Article

MHD Stagnation Point Flow of Nanofluid on a Plate with Anisotropic Slip

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Abstract: In this article, an axisymmetric three-dimensional stagnation point flow of a nanofluid on a moving plate with different slip constants in two orthogonal directions in the presence of uniform magnetic field has been considered. The magnetic field is considered along the axis of the stagnation point flow. The governing Navier–Stokes equation, along with the equations of nanofluid for three-dimensional flow, are modified using similarity transform, and reduced nonlinear coupled ordinary differential equations are solved numerically. It is observed that magnetic field and slip parameter increase the velocity and decrease the boundary layer thickness near the stagnation point. Also, a thermal boundary layer is achieved earlier than the momentum boundary layer, with the increase in thermophoresis parameter and Brownian motion parameter. Important physical quantities, such as skin friction, and Nusselt and Sherwood numbers, are also computed and discussed through graphs and tables.

Keywords: stagnation point flow; numerical solution; magnetic field; nanofluid

1. Introduction

The phenomenon of stagnation point flow has various uses in and aerodynamic industries. Such flows mainly compact with the movement of fluid close to the stagnated region of a rigid surface flowing in the fluid material, or retained with dynamics of fluid. Stagnation point has been studied by many researchers in the past because of its wide range of applications in engineering. Initially, stagnation point flow was analyzed by Hiemenz in 1911. He studied the two-dimensional stagnation point flow on a stationary plate. Stagnation point flow applications include cooling of electronic devices by fans, cooling of nuclear reactors, polymer extrusion, wire drawing, drawing of plastic sheets, and many hydrodynamic processes in engineering applications. Stagnation point flow possesses much physical significance, as it is used to calculate the velocity gradients and the rate of heat and mass transfer abutting to stagnation area of frames in high-speed flows, cooling of transpiration, rustproof designs of bearings, etc.

Recently, Borrelli et al. [1] deliberated over the impact of buoyancy on three-dimensional (3D) stagnation point flow. They stated that the buoyancy forces tend to favor an opposite flow. Later, Lok et al. [2] expanded on the work of Weidman [3] with buoyancy forces. They observed the discrete results for free convection and forced convection due to a singularity rising in the convection term. Steady oblique stagnation point flow of a viscous fluid was studied by Grosan et al. [4]. They solved the nonlinear coupled differential equation numerically using the Runge–Kutta method. It is observed that the location of the stagnation point depends strongly on the value of the shear parameter and magnetic parameter. Wang [5–7] discussed the three-dimensional stagnation flow in the absence of MHD and nanofluids on a flat plate, shrinking disk, and rotating disk. Two-dimensional (2D) stagnation flow was discussed by Nadeem et al. [8] using HAM on a stretchable surface.
A fluid, heated by electric current in the presence of strong magnetic field, for example crystal growth in melting, has relevance in manufacturing industries. During the fluid motion, the association of electric current and magnetic field produces a divergence of Lorentz forces. This phenomenon prevents the convective motion of fluid and heat transfer characteristic changes accordingly. Ariel [9] investigated the flow near the stagnation point numerically for small magnetic fields; for large magnetic numbers, the perturbation technique was used. Raju and Sundeep [10] proved that with an increase in the magnetic number, there is an increase in the heat and mass transfer rates. They studied numerically the MHD flow of non-Newtonian fluid over a rotating cone or plate.

Generally, the size of nanoparticles is (1–100 nm). Currently, nanofluids are used for drug delivery in infected areas of the human body. Self-propagating objects containing drugs are used to remove blood clots in sensitive areas such as the brain, eye, heart, etc. Kleinstreuer [11] discussed the drug delivery system in humans at normal body temperature under the influence of some physical parameters such as nanoparticle length, artery diameter, and velocity of fluid. Recently, a mathematical model of nanofluid was developed by Choi [12]. Later, a contribution to heat transfer analysis in nanofluid was made by Buongiorno [13]. His mathematical model dealt with the non-homogeneous model for transport phenomena and heat transfer in nanofluids with applications to turbulence. Saleem et al. [14] discussed the effects of Brownian diffusion and thermophoresis on non-Newtonian fluid models, using HAM in the domain of a vertical rotating cone. Bachok et al. [15] studied the three-dimensional stagnation flow of a viscous fluid numerically, analyzed the velocity and heat transfer for different physical parameters, and compared three nanoparticles, namely C₆₀, Al₂O₃, TiO₂. In [16] Ellahi et al. explored the heat and mass transfer of non-Newtonian fluid in an annulus in a porous medium using HAM. Recently, Sheikholeslami et al. [17] studied the effects of thermal radiation on steady viscous nanofluid in the presence of MHD numerically. Khan [18] explored Brownian diffusion and thermophoresis on stagnation point flow. He considered dual solutions for shrinking/stretching parameters and heat transfer in the presence of buoyancy forces on a stretchable surface. Mustafa et al. [19] investigated 3D nanofluid flow and heat transfer in two opposite directions on a plane horizontal stretchable surface. Thermal and momentum boundary layers were discussed using physical parameters such as Brownian motion and thermophoretic forces. Some more useful studies related to nanofluids can be found in [20–29].

In this article, an axisymmetric 3D stagnation point flow of a nanofluid on a moving plate with different slip constants in two orthogonal directions in the presence of uniform magnetic field has been considered and solved numerically.

2. Mathematical Formulation

Consider a stagnation point flow of a nanofluid over a plate with anisotropic slip in a Cartesian coordinate system, so that the x-axis is taken along the corrugations of plates, the y-axis is normal to the corrugations, and the z-axis is considered with the axis of stagnation flow. The velocities of the moving plate are (u, v) in (x, y) directions, respectively. A constant magnetic field is applied perpendicular to the corrugation along the axis of the stagnation flow in such a way that the magnetic Reynolds number is small. According to Wang [5], the potential flow far from the plate is defined as:

\[
\begin{align*}
\frac{u u_x + u v_y + u w_z}{\rho} &= -\frac{p_x}{\rho} + v \left(u_{xx} + u_{yy} + u_{zz}\right) - \frac{B_0^2}{\rho} u, \\
\frac{u v_x + v v_y + v w_z}{\rho} &= -\frac{p_y}{\rho} + v \left(v_{xx} + v_{yy} + v_{zz}\right) - \frac{B_0^2}{\rho} v, \\
\frac{u w_x + v w_y + w w_z}{\rho} &= -\frac{p_z}{\rho} + v \left(w_{xx} + w_{yy} + w_{zz}\right),
\end{align*}
\]
\[
\frac{\partial T}{\partial x} + \frac{w}{u} \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\rho C}{(\rho C)_f} \left[ \frac{D_B}{\lambda} \left( \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} + \frac{\partial C}{\partial z} \right) \right] + \frac{D_T}{T_\infty} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right],
\]

(4)

\[
\frac{\partial C}{\partial x} + \frac{v}{u} \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{D_B}{\lambda} \left[ \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right] + \frac{D_T}{T_\infty} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right].
\]

(5)

and the boundary conditions are:

\[
u - V = N_2 \mu \frac{\partial w}{\partial z}, \quad T = T_\infty, \quad C = C_\infty \quad \text{at} \quad z = 0,
\]

\[
u \to ax, \quad v \to ay, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at} \quad z \to \infty.
\]

(6)

where \((u, v)\) are the velocity components in the \((x, y)\) directions, \(\nu\) is the kinematic viscosity, \(T\) is the temperature, \(\alpha_m\) is the thermal diffusivity, \(C\) is the volume of nanoparticles, \((\rho C)_f\) is the heat capacity of fluid, \(D_B\) is the Brownian diffusion coefficient and \(D_T\) is the thermophoresis diffusion coefficient. For the non-dimensionalization, we use the following similarity variables:

\[
u = ax f'(\eta) + Uh(\eta),
\]

\[v = ay g'(\eta) + Vk(\eta),
\]

\[w = -\sqrt{\alpha v} [f(\eta) + g(\eta)].
\]

(7)

where \(\eta = \sqrt{\alpha v} z\). Using Equation (7) in Equations (5) and (6) finally we get:

\[
f''' + f'' (f + g) - (f')^2 - M^2 f' = -(1 + M^2)
\]

(8)

\[g''' + g'' (f + g) - (g')^2 - M^2 g' = -(1 + M^2)
\]

(9)

\[h'' + h' (f + g) - hf' - M^2 h = 0
\]

(10)

\[k'' + k' (f + g) - kg' - M^2 k = 0
\]

(11)

\[\theta'' + P_r (f + g) \theta' + P_r \left[ N_t \theta' + N_b \theta'^2 \right] = 0
\]

(12)

\[\phi'' + S_e (f + g) \phi' + \frac{N_t}{N_b} \phi'' = 0.
\]

(13)

and boundary conditions are:

\[f'(0) = \lambda_1 f''(0), \quad g'(0) = \lambda_2 g''(0), \quad h(0) = 1 + \lambda_1 h'(0), \quad k(0) = 1 + \lambda_2 k'(0), \quad f(0) = 0, \quad g(0) = 0,
\]

\[f'(\infty) \to 1, \quad g'(\infty) \to 1, \quad h(\infty) \to 1, \quad k(\infty) \to 1, \quad \theta(0) = 1, \quad \theta(\infty) \to 0, \quad \phi(0) = 1, \quad \phi(\infty) \to 0.
\]

(14)

here \(\lambda_1\) and \(\lambda_2\) are the slip parameters, \(P_r\) the prandtl number, \(S_e\) the Schmidt number, \(N_t\) and \(N_b\) are thermophoresis parameter, Brownian motion parameters, respectively.

The expression for the skin friction coefficient, the local Nusselt number, and Sherwood number for second-grade fluid are defined as:

\[Re_x^{1/2} C_f = f''(0), \quad Nux Re_x^{-1/2} = -\theta'(0), \quad Sh_x Re_x^{-1/2} = -\phi'(0),
\]

(15)
where $Re_x = \frac{U_x}{\nu}$ is the local Reynolds number. The solution of above coupled nonlinear differential equations are found numerically and discussed in the following section.

3. Result and Discussion

A system of nonlinear ordinary differential Equations (8)–(13) subject to the boundary conditions of Equation (14) are solved numerically using the Richardson extrapolation enhancement method. Richardson extrapolation is generally faster, and capable of handling BVP systems with unknown parameters. The values of these parameters can be determined under the presence of a sufficient number of boundary conditions. The solutions are discussed through graphs from Figures 1–10, and values of physical quantities, such as skin friction and Nusselt and Sherwood numbers, are presented in Tables 1–3.

Figures 1 and 2 show the variation of velocity profile $f'$ and $g'$ against $\eta$ for different values of magnetic field $M$ and slip parameter $\lambda_1$. It was observed that increasing in the values of $M$ and $\lambda_1$ causes increase in the velocity profile, while boundary layer thickness reduces. Thus, these parameters cause a reduction in the momentum boundary layer. Analysis shows that increasing the values of these parameters to a sufficiently large level shows the monotonic behavior of velocity throughout the whole domain. Figures 3 and 4 shows the opposite behavior of $h$ and $k$ with the increment of $M$ and $\lambda_1$, such that with the increase in value of these parameters, $h$ and $k$ decreases.

The temperature profile for the nanofluid against different values of thermophoresis parameter $N_t$ and Brownian motion parameter $N_b$ are plotted in Figures 5 and 6. As the temperature increase within the boundary layer, the values of these parameters increase. The thermal boundary layer is achieved earlier than the momentum boundary layer. The variation of nanoconcentration for different values of Schmidt number $S_c$ and $N_t$ is presented in Figures 7 and 8, respectively. It is observed that nanoconcentration $\phi$ decreases as the increase in $S_c$ and boundary layer thickness decreases. Also, with the increase in $N_t$, the nanoconcentration decreases. Figures 9 and 10 show the velocity profile for different values of magnetic parameter $M = 0$ and for $M = 2$. It is observed that in the absence of magnetic parameter $M$, the boundary layer thickness is larger than while $M$ is present. $M = 0$ in Figures 11 and 12 represents the results of Wang [5]. The slip parameter ratio can be defined as $\gamma = \frac{\lambda_2}{\lambda_1}$. Figures 13 and 14 describe the $f'(\eta)$, $g'(\eta)$ for $\gamma = 0.5$. The range of $\gamma$ varies from 0.2 to 10. $\gamma = 1$ represents the isotropic case where $f'(\eta) = g'(\eta)$ and $h(\eta) = k(\eta)$.

Table 1 shows local Nusselt number $Nu_x$ and local Sherwood number $Sh_x$ for the variation of $Pr$ and thermophoresis parameter $N_t$. Here we see that with the increase of $Pr$, the local Nusselt number decreases, while local Sherwood number gives opposite results, meaning $Sh_x$ increases. Moreover, with the increase of $N_t$, the results are again the opposite for $Nu_x$ and $Sh_x$. Table 2 shows local Nusselt number and local Sherwood number for variations of slip parameter $\lambda_1$ and Brownian motion $Nb$. Here it is observed that with the increase of $\lambda_1$ both Nusselt number and local Sherwood number increase. Table 3 shows the skin friction coefficient $C_f$ for different values of $\lambda_1$ and magnetic parameter $M$. Note that with the increment in $\lambda_1$, the value of skin friction decreases. A high value of $M$ gives larger values of skin friction.
Figure 1. Variation of $f'(\eta)$ for different $M$.

Figure 2. Variation of $g'(\eta)$ for different $M$.
Figure 3. Variation of $f'(\eta)$ for different $\lambda_1$.

Figure 4. Variation of $g'(\eta)$ for different $\lambda_2$. 

$N_b = 0.5, N_t = 0.5, M = 2, \lambda_2 = 1, Pr = 6.2, S_c = 2$
Figure 5. Variation of $h(\eta)$ for different $M$.

Figure 6. Variation of $k(\eta)$ for different $M$. 

$N_b = 0.5, N_f = 0.5, \lambda_1 = 1, \lambda_2 = 1, Pr = 6.2, S_c = 2$
Figure 7. Variation of $h(\eta)$ for different $\lambda_1$.

Figure 8. Variation of $k(\eta)$ for different $\lambda_2$. 
Figure 9. Variation of $\theta(\eta)$ for different $N_b$.

$N_t = 0.5, \lambda_1 = 1, \lambda_2 = 1, Pr = 6.2, M = 2, S_c = 2$

Figure 10. Variation of $\theta(\eta)$ for different $N_t$.

$N_b = 0.5, \lambda_1 = 1, \lambda_2 = 1, M = 2, Pr = 6.2, S_c = 2$
Figure 11. Variation of $\phi(\eta)$ for different $N_t$.

$N_b = 0.5, \lambda_1 = 1, \lambda_2 = 1, Pr = 6.2, M = 4, S_c = 0.1$

Figure 12. Variation of $\phi(\eta)$ for different $S_c$.

$N_b = 0.5, N_t = 0.5, \lambda_1 = 1, \lambda_2 = 1, Pr = 6.2, M = 2$
Figure 13. $f'(\eta)$ solid curves and $g'(\eta)$ dashed curves for $\gamma = \frac{\lambda_2}{\lambda_1} = 0.5$. From top: $\lambda_1 = 10, 1, 0.1$.

Figure 14. $h(\eta)$ solid curves and $k(\eta)$ dashed curves for $\gamma = \frac{\lambda_2}{\lambda_1} = 0.5$. From top: $\lambda_1 = 10, 1, 0.1$. 
Table 1. Variation of Local Nusselt number $Nu_x$ and Sherwood number $Sh_x$ for different $Nb$ and $Pr$.

<table>
<thead>
<tr>
<th>$Nb$ = 0.1</th>
<th>$Nb$ = 0.3</th>
<th>$Nb$ = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Pr$</td>
<td>$Nu_x$</td>
<td>$Sh_x$</td>
</tr>
<tr>
<td>5.5</td>
<td>0.67084</td>
<td>1.47332</td>
</tr>
<tr>
<td>5.6</td>
<td>0.66527</td>
<td>1.47440</td>
</tr>
<tr>
<td>5.7</td>
<td>0.65976</td>
<td>1.47546</td>
</tr>
<tr>
<td>5.8</td>
<td>0.65431</td>
<td>1.47651</td>
</tr>
<tr>
<td>5.9</td>
<td>0.64890</td>
<td>1.47754</td>
</tr>
<tr>
<td>6.0</td>
<td>0.64355</td>
<td>1.47856</td>
</tr>
<tr>
<td>6.1</td>
<td>0.63826</td>
<td>1.47956</td>
</tr>
<tr>
<td>6.2</td>
<td>0.63302</td>
<td>1.48055</td>
</tr>
<tr>
<td>6.3</td>
<td>0.62784</td>
<td>1.48152</td>
</tr>
<tr>
<td>6.4</td>
<td>0.62272</td>
<td>1.48248</td>
</tr>
<tr>
<td>6.5</td>
<td>0.61766</td>
<td>1.48342</td>
</tr>
</tbody>
</table>

Table 2. Variation of Local Nusselt number $Nu_x$ and Sherwood number $Sh_x$ for different $M$ and $\lambda_1$.

<table>
<thead>
<tr>
<th>$M$ = 2</th>
<th>$M$ = 4</th>
<th>$M$ = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$Nu_x$</td>
<td>$Sh_x$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25129</td>
<td>1.78860</td>
</tr>
<tr>
<td>0.6</td>
<td>0.25465</td>
<td>1.79657</td>
</tr>
<tr>
<td>0.7</td>
<td>0.25737</td>
<td>1.80301</td>
</tr>
<tr>
<td>0.8</td>
<td>0.25962</td>
<td>1.80831</td>
</tr>
<tr>
<td>0.9</td>
<td>0.26152</td>
<td>1.81276</td>
</tr>
<tr>
<td>1.0</td>
<td>0.26314</td>
<td>1.81655</td>
</tr>
<tr>
<td>1.1</td>
<td>0.26435</td>
<td>1.81980</td>
</tr>
<tr>
<td>1.2</td>
<td>0.26575</td>
<td>1.82263</td>
</tr>
<tr>
<td>1.3</td>
<td>0.26682</td>
<td>1.82511</td>
</tr>
<tr>
<td>1.4</td>
<td>0.26776</td>
<td>1.82731</td>
</tr>
<tr>
<td>1.5</td>
<td>0.26861</td>
<td>1.82926</td>
</tr>
</tbody>
</table>

Table 3. Variation of Skin friction coefficient for different $M$ and $\lambda_1$.

| $\lambda_2 = 1$, $S_c = 2$, $Nb = 0.5$, $Nt = 0.5$, $Pr = 6.2$ |
|---------|---------|---------|
| $M$ = 2 | $M$ = 4 | $M$ = 6 |
| $\lambda_1$ | $C_f$ | $C_f$ | $C_f$ |
| 0.5 | 1.12177 | 1.36687 | 1.51354 |
| 0.6 | 1.00998 | 1.20285 | 1.31469 |
| 0.7 | 0.91823 | 1.07391 | 1.16200 |
| 0.8 | 0.84163 | 0.96991 | 1.04108 |
| 0.9 | 0.77675 | 0.88425 | 0.94294 |
| 1.0 | 0.72109 | 0.81248 | 0.86171 |
| 1.1 | 0.67283 | 0.75148 | 0.79336 |
| 1.2 | 0.63060 | 0.69899 | 0.73505 |
| 1.3 | 0.59334 | 0.65335 | 0.68473 |
| 1.4 | 0.56022 | 0.61330 | 0.64086 |
| 1.5 | 0.53059 | 0.57788 | 0.60227 |

4. Conclusions

The current paper investigated the effects of uniform magnetic field of axisymmetric three-dimensional stagnation point flow of a nanofluid on a moving plate with different slip constants.
The governing equations were made dimensionless and then solved using the Richardson extrapolation enhancement method. The following are the findings of the above work:

- An increase in the magnetic field \( M \) and slip parameter \( \lambda_1 \) causes an increase in the velocity profile and decrease in the boundary layer thickness near the stagnation point.
- It is observed that in the absence of magnetic parameter \( M \) the boundary layer thickness is larger than while \( M \) is present.
- The thermal boundary layer increases with an increase in the thermophoresis parameter \( N_t \) and brownian motion parameter \( N_b \). It is observed that the thermal boundary layer is achieved earlier compared to the momentum boundary layer.
- It is observed that with the increase in \( S_c \) and \( N_t \) the nanoconcentration \( \phi \) decreases and vice versa.

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**Abbreviations**

The following abbreviations are used in this manuscript:

- \((u, v)\) velocity Components
- \(\nu\) kinematic viscosity
- \(N_1, N_2\) slip coefficient
- \(T\) temperature
- \(\alpha_m\) thermal diffusivity
- \(C\) volume of nano particles
- \((\rho C)_f\) heat capacity of fluid
- \(D_B\) Brownian diffusion coefficient
- \(D_T\) thermophoretic diffusion coefficient
- \(\lambda_1, \lambda_2\) slip parameters
- \(N_t\) thermophoresis parameter
- \(N_b\) browning motion parameter
- \(C_f\) skin friction coefficient
- \(Nu_x\) local Nusselt number
- \(Sh_x\) Sherwood number
- \(Re_x\) local Reynolds number
- \(S_c\) Schmidt number
- \(Pr\) prantle number
- \(\gamma\) ratio of slip parameters
- \(\phi\) nano concentration
- \(M\) magnetic parameter

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