Abstract: The growing tendency for suppliers to encroach on the retailers’ market has forced the retailers, being independent entities, to distort shared information to maintain their information superiority. Previous studies on asymmetric information assumed that retailers share information truthfully or that demand satisfies a two-point distribution, which does not always conform to the reality of the dual-channel supply chain. Considering the potential information leakage problem, this paper studied the optimal strategies of the participants and focused on the strategic information management of the dual-channel supply chain. By introducing the retailers’ adverse selection behavior, a sequential game model under general uncertain demand was established, which replaced the classic high-low demand model. The perfect Bayesian Nash equilibrium was characterized, which depended on stochastic demand disturbance, product heterogeneity, supply chain structure, and market investigation cost. The results showed that asymmetric information made the supply chain management inefficient. When the demand disturbance was within the threshold, the retailer distorted order quantity to maintain the information advantage under potential information leakage, and information acquisition was not always good for the retailer—in some cases due to adverse selection problems. A numerical example and a sensitivity analysis were done to validate the model. Our work provides participants in the dual-channel supply chain with decision-making support and direction for improving information management.

Keywords: supply chain management; asymmetric information; information leakage; dual-channel supply chain; decision support systems

1. Introduction

With the emergence of e-commerce and the growing trend of customers online shopping, many traditional suppliers have increased internet channels of direct sales to occupy more market share. A combination of traditional B2B (business-to-business) and modern B2C (business-to-customer) has been a popular market structure (dual-channel supply chain) that has extended to various fields. Currently, according to the authoritative report, 42% of the top suppliers such as IBM, Dell, Wal-Mart, and Taco-Bell sell their products to customers through dual-channel structures [1]. In a dual-channel supply chain, there are not only vertical competitions but also horizontal competitions between the suppliers and retailers [2]. Under this competition structure market, demand information is an important factor for the participants, especially for the suppliers. An accurately forecasted demand state not only makes the upstream suppliers respond quickly to the market and alleviate the bullwhip effect of the dual-channel supply chains but reduces inventory management costs due to the mismatch between supply and demand [3,4]. Because the retailers are closer to consumers and markets than the suppliers, the retailers can use market research to forecast market demand more accurately than the suppliers, i.e., information is asymmetric [5]. A large amount of academic literature has analyzed the value of information and proved that information sharing can yield more profits to a supply chain [6–9].
However, once the retailers share their valuable proprietary information with their competitors, the ability to control the information is seriously damaged, and the dark side of information sharing, or information leakage, appears [7].

Thus, one critical issue faced by the retailers is how to manage their private information strategically. To the suppliers, the retailers’ strategy (traditional-channel order quantity) greatly impacts the suppliers’ decision (direct-channel order quantity). If the retailers (Stackelberg leader) determined the order strategy according to the “true” demand state, the suppliers (Stackelberg follower) would infer the demand state accurately due to the retailers’ order signal, and the retailers would use the value of private information in exchange for first-mover advantages. If the retailers were to determine the order strategy that conveys a “mendacious” demand state, once the suppliers did not credit it, the retailers would incur great loss resulting from the misleading order strategy. In this case, the retailers would rethink the optimal strategy of how to convey demand information.

Concerning information sharing and supply chain coordination, there are some fruitful results in traditional supply chain [7–10]. For example, Lee et al. [7] researched the profits of demand sharing for the supplier and the retailer under a traditional supply chain and identified the drivers of the magnitudes of these benefits. Based on this proposed model, Zhao and Simchilevi [9] analyzed the value of information sharing in a traditional supply chain over a finite time horizon. According to the value of information sharing, some literature adopts the supplier’s point of view to design contracts that give the retailers incentives to share information [11–13], such as revenue-sharing contracts or risk-sharing contracts. Most of the literature, from the perspective of maximum supplier profits, researched the value of demand information and mainly focused on designing contracts to “force” the retailers to share information with the supplier. A key assumption in the traditional supply chain of these models is that when the retailer decides to share the demand information with the supplier, the information is “true” or can be detected by an outside agency. However, this assumption is not realistic in a dual-channel supply chain. Compared with the traditional supply chain, for the retailers, the suppliers are not only their upstream production suppliers but also their competitors. The retailers have incentives to transmit “mendacious” demand signals—that is, adverse selection behavior [14–16]. While the literature on such information sharing is proliferating [17–19], it is not clear how to research strategic information management in a dual-channel supply chain.

The optimal pricing/ordering strategy has been studied in dual-channel supply chain management literatures [20–22]. These literatures typically considered the duopoly competition using different market scenarios. Cai et al. [20] considered the price competition between a supplier and a retailer and compared the effects of price discounts contracts and pricing schemes on supply chain coordination. Based on cost disruption, Song et al. [21] studied a production problem. The results showed that rapid response was always good for the suppliers while not always good for the retailers. In recent years, more and more scholars have studied the problem of information sharing in dual-channel supply chain. For instance, Yue and Liu [23] analyzed the profits of demand forecast information sharing in a dual-channel supply chain while focusing on comparing the value of information sharing under different supply chain structures. In Yue and Liu’s studies, the demand information shared by the retailer needed to be true, and there was no information leakage in the market. Yan and Pei [24] researched a special dual-channel market in which each retailer had direct and traditional channels to sell competitive products to consumers. They found that the retailers were motivated to exaggerate their predictions when sharing information, which made the retailers’ profits decline. By focusing on demand forecasting information sharing, these models do not address the problem of strategic information sharing. Our paper contributes to the existing literature by proposing strategic information sharing in dual-channel supply chain and analyzing the impact of adverse selection behavior on supply chain efficiency.

To supplement the research gap and take into account the information leakage, this study first analyzed the optimal quantity strategy in a dual-channel supply chain under asymmetric information and focused specifically on the role of the retailers’ adverse selection behaviors. Distinct from
traditional supply chain, products in the dual-channel supply chain are heterogeneous. The stochastic demand disturbance greatly influences the strategies of the suppliers and the retailers, especially the retailers’ strategic information management. In addition, we firstly considered demand information acquisition as a decision variable of the retailers instead of a default exogenous variable. From a retailer’s point of view, we built an optimal quantity decision model for the dual-channel supply chain that consisted of one supplier and one retailer. When the demand information existed asymmetrically, we answered the following questions:

(1) Should the retailer invest in demand information acquisition?
(2) Faced with the strategic supplier and retailer, what is the optimal strategy in dual-channel supply chain?
(3) To the retailer, how to strategically share demand information with the supplier?
(4) To the supplier, what is the credible information revelation mechanism?

Our paper makes several contributions. Stochastic demand disturbance has a great influence on the optimal decision and information transmission of dual-channel supply chain. Firstly, stochastic disturbance of the market demand determines the existence of adverse selection behavior. When the stochastic disturbance of the market demand is beyond the threshold, there is no adverse selection in the decision-making game, and information transmission is not distorted. Accurate demand information is valuable to both the retailer and the supplier. When the stochastic disturbance of the market demand is within the threshold, the results become interesting. The material is distorted due to the adverse selection of the retailer. The retailer maintains information superiority by not changing order quantity. The threshold is related to the supply chain structure and product heterogeneity. Besides, information acquisition is not always good for the retailer in this case.

The rest of this paper proceeds as follows. Section 2 solves the dual-channel supply chain equilibrium under asymmetric information and obtains the conditions for the retailer to conduct market investigation. Section 3 researches the impact of stochastic disturbance of the market demand on the optimal strategy and offers case studies to verify the proposed model. Section 4 summarizes the central findings.

2. The Model

2.1. Dual-Channel Supply Chain Structure

This paper considers a dual-channel supply chain consisting of one supplier and one retailer. The supplier sells products or services directly by the direct-channel, such as B2C, and can also sell products at the wholesale price \( w \) through the traditional channel, such as B2C. We assume that the supplier needs a long raw materials purchase lead-time to produce product, and the supply chain structure is shown as Figure 1. The decision-making process can be seen as a sequential game, and the timing of the game is as follows: (i) the retailer decides (Stackelberg leader) whether to conduct market investigation or not and determines the order quantity \( q_r \); (ii) the supplier (Stackelberg follower) infers the market demand according to the retailer’s order quantity \( q_r \) in traditional-channel and determines the order quantity \( q_s \) in direct-channel; (iii) the supplier and the retailer sell products in the market simultaneously, and the market prices \( p_r, p_s \) in two channels are determined by the actual demand function.
Since the customers are heterogeneous in the dual-channel supply chain, we assume that the inverse demand functions in the dual-channel supply chain are formulated as follows [25–27]:

\[ p_r(q_r, q_s, \theta) = a_r - b_r q_r - dq_s + \theta \]  
(1)

\[ p_s(q_s, q_r, \theta) = a_s - b_s q_s - dq_r + \theta \]  
(2)

Equation (1) represents the actual market price \( p_r \) in traditional-channel, which is a linear form of self-quantity \( q_r \) and cross-quantity \( q_s \). \( a_r \) is a constant, which denotes the highest price to customers in traditional-channel. \( b_r \) is a parameter for price sensitivity in traditional-channel market, and cross-price sensitivity parameter \( d \) reflects the degree of substitution of products from two channels. Similarly, Equation (2) is the market price in direct-channel. \( a_s \) denotes the highest price to customers in direct-channel, and \( b_s \) is a parameter for price sensitivity in direct-channel market. The stochastic disturbance \( \theta \) represents demand uncertainty with zero mean and variance \( \sigma^2 \). Therefore, the profits of the supplier and the retailer are as follows:

\[ \pi_s = (p_s - c)q_s + (w - c)q_r \]  
(3)

\[ \pi_r = (p_r - w)q_r \]  
(4)

where \( w \) is the supplier’s wholesale price to the retailer, and \( c \) represents the unit manufacturing cost. To simplify the model, we made some reasonable assumptions consistent with previous research [28,29]. Firstly, for ease of calculation, we assume that manufacturing cost \( c \) equals 0. Then, we assume that the direct-channel price \( p_s \) is always higher than the wholesale price \( w \) (\( p_s \geq w \)), which prevents the retailer from speculating. The retailer is a price-taker of the wholesale price, and the retailer’s price \( p_r \) is higher than the wholesale price, which ensures that the retailer’s profit is more than 0 (\( p_r \geq w \)).

The retailer can privately know the exact demand disturbance \( \theta \) through market investigation, while the supplier can only infer from the retailer’s decision-making. This setting is in line with previous literature [30,31], which demonstrates that the retailers can get more information because of their expertise. The cost of market research is assumed to be \( C_{\text{invest}} \).

**Lemma 1 (PBNE without information acquisition).** In the absence of information acquisition, the Perfect Bayesian Nash Equilibrium (PBNE) can be seen as follows:

\[ q_r^* = \frac{2b_r a_r - da_s - 2b_s w}{4b_r b_s - 2d^2} \]  
(5)

\[ q_s^* = \frac{4b_s b_r a_s - d(2b_r a_r + da_s - 2b_s w)}{2b_r (4b_r b_s - 2d^2)} \]  
(6)

\[ E(\pi_r^*) = \frac{(2b_r a_r - da_s - 2b_s w)^2}{4b_s (4b_r b_s - 2d^2)} \]  
(7)
Proof. According to the inverse method, the optimal response curve of the supplier is:

$$q_s(q_r) = \arg\max_{q_r} \int [a_s - b_s q_s - d q_s(\theta) q_s + w q_r] d\theta = \frac{a_s - d q_r}{2b_s} \tag{8}$$

Equation (8) is substituted into the expected profits of the retailer $E(\pi_r)$. With the derivative of function $E(\pi_r)$ to $q_r$, we understand that the retailer’s optimal quantity $q_r^*$ satisfies the following:

$$q_n^* = \arg\max_{q_r} \int (a_r - b_r q_r - d q_s(\theta) q_r + \theta - w) q_s d\theta = \frac{2a_r b_s - da_s - 2b_s w}{4b_r b_s - d^2} \tag{9}$$

By substituting Equation (9) into Equation (8), we obtain the supplier’s optimal order quantity $q_s^*$. By substituting Equation (9) and Equation (6) into Equation (4), we obtain Equation (7). □

2.2. Optimal Strategy under Potential Information Leakage

In this section, we first analyze the optimal decision-making and expected profits of the dual-channel supply chain when the retailer already has demand information. Then, the value of demand information is obtained by comparing profits of Lemma 1. When the selling season approaches and the retailer already knows the demand disturbance $\theta$ through market research, the retailer’s order decision problem can be reduced to the following optimization problems:

$$\pi_r = \max_{q_r} \left\{ (a_r - b_r q_r - d q_s(\theta) q_r + \theta - w) q_r \right\} \tag{10}$$

s.t. $$(a_r - b_r q_r - d q_s(\theta) q_r + \theta - w) q_r^* \leq (a_r - b_r q_r - d q_s(\theta) q_r + \theta - w) q_r \forall q_r \tag{11}$$

$$(a_r - b_r q_r - d q_s(\theta) q_r + \theta - w) q_r \geq 0 \tag{12}$$

$$q_s = \begin{cases} \frac{a_s - d q_r + \theta(q_r)}{2b_s} & q_r \neq q_r^* \\ \frac{a_s - d q_r}{2b_s} & q_r = q_r^* \end{cases} \tag{13}$$

The objective function Equation (10) denotes the maximum expected profit of the retailer when there exists information leakage. In this case, both the retailer and the supplier can accurately obtain the demand price information $\theta$. Constraints in Equation (11) are incentive compatibility constraints, which indicate that the retailer’s maximum profit from owning demand information is more than that from not acquiring information. This constraint is to prevent the retailer from adverse selection behavior. That is, the retailer observes stochastic disturbance ($\theta \neq 0$) but knows that changing order quantity according to demand disturbance will lead to demand information leakage, which will reduce his own profit. Therefore, a rational retailer chooses not to change the order quantity to prevent information leakage. Equation (12) is the participation constraint of the retailer to ensure that the expected profit of the retailer is greater than 0. Equation (13) represents the supplier’s optimal response curve to the retailer’s order quantity under potential information leakage.

It is noteworthy that Equation (11) is a tight constraint only when $\theta > 0$. When the retailer observes the negative stochastic disturbance ($\theta < 0$), the rational retailer must disclose the information to the supplier. If not, the retailer will bear losses from two sides. On the one hand, he needs to bear the economic losses caused by the untimely adjustment of the order quantity according to stochastic demand disturbance, and on the other hand, because the supplier does not reduce his order quantity in time, there is consequent excessive demand in the market that further reduces the market price. To solve this optimization problem, we obtained the PBNE with potential information leakage as follows.
Proposition 1 (PBNE under potential information leakage). When the retailer already has demand information, the optimal order quantity under the dual-channel supply chain is as follows:

\[
\hat{q}_r = \begin{cases} 
\frac{2\lambda b_q - da_q - 2b_v w + \theta (2b_v - d)}{4b_v b_q - d^2} & \theta < 0 \text{ or } \theta \geq \hat{\theta} \\
\frac{2\lambda b_q - da_q - 2b_v w}{4b_v b_q - d^2} & 0 \leq \theta < \hat{\theta}
\end{cases}
\]  

(14)

\[
\hat{q}_s = \begin{cases} 
\frac{2\lambda b_q - da_q}{2b_v} + \frac{da_s + 2b_v (w - a) + (4b_v b_q - 2d^2)q_r}{2b_v (2b_q - d)} & \theta < 0 \text{ or } \theta \geq \hat{\theta} \\
\frac{2\lambda b_q - da_q}{2b_v} & 0 \leq \theta < \hat{\theta}
\end{cases}
\]  

(15)

**Proof.** The Lagrangian for the above formulation (Equations (10)–(13)) is:

\[
L(q_r, \lambda) = (a_r - b_r q_r - dq_r(q_r) + \theta - w)q_r + \lambda \left[(a_r - b_r q_r - dq_r(q_r) + \theta - w)q_r - (a_r - b_r q_r^* - dq_r^* + \theta - w)q_r^*\right]
\]  

(16)

The first-order Karush-Kuhn-Tucker (KKT) conditions for the Lagrangian are:

\[
\frac{\partial L(q_r, \lambda)}{\partial q_r} \leq 0 \text{ and } q_r \frac{\partial L(q_r, \lambda)}{\partial q_r} = 0 \text{ by complementary slackness.}
\]  

(17)

\[
\frac{\partial L(q_r, \lambda)}{\partial \lambda} \geq 0 \text{ and } \lambda \frac{\partial L(q_r, \lambda)}{\partial q_r} = 0 \text{ by complementary slackness.}
\]  

(18)

Solving the above system, we get:

When the constraint condition is loose (\(\lambda = 0\), we have

\[
\begin{align*}
(a_r - 2b_r q_r - d \left(\frac{a_r - 2a q_r + \theta}{2q_r}\right) + \theta - w & = 0 \\
(a_r - b_r q_r - dq_r(q_r) + \theta - w)q_r - (a_r - b_r q_r^* - dq_r^* + \theta - w)q_r^* - v & = 0 \\
\lambda & = 0 \\
v & \geq 0
\end{align*}
\]  

(19)

By solving Equation (19), we get the optimal retail quantity \(\hat{q}_r\), where \(\hat{\theta}\) is the positive root of Equation (20).

\[
(a_r - b_r \hat{q}_r - dq_r(q_r) + \theta - w)q_r - (a_r - b_r q_r^* - dq_r^* + \theta - w)q_r^* = 0
\]  

(20)

Algebraic manipulation using Equation (20) results \(\hat{\theta}\) as shown in Equation (21):

\[
\hat{\theta} = \frac{2[4(2b_v b_q - d^2)q_r^* + (4b_s - 2d)w + 2(a_s + a_r)w - 4a_r b_s]b_s - 2a_r d^2}{(2b_s - d)^2}
\]  

(21)

When the constraint condition is tight (\(\lambda > 0\), we have:

\[
\hat{q}_r = q_r^* = \frac{2a_r b_s - da_s - 2b_v w}{4b_v b_q - d^2}
\]  

(22)

By substituting Equation (14) into the optimal response curve of the supplier Equation (13), we get the optimal quantity \(\hat{q}_s\) as Equation (15) and the retailer’s profit is:

\[
\hat{\pi}_r = (a_r - b_r \hat{q}_r - dq_r(q_r) + \theta - w)\hat{q}_r
\]  

(23)

□

Proposition 1 shows the optimal decision under different demand disturbances \(\theta\) based on potential information leakage. When the demand disturbance \(\theta\) is positive (\(\theta > 0\)), the demand information has two effects on the retailer’s decision-making—positive effect and negative effect. On
the one hand, if the retailer increases the order quantity \( q_r \), more retailer profits will ensue, which is the positive effect. On the other hand, if the retailer increases the order quantity \( q_r \), the supplier will infer the high demand \( \theta \) from the order quantity \( q_r \) and increase the order quantity \( q_s \) under the direct channel, which intensifies competition among channels. This is the negative effect. Whether the retailer leaks information or not depends on which effect is larger. When the stochastic demand disturbance is large enough (\( \theta > \hat{\theta} \)), the positive effect of increasing the order quantity is greater than the negative one, which makes the retailer share demand information with the supplier rather than retain it. When the stochastic demand disturbance is small (\( 0 < \theta < \hat{\theta} \)), the negative effect of increasing the order quantity is greater than the positive effect. In order to protect the demand information, the retailer chooses the order quantity \( q_r^* \), which makes the supplier unable to judge the real demand information.

When the stochastic demand disturbance is negative (\( \theta < 0 \)), the result is different. The demand information has two positive effects on the retailer’s decision-making, and the retailer will choose to reduce the order quantity \( q_r \) and disclose information to the supplier. On the one hand, the retailer can reduce losses by reducing the order quantity \( q_r \). On the other hand, leaking information can also reduce the order quantity \( q_s \) under direct channel, which reduces the intensity of competition among channels and further reduces losses.

We assumed the probability density of the stochastic demand disturbance \( \theta \) is \( f(\theta) \). Given Proposition 1, we get the expected profit \( E(\hat{\pi}_r) \) of the retailer when the retailer already has demand information as follows:

\[
E(\hat{\pi}_r) = \int \hat{\pi}_r f(\theta) d\theta \quad (24)
\]

**2.3. Conditions for the Retailer to Conduct Market Research**

In Section 2, we assume that the retailer has demand information by default. However, in practice, it takes a lot of time and economic cost for the retailer to collect data and forecast demand based on experience (such as market survey, data analysis, etc.). Therefore, it is an important issue for the retailer to decide whether to conduct market investigation or not, which is closely related to the market investigation cost and the value of demand information. Assuming that the cost of market investigation \( C_{\text{invest}} \) is a fixed constant, Proposition 2 gives the conditions for conducting market investigation.

**Proposition 2.** If the cost of market investigation exceeds the threshold (\( C_{\text{invest}} > \tau \)), the rational retailer’s optimal decision is not to conduct market investigation. Otherwise, if the cost of investigation is less than the threshold (\( C_{\text{invest}} \leq \tau \)), conducting market investigation to accurately know the demand information is the optimal decision, where the threshold:

\[
\tau = E(\hat{\pi}_r) - \frac{(2b_r a_r - d a_s - 2b_s w)^2}{4b_s (4b_r b_s - 2a_r^2)} \quad (25)
\]

**Proof.** Based on the previous assumptions that \( \theta \sim f(\theta) \), \( E(\theta) = 0 \) and the conclusion of Proposition 1, the expected profit of the retailer from market investigation \( E(\overline{\pi}_r) \) is as follows:

\[
E(\overline{\pi}_r) = E(\hat{\pi}_r) - c \quad (26)
\]

The expected profit of the retailer in the absence of information acquisition is:

\[
E(\pi^*_r) = \int_0^\infty (a_r - b_r q^*_r - d q^*_s + \theta - w) f(\theta) q^*_s d\theta \quad (27)
\]

By making Equation (26) equal to Equation (27) and by substituting Lemma 1, the threshold \( \tau \) is obtained. \( \square \)
In summary, whether the retailer decides to conduct a demand investigation or not depends on the cost of the investigation. When the investigation cost exceeds the expected profits from acquiring demand information, the retailer will give up market investigation and order $q^*_r$. On the contrary, the retailer spends cost $c$ to conduct demand investigation and observe demand stochastic disturbance $\theta$ accurately.

3. Numerical Examples

3.1. Optimal Decision with Uniform Distribution of Demand Disturbance

For the ease of exposition, we consider a linear demand function $p_r(q_r, q_s, \theta) = a_r - q_r - q_s + \theta$ in traditional-channel and $p_s(q_s, q_r, \theta) = a_s - q_s - q_r + \theta$ in direct-channel, where $a_r > 0$ and $a_s > 0$ are constants, representing the respective demand scenarios in dual-channel supply chain. The exogenous variable is $b_r = b_s = d = 1$, which means that the two channels have equal price sensitivity. The stochastic demand disturbance $\theta$ is a uniform distribution of $[-200, 200]$. The exogenous wholesale price $w$ is 20 yuan, and all parameters introduced are general knowledge to all participants in this section.

Table 1 shows the results of a case where demand scenario is $a_r = 100$ and $a_s = 80$, respectively. According to Lemma 1, the PBNE of the dual-channel supply chain can be obtained without demand information acquisition as $q^*_r = \frac{2b_1a_2 - a_0 - 2bw}{4b_1b_2 - 2d_2} = 40$ and $q^*_s = \frac{4b_1b_2a_2 - d(2b_1a_2 + a_0 - 2bw)}{2b_1(4b_1b_2 - 2d_2)} = 20$. The expected profit of the retailer is $E(\pi^*_r) = \int_{-200}^{200}(a_r - q^*_r - q^*_s + \theta - w)q^*_r f(\theta)d\theta = 800$. We randomly select seven numbers in the interval $[-200, 200]$ to represent true demand disturbance, which is private information only precisely known to the retailer. Substituting $q^*_s$ into Equation (21), two thresholds $\hat{\theta} = 0$ and $\hat{\theta} = 80/3$ are obtained, indicating that the retailer has the same profit whether or not he has demand information. When the demand disturbance is $\theta$ within the threshold $[0, 80/3]$, the optimal order quantity of the retailer $\hat{q}_r$ equals $\left\{240 - \sqrt{(80 - \theta)(400 - \theta)}\right\}/2$ according to Proposition 1. Otherwise, the optimal order quantity $\hat{q}_r$ is equal to $q^*_r + \theta/2$. The optimal response curve of the supplier is $\hat{q}_s = (a_s - \hat{q}_r)/2$. To prove the value of demand information, we compare the profits of the retailer when he has and does not have demand information. The profit of the retailer from possessing demand information is $\pi_r = (a_r - \hat{q}_r - \hat{q}_s + \theta - w)\hat{q}_r$, while that without demand information is $\pi^*_r = (a_r - q^*_r - q^*_s + \theta - w)q^*_r$. Therefore, the value of information is the difference between two profits $\pi_r - \pi^*_r$. The results are shown in Table 1.

<table>
<thead>
<tr>
<th>Demand Disturbance $\theta$</th>
<th>Optimal Quantity $\hat{q}_r$</th>
<th>Profit with Demand Information $\pi_r$</th>
<th>Profit with no Demand Information $\pi^*_r$</th>
<th>Information Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17.7201</td>
<td>31.1399</td>
<td>208.9</td>
<td>91.2</td>
<td>117.7507</td>
</tr>
<tr>
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<td>39.4151</td>
<td>753.7</td>
<td>753.2</td>
<td>0.5132</td>
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<td>0</td>
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<td>800.0</td>
<td>800.0</td>
<td>0</td>
</tr>
<tr>
<td>3.7505</td>
<td>31.2141</td>
<td>878.5</td>
<td>990.0</td>
<td>-71.5472</td>
</tr>
<tr>
<td>10.5887</td>
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<td>1116.0</td>
<td>1223.5</td>
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<td>1807.1</td>
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</tr>
<tr>
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<td>1866.7</td>
<td>1866.7</td>
<td>0</td>
</tr>
<tr>
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<td>58.5956</td>
<td>2806.3</td>
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<td>518.6917</td>
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</table>

Observing Table 1, we find that the higher the demand disturbance $\theta$ is, the higher the retailer’s profit is, and it does not matter if he has demand information or not. However, the retailer with demand information does not necessarily generate more profits. When demand disturbance is between two thresholds $\theta \in [0, 80/3]$, possessing demand information will bring negative effects to the retailer, which is consistent with the conclusion of Proposition 1.
3.2. Impact of Adverse Selection Behavior on the Optimal Quantity

Adverse selection behavior of the retailer is a major factor that affects strategic behaviors in the dual-channel supply chain. In practice, because the retailer has incentives to transmit "mendacious" demand information, there will be distortion of the optimal order quantity in the dual-channel supply chain. Thus, in this section, we study the influence of the retailer’s adverse selection behavior on the optimal order strategy. Similarly, we compute the optimal order strategy for the supplier and the retailer according to Proposition 1, and the results can be seen based on different stochastic demand disturbances as Figure 2 (selected representative region $\theta \in [-40, 40]$).

![Figure 2](image_url)

**Figure 2.** (a) Optimal ordering quantity in the dual-channel supply chain under different demand disturbance; (b) profits comparison of the retailer with and without demand information.

It can be seen from Figure 2a that the optimal order quantity $q_r$ of the retailer (the blue solid line) increases when the demand disturbance $\theta$ increases, while the optimal order quantity $\hat{q}_s$ for the supplier (the red solid line) decreases when the demand disturbance increases. From the microeconomic view, with the increase of demand disturbance, it is particularly important to adjust the order quantity in time according to actual market demand $\theta$. Due to the first-mover advantage of the retailer, the retailer preferentially occupies the increased demand market and occupies the supplier’s market, which leads to a decline in the market share of the supplier. The optimal order quantity of the retailer $q_r$ equals $40 + \theta/2$. Correspondingly, the optimal order quantity of the supplier $\hat{q}_s$ equals $20 - \theta/4$ in this case.

When demand disturbance $\theta$ is in between 0 and $80/3$, things get interesting. The retailer has a first-mover disadvantage in this interval. If his order strategy truly reflects the market demand disturbance (the blue dotted line in Figure 2a), the total quantity ordered by the supplier and the
retailer will reduce the market prices \( p_s, p_r \), leading to a decline in the retailer’s profit. Therefore, the rational retailer will distort the order quantity \( q_r \) to stop, as shown in Figure 2. The optimal order quantity of the retailer \( \hat{q}_r \) in this case equals \( \left[ 240 - \theta - \sqrt{(80 - \theta)(400 - \theta)} \right] / 2 \), which is less than \( 40 + \theta / 2 \) within the interval \([0, 80/3]\).

Then, we discuss the effect of demand information on the retailer’s profits. As can be seen from Figure 2b, the demand information value for the retailer is positive except within the interval \([0, 80/3]\). The red solid line is the profit of the retailer from having the demand information \( \hat{\pi}_r \), while the blue solid line is the profit of the retailer from not having the demand information \( \pi^*_r \). Therefore, the blue region in the Figure 2b is the value of demand information \( (\hat{\pi}_r - \pi^*_r) \). The value of demand information increases with the increase of demand disturbance \( \theta \). When demand disturbance is within the interval \([0, 80/3]\), the reverse selection behavior of the retailer leads to distorted order quantity of the retailer, which makes the information value negative. As the demand disturbance obeys uniform distribution, we get the expected profits of obtaining demand information

\[
E(\hat{\pi}_r) = \int_{\theta=0}^{\theta=80} \left(100 - \theta - \hat{\theta} + \theta - 20\hat{\theta} \right) d\theta = 2475.013.
\]

According to Proposition 2, we derive that the expected value of information is \( \bar{\varepsilon} = E(\hat{\pi}_r) - E(\pi^*_r) = 1675.013 \). When market investigation cost is less than threshold \( \varepsilon \), it is the optimal strategy for the retailer to conduct market investigation. Otherwise, the retailer chooses not to conduct market research.

3.3. Impact of Product Heterogeneity and Wholesale Price on the Distortion Threshold

In previous chapters, we assume that product heterogeneity \( d \) is equal to 1. However, product heterogeneity \( d \) and wholesale price \( w \) are two key factors affecting ordering quantity distortions. In the dual-channel supply chain, product heterogeneity \( d \) affects the competitiveness of the two channels, while the wholesale price \( w \) directly affects the profit of the retailer in the traditional-channel. Although we set these two variables as exogenous variables in this paper, the two factors have important impacts on the market equilibrium and the range of distortion. Thus, in this section, we examine the effects of the two factors on the distortion interval \([0, \hat{\theta}]\).

On the one hand, to analyze the sensitivity of the wholesale price to the threshold, we solve the first-order partial derivatives and the second-order partial derivatives of wholesale price \( w \) to the threshold \( \hat{d}(d, w) \). Calculated by Equation (21) and Lemma 1, the results are shown in the first two rows of Table 2. As can be seen from Table 2, the threshold \( \hat{\theta} \) decreases linearly with the wholesale price \( w \) (\( \frac{\partial \hat{d}}{\partial w} < 0, \frac{\partial^2 \hat{d}}{\partial w^2} = 0 \)). On the other hand, the sensitivity of the product heterogeneity \( d \) to the threshold \( \hat{\theta} \) is relatively complex. As can be seen from the results in the third to sixth rows of Table 2, the threshold \( \hat{\theta} \) decreases with the product heterogeneity \( d \), that is \( \frac{\partial \hat{d}}{\partial d} < 0 \), and the convexity and concavity of the function \( \hat{d}(d, w) \) are related to the market structure. When \( 2b_1b_3 > d^2 \), \( \hat{d}(d, w) \) is a convex function of product heterogeneity \( d \), that is \( \frac{\partial^2 \hat{d}}{\partial d^2} > 0 \). Conversely, \( \hat{d}(d, w) \) is a concave function of product heterogeneity \( d \), that is \( \frac{\partial^2 \hat{d}}{\partial d^2} < 0 \). However, theoretical analysis is not intuitive, thus we do empirical research to verify our theoretical results.

According to the theoretical results, we choose two representative market structures for empirical research. Let \( w \in [20, 35], d \in [0, 4], b_3 = 1, b_1 = 1, a_t = 80, a_r = 100 \), and according to Equation (21), the functions \( \hat{d}(d, w) \) are drawn by MATLAB, as shown in the Figure 3. Figure 3a,b show the combined effect of wholesale price and product heterogeneity on the threshold \( \hat{d}(d, w) \). It can be seen that the threshold \( \hat{\theta}(d, w) \) decreases with the decrease of wholesale price \( w \) and product heterogeneity \( d \). However, the growth rate of the threshold \( \hat{\theta}(d, w) \) is related to market structure. Therefore, based on the two market structures \( 2b_1b_3 > d^2 \) and \( 2b_1b_3 < d^2 \), we carry out a comparative analysis.
Table 2. Summary of partial derivatives of product heterogeneity and wholesale price to threshold \( \hat{\theta} \).

<table>
<thead>
<tr>
<th>Notations</th>
<th>Mathematical Expression</th>
<th>Definition</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial \theta}{\partial d} )</td>
<td>-2(b_2 b_3) \left{ \begin{array}{ll} &lt; 0, &amp; 2b_2 b_3 &gt; d^2 \ &gt; 0, &amp; 2b_2 b_3 &lt; d^2 \end{array} \right.</td>
<td>\frac{\partial q^*}{\partial d}</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial \theta}{\partial w} )</td>
<td>\frac{-4b_2 d}{(2b_2 - d)^2}</td>
<td>\frac{\partial q^*}{\partial w}</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial^2 \theta}{\partial d^2} )</td>
<td>\frac{4(2b_2 b_3 - d b_2 - 2b_3 w) - 2(2b_2 b_3 - d^2)}{(4b_2 - d^2)^2}</td>
<td>&lt; 0</td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial^2 \theta}{\partial w^2} )</td>
<td>\frac{4(2b_2 b_3 - d b_2 - 2b_3 w) + 2a_d (4b_2 b_3 - d^2)}{(4b_2 - d^2)^2} (d - 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial \theta}{\partial \alpha} )</td>
<td>( \theta^* )</td>
<td>( \frac{\partial q^*}{\partial \alpha} )</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial^2 \theta}{\partial \alpha^2} )</td>
<td>( \frac{2b_2 4(2b_2 b_3 - d b_2 - 2b_3 w) - 2a_d + 2(a_2 + a_3) - 8a_d q^*}{(4b_2 - d^2)^2} - 4a_d, d )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Impact of product heterogeneity \( d \) and wholesale price \( w \) on the distortion threshold \( \hat{\theta} \).

(a) Combined effect based on \( 2b_2 b_3 > d^2 \); (b) Combined effect based on \( 2b_2 b_3 < d^2 \); (c) Product heterogeneity sensitivity based on \( 2b_2 b_3 > d^2 \); (d) Product heterogeneity sensitivity based on \( 2b_2 b_3 < d^2 \); (e) Wholesale price sensitivity based on \( 2b_2 b_3 > d^2 \); (f) Wholesale price sensitivity based on \( 2b_2 b_3 < d^2 \).

On the one hand, fixed wholesale price stays at 20, 25, 30, and 35, respectively, and other parameters remain unchanged. According to Proposition 1, the numerical results of function \( \hat{\theta}(d, w) \) in intervals \([0, 1]\) and \([3, 4]\) are shown in the Figure 3c,d. From Figure 3c,d, we find that the higher the product heterogeneity \( d \) of the two channels, the wider the scope of quantity distortions (the higher the threshold \( \hat{\theta} \)), regardless of the market structure. Product heterogeneity indicates the substitutability...
of direct-channel products and traditional-channel products. The higher the product heterogeneity $d$, the fiercer the competition between the supplier and the retailer. Therefore, an accurate grasp of demand information is more valuable for both sides, which leads the retailer to maintain information superiority by distorting ordering quantity. However, the growth rate of threshold $\hat{\theta}$ is related to the market structure, that is, the sensitivity of the retailer and the supplier to the market $b_r, b_s$. The more sensitive the retailer and the supplier are to market prices ($2b_r b_s > d^2$), the faster the value of demand information increases.

On the other hand, we analyze the sensitivity of thresholds $\hat{\theta}(w, d)$ to wholesale prices $w$ under two market structures. Firstly, exogenous variables $d = 0.1, 0.2, 0.3, 0.4$ are selected in interval $[0, 1]$ to represent market structure $2b_r b_s > d^2$. $d = 3.0, 3.3, 3.6, 4.0$ are selected in interval $[3, 4]$ to represent market structure $2b_r b_s < d^2$. According to Proposition 1, the numerical results of function $\hat{\theta}(w, d)$ are shown in Figure 3e,f. From Figure 3e,f, we can conclude that the threshold $\hat{\theta}$ decreases linearly with the wholesale price $w$. The lower the wholesale price, the lower the retailer’s cost. On the one hand, similar to the analysis of product heterogeneity, the decline of the wholesale price alleviates the competition between the supplier and the retailers, thus reducing the threshold $\hat{\theta}$. On the other hand, the retailer’s purchase cost decreases, which increases the retailer’s optimal ordering quantity. In this way, the retailer is more likely to meet market demand than to protect demand information by distorting ordering quantity, leading to a drop in the threshold $\hat{\theta}$.

3.4. Discussion

Considering potential information leakage, this paper proposes the optimal strategies of the retailer and the supplier in a dual-channel supply chain (Proposition 1). The analysis results show that potential information leakage has a great impact on the decision-making of the dual-channel supply chain. Moreover, information asymmetry leads to quantity distortion, which brings difficulties to supply chain management. Previous studies on optimal strategies of dual-channel supply chain under asymmetric information can be divided into two categories. One is to assume that there is no information leakage—that is, the information shared by the retailer must be true. The other takes into account information leakage with the demand disturbance satisfying a two-point distribution. Thus, we analyze the contributions of this paper in two aspects and compare our conclusions with previous classical studies.

In reality, the development of modern information technology and the inherent characteristics of dual-channel supply chain make information leakage a common phenomenon. Our results show that, compared with the optimal decision-making studies without considering information leakage (Lemma 1) [7,9,11], potential information leakage led to quantitative distortion in a distorted interval $[0, \hat{\theta}(d, w)]$. Based on sensitivity analysis, we analyzed the influence of demand disturbance, product heterogeneity, and wholesale price on the distorted interval. This quantitative distortion led to inefficiency of the dual-channel supply chain management. Therefore, contracts or regulatory agencies should be proposed to ensure that the retailer shares true demand information with the supplier.

One the other hand, compared with the research based on a two-point distribution demand disturbance (high-low demand) [5,6], our research extends it to a general distribution of demand disturbance, which is more in line with reality. Previous research shows that the distortion is related to the demand level [6]. When the gap between high demand and low demand is higher than the threshold, there will be distortion. Otherwise, there will be no distortion [26]. This is a special case of our results. Our results show that if the demand distribution is continuous, distortion will always exist. The distorted interval is related to the distribution function of demand. When the demand disturbance satisfies a discrete distribution, the results are consistent with previous studies.

In addition, there are some restrictions in our paper. We obtained the optimal strategy of the dual-channel supply chain, but in practice, when the retailers do not have incentives to conduct market research, the suppliers may coordinate the dual-channel supply chains through contracts or subsidies, thus the model could be further improved in this respect.
4. Conclusions

With the development of the dual-channel supply chain, how to make decisions and manage information strategies is a significant issue in supply chain management. Given potential information leakage, this paper studies the optimal strategy in a dual-channel supply chain that considers potential information leakage. We derive the Perfect Bayesian Nash equilibrium of the sequential game in the dual-channel supply chain. In particular, the adverse selection behavior is considered on the market equilibrium, and numerical examples are provided to show the conclusions and inferences intuitively.

The paper has two main contributions. Firstly, we obtain the PBNE of the sequential game in the dual-channel supply chain. The optimal order quantity depends on the wholesale price, market structure, and product heterogeneity. Then, our research finds that stochastic demand disturbance has a great effect on the optimal decision and information transmission of the dual-channel supply chain. When stochastic demand disturbance goes beyond the threshold, accurate demand information is valuable to both the retailer and the supplier, which is consistent with classical research [25,26]. On the contrary, the retailer’s adverse selection behavior reduces the value of demand information. The order strategy is distorted due to adverse selection behavior, and in this case, information acquisition is not always good for the retailer.

In addition, the model can be further improved in several aspects. We propose the optimal bidding strategy under potential information leakage. In future research, it will be a hot issue to design optimal contracts and incentives in order to coordinate dual-channel supply chain under potential information leakage.

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