situation \(S_{2\theta}\).

For a clear understanding, the procedures of the new proposed method are summarized as the following steps:

1. Based on the information of \(R_{\theta m}\) and \(E_{\delta m}\), gains and losses can be calculated by using the equations provided in Tables 3 and 4, respectively.
2. The gain and loss matrix \(GM_{\theta}\), \(LM_{\delta}\) can be formed on the basis of the obtained gains and losses, respectively. Then, the value matrix \(VM_{\theta}\) and its normalized form \(VM_{\theta}^n\) can be obtained by using Equations (2) and (3), respectively. Afterwards, the overall prospect value \(O_{\theta\delta}\) can be calculated by Equation (4).
3. Based on the overall prospect value \(O_{\theta\delta}\) and the cost of each alternative, the payoffs of DM and EE can be determined by Equations (5) and (6), respectively.
4. Based on the obtained payoffs of DM and EE, the DM can select the optimal strategies for dealing with all possible emergency situations according to Equation (7).

4. Case Study and Comparison

4.1. Case Study

This part will provide a case study on a typhoon emergency event to demonstrate the validity and rationality of the proposed method.

In summer, it is quite common for coastal cities to suffer from different kinds of losses (lives, property, environment, etc.) caused by typhoons. In order to take effective measures to reduce the losses caused by typhoon as much as possible in the real world, this section takes typhoon landfall as an application background to demonstrate the validity and rationality of our proposal. Suppose that a typhoon is approaching and will possibly make landfall at one city located on the southeast coast of China. When it makes landfall, it might cause various losses, such as lives, properties, environment damages, etc. Thus, the following criteria are concerned in this case study:

\(c_1\): The number of casualties.
$c_2$: Property losses (in 1000$).
$c_3$: The negative effects on the environment on a scale of 0–100 (0: no negative effect; 100: serious negative effect).

The emergency alternatives are described as follows:
Regarding the coming typhoon, the following alternatives can be carried out:

$S_{11}$: Broadcast and send short messages to remind citizens regarding the coming typhoon and suggest that citizens prepare food, water, medicine and other daily necessities in advance; furthermore, local government organizes related departments to check the evacuation solutions and paths to ensure the citizens’ safety as much as possible;

$S_{12}$: Based on $S_{11}$, inform schools and plants to check the safety issues; classes and work can be stopped if necessary. Meanwhile, employees in ocean transport, fishermen and mariculture are required to come back to or go closer to harbors to take shelter from the typhoon. In addition, check the stability of high-altitude facilities and dangerous buildings.

$S_{13}$: Based on $S_{12}$, telecom operators and power supply departments strengthen their checking and maintenance to ensure all different lines of communication and power supply are open. Meanwhile, check the urban drainage pipelines to avoid urban waterlogging.

$S_{14}$: Based on $S_{13}$, vindicate public security in preventing criminal issues from occurring; meanwhile, hospitals prepare enough ambulances and staff to ensure that injured citizens can be rescued and treated immediately. Furthermore, the reservoirs and hydropower stations near the city should make reasonable schedules to avoid floods.

$C_\delta$ is the cost of the $\delta$-th alternative (in 1000$). The criteria weights of each criterion are provided by DM in this case study. The pre-defined effective control scope $E_{\delta m}$, the cost $C_\delta$ and related weights $w_{X_m}$ are given in Table 5.

![Table 5. The $E_{\delta m}$, $C_\delta$ and $w_{X_m}$ of the typhoon emergency.](image)

Analyzing by the weather forecast and historical data, there are four possible situations of a typhoon in the coming 72 h, as follows:

$S_{21}$: The typhoon will not make landfall at the city, and it just brings light rain and wind;

$S_{22}$: The typhoon will make landfall at part of the area of the city and bring moderate rain and gales;

$S_{23}$: The typhoon will make landfall over the entire city and bring rainstorms and strong wind;

$S_{24}$: The typhoon will have a front landfall over the entire city and bring downpours and blustery weather;

The reference points $R_{\theta m}$ regarding the four possible emergency situations provided by DM are shown in Table 6.
Table 6. Reference points (RPs) regarding the four emergency situations.

<table>
<thead>
<tr>
<th>Situations</th>
<th>Criteria c1</th>
<th>Criteria c2</th>
<th>Criteria c3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{21}$</td>
<td>[5,8]</td>
<td>[100,300]</td>
<td>[20,35]</td>
</tr>
<tr>
<td>$S_{22}$</td>
<td>[5,12]</td>
<td>[300,500]</td>
<td>[35,45]</td>
</tr>
<tr>
<td>$S_{23}$</td>
<td>[12,18]</td>
<td>[600,800]</td>
<td>[45,55]</td>
</tr>
<tr>
<td>$S_{24}$</td>
<td>[18,20]</td>
<td>[800,100]</td>
<td>[55,65]</td>
</tr>
</tbody>
</table>

According to the information shown in Tables 5 and 6, the positional relationship between $R_{\theta m}$ and $E_{\delta m}$ in Table 2 and the equations provided in Tables 3 and 4, the gain and loss matrix $GM_{\theta}$, $LM_{\theta}$ can be obtained as follows,

$$GM_1 = \begin{bmatrix} 0 & 0 & 2.5 \\ 3 & 700 & 10 \\ 9 & 1050 & 30 \\ 13.5 & 1350 & 40 \end{bmatrix}, \quad GM_2 = \begin{bmatrix} 0 & 0 & 10 \\ 1 & 500 & 10 \\ 5 & 850 & 20 \\ 9.5 & 1150 & 30 \end{bmatrix},$$

$$GM_3 = \begin{bmatrix} 0 & 0 & 2.5 \\ 1 & 550 & 10 \\ 3.5 & 850 & 20 \end{bmatrix}, \quad GM_4 = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 350 & 2.5 \end{bmatrix};$$

$$LM_1 = \begin{bmatrix} -1 & -50 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad LM_2 = \begin{bmatrix} -1 & -100 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$LM_3 = \begin{bmatrix} -8 & -300 & -2.5 \\ 0 & 0 & 0 \end{bmatrix}, \quad LM_4 = \begin{bmatrix} -8 & -50 & -10 \\ 0 & 0 & 0 \end{bmatrix};$$

Based on $GM_{\theta}$ and $LM_{\theta}$, the value matrix $VM_{\theta}$ and its normalized form $\overline{VM}_{\theta}$ can be obtained according to Equations (2) and (3), respectively, i.e.,

$$VM_1 = \begin{bmatrix} -2.25 & -70.35 & 7.59 \\ 2.63 & 318.92 & 13.96 \end{bmatrix}, \quad VM_2 = \begin{bmatrix} -2.25 & -129.47 & 2.24 \\ 2.63 & -123.19 & 7.59 \end{bmatrix};$$

$$VM_3 = \begin{bmatrix} -14.02 & -340.45 & -5.04 \\ 3.01 & 57.94 & 7.59 \end{bmatrix}, \quad VM_4 = \begin{bmatrix} -22.95 & -533.67 & -17.07 \\ 3.01 & 173.29 & 7.59 \end{bmatrix};$$

$$\overline{VM}_1 = \begin{bmatrix} -2.25 & -70.35 & 7.59 \\ 2.63 & 318.92 & 13.96 \end{bmatrix}, \quad \overline{VM}_2 = \begin{bmatrix} -2.25 & -129.47 & 2.24 \\ 2.63 & -123.19 & 7.59 \end{bmatrix};$$

$$\overline{VM}_3 = \begin{bmatrix} -22.95 & -533.67 & -17.07 \\ 3.01 & 173.29 & 7.59 \end{bmatrix}, \quad \overline{VM}_4 = \begin{bmatrix} -22.95 & -533.67 & -17.07 \\ 3.01 & 173.29 & 7.59 \end{bmatrix};$$

According to Equation (4), the overall prospect values $O_{\theta \delta}$ of the $\theta$-th alternatives in $\delta$-th emergency situation are calculated and shown in Table 7.

Table 7. The overall prospect values $O_{\theta \delta}$ of the $\theta$-th alternatives in the $\delta$-th emergency situation.

<table>
<thead>
<tr>
<th>Situations</th>
<th>$S_{21}$</th>
<th>$S_{22}$</th>
<th>$S_{23}$</th>
<th>$S_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{11}$</td>
<td>0.0710</td>
<td>-0.1927</td>
<td>-0.8152</td>
<td>-1.000</td>
</tr>
<tr>
<td>$O_{12}$</td>
<td>0.4092</td>
<td>0.2841</td>
<td>-0.1008</td>
<td>-0.3524</td>
</tr>
<tr>
<td>$O_{13}$</td>
<td>0.7444</td>
<td>0.6508</td>
<td>0.3419</td>
<td>0.0238</td>
</tr>
<tr>
<td>$O_{14}$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.6074</td>
<td>0.2511</td>
</tr>
</tbody>
</table>

Based on Equations (5) and (6) and the results of $O_{\theta \delta}$ shown in Table 7, the payoff matrix of EE and DM is provided in Table 8.
Table 8. The payoff matrix of EE and DM.

<table>
<thead>
<tr>
<th></th>
<th>$S_{21}$</th>
<th>$S_{22}$</th>
<th>$S_{23}$</th>
<th>$S_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EE</strong></td>
<td>$S_{11}$ $(-0.0071, 0.0071)$</td>
<td>$S_{12}$ $(-0.0193, 0.0193)$</td>
<td>$S_{13}$ $(-0.0815, 0.0815)$</td>
<td>$S_{14}$ $(-0.0100, 0.0100)$</td>
</tr>
<tr>
<td><strong>DM</strong></td>
<td>$S_{12}$ $0.0136, -0.0136$</td>
<td>$S_{13}$ $0.0093, -0.0093$</td>
<td>$S_{14}$ $0.0049, -0.0049$</td>
<td>$S_{14}$ $0.0003, -0.0003$</td>
</tr>
</tbody>
</table>

Then, based on Table 8 and Equation (7), the best strategy of DM for different possible situations can be obtained as follows:

If the EE has selected the strategy $S_{21}$, the best strategy of DM is the one with the biggest payoff value, which can be obtained by using Equation (7):

$$P_1(S_{21}, S_{1\delta}) = \max_{S_{1\delta} \in k_1} P_1(S_{21}, S_{1\delta})$$

$$= \max \{ P_1(S_{21}, S_{11}), P_1(S_{21}, S_{12}), P_1(S_{21}, S_{13}), P_1(S_{21}, S_{14}) \}$$

$$= \max \{ -0.0071, 0.0136, 0.0106, 0.0077 \}$$

$$= 0.0136$$

That is $P_1(S_{21}, S_{1\delta}) = P_1(S_{21}, S_{12})$, which means if the EE has selected the strategy $S_{21}$, the best strategy for DM is $S_{12}$.

Similarly, the best strategies of DM regarding different possible situations are the ones with the biggest payoffs, which are underlined and bolded in Table 9.

Table 9. Best strategies of DM.

<table>
<thead>
<tr>
<th></th>
<th>$S_{21}$</th>
<th>$S_{22}$</th>
<th>$S_{23}$</th>
<th>$S_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EE</strong></td>
<td>$S_{11}$ $(-0.0071, 0.0071)$</td>
<td>$S_{12}$ $(-0.0193, 0.0193)$</td>
<td>$S_{13}$ $(-0.0815, 0.0815)$</td>
<td>$S_{14}$ $(-0.0100, 0.0100)$</td>
</tr>
<tr>
<td><strong>DM</strong></td>
<td>$S_{12}$ $0.0136, -0.0136$</td>
<td>$S_{13}$ $0.0093, -0.0093$</td>
<td>$S_{14}$ $0.0049, -0.0049$</td>
<td>$S_{14}$ $0.0003, -0.0003$</td>
</tr>
</tbody>
</table>

The four optimal solutions with respect to each emergency situation are $(S_{21}, S_{12})$, $(S_{22}, S_{12})$, $(S_{23}, S_{13})$ and $(S_{24}, S_{14})$, which means if the EE has selected $S_{21}$, the best strategy of DM is to select $S_{12}$; if the EE has selected $S_{22}$, the best strategy of DM is to select $S_{12}$; if the EE has selected $S_{23}$, the best strategy of DM is to select $S_{13}$; the EE has selected $S_{24}$, the best strategy of DM is to select $S_{14}$.

4.2. Comparison with Other Methods

In order to demonstrate the superiority and novelty of our proposal, a comparison with other methods will be conducted. Because there are no existing approaches that are based on PT and GT simultaneously, thus, some characteristics have been studied to highlight the superiority of our proposal; see Table 10.

Table 10. Comparison with other emergency decision making (EDM) methods.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Considering DM’s Psychological Behaviors</th>
<th>Considering Different Emergency Situations</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4–8,40,46]</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>[1,15–18]</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>[20–24]</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Our proposal</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
According to Table 10, it can be seen clearly that our proposal considers not only the DM’s psychological behavior, but also the coping with the different emergency situations. The proposed EDM method is closer to the real-world situations than other EDM methods.

5. Conclusions and Future Works

A new EDM method based on GT and PT is proposed in this paper aiming at overcoming the limitations in previous EDM approaches. Due to the inadequate and incomplete information about EEs, interval values are employed in our proposal to estimate the possible losses caused by different situations. DM’s psychological behavior and coping with different emergency situations have been considered simultaneously, which is the significant difference between our proposal and the existing EDM approaches. An example about a typhoon and related comparison with existing EDM approaches have been conducted to demonstrate the novelty and rationality of our proposal. It is hoped that our proposed method can be applied to solve real-word problems in the near future.

The research in the near future should consider the different types of information in the game process, such as linguistic information, hesitant fuzzy linguistic information, and so on, which are common information types in the real world when DM hesitates in his/her assessments.

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