Abstract: Water scarcity is an important issue in many countries, and it is therefore necessary to improve the efficiency and equality of water resource allocation for decision makers. Based on game theory (GT), a bi-level optimization model is developed from the perspective of a leader-follower relationship among agents (stakeholders) of a river basin in this study, which consists of a single-agent GT-based optimization model of common interest and a multi-agent cooperative GT-based model. The Hanjiang River Basin is chosen as a case study, where there are conflicts among different interest agents in this basin. The results show that the proposed bi-level model could attain the same improvement of common interest by 8%, with the conventional optimal model. However, different from the conventional optimal model, since the individual interests have been considered in the bi-level optimization model, the willingness of cooperation of individuals has risen from 20% to 80%. With a slight decrease by 3% of only one agent, the increases of interest of other agents are 14%, 18%, 7%, and 14%, respectively, when using the bi-level optimization model. The conclusion could be drawn that the proposed model is superior to the conventional optimal model. Moreover, this study provides scientific support for the large spatial scale water resource allocation model.

Keywords: multi-agent of river basin; game theory; water resources allocation

1. Introduction

Water is essential for human well-being and all activities [1]. Owing to the impact of climate change and human activities, water scarcity has become a common problem in many countries, especially in developing countries [2–4]. Since the imbalance between the supply and demand of water resources is getting more and more prominent, it is urgent for decision makers to solve the conflicts arisen from ineffective and unfair water resource allocation [5].

To enhance the effectiveness and benefits of water resource allocation schemes, a large group of scholars have suggested the use of optimization models. Optimization techniques, such as linear programming, mixed-integer linear programing, dynamic programming, evolutionary computation, artificial neural networks, and so on [5–8], have been trying to find the optimal schemes of water resource allocation. However, these conventional optimization methods usually convert the multi-decision-maker problems of the whole system into a single-decision-maker problem, with a single composite objective [5]. Consequently, based on perfect cooperation, the ideal top-down schemes attained by conventional optimization methods, only emphasize the common interest of the system and ignore individual interests. Nevertheless, in fact, the ideal optimal scheme can’t be realized without the willingness to cooperation of individuals [9–11].

Taking individual willingness into consideration, game theory was introduced as a solution to the conflicts caused by ineffective and unfair water resource allocation among multi decision makers. Game
theory (GT) has been applied to different water or cost/benefit allocation situations, among users in water resources. Both non-cooperative and cooperative game theory methods have been used to solve water conflicts [12,13]. While non-cooperative game theory is useful in providing strategic insights into conflicts, cooperative game theory is normally helpful in providing an alternative framework for fair and efficient allocation of the incremental benefits of cooperation, among multi decision makers [14]. Using the idea of game theory, Adams et al. [15] advanced a new framework for noncooperative, multilateral bargaining, which can be used to conceptualize negotiation processes. The cooperative water allocation model was designed by Wang et al. [2], aiming at modeling equitable and efficient water allocation among competing users. A multi-objective game-theory model, which could balance economic and environmental concerns in reservoir watershed management, was developed by Lee [3]. Furthermore, Madani et al. [14] proposed a new framework for resolving conflicts over transboundary rivers using bankruptcy methods. However, there are few studies about the conflicts among multi-agent (i.e., water user) at the river basin level, and no strict systems are available to guide practical problems.

The aim of this study is to develop a bi-level optimization model, which consists of the optimization model of water resources allocation among the superiors and the optimization model of water resources allocation among the subordinates. In the bi-level optimization model, not only the maximization of the common interest is realized, but also the individual interest is taken into consideration. In other words, our model is featured by top-down coordination and bottom-up feedback, aiming at proposing an optimal scheme that could be accepted by superiors and subordinates. The Hanjiang River Basin is used as a case study to prove the equality and effectiveness of this bi-level optimization model. Thereby, our model could provide a fundamental basis for water resource allocation on a large scale.

2. Model Description

The bi-level optimization model begins with the framework of the model (Section 2.1), followed by the mathematical formulae of the model, which are presented in Section 2.2.

2.1. Model Framework

As shown in Figure 1, the bi-level optimization model (i.e., multi-agent cooperative game-based optimization model of water resources allocation) consists of the optimization model of water resource allocation among the superiors (in brief, the superiors model), and the optimization model of water resource allocation among the subordinates (in brief, the subordinates model). The former is a single-agent GT-based optimization model of common interest, which is designed to solve the multi-objective optimization problems of water resource management agents. The latter is a multi-agent cooperative GT-based model, which intends to solve multi-agent cooperation problems in water usage. Due to the existence of the cooperation agreement between the superiors and subordinates, the common interest of the superiors could be dealt as an equality constraint on the subordinate model. Both models have their separate objective functions and constrains, and any solution to the subordinates model depends on the corresponding solution to the superiors model.
zones on the basis of meteorological and hydrological features and natural geographical conditions. Zone 1 is located in the upper and middle reaches of the river’s main stream. Zone 2 is located in a tributary of the river basin. Zone M is the most downstream area of the basin, where the tributaries merge with the main stream. According to the spatial distribution, there are mainly three types among these interest agents, including consumptive water users outside the river, users with ecological water needs inside the river, and water energy users inside the river. Water transfer to other basins is also considered in this model. It is assumed that there are L water users outside the river, and the total number of interest agents in zone M is denoted as N_M. R represents the outflow of each zone, and these interest agents are correlated with each other through the runoff.

2.2. Model Formulation

The generalization of multi-agents in a river basin must be the first step. As shown in Figure 2 for demonstration, the whole basin is divided into M zones on the basis of meteorological and hydrological features and natural geographical conditions. Zone 1 is located in the upper and middle reaches of the river’s main stream. Zone 2 is located in a tributary of the river basin. Zone M is the most downstream area of the basin, where the tributaries merge with the main stream. According to the spatial distribution, there are mainly three types among these interest agents, including consumptive water users outside the river, users with ecological water needs inside the river, and water energy users inside the river. Water transfer to other basins is also considered in this model. It is assumed that there are L water users outside the river, and the total number of interest agents in zone M is denoted as N_M. R represents the outflow of each zone, and these interest agents are correlated with each other through the runoff.

2.2.1. The Superiors Model

The optimization of the common interest of water resource management agents in the superior model is described by the following equation:

\[
\text{Max } B^u = \frac{1}{\sum_{z=1}^{M} N_z} \sum_{z=1}^{M} \sum_{i=1}^{N_z} B_{zi}^u = \frac{1}{\sum_{z=1}^{M} N_z} \sum_{z=1}^{M} \sum_{i=1}^{N_z} \left[ 1 - \left( \frac{D_{zi} - x_{zi}}{D_{zi}} \right)^2 \right]
\]  

(1)
In Equation (1), the objective function of each agent \( B_{zi}^{u} \) represents the water requirement satisfaction index of agents \( i \) in Zone \( z \) \((z = 1, \cdots, M; i = 1, \cdots, N_z)\). \( M \) represents the number of zones in the basin, and \( N_z \) represents the number of agents in Zone \( z \) \((z = 1, \cdots, M)\). \( \sum_{z=1}^{M} N_z \) represents the number of agents in the whole river basin. \( x_{zi} \) represents the water allocated to agent \( i \) in Zone \( z \), which is also a decisive variable of this model. \( D_{zi} \) represents the water demand of each agent. \( B_{zi}^{u} \) represents the standard benefit value (i.e., [0, 1]) of agent \( i \) in Zone \( z \), which is also the goal of individual interests. \( B^u \) represents the total benefits of interest agents, which is also the optimization goal of the common interest in the superior model.

The constraints are as follows:

1. Water balance equation

\[
R_{z} = W_{z} + \sum_{k=1}^{K} R_{z-k} - (1 - \varphi_{zl}) \sum_{l=1}^{L} x_{zl} - W_{zd} - \Delta S_{z} \quad (2)
\]

In Equation (2), \( R_{z} \) represents the outflow from Zone \( z \); \( W_{z} \) represents the inflow of Sub-basin \( z \); \( R_{z-k} \) represents the outflow from Zone \( z - k \); \( k \) represents the upstream zones who are related to the flow in Zone \( z \) \((k = 1, \cdots, K)\); \( \varphi_{zl} \) represents the water receding coefficient of the user \( l \) \((l = 1, \cdots, L)\) outside the river in Zone \( z \); \( x_{zl} \) represents the water allocated to user \( l \) outside the river in Zone; \( W_{zd} \) represents the water transfer from reservoirs in Zone \( z \) (the positive value represents the outflow, whereas the negative value represents the inflow); and \( \Delta S_{z} \) represents the variation in the poundage of reservoir in Zone \( z \) at the specified period.

2. Constraints on the water available to each agent

\[
x_{zi} \leq W_{z} + \sum_{k=1}^{K} R_{z-k} - W_{zd} - \Delta S_{z} \quad (3)
\]

3. Water demand constraints

\[
0 \leq x_{zi} \leq D_{zi} \quad (4)
\]

4. Constraints on the total amount of water available

\[
\sum_{z=1}^{M} \sum_{l=1}^{L} x_{zl} \leq W_{k} \quad (5)
\]

In these equations, \( W_{k} \) represents the total amount of water available to users outside the river basin.

2.2.2. The Subordinates Model

The objective function of the lower multi-agent cooperative game-based optimization model [7] of water resource allocation, based on the Nash bargaining model [8,16,17], is obtained as follows:

\[
\text{Max } B^{d} = \prod_{z=1}^{M} \prod_{i=1}^{N_z} \left( B_{zi}^{d} - N_{zi} \right) \quad (6)
\]

The constraints are as follows:

\[
\frac{1}{M} \sum_{z=1}^{M} \sum_{i=1}^{N_z} B_{zi}^{d} = B^{u} \quad (7)
\]

\[
N_{zi} = \text{Max } S_{zi} \quad (8)
\]
In this model, \(x_{zi}\), i.e., the water allocated to agent \(i\) in Zone \(z\), is the decisive variable. \(B_{zi}^u\), \(R_z\), \(R_{z-k}\), \(x_{zl}\), and \(B_{zi}^d\) are also variables in this model, while others are parameters. With a cooperative alliance of all agents, the optimal solution is solved on the condition that the Nash product
\[
\prod_{z=1}^{M} N_z \prod_{i=1}^{N_z} (B_{zi}^d - N_{zi})
\]
reaches the maximum. \(B_{zi}^d\) and \(N_{zi}\) are the agent’s benefit in the cooperation mode, and the agent’s benefit in the individual (non-cooperative) mode, respectively. \(B_{zi}^d - N_{zi}\) is the gain from cooperation. By using the Nash bargaining model, the bargaining process can be simulated among the agents. A compromise among the conflicting objectives could be found, in which agents have considered their own benefits and know that they can get more benefits in the coalition on the whole [17,18].

Constraint (7) denotes the solution of the superior model as the equality constraint of the model.

In Equation (8), \(N_{zi}\) represents the benefit of each agent in the non-cooperative mode. The optimization model of individual interests covers objective function (8) and constraints (2)–(4). When the benefit of the most upstream agent with first-mover advantage reaches the maximum \(N_{zi} = 1\) in the non-cooperative mode, \(N_{zi} = B_{zi}^u\) will facilitate cooperation.

Equation (9) indicates a rational requirement for individuals to participate in cooperation, which is the “bottom line”. Failure to reach the bottom line may cause the agent’s unwillingness to cooperate and the collapse of their cooperative alliance. The optimization solution of the lower multi-agent cooperative game-based decision-making model for water resource allocation involves identifying appropriate allocation strategies to maintain the cooperative alliance, where the benefit of each individual is not less than that in the non-cooperative mode (or when optimizing the superior model).

Equations (1)–(9) provide the GT-based optimization model that combines the superior and subordinate models of water resource allocation among interest agents, and the optimization model can be solved by the dynamic programming algorithm [6,19]. This optimization describes the carrying capacity of spatially-related water resources using the water available to zones in the same watershed and the pressure load on water resources in each zone using the requirements of interest agents. The model also proposes to achieve the load balance of the entire basin on a large scale through the downward constraints in the optimization of the common interest of the superiors and the fair allocation of the common interest of the subordinates.

3. Case Study

3.1. Study Area

The Hanjiang River basin, with an area of 159,000 km², is the largest and most developed tributary of the Yangtze River Basin [20]. Being the ninth largest river in China, with a total length of 1570 km, the Hanjiang River flows through the south of Shanxi Province, the northwest and central region of Hubei Province, and runs into Yangtze River at Wuhan city [21]. After the height of the Danjiangkou Dam being increased from 162.0 m to 176.7 m, Hanjiang River has served as an important water source for the middle route of the South-to-North Water Diversion Project in China, and has formed a connection between north and south China [22–24]. Meanwhile, Hanjiang River is an important channel, which connects the inland area in the northwest with the marine coastal area in the east, and nurtures the natural and human environment of the basin [21].

Because of the rapid growth of population and economy, water demand is expanding and conflicts over water use among interest agents in the Hanjiang River Basin are intensifying [25]. The bi-level optimization model is established and applied to the Hanjiang River Basin, trying to alleviate conflicts among different individual interests.

3.2. Data and Primary Analyses

The Hanjiang River Basin is divided into three zones (\(M = 3\)), according to the three-tier zoning of nationwide water resources. Zone 1 is located above the Danjiangkou reservoir, as Danjiangkou
Reservoir is the boundary of the upper and mid-lower reaches of the Hanjiang River Basin. In Zone 1, the interest agents could be conceptualized into one type \( (N_1 = 1) \), namely, the consumptive water user outside of the river. What needs to be explained is that the water transfer from Danjiangkou reservoir is considered, and the total volume of the transferred water is assumed to be 9.5 billion m\(^3\) per year, through the middle route of South to the North Water Transfer Project \([23,26]\). Zone 2 is located in the Tangbai river, which is the main tributary of the Hanjiang river. Similarly, the interest agents could be conceptualized into one type \( (N_2 = 1) \), namely, the consumptive water user outside of the river. Zone 3 is located below the Danjiangkou reservoir, in which the interest agents should be conceptualized into three types \( (N_3 = 3) \). The first is the users with ecological water needs inside the river, since the flow in Xiangyang is both the spawning ground of the Asian carp and control node of the ecological condition of the main stream of the Hanjiang River. The second is the consumptive water user outside of the Tangbai river. The third is the users with ecological water needs inside the river, because the flow in Xiantao is the critical flow whether the phytoplankton blooms would occur in the river below Qianjiang, and another control node of the ecological condition of the main stream of the Hanjiang River.

As shown in Table 1, the amount of water resources in different zone is presented.

<table>
<thead>
<tr>
<th>Content</th>
<th>Unit (Billion m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total of the Hanjiang River Basin</td>
<td>14.7</td>
</tr>
<tr>
<td>Annual inflow of Zone 1 based on a guaranteed rate of 95%</td>
<td>16.7</td>
</tr>
<tr>
<td>Annual inflow of Zone 2 based on a guaranteed rate of 95%</td>
<td>1.5</td>
</tr>
<tr>
<td>Local inflow of Zone 3</td>
<td>5.5</td>
</tr>
</tbody>
</table>

In Table 2, according to the flow, the annual water demands and water receding coefficients of the three consumptive water users outside the river (i.e., Agents 1, 2 and 4) are presented.

<table>
<thead>
<tr>
<th>Agent 1</th>
<th>Agent 2</th>
<th>Agent 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual water demand (million m(^3)/s)</td>
<td>water receding coefficient</td>
<td>annual water demand (million m(^3)/s)</td>
</tr>
<tr>
<td>510</td>
<td>0.45</td>
<td>560</td>
</tr>
</tbody>
</table>

In Table 3, the ecological flow requirement of Agents 3 and 5 are presented, which is the minimum flow of the river to avoid the occurrence of the phytoplankton blooms.

<table>
<thead>
<tr>
<th>Xiangyang Section</th>
<th>Xiantao Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>May–October</td>
<td>May–October</td>
</tr>
<tr>
<td>November–Next April</td>
<td>November–Next April</td>
</tr>
<tr>
<td>632.3</td>
<td>625.7</td>
</tr>
<tr>
<td>379.4</td>
<td>374.3</td>
</tr>
<tr>
<td>February–March</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

4. Results and Discussion

As mentioned in Section 3.2, interest agents are correlated with each other through the runoff. It has been mentioned above that there are five agents in the Hanjiang River Basin. When the basics of the basin are put into the model, and are solved by the bi-level optimization model with dynamic programming, we see the results as shown in Figure 3.
In Figure 3, the bar in different colors represent different models. Compared to the single-objective individual optimization model (in brief, Scheme 1), the total benefits of the upper model (in brief, Scheme 2) increased by 8%. Since Agent 1 is located in the most upstream river, which could allow for it to take advantage of its spatial location, assuming its individual interests reach the maximum in having enough water resources. Except for Agent 2, whose individual interests increased by 49%, the individual interests of Agents 1, 3, 4, and 5 are decreased by 8%, 10%, 12%, and 2%, respectively, when comparing Scheme 1 with Scheme 2. Although the common interest of the upper optimization model is improved, it is at the sacrifice of some individual interests. In conclusion, only Agent 2 is willing to cooperate, while the rest of the agents are not.

Compared to Scheme 1, the total benefits of the bi-level optimization model (in brief, Scheme 3) also increased by 8%. From the perspective of individual interests, except for Agent 1, whose individual interests decreased by 3%, the individual interests of Agents 2, 3, 4, and 5 are increased by 14%, 18%, 7%, and 14%, respectively, comparing Scheme 1 with Scheme 3. Evidently, Scheme 3, which not only emphasizes the common interest but also considers individual interests, is in favor of the cooperation of the agents in the basin, because a slight decline in individual interests can contribute greatly to great increases of other individual and common interests.

When comparing Scheme 2 with Scheme 3, it can be found that the total benefits are the same. However, in Scheme 2, 4 out of 5 of the individuals are unwilling to cooperate due to their damaged interests. Whereas, 4 out of 5 of the individuals tend to cooperate in Scheme 3. Scheme 2 emphasizes the common interest but neglects individual interests, thereby causing in efficiency and inequity. On the contrary, Scheme 3 realized the optimization of the common interest, considering individual interests simultaneously, through bottom-up feedback with optimal individual interests optimized and top-down coordination with the optimal common interest.

5. Conclusions

In this study, the bi-level optimization model is proposed for water resource allocation. Based on the cooperative game-based theory, the bi-level model consists of the superiors model and subordinates model, which could realize a win-win cooperation of common and individual interests in water
resource allocation. The following conclusions are drawn, based on the results of the case study of the Hanjiang River Basin.

The results of the case study indicate the effectiveness and fairness of the bi-level optimization model. Compared to the single-objective individual optimization model, our model can bring more common interest, by 8% in total. Four out of five of the interest agents can get more benefits, by 7%–18%, although one out of five of the interest agents’ benefit decreases, only by 3%. Compared to the upper optimization model, from the perspective of the common interest, there is no improvement. However, it must be realized that in the upper model, only one out of five of the interest agents can get more benefits, so that this model is only ideal, since most of the interest agents will not choose to cooperate. In a word, both common and individual interests are improved by our model, and these agents are more likely to cooperate to get more benefits.

From the case study, there are some recommendations for decision and policy makers. Before making decisions, policy makers are advised to realize the needs and acceptable values of agents adequately, and adjust the decisions according to the feedback from agents. Afterwards, the allocation could be efficient and equitable.

The bi-level optimization model can be easily applied into other basins, and is possible to solve more problems related to water resources. However, there are still some areas to improve, which require further research.

Author Contributions: Conceptualization, Q.H. and X.F.; Methodology, Q.H. and X.F.; Software, Q.H. and X.F.; Validation, G.T., Y.M. and Z.Y.; Formal Analysis, Z.Y.; Investigation, G.T.; Resources, Q.H. and X.F.; Data Curation, Q.H. and X.F.; Writing-Original Draft Preparation, Q.H.; Writing-Review & Editing, X.F. and G.T.; Visualization, Z.Y.; Supervision, Y.M.; Project Administration, Y.M.; Funding Acquisition, X.F.

Funding: This study is financially supported by the National Key Research and Development Project of China (Grant No. 2016YFC0401306).

Conflicts of Interest: The authors declare no conflict of interest.

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