Pythagorean Fuzzy Dombi Aggregation Operators and Their Application in Decision Support System

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Abstract: Keeping in mind the importance and well growing Pythagorean fuzzy sets, in this paper, some novel operators for Pythagorean fuzzy sets and their properties are demonstrated. In this paper, we develop a comprehensive model to tackle decision-making problems where strong points of view are in the favour and against the some projects, entities or plans. Therefore, a new approach, based on Pythagorean fuzzy set models by means of Pythagorean fuzzy Dombi aggregation operators is proposed. An approach to deal with decision-making problems using Pythagorean Dombi averaging and Dombi geometric aggregation operators is established. This model has a stronger capability than existing averaging, geometric, Einstein, logarithmic averaging and logarithmic geometric aggregation operators for Pythagorean fuzzy information. Finally, the proposed method is demonstrated through an example of how the proposed method helps us and is effective in decision-making problems.

Keywords: Pythagorean fuzzy sets; Dombi operations; Pythagorean fuzzy Dombi aggregation operators; decision support system

1. Introduction

This universe is loaded with qualms, imprecision and unclearness. In reality, the greater part of the ideas we encounter in daily life are more unclear than exact. Managing with qualm or uncertainty is a noteworthy issue in numerous territories—for example, economics, engineering, natural science, medicinal science and sociology. Recently, many authors have become keen on demonstrating unclearness. Traditional speculations like fuzzy sets [1], intuitionistic fuzzy sets [2], and Pythagorean fuzzy sets [3] are notable and assume vital jobs in demonstrating uncertainty.

Notion of fuzzy sets introduced by Zadeh [1] revolutionized not only mathematics and logic but also science and technology. It is a very nice tool to handle uncertainty. Here, some membership grade is assigned to an object of a fuzzy sets. In many situations in the real world, apart from the grade of membership, the grade of non-membership is also required. To handle such conditions, Atanassov in [2] initiates the notion of intuitionistic fuzzy sets (IFSs), which are a significant improvement on fuzzy sets. In IFSs, the sum of membership grade and non-membership grade of an object is always from the unit interval. However, the fascinating scenario emerges when the membership and non-membership of an object is given from the unit interval, but their sum exceeds. Ordinary IFSs fail to handle such situations. Therefore, a more comprehensive model is required for such situations.

Yager enquired about this scenario in [3,4] and improved the concept of IFSs to Pythagorean fuzzy sets (PFSs), which could be considered as a generalization of IFSs. The main difference between IFSs
and PFSs is that, in IFSs, the sum of membership and non-membership is always from unit closed interval, but, in PFSs, the sum of squares of membership grade and non-membership grade are real numbers between 0 and 1.


In 1982, Dombi [12] defined Dombi triangular-norm and Dombi triangular-conorm operations, which have made the preference of variability with the operation of parameters. For this advantage, Liu et al. [13] used Dombi operations to intuitionistic fuzzy sets and developed multiple attribute group decision-making problem using a Dombi Bonferroni mean operator under the intuitionistic fuzzy information. Chen and Ye [14] proposed a multiple attribute decision-making problem utilizing Dombi aggregations operators in the single-valued neutrosophic information. Shi et al. [15] extend Dombi operations to neutrosophic cubic sets and use it for travel decision-making problems. Lu and Ye [16] firstly defined a Dombi aggregation operator for linguistic cubic variables, and a multiple attribute decision-making (MADM) method is developed in linguistic cubic setting. He [17] introduced Typhoon disaster assessment based on Dombi hesitant fuzzy information aggregation operators. Jana et al. [18] exhibit some aggregation operators under picture fuzzy data for assessing the distinct priorities of the choices amid the decision-making process. Jana et al. [19] defined some bipolar fuzzy Dombi aggregation operators on the basis of traditional arithmetic, geometric operations and Dombi operations. Wei and Wei [20] presented some combination of operations of prioritized aggregation operators and Dombi operations of SVNNs that consider the prioritized relationship between the SVNNs and proposed some single-valued neutrosophic Dombi prioritized weighted aggregating operators for the aggregation of SVNNs and also investigate their properties.

Motivated by the above discussion, we propose the novel aggregation operators for Pythagorean fuzzy sets using Dombi t-norm and Dombi t-conorm. In the decision-making process, the aggregation operators play the vital role in aggregating the fuzzy information. Thus, in this manner, we propose a series of novel aggregation operators, namely, Dombi weighted average, Dombi weighted geometric, Dombi ordered weighted average, Dombi ordered weighted geometric, Dombi hybrid weighted average and Dombi hybrid weighted geometric aggregation operators for Pythagorean fuzzy information. After that, launch the algorithm to deal with the decision-making problems based on the proposed Dombi aggregation operators. A numerical example demonstrates how our proposed technique helps and is effective in decision-making problems using Pythagorean fuzzy information.

The rest of this study is designed as follows. Section 2 briefly introduces the basic knowledge of the extension of fuzzy sets. The novel Dombi aggregations operators are presented in Section 3. Section 4 presents some discussions on the application of the proposed method. Section 5 discussed the advantages of the proposed work and essential conclusions.

2. Preliminaries

This section consists of a brief review of norms, fuzzy sets and their generalization such as intuitionistic fuzzy sets, Pythagorean fuzzy sets and their basic properties and results.
Definition 1 ([21,22]). A mapping $\hat{T} : \Theta \times \Theta \to \Theta$ is said to be triangular-norm if, for each element, $\hat{T}$ satisfies that

1. $\hat{T}$ is commutative, monotonic and associative,
2. $\hat{T}(\nu^*, 1) = \nu^*$, each $\nu^* \in \hat{T}$,

where $\Theta = [0, 1]$ is the unit interval.

Definition 2 ([21,22]). A mapping $\hat{S} : \Theta \times \Theta \to \Theta$ is said to be triangular-conorm if, for each element, $\hat{S}$ satisfies that

1. $\hat{S}$ is commutative, monotonic and associative,
2. $\hat{S}(\nu^*, 0) = \nu^*$, each $\nu^* \in \hat{S}$,

where $\Theta = [0, 1]$ is the unit interval.

Now, the following is a list of different types of t-norm and t-conorm with generators.

<table>
<thead>
<tr>
<th>Name</th>
<th>t-norm</th>
<th>Additive Generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic</td>
<td>$\hat{T}_A (\hat{d}, \hat{c}) = \hat{d} \hat{c}$</td>
<td>$t (\nu) = - \log \nu$</td>
</tr>
<tr>
<td>Einstein</td>
<td>$\hat{T}_E (\hat{d}, \hat{c}) = \frac{\hat{d} \hat{c}}{1 + (1-\nu)(1-\hat{c})}$</td>
<td>$t (\nu) = \log \frac{2 - \nu}{1 - \nu}$</td>
</tr>
<tr>
<td>Hamacher</td>
<td>$\hat{T}_H (\hat{d}, \hat{c}) = \frac{\hat{d} \hat{c}}{1 + (1-\nu)(1-\hat{c})}$</td>
<td>$t (\nu) = \log \frac{\gamma + (1-\nu)\gamma}{1 - \nu}$, $\gamma &gt; 0$</td>
</tr>
<tr>
<td>Frank</td>
<td>$\hat{T}<em>F (\hat{d}, \hat{c}) = \log</em>\gamma \left(1 + \frac{(1-\gamma)(1-\hat{c})}{\gamma - 1}\right)$</td>
<td>$\gamma = 1$, $t (\nu) = - \log \nu$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>t-conorm</th>
<th>Additive Generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic</td>
<td>$\hat{S}_A (\hat{d}, \hat{c}) = \hat{d} + \hat{c} - \hat{d} \hat{c}$</td>
<td>$s (\nu) = - \log (1 - \nu)$</td>
</tr>
<tr>
<td>Einstein</td>
<td>$\hat{S}_E (\hat{d}, \hat{c}) = \frac{\hat{d} + \hat{c} - \hat{d} \hat{c}}{1 + \hat{d} \hat{c}}$</td>
<td>$s (\nu) = \log \frac{1 + \nu}{1 - \nu}$</td>
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<td>$\gamma = 1$, $s (\nu) = - \log (1 - \nu)$</td>
</tr>
</tbody>
</table>

Definition 3 ([2]). Let us consider a universal set $E$. An intuitionistic fuzzy set $R$ on a set $E$ consists of two mappings, which are defined as:

$$R = \{(P_\Theta (\epsilon), N_\Theta (\epsilon)) \mid \epsilon \in E\},$$

such that the mappings $P_\Theta : E \to \Theta$ and $N_\Theta : E \to \Theta$ represent the positive and negative grades $\epsilon \in E$ to the set $\Theta$, and $\Theta = [0, 1]$ is the unit interval. Having the condition that $0 \leq P_\Theta (\epsilon) + N_\Theta (\epsilon) \leq 1$, for all $\epsilon \in E$, then $R$ is said to be an intuitionistic fuzzy set in $E$.

Definition 4 ([3]). Let us consider a universal set $E$. A Pythagorean fuzzy set $C$ on a set $E$ consists of two mappings which are defined as:

$$C = \{(P_\Theta (\epsilon), N_\Theta (\epsilon)) \mid \epsilon \in E\},$$

such that the mappings $P_\Theta : E \to \Theta$ and $N_\Theta : E \to \Theta$ represent the positive and negative grades $\epsilon \in E$ to the set $C$, and $\Theta = [0, 1]$ is the unit interval. Having the condition that $0 \leq P_\Theta^2 (\epsilon) + N_\Theta^2 (\epsilon) \leq 1$, for all $\epsilon \in E$, then $C$ is said to be the Pythagorean fuzzy set in $E$.

$$\chi_\Theta (\epsilon) = \sqrt{1 - (P_\Theta^2 (\epsilon) + N_\Theta^2 (\epsilon))}$$

is known to be a hesitancy degree of $\epsilon \in E$ to the set $C$.

Yager [3] proposed the basic operations of the Pythagorean fuzzy set as follows:

Definition 5 ([3]). For any two PFNs, $C_1 = \langle P_{\Theta_1} (\epsilon), N_{\Theta_1} (\epsilon) \rangle$ and $C_2 = \langle P_{\Theta_2} (\epsilon), N_{\Theta_2} (\epsilon) \rangle$ in $E$. The union, intersection and compliment proposed as:
If $C_1 \subseteq C_2$ iff $\forall e \in E, P_{\theta_1}(e) \leq P_{\theta_2}(e)$ and $N_{\theta_1}(e) \geq N_{\theta_2}(e)$;
(2) $C_1 = C_2$ iff $C_1 \subseteq C_2$ and $C_2 \subseteq C_1$;
(3) $C_1 \cup C_2 = \{ \max (P_{\theta_1}, P_{\theta_2}), \min (N_{\theta_1}, N_{\theta_2}) \}$;
(4) $C_1 \cap C_2 = \{ \min (P_{\theta_1}, P_{\theta_2}), \max (N_{\theta_1}, N_{\theta_2}) \}$;
(5) $C_1 = \langle N_{\theta_1}, P_{\theta_1} \rangle$.

**Definition 6 ([3]).** For any two PFNs, $C_1 = \langle P_{\theta_1}(e), N_{\theta_1}(e) \rangle$ and $C_2 = \langle P_{\theta_2}(e), N_{\theta_2}(e) \rangle$ in $E$ and $\beta \geq 0$. Then, the operations of PFNs are proposed as,

1. $C_1 \oplus C_2 = \{ \sqrt{p^2_{\theta_1} + p^2_{\theta_2} - p^2_{\theta_1} \cdot p^2_{\theta_2}}, N_{\theta_1} \cdot N_{\theta_2} \}$;
2. $C_1 \cdot C_2 = \{ \sqrt{1 - (1 - P^2_{\theta_1})^\beta}, (N_{\theta_1})^\beta \}$;
3. $C_1 \otimes C_2 = \{ P_{\theta_1} \cdot P_{\theta_2}, \sqrt{N^2_{\theta_1} + N^2_{\theta_2} - N^2_{\theta_1} \cdot N^2_{\theta_2}} \}$;
4. $C_1^\beta = \{ (P_{\theta_1})^\beta, \sqrt{1 - (1 - N^2_{\theta_1})^\beta} \}$.

Yager [3] introduced some properties of the operational laws of Pythagorean fuzzy sets are as follows:

**Theorem 1.** For any three SFNs, $C_1 = \langle P_{\theta_1}(e), N_{\theta_1}(e) \rangle$, $C_2 = \langle P_{\theta_2}(e), N_{\theta_2}(e) \rangle$ and $C_3 = \langle P_{\theta_3}(e), N_{\theta_3}(e) \rangle$ in $E$ and $\beta_1, \beta_2 \geq 0$. Then,

1. $C_1 \oplus C_2 = C_2 \oplus C_1$;
2. $C_1 \otimes C_2 = C_2 \otimes C_1$;
3. $\beta_1(C_1 \oplus C_2) = \beta_1C_1 + \beta_1C_2, \ \beta_1 > 0$;
4. $(C_1 \otimes C_2)^\beta_1 = C_1^{\beta_1} \otimes C_2^{\beta_1}, \ \beta_1 > 0$;
5. $\beta_1C_1 \oplus \beta_2C_2 = (\beta_1 + \beta_2)C_1, \ \beta_1 > 0, \beta_2 > 0$;
6. $C_1^{\beta_1} \otimes C_2^{\beta_2} = C_1^{(\beta_1 + \beta_2)}, \ \beta_1 > 0, \beta_2 > 0$;
7. $(C_1 \oplus C_2) \otimes C_3 = C_1 \otimes (C_2 \oplus C_3)$;
8. $(C_1 \otimes C_2) \otimes C_3 = C_1 \otimes (C_2 \otimes C_3)$.

**Definition 7 ([3]).** For any PFN $C_p = \langle P_{\theta_p}(e), N_{\theta_p}(e) \rangle$ in $E$. Then, score and accuracy values are defined as

1. $S(C_p) = P^2_{\theta_p} - N^2_{\theta_p} \in [0,1]$;
2. $\tilde{A}(C_p) = P^2_{\theta_p} + N^2_{\theta_p} \in [0,1]$.

**Definition 8.** For any PFNs, $C_p = \langle P_{\theta_p}(e), N_{\theta_p}(e) \rangle$ $(p = 1,2)$ in $E$. Then, comparison technique proposed as,

1. If $\tilde{S}(C_1) < \tilde{S}(C_2)$ then $C_1 < C_2$,
2. If $\tilde{S}(C_1) > \tilde{S}(C_2)$ then $C_1 > C_2$,
3. If $\tilde{S}(C_1) = \tilde{S}(C_2)$ then
   a. $\tilde{A}(C_1) < \tilde{A}(C_2)$ then $C_1 < C_2$,
   b. $\tilde{A}(C_1) > \tilde{A}(C_2)$ then $C_1 > C_2$,
   c. $\tilde{A}(C_1) = \tilde{A}(C_2)$ then $C_1 \approx C_2$.

Garg [23,27] proposed that Pythagorean fuzzy aggregation operators are as follows:

**Definition 9 ([23]).** For any collection of PFNs, $C_p = \langle P_{\theta_p}(e), N_{\theta_p}(e) \rangle$ $(p = 1,2,\ldots,n)$ in $E$. The structure of Pythagorean fuzzy weighted averaging (PFWA) operator is

$$\text{PFWA} (C_1, C_2, \ldots, C_n) = \sum_{p=1}^{n} \beta_p C_p,$$
where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

**Definition 10** ([23]). For any collection of PFNs, $C_p = \langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \rangle$ ($p = 1, 2, ..., n$) in $E$. The structure of Pythagorean fuzzy order weighted averaging (PFOWA) operator is

$$PFOWA (C_1, C_2, ..., C_n) = \sum_{p=1}^{n} \beta_p C_{\theta_p(p)}$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\theta_p(p)}$ consequently by total order $C_{\theta_p(1)} \geq C_{\theta_p(2)} \geq ... \geq C_{\theta_p(n)}$.

**Definition 11** ([23]). For any collection of PFNs, $C_p = \langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \rangle$ ($p = 1, 2, ..., n$) in $E$. The structure of Pythagorean fuzzy hybrid weighted averaging (PFHWA) operator is

$$PFHWA (C_1, C_2, ..., C_n) = \sum_{p=1}^{n} \beta_p C_{\theta_p(p)} *, p \in N$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\theta_p(p)}$ consequently by total order $C_{\theta_p(1)} \geq C_{\theta_p(2)} \geq ... \geq C_{\theta_p(n)}$. In addition, associated weights are $\omega = (\omega_1, \omega_2, ..., \omega_n)$ with $\omega_p \geq 0$, $\sum_{p=1}^{n} \omega_p = 1$.

**Definition 12** ([23]). For any collection of PFNs, $C_p = \langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \rangle$ ($p = 1, 2, ..., n$) in $E$. The structure of Pythagorean fuzzy weighted geometric (PFWG) operator is

$$PFWG (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} (C_p)^{\beta_p}$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

**Definition 13** ([23]). For any collection of PFNs, $C_p = \langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \rangle$ ($p = 1, 2, ..., n$) in $E$. The structure of Pythagorean fuzzy order weighted geometric (PFOWG) operator is

$$PFOWG (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} \left( C_{\theta_p(p)} \right)^{\beta_p}$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\theta_p(p)}$ consequently by total order $C_{\theta_p(1)} \geq C_{\theta_p(2)} \geq ... \geq C_{\theta_p(n)}$.

**Definition 14** ([23]). For any collection of PFNs, $C_p = \langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \rangle$ ($p = 1, 2, ..., n$) in $E$. The structure of Pythagorean fuzzy hybrid weighted geometric (PFHGW) operator is

$$PFHGW (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} \left( C_{\theta_p(p)} \right)^{\beta_p}$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\theta_p(p)}$ consequently by total order $C_{\theta_p(1)} \geq C_{\theta_p(2)} \geq ... \geq C_{\theta_p(n)}$. In addition, associated weights are $\omega = (\omega_1, \omega_2, ..., \omega_n)$ with $\omega_p \geq 0$, $\sum_{p=1}^{n} \omega_p = 1$. 

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*Symmetry* 2019, 11, 383 5 of 19
Definition 15 ([12]). Suppose that \((g, d) \in (0, 1) \times (0, 1)\) are any real numbers with \(\beta \geq 1\). Then, Dombi norms are defined as
\[
\tilde{T}(g, d) = \frac{1}{1 + \left\{ \left( \frac{1-g}{g} \right)^{\beta} + \left( \frac{1-d}{d} \right)^{\beta} \right\}^{\frac{1}{\beta}}},
\]
(1)
\[
\tilde{S}(g, d) = 1 - \frac{1}{1 + \left\{ \left( \frac{g}{1-g} \right)^{\beta} + \left( \frac{d}{1-d} \right)^{\beta} \right\}^{\frac{1}{\beta}}}.\]
(2)

Example 1. Suppose that we take \(g = 0.7, d = 0.3\) and \(\beta = 3\). Then,
\[
\tilde{T}(g, d) = \frac{1}{1 + \left\{ \left( \frac{1-0.7}{0.7} \right)^{3} + \left( \frac{1-0.3}{0.3} \right)^{3} \right\}^{\frac{1}{3}}} = 0.2995,
\]
\[
\tilde{S}(g, d) = 1 - \frac{1}{1 + \left\{ \left( \frac{0.7}{1-0.7} \right)^{3} + \left( \frac{0.3}{1-0.3} \right)^{3} \right\}^{\frac{1}{3}}} = 0.7005.
\]

3. Pythagorean Fuzzy Dombi Operators

Now, we propose novel Pythagorean fuzzy Dombi basic operations based on Definition 6.

Definition 16. For any two PFNs, \(C_1 = \langle P_{\theta_1}(\varepsilon), N_{\theta_1}(\varepsilon) \rangle\) and \(C_2 = \langle P_{\theta_2}(\varepsilon), N_{\theta_2}(\varepsilon) \rangle\) in \(E\) and \(\beta \geq 0\). Then, the operations of PFNs based on Dombi operation are introduced as

(1) \(C_1 \oplus C_2 = \left( \frac{1}{1 + \left\{ \left( \frac{1-P_{\theta_1}(\varepsilon)}{P_{\theta_1}(\varepsilon)} \right)^{2\beta} + \left( \frac{1-N_{\theta_1}(\varepsilon)}{N_{\theta_1}(\varepsilon)} \right)^{2\beta} \right\}^{\frac{1}{\beta}}} \right)^{\frac{1}{2}}\);

(2) \(\beta \cdot C_1 = \left( \frac{1}{1 + \left\{ \beta \left( \frac{1-P_{\theta_1}(\varepsilon)}{P_{\theta_1}(\varepsilon)} \right)^{2\beta} \right\}^{\frac{1}{\beta}}} \right)^{\frac{1}{2}}\);

(3) \(C_1 \otimes C_2 = \left( \frac{1}{1 + \left\{ \left( \frac{1-P_{\theta_1}(\varepsilon)}{P_{\theta_1}(\varepsilon)} \right)^{2\beta} + \left( \frac{1-N_{\theta_2}(\varepsilon)}{N_{\theta_2}(\varepsilon)} \right)^{2\beta} \right\}^{\frac{1}{\beta}}} \right)^{\frac{1}{2}}\);

(4) \(C_1^{\beta} = \left( \frac{1}{1 + \left\{ \beta \left( \frac{1-P_{\theta_1}(\varepsilon)}{P_{\theta_1}(\varepsilon)} \right)^{2\beta} \right\}^{\frac{1}{\beta}}} \right)^{\frac{1}{2}}\).

3.1. Pythagorean Fuzzy Dombi Weighted Averaging Operators

Based on the defined Dombi operators for PFNs, we defined the following weighted averaging aggregation operators.
Definition 17. For any collection of PFNs, $C_p = \langle P_{\theta_p}(\varepsilon), N_{\theta_p}(\varepsilon) \rangle$ $(p = 1, 2, ..., n)$ in $E$. The structure of Pythagorean fuzzy Dombi weighted averaging (PFDWA) operator is

$$PFDWA(C_1, C_2, ..., C_n) = \sum_{p=1}^{n} \beta_p C_p,$$

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

Theorem 2. For any collection of PFNs, $C_p = \langle P_{\theta_p}(\varepsilon), N_{\theta_p}(\varepsilon) \rangle$ $(p = 1, 2, ..., n)$ in $E$. Then, the structure of Pythagorean fuzzy Dombi weighted averaging (PFDWA) operator is defined using Dombi operations with $\sigma > 0$;

$$PFDWA(C_1, C_2, ..., C_n) = \left( \frac{1 - \frac{1}{1 + \sum_{p=1}^{n} \beta_p \left( \frac{1}{P_{\theta_p}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\theta_p}} - 1 \right)^{2\sigma} \frac{1}{\sigma} }{1 + \sum_{p=1}^{n} \beta_p \left( \frac{1}{P_{\theta_p}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\theta_p}} - 1 \right)^{2\sigma} \frac{1}{\sigma} } \right)^{1/\sigma},$$

(3)

where $\beta_p (p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

Proof. Using mathematical induction to prove (3), we therefore proceed as:

(a) For $n = 2$, since

$$\beta_1 C_1 = \left( \frac{1 - \frac{1}{1 + \beta_1 \left( \frac{1}{P_{\theta_1}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \beta_1 \left( \frac{1}{N_{\theta_1}} - 1 \right)^{2\sigma} \frac{1}{\sigma} }{1 + \beta_1 \left( \frac{1}{P_{\theta_1}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \beta_1 \left( \frac{1}{N_{\theta_1}} - 1 \right)^{2\sigma} \frac{1}{\sigma} } \right)^{1/\sigma},$$

and

$$\beta_2 C_2 = \left( \frac{1 - \frac{1}{1 + \beta_2 \left( \frac{1}{P_{\theta_2}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \beta_2 \left( \frac{1}{N_{\theta_2}} - 1 \right)^{2\sigma} \frac{1}{\sigma} }{1 + \beta_2 \left( \frac{1}{P_{\theta_2}} - 1 \right)^{2\sigma} \frac{1}{\sigma} \beta_2 \left( \frac{1}{N_{\theta_2}} - 1 \right)^{2\sigma} \frac{1}{\sigma} } \right)^{1/\sigma}.$$
then,

\[
PFDWA (C_1, C_2) = \beta_1 C_1 + \beta_2 C_2
\]

\[
= \left( 1 - \frac{1}{\sqrt{1 + \left\{ \frac{1}{N_{\beta_1}} \right\} - \left\{ \frac{1}{N_{\beta_2}} \right\} + \beta_2 \left( \frac{1}{N_{\beta_2}} \right)^{2^p}} \right)^{\frac{1}{2^p}}
\]

\[
= \left( 1 + \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^{2^p} \right)^{-\frac{1}{2^p}}
\]

(b) Now, (3) is true for \( n = k \),

\[
PFDWA (C_1, C_2, ..., \tilde{C}_{\tilde{n}}) = \left( \frac{1 - \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^{2^p}}{1 + \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^{2^p}} \right)^{\frac{1}{2^p}}
\]

(c) Now, we prove that (3) for \( n = k + 1 \), that is, \( PFDWA (C_1, C_2, ..., C_{k+1}) = \sum_{p=1}^{k} \beta_p C_p + \beta_{k+1} C_{k+1} \)

\[
PFDWA (C_1, C_2, ..., C_{k+1}) = \left( 1 + \sum_{p=1}^{k} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^{2^p} \right)^{-\frac{1}{2^p}} + \left( 1 + \beta_{k+1} \left( \frac{1}{N_{\beta_{k+1}}} - 1 \right)^{2^p} \right)^{-\frac{1}{2^p}}
\]

\[
= \left( 1 - \frac{1}{\sqrt{1 + \left\{ \frac{1}{N_{\beta_1}} \right\} - \left\{ \frac{1}{N_{\beta_2}} \right\} + \beta_2 \left( \frac{1}{N_{\beta_2}} \right)^{2^p}} \right)^{\frac{1}{2^p}}
\]
Thus, (3) is true for \( n = z + 1 \). Hence, it satisfies for all \( n \). Therefore,

\[
PFDWA (C_1, C_2, \ldots, C_n) = \left\{ \frac{1}{1 + \left( \sum_{p=1}^{n} \beta_p \left( \frac{1}{\beta_p} - 1 \right)^{2^z} \right)^{\frac{1}{\sigma}}} \right\},
\]

which completed the proof. \( \square \)

**Properties:** PFDWA operator that satisfies some properties are enlisted below:

1. **Idempotency:** For any collection of PFNs, \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \). Then, if the collection of PFNs \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) are identical, which is,

\[
PFDWA (C_1, C_2, \ldots, C_n) = C.
\]

2. **Boundedness:** For any collection of PFNs, \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \). \( C_p = \langle \min_p p_{b_p}, \max_p n_{b_p} \rangle \) and \( C_p = \langle \max_p p_{b_p}, \min_p n_{b_p} \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \); therefore,

\[
C_p \subseteq PFDWA (C_1, C_2, \ldots, C_n) \subseteq C_p^*.
\]

3. **Monotonicity:** For any collection of PFNs, \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \). If \( C_p \subseteq C_p^* \) for \( (p = 1, 2, \ldots, n) \), then

\[
PFDWA (C_1, C_2, \ldots, C_n) \subseteq PFDWA (C_1^*, C_2^*, \ldots, C_n^*).
\]

**Definition 18.** For any collection of PFNs, \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \). The structure of Pythagorean fuzzy Dombi order weighted averaging (PFDOWA) operator is

\[
PFDOWA (C_1, C_2, \ldots, C_n) = \sum_{p=1}^{n} \beta_p C_{\theta_{w(p)}},
\]

where \( \beta_p \) \( (p = 1, 2, \ldots, n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \) th biggest weighted value is \( C_{\theta_{w(p)}} \) consequently by total order \( C_{\theta_{w(1)}} \geq C_{\theta_{w(2)}} \geq \cdots \geq C_{\theta_{w(n)}} \).

**Theorem 3.** For any collection of PFNs, \( C_p = \langle p_{b_p} (\epsilon), n_{b_p} (\epsilon) \rangle \) \( (p = 1, 2, \ldots, n) \) in \( E \). Then, the structure of Pythagorean fuzzy Dombi order weighted averaging (PFDOWA) operator is defined using Dombi operations with \( \sigma > 0 \);

\[
PFDOWA (C_1, C_2, \ldots, C_n) = \left\{ \frac{1}{1 + \left( \sum_{p=1}^{n} \beta_p \left( \frac{1}{\beta_p} - 1 \right)^{2^z} \right)^{\frac{1}{\sigma}}} \right\},
\]

where \( \beta_p \) \( (p = 1, 2, \ldots, n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \) th biggest weighted value is \( C_{\theta_{w(p)}} \) consequently by total order \( C_{\theta_{w(1)}} \geq C_{\theta_{w(2)}} \geq \cdots \geq C_{\theta_{w(n)}} \).
Theorem 4. For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. Then, if the collection of PFNs $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ are identical, which is

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) = C.$$ 

Properties: PFDOWA operator satisfies some properties that are enlisted below;

1. Idempotency: For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. Then, if the collection of PFNs $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ are identical, which is

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) = C.$$ 

2. Boundedness: For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. Then, if the collection of PFNs $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ are identical, which is

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) = C.$$ 

3. Monotonicity: For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. If $C_p \subseteq C_p$ for $(p = 1, 2, \ldots, n)$, then

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) \subseteq PFDOWA \left( C_1', C_2', \ldots, C_n' \right).$$

Definition 19. For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. The structure of Pythagorean fuzzy Dombi hybrid weighted averaging (PFDOWA) operator is

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) = \sum_{p=1}^{n} \beta_p C_{\omega(p)},$$

where $\beta_p$ $(p = 1, 2, \ldots, n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\omega(p)}^\ast \left( C_{\omega(p)}^\ast = n \beta_p C_{\omega(p)} \right)$, $P \in N$ consequently by total order $C_{\omega(1)}^\ast \geq C_{\omega(2)}^\ast \geq \ldots \geq C_{\omega(n)}^\ast$. In addition, associated weights are $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ with $\omega_p \geq 0$, $\sum_{p=1}^{n} \omega_p = 1$.

Theorem 4. For any collection of PFNs, $C_p = \langle P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \rangle$ $(p = 1, 2, \ldots, n)$ in $E$. Then, the structure of Pythagorean fuzzy Dombi hybrid weighted averaging (PFDOWA) operator is defined using Dombi operations with $\sigma > 0$;

$$PFDOWA \left( C_1, C_2, \ldots, C_n \right) = \left( \frac{1}{\sum_{p=1}^{n} \beta_p \left( \frac{1}{\omega_p(1)} - 1 \right)^{2^{\sigma}}} \right)^{\frac{1}{2}},$$

where $\beta_p$ $(p = 1, 2, \ldots, n)$ are weight vectors with $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ and $p$th biggest weighted value is $C_{\omega(p)}^\ast \left( C_{\omega(p)}^\ast = n \beta_p C_{\omega(p)} \right)$, $P \in N$ consequently by total order $C_{\omega(1)}^\ast \geq C_{\omega(2)}^\ast \geq \ldots \geq C_{\omega(n)}^\ast$. In addition, associated weights are $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ with $\omega_p \geq 0$, $\sum_{p=1}^{n} \omega_p = 1$.

Proof. The procedure is similar to Theorem 2. □
Theorem 5.
For any collection of PFNs, \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) in \( E \). Then, if collection of PFNs \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) are identical, which is
\[
PFDHWA (C_1, C_2, ..., C_n) = C.\]

(2) Boundedness: For any collection of PFNs, \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) in \( E \). \( C_p = \left\langle \min_p P_{\theta_p}, \max_p N_{\theta_p} \right\rangle \) and \( C_p^+ = \left\langle \max_p P_{\theta_p}, \min_p N_{\theta_p} \right\rangle (p = 1, 2, ..., n) \) in \( E \), therefore
\[
C_p^- \subseteq PFDHWA (C_1, C_2, ..., C_n) \subseteq C_p^+.\]

(3) Monotonicity: For any collection of PFNs, \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) in \( E \). If \( C_p \subseteq C_p^+ \) for \( (p = 1, 2, ..., n) \), then
\[
PFDHWA (E_{a_1}, C_2, ..., C_n) \subseteq PFDHWA (C_1^+, C_2^+, ..., C_n^+).\]

3.2. Pythagorean Fuzzy Dombi Weighted Geometric Operators

Based on the defined Dombi operators for PFNs, we defined the following weighted geometric aggregation operators.

**Definition 20.** For any collection of PFNs, \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) in \( E \). The structure of Pythagorean fuzzy Dombi weighted geometric (PFDWG) operator is
\[
PFDWG (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} (C_p)^{\beta_p},\]
where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \).

**Theorem 5.** For any collection of PFNs, \( C_p = \left\langle P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right\rangle (p = 1, 2, ..., n) \) in \( E \). Then, the structure of Pythagorean fuzzy Dombi weighted geometric (PFDWG) operator is defined using Dombi operations with \( \sigma > 0 \):
\[
PFDWG (C_1, C_2, ..., C_n) = \left( \frac{1}{1 + \left\{ \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\theta_p} \sigma} - 1 \right)^{2\sigma} 1 \right\}^{\frac{1}{2}}} \right),\]
where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \).

**Proof.** Using mathematical induction to prove (3), we therefore proceed as:

(a) For \( n = 2 \), since
\[
\beta_1 C_1 = \left( \frac{1}{1 + \left\{ \beta_1 \left( \frac{1}{N_{\theta_1} \sigma} - 1 \right)^{2\sigma} 1 \right\}^{\frac{1}{2}}} \right),\]
and

$$\beta_2 C_2 = \left( \frac{1}{1 + \left\{ \beta_1 \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)^{\frac{1}{2}},$$

then,

$$PFDWG (C_1, C_2) = (C_1)^{\beta_1} + (C_2)^{\beta_2}$$

$$= \left( \frac{1}{1 + \left\{ \beta_1 \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)^{\frac{1}{2}} \left( \frac{1}{1 + \left\{ \beta_2 \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)^{\frac{1}{2}}$$

$$= \left( \frac{1 + \left\{ \sum_{p=1}^{1} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} {1 + \left\{ \sum_{p=1}^{1} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)$$

(b) Now, (3) is true for \( n = k \),

$$PFDWG (C_1, C_2, ..., C_k) = \left( \frac{1 + \left\{ \sum_{p=1}^{k} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} {1 + \left\{ \sum_{p=1}^{k} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)$$

(c) Now, we prove that (3) is true for \( n = k + 1 \), which is

$$PFDWG (C_1, C_2, ..., C_{k+1}) = \prod_{p=1}^{k} (C_k)^{\beta_k} + (C_{k+1})^{\beta_{k+1}}$$

$$PFDWG (C_1, C_2, ..., C_{k+1})$$

$$= \left( \frac{1}{1 + \left\{ \sum_{p=1}^{k+1} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right) + \left( \frac{1}{1 + \left\{ \beta_{k+1} \left( \frac{\alpha_p}{\left( \frac{1}{N_{k+1}} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)$$

$$= \left( \frac{1 + \left\{ \sum_{p=1}^{k+1} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} {1 + \left\{ \sum_{p=1}^{k+1} \beta_p \left( \frac{\alpha_p}{\left( \frac{1}{N_p} - 1 \right)^{2 \nu}} \right) \right\}^{\frac{1}{2}}} \right)$$
Thus, (5) is true for \( n = z + 1 \). Hence, it satisfies all \( n \). Therefore,

\[
PFDWG (C_1, C_2, ..., C_n) = \left( \frac{1}{1 + \left\{ \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^2 \right\}^{\frac{1}{2}}} \right),
\]

which completed the proof. \( \square \)

**Properties:** \( PFDWG \) operator satisfies some properties are enlisted below;

1. **Idempotency:** For any collection of PFNs, \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) in \( E \). Then, if collection of PFNs \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) are identical, which is

\[
PFDWG (C_1, C_2, ..., C_n) = C.
\]

2. **Boundedness:** For any collection of PFNs, \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) in \( E \). \( C_p = \left( \min_p P_{\theta_p}, \max_p N_{\theta_p} \right) \) and \( C'_p = \left( \max_p P_{\theta_p}, \min_p N_{\theta_p} \right) \) (\( p = 1, 2, ..., n \)) in \( E \), therefore

\[
C_p \subseteq PFDWG (C_1, C_2, ..., C_n) \subseteq C'_p.
\]

3. **Monotonicity:** For any collection of PFNs, \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) in \( E \). If \( C_p \subseteq C'_p \) for (\( p = 1, 2, ..., n \)), then

\[
PFDWG (C_1, C_2, ..., C_n) \subseteq PFDWG (C'_1, C'_2, ..., C'_n).
\]

**Definition 21.** For any collection of PFNs, \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) in \( E \). The structure of Pythagorean fuzzy Dombi order weighted geometric (\( PFDWG \)) operator is

\[
PFDOWG (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} \left( C_{\theta_p (p)} \right)^{\beta_p},
\]

where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \)th biggest weighted value is \( C_{\theta_p (p)} \) consequently by total order \( C_{\theta_p (1)} \geq C_{\theta_p (2)} \geq ... \geq C_{\theta_p (n)} \).

**Theorem 6.** For any collection of PFNs, \( C_p = \left( P_{\theta_p} (\epsilon), N_{\theta_p} (\epsilon) \right) \) (\( p = 1, 2, ..., n \)) in \( E \). Then, the structure of Pythagorean fuzzy Dombi order weighted geometric (\( PFDOWG \)) operator is defined using Dombi operations with \( \sigma > 0 \);

\[
PFDOWG (C_1, C_2, ..., C_n) = \left( \frac{1}{1 + \left\{ \sum_{p=1}^{n} \beta_p \left( \frac{1}{N_{\beta_p}} - 1 \right)^{2\sigma} \right\}^{\frac{1}{2}}} \right),
\]

where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \) and \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \)th biggest weighted value is \( C_{\theta_p (p)} \) consequently by total order \( C_{\theta_p (1)} \geq C_{\theta_p (2)} \geq ... \geq C_{\theta_p (n)} \).
Theorem 7. For any collection of PFNs, \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) in E. Then, if collection of PFNs \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) are identical, which is

\[
PFDOWG (C_1, C_2, ..., C_n) = C.
\]

(2) Boundedness: For any collection of PFNs, \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) in E. \( C_p = \left( \min_p P_{\theta_p}, \max_p N_{\theta_p} \right) \) and \( C_p^+ = \left( \max_p P_{\theta_p}, \min_p N_{\theta_p} \right) (p = 1, 2, ..., n) \) in E, therefore

\[
C_p^+ \subseteq PFDOWG (C_1, C_2, ..., C_n) \subseteq C_p^-.
\]

(3) Monotonicity: For any collection of PFNs, \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) in E. If \( C_p \subseteq C_p^* \) for \( (p = 1, 2, ..., n) \), then

\[
PFDOWG (C_1, C_2, ..., C_n) \subseteq PFDOWG (C_1^*, C_2^*, ..., C_n^*).
\]

Definition 22. For any collection of PFNs, \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) in E. The structure of Pythagorean fuzzy Dombi hybrid weighted geometric (PFDHWG) operator is

\[
PFDHWG (C_1, C_2, ..., C_n) = \prod_{p=1}^{n} \left( C_{\omega(p)}^{*} \right) \beta_p,
\]

where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \), \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \)th biggest weighted value is \( C_{\omega(p)}^{*} = n\beta_p C_{\omega(p)}^{*} (p \in N) \) consequently by total order \( C_{\omega(1)}^{*} \geq C_{\omega(2)}^{*} \geq ... \geq C_{\omega(n)}^{*} \). In addition, associated weights are \( \omega = (\omega_1, \omega_2, ..., \omega_n) \) with \( \omega_p \geq 0 \), \( \sum_{p=1}^{n} \omega_p = 1 \).

Theorem 7. For any collection of PFNs, \( C_p = \left( P_{\theta_p}(\epsilon), N_{\theta_p}(\epsilon) \right) (p = 1, 2, ..., n) \) in E. Then, the structure of Pythagorean fuzzy Dombi hybrid weighted geometric (PFDHWG) operator is defined using Dombi operations with \( \sigma > 0 \);

\[
PFDHWG (C_1, C_2, ..., C_n) = \left( \frac{1}{1 + \left( \sum_{p=1}^{n} \beta_p \left( \frac{1}{C_{\omega(p)}^{*}} \right)^{-1} \right)^{2\sigma}} \right)^{\frac{1}{q}},
\]

where \( \beta_p (p = 1, 2, ..., n) \) are weight vectors with \( \beta_p \geq 0 \), \( \sum_{p=1}^{n} \beta_p = 1 \) and \( p \)th biggest weighted value is \( C_{\omega(p)}^{*} = n\beta_p C_{\omega(p)}^{*} (p \in N) \) consequently by total order \( C_{\omega(1)}^{*} \geq C_{\omega(2)}^{*} \geq ... \geq C_{\omega(n)}^{*} \). In addition, associated weights are \( \omega = (\omega_1, \omega_2, ..., \omega_n) \) with \( \omega_p \geq 0 \), \( \sum_{p=1}^{n} \omega_p = 1 \).

Proof. The procedure is similar to Theorem 5. □

Properties: The PFDHWG operator satisfies some properties that are listed below;
Assume that

\( (2) \) Boundedness: For any collection of PFNs, \( C_p = \langle P_{\theta_p}(\varepsilon), N_{\theta_p}(\varepsilon) \rangle (p = 1, 2, ..., n) \) in \( E \). Then, the collection of PFNs \( C_p = \langle P_{\theta_p}(\varepsilon), N_{\theta_p}(\varepsilon) \rangle (p = 1, 2, ..., n) \) is identical, which is

\[
PFDHWG (C_1, C_2, ..., C_n) = C.
\]

\( (3) \) Monotonicity: For any collection of PFNs, \( C_p = \langle P_{\theta_p}(\varepsilon), N_{\theta_p}(\varepsilon) \rangle (p = 1, 2, ..., n) \) in \( E \). If \( C_p \subseteq C_p^* \) for \( (p = 1, 2, ..., n) \), then

\[
PFDHWG (C_1, C_2, ..., C_n) \subseteq PFDHWG (C_1^*, C_2^*, ..., C_n^*).
\]

4. Algorithm for Multi-Attribute Decision-Making Using Pythagorean Fuzzy Information

In this section, a novel approach for decision problems using Pythagorean fuzzy information is proposed. In this approach, the decision makers give the information in the form of Pythagorean fuzzy sets.

Let \( H = (h_1, h_2, ..., h_m) \) be a distinct set of \( m \) probable alternatives and \( \tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n) \) be a finite set of \( n \) criteria, where \( h_i \) indicates the \( i \)th alternatives and \( \tilde{c}_j \) indicates the \( j \)th criteria. Let \( D = (d_1, d_2, ..., d_l) \) be a finite set of \( l \) experts, where \( d_k \) indicates the \( k \)th expert. The expert \( d_k \) supplies her appraisal of an alternative \( h_i \) on an attribute \( \tilde{c}_j \) as a PFN \( (i = 1, 2, ..., m; j = 1, 2, ..., n) \). The expert information is represented by the Pythagorean fuzzy set decision-making matrix \( D^\psi = \left[ E_{ip}^{(\psi)} \right]_{m \times n} \). Assume that \( \beta_p(p = 1, 2, ..., m) \) is weight vector of the attribute \( \tilde{c}_j \) such that \( 0 \leq \beta_p \leq 1 \) and \( \sum_{p=1}^{m} \beta_p = 1 \) and \( \psi = (\psi_1, \psi_2, ..., \psi_m) \) is the weight vector of the decision makers \( d_k \) such that \( \psi_k \leq 1 \), \( \sum_{k=1}^{n} \psi_k = 1 \).

We construct the Pythagorean fuzzy decision-making matrices, \( D^\psi = \left[ E_{ip}^{(\psi)} \right]_{m \times n} \) for decisions. Basically, criteria have two types: one is benefit criteria and the other one is cost criteria. If the Pythagorean fuzzy decision matrices have cost type criteria matrices, \( D^\psi = \left[ E_{ip}^{(\psi)} \right]_{m \times n} \) can be converted into the normalized Pythagorean fuzzy decision matrices, \( R^\psi = \left[ r_{ip}^{(\psi)} \right]_{m \times n} \), where \( r_{ip}^{(\psi)} = \begin{cases} E_{ip}^{(\psi)} & \text{for benefit criteria } A_p, \\ \frac{E_{ip}^{(\psi)}}{E_{ip}} & \text{for cost criteria } A_p, \end{cases} \) for \( j = 1, 2, ..., n \), and \( T_{ip}^{(\psi)} \) is the complement of \( E_{ip}^{(\psi)} \). If all the criteria have the same type, then there is no need for normalization.

**Step 1:** In this step, we get the collective Pythagorean fuzzy information and using proposed Dombi operators to evolve the alternative preference values with associated weights, which are \( \omega = (\omega_1, \omega_2, ..., \omega_n) \) with \( \omega_p \geq 0, \sum_{p=1}^{n} \omega_p = 1 \).

**Step 2:** We find the score value \( \tilde{S}(C_p) \) and the accuracy value \( \tilde{A}(C_p) \) of the cumulative overall preference value \( h_i \) \( (i = 1, 2, ..., m) \).

**Step 3:** By the definition, rank the alternatives \( h_i \) \( (i = 1, 2, ..., m) \) and choose the best alternative which has the maximum score value.

4.1. Numerical Example

Assume that a fund manager Mr. M in a wealth management firm is assessing five potential investment opportunities \( R_1, R_2, R_3, R_4, R_5 \) (let \( R = \{r_1, r_2, r_3, r_4, r_5\} \)). The firm mandates that the fund manager has to evaluate the following five parameters (criteria) \( Y_1, Y_2, Y_3, Y_4, Y_5 \) (let \( Y = \)
Now, we use PFDWG to evaluate collective performance with weight vectors \( \beta = (0.15, 0.25, 0.35, 0.10, 0.15)^T \) and \( \sigma = 1 \) as follows in Table 2:

### Table 2. Aggregated Pythagorean information matrix (PFDWG).

<table>
<thead>
<tr>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( R_4 )</th>
<th>( R_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.06568, 0.98699)</td>
<td>(0.57562, 0.99870)</td>
<td>(0.42181, 0.99777)</td>
<td>(0.24911, 0.99898)</td>
<td>(0.16873, 0.99701)</td>
</tr>
</tbody>
</table>

Similarly, we can find for \( \sigma = 2, 3, 4, 5, 6, 7, 8, 9, 10. \)

### Steps 2 and 3:
Now, we find the score value of each alternative and their ranking as shown in Table 3.

### Table 3. Ranking using PFDWG operator.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \hat{S}(R_1) )</th>
<th>( \hat{S}(R_2) )</th>
<th>( \hat{S}(R_3) )</th>
<th>( \hat{S}(R_4) )</th>
<th>( \hat{S}(R_5) )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.9698</td>
<td>-0.6660</td>
<td>-0.8176</td>
<td>-0.9359</td>
<td>-0.9655</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>2</td>
<td>-0.9789</td>
<td>-0.6917</td>
<td>-0.8375</td>
<td>-0.9498</td>
<td>-0.9749</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>3</td>
<td>-0.9824</td>
<td>-0.7090</td>
<td>-0.8527</td>
<td>-0.9545</td>
<td>-0.9796</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>4</td>
<td>-0.9841</td>
<td>-0.7202</td>
<td>-0.8632</td>
<td>-0.9568</td>
<td>-0.9821</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>5</td>
<td>-0.9851</td>
<td>-0.7275</td>
<td>-0.8703</td>
<td>-0.9582</td>
<td>-0.9836</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>6</td>
<td>-0.9857</td>
<td>-0.7326</td>
<td>-0.8752</td>
<td>-0.9590</td>
<td>-0.9845</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>7</td>
<td>-0.9862</td>
<td>-0.7363</td>
<td>-0.8787</td>
<td>-0.9597</td>
<td>-0.9852</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>8</td>
<td>-0.9865</td>
<td>-0.7391</td>
<td>-0.8813</td>
<td>-0.9601</td>
<td>-0.9856</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>9</td>
<td>-0.9867</td>
<td>-0.7412</td>
<td>-0.8833</td>
<td>-0.9605</td>
<td>-0.9860</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
<tr>
<td>10</td>
<td>-0.9869</td>
<td>-0.7429</td>
<td>-0.8848</td>
<td>-0.9608</td>
<td>-0.9863</td>
<td>( R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1 )</td>
</tr>
</tbody>
</table>

(Case 2)

### Step 1:
Now, we use PFDWA to evaluate collective performance with weight vectors \( \beta = (0.15, 0.25, 0.35, 0.10, 0.15)^T \) and \( \sigma = 1 \) as follows in Table 4:

### Table 4. Aggregated Pythagorean information matrix (PFDWA).

<table>
<thead>
<tr>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( R_4 )</th>
<th>( R_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.99784, 0.17076)</td>
<td>(0.99784, 0.16076)</td>
<td>(0.90668, 0.06663)</td>
<td>(0.96847, 0.04495)</td>
<td>(0.98566, 0.077166)</td>
</tr>
</tbody>
</table>
Similarly, we can find for \( \sigma = 2, 3, 4, 5, 6, 7, 8, 9, 10 \).

**Steps 2 and 3:** Now, we find the score value of each alternative and their ranking as follows:

Looking at the illustration above, it is evident that, though overall ranking values of the alternatives are dissimilar, due to the usage of two Dombi aggregation operators, the ranking order regarding the alternatives are analogous, and the most desirable alternative is \( \mathcal{R}_2 \) in order to analyze the consequence of parameter \( \sigma \in [1, 10] \) on the ranking of the alternatives in the PFDWG and PFDWA operators, which are exposed in Tables 3 and 5.

**Table 5.** Ranking using a PFDWA operator.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \hat{S}(\mathcal{R}_1) )</th>
<th>( \hat{S}(\mathcal{R}_2) )</th>
<th>( \hat{S}(\mathcal{R}_3) )</th>
<th>( \hat{S}(\mathcal{R}_4) )</th>
<th>( \hat{S}(\mathcal{R}_5) )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9664</td>
<td>0.9698</td>
<td>0.81763</td>
<td>0.9359</td>
<td>0.9655</td>
<td>( \mathcal{R}_2 &gt; \mathcal{R}_1 &gt; \mathcal{R}_3 &gt; \mathcal{R}_4 &gt; \mathcal{R}_5 )</td>
</tr>
<tr>
<td>2</td>
<td>0.9789</td>
<td>0.6917</td>
<td>0.8375</td>
<td>0.9498</td>
<td>0.9749</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>3</td>
<td>0.9824</td>
<td>0.7090</td>
<td>0.8527</td>
<td>0.9545</td>
<td>0.9796</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>4</td>
<td>0.9841</td>
<td>0.7202</td>
<td>0.8632</td>
<td>0.9568</td>
<td>0.9821</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>5</td>
<td>0.9851</td>
<td>0.7275</td>
<td>0.8703</td>
<td>0.9582</td>
<td>0.9836</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>6</td>
<td>0.9857</td>
<td>0.7326</td>
<td>0.8752</td>
<td>0.9590</td>
<td>0.9845</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>7</td>
<td>0.9862</td>
<td>0.7363</td>
<td>0.8787</td>
<td>0.9597</td>
<td>0.9852</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>8</td>
<td>0.9865</td>
<td>0.7391</td>
<td>0.8813</td>
<td>0.9601</td>
<td>0.9856</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>9</td>
<td>0.9867</td>
<td>0.7412</td>
<td>0.8833</td>
<td>0.9605</td>
<td>0.9860</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
<tr>
<td>10</td>
<td>0.9869</td>
<td>0.7429</td>
<td>0.8848</td>
<td>0.9608</td>
<td>0.9863</td>
<td>( \mathcal{R}_1 &gt; \mathcal{R}_5 &gt; \mathcal{R}_4 &gt; \mathcal{R}_3 &gt; \mathcal{R}_2 )</td>
</tr>
</tbody>
</table>

4.2. Analyzing the Consequence of Parameter \( \sigma \) on Decision-Making Results

To describe the effect of the parameters \( \sigma \) on multi attribute decision-making outcomes, we have utilized dissimilar values of \( \sigma \) to rank the alternatives. The results of score function and ranking order of the alternatives \( H_i (i = 1, 2, 3, 4, 5) \) in the range of \( 1 \leq \sigma \leq 10 \) based on PFDWG and PFDWA operators are presented in Tables 3 and 5 correspondingly. When \( \sigma \in [1, 10] \) using an PFDWG aggregation operator, we obtained a rank of alternatives as \( \mathcal{R}_2 > \mathcal{R}_3 > \mathcal{R}_4 > \mathcal{R}_5 > \mathcal{R}_1 \); here, \( \mathcal{R}_2 \) is the best choice, but, when using a PFDWA aggregation operator, we obtained two different ranks. When \( \sigma = 1 \), we obtained \( \mathcal{R}_2 > \mathcal{R}_1 > \mathcal{R}_5 > \mathcal{R}_4 > \mathcal{R}_3 \) and when \( \sigma \in [2, 10] \), we get \( \mathcal{R}_1 > \mathcal{R}_5 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_2 \). Hence, the overall best rank is \( \mathcal{R}_2 \).

To these MADM problems based on PFDWG and PFDWA operators, we realize that the different values of parameters \( \sigma \) can change corresponding ranking orders of the alternatives for PFDWA operator, which is more reactive to \( \sigma \) in this MADM procedure, even though for numerous values of the parameters \( \sigma \) might be reformed making arrangements corresponding to PFDWG operators, which is less responsive to \( \sigma \) in this multi-attribute decision-making (MADM) procedure.

4.3. Comparison Analysis

This section deals with comparison analysis of the proposed Dombi aggregation operators under Pythagorean fuzzy numbers with other well known aggregation operators. We compared this proposed Dombi aggregation Operators with O-PFWA [3], O-PFPWA [3], PFWA [8], PFOWA [8], SPFWA [24], PFEWA [23], PFEOWA [23], CPFWA [25], L-PFWA [26], and L-PFOWA [26,27]. Their results are summarized as follows.
Comparison analysis of existing operators:

<table>
<thead>
<tr>
<th>Averaging Operators</th>
<th>Ranking</th>
<th>Geometric Operators</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-PFWA [3]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>O-PFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>O-PFWAPA [3]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>O-PFWPG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFWA [8]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>PFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFOWA [8]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>PFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>SPFWA [24]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>SPFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFEWA [23]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>PFEWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFEOWA [23]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>PFEOWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>CPFWA [25]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>CPFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>L-PFWA [26]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>L-PFWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>L-PFOWA [26]</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
<td>L-PFOWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
</tbody>
</table>

Comparison analysis of proposed operators:

<table>
<thead>
<tr>
<th>Averaging Operators</th>
<th>Ranking</th>
<th>Geometric Operators</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>PFDWA</td>
<td>$R_2 &gt; R_3 &gt; R_5 &gt; R_4 &gt; R_1$</td>
<td>PFDWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFDOWA</td>
<td>$R_2 &gt; R_3 &gt; R_5 &gt; R_4 &gt; R_1$</td>
<td>PFDOWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
<tr>
<td>PFDHWA</td>
<td>$R_2 &gt; R_3 &gt; R_5 &gt; R_4 &gt; R_1$</td>
<td>PFDHWG</td>
<td>$R_2 &gt; R_3 &gt; R_4 &gt; R_5 &gt; R_1$</td>
</tr>
</tbody>
</table>

From the above comparative analysis table, we say that our proposed Dombi Pythagorean fuzzy aggregation operators are more effective and reliable than previous aggregation operators.

5. Conclusions

In this paper, we have proposed novel aggregation operators, namely, Dombi weighted average/geometric, ordered weighted average/geometric and Dombi hybrid weighted average/geometric for Pythagorean fuzzy numbers. In addition, we gave the comparison of proposed and existing aggregation operators and discussed how our proposed technique is more effective than other existing operators for aggregation. Finally, we provided an approach to deal with the decision problems using the proposed Dombi operators. A numerical example shows how our proposed technique helped us with being effective in decision-making problems.

Author Contributions: Conceptualization, S.A. (Shahzaib Ashraf) and S.A. (Saleem Abdullah); methodology, S.A. (Shahzaib Ashraf); software, S.A. (Shahzaib Ashraf); validation, A.A.K. (Arshad Ahmad Khan), S.A. (Shahzaib Ashraf), S.A. (Saleem Abdullah), M.Q. (Muhammad Qiyas) and J.L. (Jianchao Luo); investigation, S.A. (Shahzaib Ashraf); writing—original draft preparation, S.A. (Shahzaib Ashraf); writing—review and editing, S.A. (Saleem Abdullah), M.Q.; visualization, S.A. (Saleem Abdullah); supervision, S.A. (Saleem Abdullah); funding acquisition, A.A.K., J.L.

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Conflicts of Interest: The authors declare no conflict of interest.

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