A Multi-Criteria Group Decision Making Model for Green Supplier Selection under the Ordered Weighted Hesitant Fuzzy Environment

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Received: 24 November 2018; Accepted: 20 December 2018; Published: 25 December 2018

Abstract: The green supplier selection (GSS) problem is one of the most pressing issues that can directly affect manufacturer performance. GSS has been studied in previous literature, which is considered to be a typical multiple criteria group decision making (MCGDM) problem. The ordered weighted hesitant fuzzy MCGDM method can present the importance of each possible value, and the priority relationship among criteria has rarely been studied. In this study, we first extend the prioritized average (PA) operator to the ordered weighted hesitant fuzzy set (OWHFS) for solving the both problems. The generalized ordered weighted hesitant fuzzy prioritized weighted average operator (GOWHFPWA) is recommended, and some desirable properties are discussed. Based on this operator, a novel MCGDM method for GSS is developed. A numerical example of GSS is then given to prove the robustness of the proposed approach, and a sensitivity analysis is used to identify the robustness of the proposed method. Finally, a comparative analysis based on the MCGDM approach with the hesitant fuzzy prioritized weighted average (HFPWA) operator is illustrated to indicate the validity and advantages of the proposed approach.

Keywords: green supplier selection; ordered weighted hesitant fuzzy set; GOWHFPWA operator; multi-criteria group decision making

1. Introduction

Nowadays, with the increasingly global awareness of environmental responsibility, green production has already become the development orientation of industrial production for most manufacturing firms. Growing environmental concerns mean that it is necessary for manufacturing companies to be more concerned about green supply chain management (GSCM) to reduce environmental pollution from industrial sectors [1]. The green supplier selection (GSS) is a critical link of GSCM, which can directly affect the sustainable development and performance of manufacturing enterprises [2]. GSS can be regarded as a multiple criteria group decision making (MCGDM) problem that involves many conflicting assessment criteria [3], such as cost, materials, recycling capacity, green competencies, green technology, and green certification. Essentially, the act of decision making is more complicated than in the traditional supplier selection since some environmental criteria need to be considered, and these criteria are qualitative in nature and the weights cannot be provided in advance [3,4]. Therefore, how to choose a suitable green supplier in GSCM has become a key strategic consideration.

Researchers have come up with and applied a range of multi-criteria decision making (MCDM) approaches for green supplier decision making problems [5,6]. To synthesize multiple qualitative or quantitative environmental criteria and obtain a clear evaluation result, some MCDM approaches based...
on precise information are used in green supplier decision making. Handfield et al. [7] evaluated the environmental standards of green suppliers by using the analytic hierarchy process (AHP). Likewise, Lu et al. [8] used AHP to evaluate and coordinate green suppliers. Hsu and Hu [9] applied the analytic network process (ANP) for GSS. Kuo et al. [10] integrated artificial neural network and MCDM approaches for GSS. Bai and Sarkis [11] came up with an analytical evaluation on the basis of rough set theory. Yeh and Chuang [12] introduced an optimal mathematical planning approach for selecting a green supplier. The ANP and radial basis function neural network approaches of choosing green suppliers for China chemical industries was proposed by Zhou et al. [13]. Kuo et al. [14] integrated ANP with the data envelopment analysis (DEA) to evaluate green suppliers. A mathematical model based on DEA for choosing green suppliers was proposed by Jauhar et al. [15]. Dobos and Vörösmarty [16] used a DEA approach towards environmental issues. Freeman and Chen [17] designed an approach for GSS by combining technique for order preference by similarity to ideal solution (TOPSIS), the AHP model, and entropy approach. Hashemi et al. [18] combined the GSS approach with the ANP method. Yeh and Chuang [19] recommended a novel integrated MCDM basis of selecting most suitable green suppliers. Liu et al. [20] expanded a linguistic group decision-making method in assessing big projects.

The major issue obstructing the ability to determine the right mathematical method for choosing a green supplier is the absence of the ability to handle uncertain and inadequate information which mostly happens in real-life conditions. In the practical problems of GSS, a great number of assessment detailed information is unknown, and additionally, several criteria are affected by uncertainty. Meanwhile, decision makers (DMs) usually cannot make completely reasonable judgements due to uncertain and ambiguous information. DMs judgments are usually uncertain and difficult to measure by exact numerical values, so a fuzzy set theory proposed by Zadeh [21] has become essential for solving the complications characterized by vagueness and imprecision. Recently, several studies have applied the typical MCDM methods to a range of fuzzy environments [22–25]. Chiou et al. [26] applied a fuzzy AHP for GSS in China electronic industries. Lee et al. [27] extended a fuzzy AHP decision model to identify GSS for high-tech industries. Tsai and Huang [28] came up with a fuzzy goal programming technique for GSS. Tuzkaya et al. [29] developed a hybrid fuzzy MCDM model, and Büyüközkan and Cifçi [30] recommended a unique hybrid MCDM method to evaluate green suppliers base on reference [29]. Datta et al. [31] presented a VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) method together with the interval valued fuzzy set to choose the best green supplier. Shen et al. [32] presented a fuzzy MCDM as basis for selecting green supplier with linguistic preference. Wang and Chan [33] proposed the hierarchical fuzzy TOPSIS model to choose the green supplier. Cao et al. [34] presented a unique intuitionistic fuzzy judgment matrix integrated with the TOPSIS approach to define the subjective and objective weights in green supplier assessment and selection. Kannan et al. [35] utilized a fuzzy axiomatic design method to choose the most suitable green supplier. Hamdan and Cheaitou [36] proposed fuzzy TOPSIS and AHP methods to define preference weights of respective supplier and criterion. Guo et al. [37] developed a fuzzy MCDM method to solve the GSS in apparel manufacturing.

GSS is known as a MCGDM problem that involves both several interrelated evaluation criteria and several DMs behavior characters. Moreover, the complexity of MCGDM problems is increased when several DMs might be considered in assessment of the problems [38]. Tsui et al. [39] came up with a hybrid MCGDM method based on entropy and AHP to assess GSS problems in manufacturing enterprise. Based on group decision analysis, Darabi and Heydari [40] presented an interval-valued hesitant fuzzy ranking method for selecting green suppliers. Gitinavard et al. [41] developed a unique interval-valued hesitant fuzzy group outranking method for choosing green suppliers. Qin et al. [42] recommended a comprehensive MCGDM approach for GSS in interval type-2 fuzzy sets. Tang [43] employed the hesitant fuzzy Hamacher power weighted average operator to solve the GSS complexities with hesitant fuzzy information.

As evidently shown in the above reviewed literature, various MCDM methods for GSS have been extended to intuitionistic fuzzy sets [44], linguistic fuzzy sets [32], interval-valued fuzzy sets,
However, little study has been done on GSS by using a hesitant fuzzy set (HFS), which was first introduced by Torra [45,46]. As a generalization of fuzzy sets, HFS can describe the situations that permit the expert’s preference judgment for a particular criterion that have few different values, which is a very suitable method for tackling uncertain information and for expressing DMs’ hesitancy in real group decision making [47–49]. Nowadays, to be able to solve the MCGDM problems, varieties of extensions of the HFS have been proposed by scholars, such as generalized HFS, dual hesitant fuzzy sets, hesitant fuzzy linguistic term sets, the higher order HFS, and NaP-HFS [50–56].

However, the HFS method has its own shortcomings, because it only expresses the expert’s judgment as several probable values lack considerations of their importance. In several applied MCGDM problems, especially in GSS, experts usually come from the same field, and might often make the same judgments on a given criterion. Thus, the possible value repeated many times is more significant than that displayed only one time. For this reason, Zhang and Wu [57] developed the model of weighted hesitant fuzzy set (WHFS), in which the importance of possible values provided by DMs has been considered. Farhadinia and Xu [58] modified the definition of WHFS and proposed a new extension of HFS as the ordered weighted hesitant fuzzy set (OWHFS), in which the importance of DMs’ judgments is defined as the repetition rate of the possible values. Therefore, OWHFS can not only express the experts’ judgments as several possible values but also give the importance of each possible value.

Besides the importance of DMs’ judgments, the priority relationship among criteria of GSS selection for OWHFS is one of the most critical research topics at present. To be able to aggregate the evaluation values of criteria for an alternative, Yager [59] first presented a prioritized scoring operator and prioritized average (PA) operator. Recently, several studies have concentrated on aggregation operators for HFS and their application in MCDM. Xia and Xu [60] investigated a series of aggregation operators for hesitant fuzzy information. Wei [61] developed hesitant fuzzy prioritized operators. Qua et al. [62] examined induced generalized dual hesitant fuzzy Shapley hybrid operators. Wei et al. [63] utilized Pythagorean hesitant fuzzy Hamacher aggregation operators. Farhadinia and Xu [58] first presented several aggregation operators for OWHFS and used them for MCDM. However, as far as we know, the priority relationship among criteria for OWHFS has rarely been investigated.

Moreover, by reviewing the existing literature, the criteria of GSS can usually be classified into two categories: General and green criteria [64,65]. Generally, organizations consider criteria such as cost, quality, and delivery performance when evaluating supplier performance. However, due to enterprises facing double pressures of environmental laws and regulations and the increasing demands of environmental protection, environmental performance is considered by many enterprises in selecting suppliers. To solve the complexity of GSS problems in practice, the criteria of green supplier evaluation were studied by scholars. For instance, Lee et al. [27] mentioned that quality, technology capability, environment management, and green competencies are the most commonly referred criteria in green supplier evaluation literature. Yeh and Chuang [12] developed assessment criteria for GSS such as green image, product recycling, green design, green supply chain management, pollution treatment cost, and environment performance assessment criteria. A summary of the most critical standards for GSS are shown in Table 1.

In summary, the concept of GSS is a typical MCGDM problem, of which there are two critical issues of concern. The first issue depicts the importance of DMs’ judgments. Another is mathematically expressing the priority relationship among criteria. The focus of this study is to develop a novel group decision making approach with ordered weighted hesitant fuzzy information for GSS that addresses both of the above problems.

The remainder of this study is established as follows: Section 2 briefly introduces the basic principles of OWHFS and the PA operator. Section 3 develops the generalized ordered weighted hesitant fuzzy prioritized weighted average (GOWHFPWA) operator and investigates its desirable properties. Section 4 proposes a novel MCGDM method for GSS with a GOWHFPWA operator.
Section 5 presents a numerical example of GSS to demonstrate the superiority and effectiveness of the proposed approach. Section 6 provides performance analysis and comparison, including sensitivity and validity analysis of the proposed approach. Finally, conclusions and recommendations are discussed in Section 7.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Criterion</th>
<th>Definition</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>Cost</td>
<td>Total cost of product and service</td>
<td>Yeh and Chuang [12], Govindan et al. [6], Mousakhani et al. [65]</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Quality</td>
<td>The quality of product and service</td>
<td>Omurca [66], Govindan et al. [6], Mousakhani et al. [65]</td>
</tr>
<tr>
<td>$c_3$</td>
<td>Service</td>
<td>Performance in terms of product service and social service</td>
<td>Omurca [66], Kannan et al. [35], Govindan et al. [6]</td>
</tr>
<tr>
<td>$c_4$</td>
<td>Environment</td>
<td>Environmental protection; certification and materials recycling capacity</td>
<td>Govindan et al. [6], Mousakhani et al. [65], Lee et al. [27]</td>
</tr>
<tr>
<td>$c_5$</td>
<td>Technology</td>
<td>Ability to facilitate the development of green products</td>
<td>Mousakhani et al. [65], Lee et al. [27], Govindan et al. [6]</td>
</tr>
<tr>
<td>$c_6$</td>
<td>Management</td>
<td>Capacity for environmental management</td>
<td>Kuo et al. [12], Tseng et al. [24], Mousakhani et al. [65]</td>
</tr>
<tr>
<td>$c_7$</td>
<td>Responsibility</td>
<td>Including safety production, social morality and public interest</td>
<td>Galankashi, et al. [6], Mousakhani et al. [65]</td>
</tr>
</tbody>
</table>

2. Preliminaries

In this section, some basic concepts related to ordered weighted hesitant fuzzy set (OWHFS) and PA operator are reviewed, which will be useful for later analysis.

**Definition 1.** [58] Let $X$ be the universe of discourse. An ordered weighted hesitant fuzzy set (OWHFS) on $X$ is defined as:

$$\omega H = \{ x, \omega h(x) \mid x \in X \}$$

where, $\omega h(x) = \bigcup_{1 \leq j \leq L} \{ < h^{(j)}(x), \omega^{(j)}(x) > \}$, referred to as the ordered weighted hesitant fuzzy element (OWHFE), is a set of some different values in $[0,1]$. It denotes all possible membership degrees of the element $x \in X$ to the set $\omega H$, and $\omega^{(j)}(x) \in [0,1]$ is the weight of $h^{(j)}(x)$ such that $\sum_{j=1}^{L} \omega^{(j)}(x) = 1$ for any $x \in X$.

It is worth noting that when $\omega^{(1)}(x) = \omega^{(2)}(x) = \ldots = \omega^{(L)}(x) = 1/L$, for any $x \in X$, then the OWHFS $\omega H$ will become a typical HFS. For the convenience of representation, OWHFE can be denoted by $\omega h = \omega h(x) = \bigcup_{1 \leq j \leq L} \{ < h^{(j)}, \omega^{(j)} > \}$

Suppose that the membership degrees provided by $k$ experts, of the element $x$ in the set $\omega H$, where $h^{(i)}(x)$ is given by $k_i$ experts, $i = 1, 2, \ldots, L$, $\sum_{i=1}^{L} k_i = k$. It should be noted that every expert cannot persuade other experts to change their opinions. In such a situation, the membership degree of the element $x$ in the set $\omega H$ has $L$ possible values $h^{(1)}(x), h^{(2)}(x), \ldots, h^{(L)}(x)$ associated with weights $\omega^{(1)}(x) = \frac{k_1}{k}, \omega^{(2)}(x) = \frac{k_2}{k}, \ldots, \omega^{(L)}(x) = \frac{k_L}{k}$ respectively.

**Definition 2.** [58] Let $\omega h = \bigcup_{1 \leq j \leq L} \{ < h^{(j)}, \omega^{(j)} > \}$, $\omega h_1 = \bigcup_{1 \leq j \leq L} \{ < h_1^{(j)}, \omega^{(j)} > \}$ and $\omega h_2 = \bigcup_{1 \leq j \leq L} \{ < h_2^{(j)}, \omega^{(j)} > \}$ be three OWHFEs. Then, some operations on the OWHFEs $\omega h, \omega h_1$ and $\omega h_2$ are defined as follows:

(1) $\omega h^\lambda = \bigcup_{1 \leq j \leq L} \{ (h^{(j)}^\lambda, \omega^{(j)}) \}$;
(2) $\lambda^h = \bigcup_{1 \leq j \leq L} \left\{ \left(1 - (1 - h^{(j)})^\lambda, w^{(j)} \right) \right\}$;

(3) $\omega^1_h \oplus \omega^2_h = \bigcup_{1 \leq j \leq L} \left\{ \left(h_1^{(j)} + h_2^{(j)} - h_1^{(j)} h_2^{(j)}, w_1^{(j)} + w_2^{(j)} \right) \right\}$;

where $\lambda > 0$ and $\frac{w_1^{(j)} + w_2^{(j)}}{\sum_{j=1}^L (w_1^{(j)} + w_2^{(j)})} = (j = 1, 2, \ldots, L)$.

Definition 3. [59] Let $\omega^h = \bigcup_{1 \leq j \leq L} \left\{ < h^{(j)}, w^{(j)} > \right\}$, $\omega^1_h = \bigcup_{1 \leq j \leq L} \left\{ < h_1^{(j)}, w_1^{(j)} > \right\}$ and $\omega^2_h = \bigcup_{1 \leq j \leq L} \left\{ < h_2^{(j)}, w_2^{(j)} > \right\}$ be three OWHFEs. $\Delta^{(\omega^h)} = \sum_{j=1}^L h^{(j)} w^{(j)}$ is called the score function of $\omega^h$, and $\nabla^{(\omega^h)} = \sum_{j=1}^L (\Delta^{(\omega^h)} - h^{(j)})^2 w^{(j)}$ is called the deviation function of $\omega^h$.

(1) If $\Delta^{(\omega^1_h)} > \Delta^{(\omega^2_h)}$, then $\omega^1_h \succ \omega^2_h$

(2) If $\Delta^{(\omega^1_h)} < \Delta^{(\omega^2_h)}$, then $\omega^1_h \prec \omega^2_h$

(3) If $\Delta^{(\omega^1_h)} = \Delta^{(\omega^2_h)}$, then

\[
\begin{align*}
\nabla^{(\omega^1_h)} > \nabla^{(\omega^2_h)} & \Rightarrow \omega^1_h \prec \omega^2_h \\
\nabla^{(\omega^1_h)} = \nabla^{(\omega^2_h)} & \Rightarrow \omega^1_h = \omega^2_h \\
\nabla^{(\omega^1_h)} < \nabla^{(\omega^2_h)} & \Rightarrow \omega^1_h \succ \omega^2_h
\end{align*}
\]

Definition 4. [59] Let $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria, and there is a prioritization among the criteria expressed by the linear ordering $C_1 \succ C_2 \succ \ldots \succ C_n$, which indicates that criterion $C_j$ has a higher priority than $C_i$, if $j < i$. The value $C_j(x)$ is the performance of any alternative $x$ under criterion $C_j$, and satisfies $C_j(x) \in [0, 1]$. If

\[
PA(C(x)) = \sum_{j=1}^n T_j C_j(x)
\]

where $w_j = \frac{T_j}{\sum_{i=1}^n T_i}$, $T_j = \prod_{l=1}^{j-1} C_l(x) (j = 1, 2, \ldots, n)$, $T_1 = 1$. Then $PA$ is called the prioritized average operator.

3. GOWHFPWA Operator and Its Properties

In this section, the GOWHFPWA operator is proposed to aggregate the OWHFEs, and some properties are studied.

The PA operator has been commonly used in situations where the DMs’ judgments are the exact values [59]. In this part, we shall extend the PA operator to ordered weighted hesitant fuzzy environments and define the GOWHFPWA operator.

Definition 5. Let $\omega^1_h, \omega^2_h, \ldots, \omega^n_h$ be a set of OWHFEs, then the GOWHFPWA operator is defined as follows:

\[
GOWHFPWA(\omega^1_h, \omega^2_h, \ldots, \omega^n_h) = \left( \frac{T_1}{\sum_{i=1}^n T_i} (\omega^1_h)^{1/\alpha} + \frac{T_2}{\sum_{i=1}^n T_i} (\omega^2_h)^{1/\alpha} + \cdots + \frac{T_n}{\sum_{i=1}^n T_i} (\omega^n_h)^{1/\alpha} \right)^{1/\alpha} = \left( \frac{\sum_{i=1}^n T_i (\omega^h_i)^{1/\alpha}}{\sum_{i=1}^n T_i} \right)^{1/\alpha}
\]

where, $\alpha > 0$ is a parameter of GOWHFPWA operator, $T_i = \prod_{l=1}^{i-1} \Delta^{(\omega^l h)} (i = 1, 2, \ldots, n)$, $T_1 = 1$ and $\Delta^{(\omega^h h)}$ is the score function of $\omega^h$. 
Theorem 1. Let $\omega h_1, \omega h_2, \cdots, \omega h_n$ be a set of OWHFEs, then their aggregated value by using the GOWHFPWA operator is also an OWHFE, and

$$
GOWHFPWA(\omega h_1, \omega h_2, \cdots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} (1 - (h_{i}^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, \left( \sum_{i=1}^{n} w_i^{(j)} \right)^{1/\alpha} \right\}
$$

where, $T_i = \prod_{l=1}^{i} \Delta(\omega h_l)(i = 1, 2, \cdots, n)$, $T_1 = 1$, $\Delta(\omega h_k)$ is the score function of $\omega h_k$, and $L$ is the number of basic units in $\omega h_i(i = 1, 2, \cdots, n)$.

Proof. For $n = 1$, the result can be obtained easily by Definition 5. In the following, we prove the equation

$$
GOWHFPWA(\omega h_1, \omega h_2, \cdots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} (1 - (h_{i}^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, \left( \sum_{i=1}^{n} w_i^{(j)} \right)^{1/\alpha} \right\}
$$

by using mathematical induction for $n(n \geq 2)$.

For $n = 2$, since

$$
\frac{T_1}{\sum T_i} \omega h_1^2 = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - (1 - (h_1^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, w_1^{(j)} \right\}
$$

$$
\frac{T_2}{\sum T_i} \omega h_2^2 = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - (h_2^{(j)})^\alpha \right)^{\frac{T_j}{\sum T_i}}, w_2^{(j)} \right\}
$$

then

$$
\frac{T_1}{\sum T_i} \omega h_1^2 \oplus \frac{T_2}{\sum T_i} \omega h_2^2 = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - (1 - (h_1^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, \left( 1 - (1 - (h_2^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, \left( \sum_{i=1}^{n} w_i^{(j)} \right)^{1/\alpha} \right\}
$$

That is, Equation (7) holds when $n = 2$.

Suppose that Equation (3) also holds when for $n = l$,

$$
GOWHFPWA(\omega h_1, \omega h_2, \cdots, \omega h_l) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{l} (1 - (h_{i}^{(j)})^\alpha) \right)^{\frac{T_j}{\sum T_i}}, \left( \sum_{i=1}^{l} w_i^{(j)} \right)^{1/\alpha} \right\}
$$
when \( n = l + 1 \), the operational laws described in Definition 2 state that

\[
GOWHFPWA(\omega h_1, \omega h_2, \ldots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} \left( 1 - \left( h_{ij}^{\delta(j)} \right)^a \right) \right)^{1/a}, L \right\}
\]

That is, Equation (3) holds for \( n = l + 1 \). Thus, Equation (3) holds for all \( n \).

Then,

\[
GOWHFPWA(\omega h_1, \omega h_2, \ldots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} \left( 1 - \left( h_{ij}^{\delta(j)} \right)^a \right) \right)^{1/a}, L \right\}
\]

Now, consider some desirable properties of the GOWHFPWA operator.

**Theorem 2.** (Idempotency). Let \( \omega h_1, \omega h_2, \ldots, \omega h_n \) be a set of OWHFs, where \( T_i = \prod_{i=1}^{l} \Delta(\omega h_i) (i = 1, 2, \ldots, n) \), \( T_1 = 1 \) and \( \Delta(\omega h_i) \) is the score function of \( \omega h_i \). If \( \omega h_1 = \omega h_2 = \ldots = \omega h_n = \omega h \), then

\[
GOWHFPWA(\omega h_1, \omega h_2, \ldots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} \left( 1 - h_{ij}^{\delta(j)} \right)^a \right) \left( \sum_{i=1}^{n} w_i^{\delta(j)} \right)^{1/a}, L \right\}
\]

**Proof.** If \( \omega h_1 = \omega h_2 = \ldots = \omega h_n = \omega h \), then

\[
GOWHFPWA(\omega h_1, \omega h_2, \ldots, \omega h_n) = \bigcup_{1 \leq j \leq L} \left\{ \left( 1 - \prod_{i=1}^{n} \left( 1 - h_{ij}^{\delta(j)} \right)^a \right) \left( \sum_{i=1}^{n} w_i^{\delta(j)} \right)^{1/a}, L \right\}
\]

**Theorem 3.** (Boundedness). Let \( \omega h_1, \omega h_2, \ldots, \omega h_n \) be a collection of OWHFES, where \( T_i = \prod_{i=1}^{l} \Delta(\omega h_i) (i = 1, 2, \ldots, n) \), \( T_1 = 1 \), \( \Delta(\omega h_i) \) is the score function of \( \omega h_i \). Let \( \omega h^- = \{ < h^+, 1 > \} \), \( \omega h^+ = \{ < h^+, 1 > \} \), \( h^- = \min \left( \min_{h_1^{\delta(j)} \in \omega h_1} (h_1^{\delta(j)}), \min_{h_2^{\delta(j)} \in \omega h_2} (h_2^{\delta(j)}), \ldots, \min_{h_n^{\delta(j)} \in \omega h_n} (h_n^{\delta(j)}) \right) \) and \( h^+ = \max \left( \max_{h_1^{\delta(j)} \in \omega h_1} (h_1^{\delta(j)}), \max_{h_2^{\delta(j)} \in \omega h_2} (h_2^{\delta(j)}), \ldots, \max_{h_n^{\delta(j)} \in \omega h_n} (h_n^{\delta(j)}) \right) \). Then

\[
\omega h^- \leq GOWHFPWA(\omega h_1, \omega h_2, \ldots, \omega h_n) \leq \omega h^+
\]
Proof. According to Definition 2, it is easy to prove that the GOWHFPWA operator satisfies Theorem 5, 6, 7, and 8, so the proof process is omitted. □

4. The MCGDM Approach with Order Weighted Hesitant Fuzzy Information

In this section, we present a novel MCGDM method based on ordered weighted hesitant fuzzy information, which utilizes the above GOWHFPWA operator to rank the alternatives of GSS. Consider a MCGDM for GSS problem, let \( X = \{x_1, x_2, \ldots, x_n\} \) be a set of suppliers, \( C = \{c_1, c_2, \ldots, c_n\} \) be a set of criteria, and \( E = \{e_1, e_2, \ldots, e_k\} \) be a set of DMs. In practice, there is a priority relationship among the GSS evaluation criteria. For example, if DMs believe that environmental protection is the most important criterion, they should take precedence over price, quality, and other criteria. Secondly, if price is more important than quality and other criteria, the priority of price is higher than quality, and so on. Such a prioritization among the criteria can be expressed by the ordering \( c_1 \succ c_2 \succ \ldots \succ c_n \), in which criterion \( c_i \) has a higher priority than \( c_j \) if \( j < i \).

For an alternative under a criterion, all the DMs provide their evaluated values anonymously. The evaluation values of alternative \( x_p \) under criteria \( c_q \) are provided by DM \( e_u(u = 1, 2, \ldots, k) \), which can be represented by an OWHFE \( \omega h_pq \). The ordered weighted hesitant fuzzy group decision matrix \( M = (\omega h_pq)_{m \times n} \) is constructed from all of these OWHFES.

In view of the above analysis, the procedure of the proposed approach is described under the following steps:

Step 1. Calculate the values of \( T_{pq}(p = 1, 2, \ldots, m; q = 1, 2, \ldots, n) \) based on Equation (11).

\[
T_{pq} = \prod_{q=1}^{n-1} \Delta^{(\omega h_{pq})}(p = 1, 2, \ldots, m, q = 1, 2, \ldots, n) \tag{11}
\]

where \( T_{p1} = 1 \).

Step 2. Aggregate the OWHFES \( \omega h_{pq} \) for each supplier \( x_p(p = 1, 2, \ldots, m) \) by the GOWHFPWA operator, then we can get the overall OWHFE \( \omega h_p(p = 1, 2, \ldots, m) \) for the supplier \( x_p(p = 1, 2, \ldots, m) \) as follows:

\[
\omega h_p = \text{GOWHFPWA}(\omega h_{p1}, \omega h_{p2}, \ldots, \omega h_{pm}) = \bigcup_{1 \leq i \leq l_p} \left\{ \left( 1 - \frac{\sum_{q=1}^{n-1} w_p \delta_q^{(i)}}{\sum_{q=1}^{n-1} w_p} \right)^{\frac{1}{\alpha}} \right\} \tag{12}
\]

Step 3. Calculate the score functions \( \Delta^{(\omega h_p)}(p = 1, 2, \ldots, m) \) of the OWHFE \( \omega h_p(p = 1, 2, \ldots, m) \) for the supplier \( x_p(p = 1, 2, \ldots, m) \), that is,

\[
\Delta^{(\omega h_p)} = \sum_{j=1}^{L_p} h_p^{\delta(j)} w_p^{\delta(j)} \tag{13}
\]

Step 4. Rank the score functions \( \Delta^{(\omega h_p)} \) in ascending order. Then, the supplier with the highest priority is the most desirable green supplier.

5. Numerical Example

In light of the above discussion, we will further illustrate the procedure of the proposed method by an example of GSS. The GSCM of manufacturing enterprises is affected by its green suppliers’ performance, and GSCM is considered as a strategic decision for manufacturing enterprises to maintain a competitive advantage in the international market. Inspired by the advantages of GSCM, there is a bus manufacturing enterprise who wants to choose the most appropriate green supplier for purchasing the key components of its new bus equipment. After initial screening, five potential suppliers \( x_i \) (\( i = 1, 2, 3, 4, 5 \)) have been determined for further assessment. In order to choose the most suitable supplier,
the company established a team of six DMs $e_u (u = 1, 2, \ldots, 6)$ from the department of purchasing, quality, and production who have abundant knowledge and experience in GSCM. Finally, four criteria are chosen from the Table 1 criteria list by experts to evaluate possible green suppliers. The four selected criteria are quality ($c_1$), technology ($c_2$), environment ($c_3$), cost ($c_4$), and the priority relationship among the criteria is $c_1 \succ c_2 \succ c_3 \succ c_4$ in the evaluation process. For a supplier under a criterion, six DMs need to give their evaluation values. As an instance, for the supplier $x_1$ under the criterion $c_1$, the evaluation values 0.3, 0.5, and 0.8 are provided by two, one and three DMs, respectively, and then an OWHFE $\omega h_{11}$ can be represented by $\{<0.3,2/6>,<0.5,1/6>,<0.8,3/6>\}$.

In the same manner, all of OWHFES $\omega h_{pq} (p = 1, 2, \ldots, 5, q = 1, 2, 3, 4)$ can be obtained, as shown in Table 2.

<table>
<thead>
<tr>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$&lt;0.3,2/6&gt;,&lt;0.5,1/6&gt;$</td>
<td>$&lt;0.3,2/6&gt;,&lt;0.6,1/6&gt;$</td>
<td>$&lt;0.4,2/6&gt;,&lt;0.5,1/6&gt;$</td>
</tr>
<tr>
<td>$1/6&gt;,&lt;0.8,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.7,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.6,3/6&gt;$</td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>$&lt;0.1,2/6&gt;,&lt;0.4,1/6&gt;$</td>
<td>$&lt;0.2,2/6&gt;,&lt;0.3,1/6&gt;$</td>
<td>$&lt;0.2,2/6&gt;,&lt;0.3,1/6&gt;$</td>
</tr>
<tr>
<td>$1/6&gt;,&lt;0.5,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.5,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.4,3/6&gt;$</td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td>$&lt;0.1,2/6&gt;,&lt;0.2,1/6&gt;$</td>
<td>$&lt;0.1,2/6&gt;,&lt;0.2,1/6&gt;$</td>
<td>$&lt;0.1,2/6&gt;,&lt;0.2,1/6&gt;$</td>
</tr>
<tr>
<td>$1/6&gt;,&lt;0.3,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.3,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.4,3/6&gt;$</td>
<td></td>
</tr>
<tr>
<td>$x_4$</td>
<td>$&lt;0.3,2/6&gt;,&lt;0.4,1/6&gt;$</td>
<td>$&lt;0.2,2/6&gt;,&lt;0.3,1/6&gt;$</td>
<td>$&lt;0.3,2/6&gt;,&lt;0.4,1/6&gt;$</td>
</tr>
<tr>
<td>$1/6&gt;,&lt;0.7,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.6,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.5,3/6&gt;$</td>
<td></td>
</tr>
<tr>
<td>$x_5$</td>
<td>$&lt;0.7,2/6&gt;,&lt;0.8,1/6&gt;$</td>
<td>$&lt;0.5,2/6&gt;,&lt;0.7,1/6&gt;$</td>
<td>$&lt;0.4,2/6&gt;,&lt;0.6,1/6&gt;$</td>
</tr>
<tr>
<td>$1/6&gt;,&lt;0.9,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.8,3/6&gt;$</td>
<td>$1/6&gt;,&lt;0.7,3/6&gt;$</td>
<td></td>
</tr>
</tbody>
</table>

**Step 1.** According to Equation (11), $T_{pq} (p = 1, 2, \ldots, 5, q = 1, 2, 3, 4)$ are calculated as follows:

$$T_{5 \times 4} = \begin{pmatrix} 1.0000 & 0.5833 & 0.3208 & 0.1764 \\ 1.0000 & 0.3500 & 0.1283 & 0.0449 \\ 1.0000 & 0.2167 & 0.0578 & 0.0125 \\ 1.0000 & 0.5167 & 0.2153 & 0.1005 \\ 1.0000 & 0.8167 & 0.5581 & 0.3255 \end{pmatrix}$$

**Step 2.** Aggregate $\omega h_{pq} (p = 1, 2, \ldots, 5, q = 1, 2, 3, 4)$ by using a GOWHPWA ($\alpha = 1$) operator to derive the overall OWHFES $\omega h_p (p = 1, 2, \ldots, 5)$ for the supplier $x_p (p = 1, 2, \ldots, 5)$.

$$\omega h_1 = \{<0.3091,2/6>,<0.5462,1/6>,<0.7470,3/6>\}$$

$$\omega h_2 = \{<0.1305,2/6>,<0.3755,1/6>,<0.4973,3/6>\}$$

$$\omega h_3 = \{<0.1000,2/6>,<0.2000,1/6>,<0.3190,3/6>\}$$

$$\omega h_4 = \{<0.2514,2/6>,<0.3866,1/6>,<0.6654,3/6>\}$$

$$\omega h_5 = \{<0.5703,2/6>,<0.7163,1/6>,<0.8233,3/6>\}$$

**Step 3.** Calculate the score functions $\Delta(\omega h_p) (p = 1, 2, \ldots, 5)$ of the OWHFES $\omega h_p (p = 1, 2, \ldots, 5)$ for the supplier $x_p (p = 1, 2, \ldots, 5)$, that is,

$$\Delta(\omega h_2) = 0.5676, \Delta(\omega h_3) = 0.3547, \Delta(\omega h_4) = 0.2262, \Delta(\omega h_5) = 0.4809, \Delta(\omega h_5) = 0.7211$$

**Step 4.** Rank all the suppliers $x_p (p = 1, 2, \ldots, 5)$ in accordance with the score functions $\Delta(\omega h_p) (p = 1, 2, \ldots, 5)$ and the priority relationship of five suppliers can be obtained, that is,

$$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$$

Thus, the most desirable green supplier is $x_5$. 

**Table 2.** Ordered weighted hesitant fuzzy decision matrix.
6. Performance Analysis and Comparison Analysis

In this section, performance analysis is provided based on the numerical example above to prove the validation and verification of the proposed method, including sensitivity analysis and effectiveness analysis. Additionally, the proposed GOWHFPWA operator is further compared with the hesitant fuzzy prioritized weighted average (HFPWA) operator suggested by Wei [61].

The sensitivity analysis is used to identify and determine the robustness of the proposed method. In Equation (2), the parameter $\alpha$ may affect the final ranking result, so the sensitivity analysis can be carried out by taking different $\alpha$. The score functions $\Delta^{(\omega)}(h_p)$ with different $\alpha$ can be calculated, and all of the results are presented in Table 3 and Figure 1.

Table 3. The results of the generalized ordered weighted hesitant fuzzy prioritized weighted average operator (GOWHFPWA) operator with different $\alpha$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5666</td>
<td>0.3527</td>
<td>0.2255</td>
<td>0.4777</td>
<td>0.7176</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5667</td>
<td>0.3529</td>
<td>0.2256</td>
<td>0.4780</td>
<td>0.7180</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5670</td>
<td>0.3535</td>
<td>0.2258</td>
<td>0.4791</td>
<td>0.7192</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>1</td>
<td>0.5676</td>
<td>0.3547</td>
<td>0.2262</td>
<td>0.4809</td>
<td>0.7211</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>2</td>
<td>0.5689</td>
<td>0.3581</td>
<td>0.2270</td>
<td>0.4847</td>
<td>0.7254</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>5</td>
<td>0.5737</td>
<td>0.3703</td>
<td>0.2309</td>
<td>0.4941</td>
<td>0.7389</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>10</td>
<td>0.5836</td>
<td>0.3854</td>
<td>0.2389</td>
<td>0.5045</td>
<td>0.7581</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
</tbody>
</table>

![Figure 1. The curve of the score function with different $\alpha$.](image)

It can be seen from Table 3 that as parameter $\alpha$ takes different values, the priority relationships of five suppliers are unchanged and the most desirable supplier is still $x_5$. Therefore, the parameter $\alpha$ is insensitive to the proposed method and the obtained result ranking is robustness. Meanwhile, it can be observed from Figure 1 that the values of the score function for each alternative will increase as $\alpha$ increases. From this point of view, the parameter $\alpha$ can be regarded as a DM’s risk attitude. As the DMs can select different $\alpha$ in accordance with their own risk preferences, the proposed GOWHFPWA operator can offer more choice opportunities for the DMs in the actual GSS problems.

Additionally, since we proposed the GOWHFPWA operator based on the HFPWA operator [61], a comparative analysis was conducted in order to illustrate the effectiveness of the proposed GOWHFPWA operator. For convenience of comparison, we apply the HFPWA operator to the above numerical example in this paper. The hesitant fuzzy decision matrix is shown in Table 4.
Table 4. Hesitant fuzzy decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>[0.3,0.5,0.8]</td>
<td>[0.3,0.6,0.7]</td>
<td>[0.3,0.6,0.7]</td>
<td>[0.4,0.5,0.6]</td>
</tr>
<tr>
<td>$x_2$</td>
<td>[0.1,0.4,0.5]</td>
<td>[0.2,0.3,0.5]</td>
<td>[0.1,0.4,0.5]</td>
<td>[0.2,0.3,0.4]</td>
</tr>
<tr>
<td>$x_3$</td>
<td>[0.1,0.2,0.3]</td>
<td>[0.1,0.2,0.4]</td>
<td>[0.1,0.2,0.3]</td>
<td>[0.1,0.2,0.4]</td>
</tr>
<tr>
<td>$x_4$</td>
<td>[0.3,0.4,0.7]</td>
<td>[0.2,0.3,0.6]</td>
<td>[0.1,0.5,0.7]</td>
<td>[0.3,0.4,0.5]</td>
</tr>
<tr>
<td>$x_5$</td>
<td>[0.7,0.8,0.9]</td>
<td>[0.5,0.7,0.8]</td>
<td>[0.4,0.6,0.7]</td>
<td>[0.5,0.6,0.7]</td>
</tr>
</tbody>
</table>

Then $t_{pq}$ ($p = 1, 2, \ldots, 5, q = 1, 2, 3, 4$) are calculated as follows:

\[
t_{5\times4} = \begin{pmatrix}
1.0000 & 0.5333 & 0.2844 & 0.1517 \\
1.0000 & 0.3333 & 0.1111 & 0.0370 \\
1.0000 & 0.2000 & 0.0467 & 0.0093 \\
1.0000 & 0.4667 & 0.1711 & 0.0741 \\
1.0000 & 0.8000 & 0.5333 & 0.3022
\end{pmatrix}
\]

We aggregate all hesitant fuzzy elements $h_{pq}$ ($p = 1, 2, \ldots, 5, q = 1, 2, 3, 4$) by using the HFPWA operator to derive the overall hesitant fuzzy elements $h_p$ ($p = 1, 2, \ldots, 5$) of the suppliers $x_p$ ($p = 1, 2, \ldots, 5$). Taking supplier $x_1$ as an example, we have $h_1 = HFPWA(h_{11}, h_{12}, h_{13}, h_{14}) = \{0.3083,0.3179,0.3295,0.3620,0.3709,0.3816,0.3879,0.3965,0.4067,0.4055,0.4138,0.4238,0.4517,0.4593,0.4685,0.4740,0.4813,0.4902\}$. The scores $s(h_p)$ ($p = 1, 2, \ldots, 5$) of the suppliers $x_p$ ($p = 1, 2, \ldots, 5$) are obtained as the following: $s(h_1) = 0.5539$, $s(h_2) = 0.3408$, $s(h_3) = 0.2080$, $s(h_4) = 0.4524$, $s(h_5) = 0.7174$. Finally, ranking all the suppliers $x_p$ ($p = 1, 2, \ldots, 5$) according to the scores $s(h_p)$ ($p = 1, 2, \ldots, 5$), we can get the priority relationship of six suppliers, that is, $x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$.

Thus, the most desirable supplier by using the HFPWA operator proposed by Wei [61] is also $x_5$. The comparative results can be shown in Table 5.

Table 5. The result of different approaches.

<table>
<thead>
<tr>
<th>Methods</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>Ranking Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOWHFPWA</td>
<td>0.5676</td>
<td>0.3547</td>
<td>0.2262</td>
<td>0.4809</td>
<td>0.7211</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
<tr>
<td>HFPWA</td>
<td>0.5689</td>
<td>0.3581</td>
<td>0.2270</td>
<td>0.4847</td>
<td>0.7254</td>
<td>$x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$</td>
</tr>
</tbody>
</table>

From Table 5, despite the evaluation result obtained by using the HFPWA operator being the same as that of the GOWHFPWA operator, the proposed method has some advantages over the previous method. Firstly, the proposed method in this paper extends a prioritized weighted average operator from HFS to OWHFS which can solve the problem of the importance of the experts' evaluation results that the previous method cannot solve. Secondly, the computational complexity of the proposed approach is much lower than that of the previous method. Therefore, the introduced model for GSS in practice is more objective and reasonable than that obtained by using the HFPWA operator proposed by Wei [61].
7. Conclusions and Further Directions

In this paper, in order to overcome the limitation of MCGDM problems with GSS in practice, we have focused on a novel MCGDM approach with a priority relationship under the ordered weighted hesitant fuzzy environment to evaluate green suppliers, which can present the importance of each DM’s judgment. Firstly, based on the ideal of the PA operator and HFPWA operator, the OWHFPWA operator was introduced and the prominent characteristics of the propose operator were studied. Secondly, we have utilized the OWHFPWA operator to develop MCGDM approaches to solve the GSS problem. Finally, a practical example of GSS in bus manufacturing enterprise was given to verify the practicality of the proposed method, meanwhile, its feasibility and effectiveness in dealing with MCGDM problems was carried out by the performance analysis and comparative analysis.

In future research, we will develop another hesitant fuzzy prioritized aggregation operator to solve the ordered weighted hesitant fuzzy MCGDM for GSS problems, namely, the generalized ordered weighted hesitant fuzzy prioritized weighted geometric (GOWHFPWG) operator. Moreover, we will combine the expanded hesitant fuzzy set (EHFS) [67] with the PA operator to deal with the MCDGM for GSS problems for future research, which take into account that a single DM gives several hesitant fuzzy elements in MCDGM problems.

Author Contributions: Authors Y.L. and L.J. conceived and designed the model for research, pre-processed and analyzed the data and the obtained inference. Authors F.Z. and L.J. processed the data collected and wrote the paper. The final manuscript has been read and approved by all authors.

Funding: The work is supported by the National Natural Science Foundation of China (NSFC) under Projects 71672182, 71711540309, U1604262 and 71272207.

Acknowledgments: The authors are grateful to the editors and the anonymous reviewers for providing us with insightful comments and suggestions throughout the revision process.

Conflicts of Interest: The authors declare no conflict of interest.

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